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Utilizing Code Orthogonality Information for Interference Suppression in UTRA downlink

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Abstract—This paper suggests the use of code orthogonality information in an iterative, zero forcing interference suppression scheme. This simple scheme is applicable in UTRA downlink conditions. It is shown that when multiple receive antennas are present, selection diversity with regard to code orthogonality information improves the system performance at a very low complexity overhead.

Keywords-UTRA downlink; zero forcing equalization; code orthogonality; selection diversity

I. INTRODUCTION

3G cellular networks have begun their roll-out across Europe, but the promise of high bit rate services, especially High Speed Downlink Packet Access (HSDPA) in the downlink has not yet fully materialized. Multiple transmit antenna techniques including STBC have been added to HSDPA specifications [1] to enhance its data throughput and performance. The Multiple Access Interference (MAI) effects, an inherent limitation in CDMA systems remain a major performance barrier. This makes it compulsory to incorporate interference mitigation mechanisms in the downlink. The conventional multi-user detection schemes (even sub-optimal ones) are currently too complex for the mobile handsets, which have strict constraints on cost, size and power consumption.

In this paper we investigate a simple Zero-Forcing (ZF) interference suppression scheme proposed by Barbarossa et. al [2], which can meet the above handset constraints. This ZF scheme can be effectively applied to pico-cell / hotspot type cellular environments, where a high Signal to Noise Ratio (SNR) can be maintained. We show that when multiple receive antennas are present, the performance of this scheme can be further enhanced by using selection diversity based on code orthogonality information of the radio channel. The above scheme is applicable to both single transmit antenna and multiple transmit antenna (through Space Time Block Codes) configurations and we analyze both cases. Additionally, the complexities of each of these options are compared in terms of clock cycles on a latest Digital Signal Processor (DSP).

II. ITERATIVE ZF SCHEME – SINGLE TRANSMITTER

The general ZF equalization of a multi-path radio channel requires the inversion of a $N \times N$ channel matrix where $N$ is the CDMA spreading code length. This has to be carried out every time the channel co-efficients change due to fast fading. If the channel matrix can be converted to a circulant Toeplitz matrix, the inversion becomes relatively straight forward as these matrices can be diagonalized by using Fast Fourier Transform (FFT) basis vectors. This idea was utilized in [3], where the channel matrix was converted to a circulant Toeplitz matrix by the addition of a cyclic prefix to the spreading codes. But such an addition incurs a rate loss and is not compatible with the UTRA standards. An alternative iterative solution was suggested in [2], which is outlined below.

A. FFT diagonalization of the channel matrix

The received signal $y(n)$ for the $n$th symbol block of a CDMA downlink can be expressed as [2];

$$y(n) = H^{(0)}F_{s}(n) + H^{(1)}F_{s}(n-1)$$

($H^{(0)}$ is a $N \times N$ Toeplitz matrix containing radio channel gains $[h_1 h_2 \ldots \ldots h_N]$ in the 1st column and $[h_1 0 \ldots \ldots 0]$ in the 1st row. The $N \times N$ Toeplitz matrix $H^{(1)}$ shows Inter Block Interference (IBI) coming from the previous data block $s(n-1)$. $H^{(1)}$ has $[0 0 \ldots \ldots 0]$ as its 1st column and $[0 \ldots h_1 h_2 \ldots h_N]$ as its 1st row. $F$ is a $N \times K$ matrix containing $N$ chip spreading codes for the $K$ active users. The AWGN contribution is ignored, assuming the system to be interference limited.

A circulant Toeplitz matrix is obtained as $H^{(0)} = H^{(0)} + H^{(1)}$ and diagonalized with FFT vectors $W = (1/N).exp(j2\pi k/N)$, giving $H^{(0)} = WAW^{H}$. Now (1) can be re-written as [2];

$$z(n) = y(n) - H^{(1)}F_{s}(n-1) = WAW^{H}F_{s}(n) - H^{(1)}F_{s}(n)$$

Ignoring $H^{(1)}F_{s}(n)$ the initial estimates for $s(n)$ are calculated.

$$s_{ZF}(n) = (W^{H}F)^{\dagger}A^{-1}W^{H}z(n)$$

The pseudo inverse $(W^{H}F)^{\dagger}$ can be pre-computed. The inversion of the diagonal matrix is straight forward. To improve upon this estimate, an iterative procedure is carried out. An estimate of the ignored term, $H^{(1)}F_{s}(n)$ is added to $z(n)$ and this updated $z(n)$ is used recursively in (3).

B. Code orthogonality based selection diversity

In delay dispersive wideband radio channels encountered in 3G Wideband CDMA (WCDMA) schemes multiple paths...
will carry the transmitted signals. When several of these paths are dominant, \(H^{(t)}\) in (2) becomes significant and the iteration process may fail to converge. In the UTRA downlink context, this multi-path channel behavior can be linked to its code orthogonality. In an ideal single path channel, the URTA downlink will not experience any intra-cell interference as the channel preserves the orthogonality of the spreading codes. In a multi-path radio channel there is a proportionate decrease in the Signal to intra-cell Interference Ratio (SIR), with the degradation of code orthogonality. We have earlier published a method to quantify code orthogonality [4], based on the above observation. These instantaneous code orthogonality estimates will be used to select the best channel when multiple receive elements are present.

C. Simulation results

Eight HSDPA data channels each with a spreading factor \(N=16\) are simulated with radio channel data gathered from a confined urban ‘hot spot’ environment. Details of this 8x4 MIMO outdoor channel measurement campaign in the 2GHz UTRA band can be found in [5]. For these simulations, a single transmit element and 2 receive elements are considered. The bit error rates (BER) for uncoded data channels with durations of 1 frame length (10ms) are calculated through repetitive simulations. The BER results for different schemes, including the traditional selection diversity scheme based on received signal power are presented in Fig.1.

The error floors in all these schemes indicate the residue interference effects on the achievable BER. The ZF schemes with 4 iterations provide little improvement over the simple Rake receiver. Selection diversity ZF schemes with 2 receivers perform notably better, reducing the \(E_b/N_0\) levels required to achieve \(10^{-2}\) uncoded BER by around 8-10dB. Selection based on the received signal power metric is seen to give the best performance under noise dominated (low \(E_b/N_0\)) conditions, but the code orthogonality based selection exceeds this in interference limited (high \(E_b/N_0\)) conditions. The threshold for this change is around \(E_b/N_0=14dB\), or SNR=2dB for \(N=16\). At high SNR levels (around 15dB), the code orthogonality based selection provides substantial gains over the traditional scheme.

To better illustrate the gains in selecting the radio channel with higher code orthogonality, the Cumulative Distribution Functions (CDF) of the code orthogonality factor are plotted in Fig.2. In addition to the single transmitter scenario, CDF curves for the code orthogonality in two transmitter systems are also presented here, but they will be taken up for discussion in section III.

![Figure 1. BER Simulations for M=1 schemes](image1)

The CDF curves for the code orthogonality of the two 1x1 channels are almost identical, with a median value of 0.55. When selection diversity is applied, the median value goes up to 0.66, indicating a 20% improvement. The shift is more prominent in radio channels with low-mid values of code orthogonality, where the delay dispersion is large. This apparent improvement in channel quality from the selection process is reflected in the improved BER performance.

III. ITERATIVE ZF SCHEME – TWO TRANSMITTERS

As shown in [2], this ZF scheme can be extended to multiple transmit antennas (M) under STBC, to yield diversity gains at the transmit end. In this section we will examine the inclusion of selection diversity, when multiple receive elements are also available. The analysis will be done on a basic 2x2 system, but the concept can be extended to higher order systems. With the current interest on multiple antenna (or MIMO) techniques, the exploration of possible gains in such a system will be of added significance.

A. FFT diagonalization of the composite channel matrix

For two transmit (\(M=2\)) systems, STBC (Alamouti codes) can be used with the symbols transmitted in the 2nd time interval \(s(n+1)\) being spread by a code set \(G = JF\), where \(J\) is the anti-diagonal matrix [2,3]. This makes FFT diagonalization possible over two symbol blocks \(s(n)\) and \(s(n+1)\), as shown below. Ignoring the noise contributions, the received signals over the two time periods \(y(n)\) and \(y(n+1)\) can be expressed as [2];
\[ y(n) = H_1^{(i)} F_{s}(n) + H_1^{(i)} F_{s}(n+1) + H_2^{(i)} G_{s}^*(n) - H_1^{(i)} G_{s}^*(n-2) \]
\[ y(n+1) = H_1^{(i)} G_{s}^*(n+1) - H_2^{(i)} G_{s}^*(n) + H_1^{(i)} F_{s}(n) - H_2^{(i)} F_{s}(n+1) \]

(4)

The Toeplitz matrices \( H_1^{(i)} \) and \( H_2^{(i)} \) contain the channel path gains representing the wanted signal and IBI respectively as in (1), for the \( i^{th} \) transmit antenna. Circulant matrices \( H_{s}^{(i)} \) can be obtained as \( H_{s}^{(i)} = H_{1}^{(i)} + H_{2}^{(i)} \) and diagonalized with FFT base vectors \( W \) to yield \( H_{s}^{(i)} = W \Lambda W^H \).

The received signals can now be expressed with FFT diagonalization with the residue terms from (4) represented by \( d(n) \) and \( d(n+1) \) [2].

\[
\begin{align*}
    y(n) &= W \Lambda W^H F_{s}(n) + W \Lambda W^H F_{s}(n+1) + d(n) \\
    y(n+1) &= W \Lambda W^H G_{s}^*(n) - W \Lambda W^H G_{s}^*(n) + d(n+1)
\end{align*}
\]

(5)

Since \( W^H F = W^T G \), pre-multiplying \( y(n) \) and \( y^*(n+1) \) by \( W^H \) and \( W^T \) respectively will result in the following simplification, when the terms \( d(n) \) and \( d(n+1) \) are initially ignored [2].

\[
\begin{bmatrix}
    W^H y(n) \\
    W^T y^*(n+1)
\end{bmatrix} = \Lambda
\begin{bmatrix}
    W^H F_{s}(n) \\
    W^T F^*(n+1)
\end{bmatrix}
\]

(6)

Where the orthogonal matrix \( \Lambda \) is a concatenation of \( \Lambda_1 \) and \( \Lambda_2 \). A diagonal matrix \( \Lambda_2 \) is generated as:

\[
\Lambda_2 = \begin{bmatrix}
|\Lambda_1|^2 & |\Lambda_2|^2 \\
0 & |\Lambda_1|^2 + |\Lambda_2|^2
\end{bmatrix}
\]

(7)

Since \( \Lambda_1 = \Lambda_1^H \Lambda_2 \), the initial estimates of \( s(n) \) and \( s(n+1) \) can be obtained by performing ZF equalization on (6).

\[
\begin{bmatrix}
    s(n) \\
    s^*(n+1)
\end{bmatrix} = \begin{bmatrix}
W^H F^+ & 0 \\
0 & W^T F^+
\end{bmatrix} \Lambda_1^H \Lambda_2 \begin{bmatrix}
W^H y(n) \\
W^T y^*(n+1)
\end{bmatrix}
\]

(8)

As with (3), the pseudo inverses \( (W^H F)^+ \) can be pre-computed, and the inversion of only a diagonal matrix is required. An iterative correction procedure is applied to improve the accuracy of initial estimates [2]. First the terms \( d(n) \) and \( d(n+1) \) are calculated with the estimates and \( s(n) \) and \( s^*(n+1) \) and subtracted from \( y(n) \) and \( y(n+1) \). The updated \( y(n) \) and \( y(n+1) \) are then used in (8), giving new estimates for \( s(n) \) and \( s^*(n+1) \).

### B. Code orthogonality calculation for \( M=2 \)

For \( M=2 \) systems also, the amount of IBI is dependent upon the number of the significant paths of the channel delay profiles and this can be attributed to code orthogonality. The calculation of code orthogonality for this composite channel follows a similar method to [4], where the signal to interference power ratio (SIR) at a Rake receiver output is considered. The signal power \( S \) in \( M=2 \) scheme can be expressed as:

\[
S = N^2 P \left( \sum_{m=1}^{M} \sum_{p=1}^{P} |h_{m,p}|^2 \right)^2
\]

(9)

Where \( N \) denotes the processing gain, \( P \) is averaged received power and \( h_{m,p} \) gives the \( p^{th} \) significant path (amongst \( R \) originating from the \( m^{th} \) antenna).

The interference term contains a linear addition of 4 terms, resulting from the interaction of the 2 radio channels’ offset paths. The self interference [4] and multiple access interference terms \( (I_s \) and \( I_m \) thus become;

\[
I_s = NP \sum_{s=1}^{2} \sum_{m=1}^{M} \sum_{p=1}^{P} \left( \sum_{l=1}^{R} 4Re^2[h_{s,p,h_{m,l}}] + |h_{s,p}|^2 \sum_{l=1}^{R} |h_{m,l}|^2 \right)
\]

(10)

\[
I_m = NP(K-1) \sum_{s=1}^{2} \sum_{p=1}^{P} |h_{1,p}|^2 + \sum_{p=1}^{P} |h_{2,p}|^2 \left( \sum_{l=1}^{R} |h_{1,l}|^2 + \sum_{l=1}^{R} |h_{2,l}|^2 \right)
\]

(11)

\( Re^2[x] \) denote the squared real part of complex \( x \). The SIR is calculated from (9-11) and compared against that of a reference uniform impulse channel (as in [4]) to give instantaneous code orthogonality values.

### C. Simulation results for \( M=2 \)

BER simulations were conducted on the same channel data, for \( M=2 \) configurations. All the simulation parameters were retained as before. The results are shown in Fig.3.

The results indicate a significant BER improvement for the ZF iterative schemes over the \( M=1 \) simulations. This is a testament to the additional spatial diversity gains provided by the multiple transmitters through STBC. The iterative ZF schemes achieve \( 10^{-4} \) uncoded BER (with \( 10^{-2} \) BER gain attributed for coding, this becomes \( 10^{-6} \) coded BER) usually regarded as sufficient for reliable data transmissions. The potential improvements with selection diversity are lower, but still the selection based upon code orthogonality out performs the selection based upon received power at high \( E_b/N_0 \) levels. The cross over threshold is at around \( E_b/N_0 =12 \text{dB} \) and there is a \( 2.5 \text{dB} \) reduction in \( E_b/N_0 \) at BER =\( 10^{-4} \) level, when employing code orthogonality based selection.
The cumulative distributions of code orthogonality for \( M=2 \) systems are included in Fig. 2. With selection diversity, the median values improve from around 0.5 to 0.58. This 16% improvement is lower than the median shift for \( M=1 \) systems, and is consistent with the lower BER gains seen over utilizing received power based selection in Fig. 3.

IV. COMPARISON OF ALGORITHM COMPLEXITIES

In this section we will compare the complexities of the detection algorithms in terms of clock cycles in a latest DSP chip, the Texas Instrument’s TMS320C6416. The comparison will include the basic ZF approach where the channel matrix is directly inverted, the FFT based ZF iterative scheme and the selection diversity ZF iterative scheme based on code orthogonality. If the selection diversity was based upon received signal power, this metric could be roughly estimated at the RF stages of the receiver chain. Hence it would not require any additional computational efforts from the DSP.

A. Basic ZF based scheme

In the basic ZF equalizer, the channel matrix \( H \) is inverted to yield an estimate of the data symbol vector \( s'(n) \). With the received signal vector \( y(n)=HFs(n) \), the ZF equalizer generates \( s'(n)=F^\dagger H^{-1}y(n) \).

Detection techniques where all users are detected by matrix inversion can be described very compactly in Matlab but are impractical to implement on DSPs due to the prohibitively high complexity in the order of \( N^3 \) steps for Gaussian elimination based methods, where \( N \) is the number of rows or columns [6]. The complexity can be reduced to order \( (N^2) \) when the matrix has a Toeplitz structure [6].

Iterative adaptive filter based algorithms (e.g. Linear Minimum Mean Squared Error (LMMSE) [7]) are generally employed in practical systems due to their lower run-time complexity. The Recursive Least Squares (RLS) update algorithm has improved convergence properties compared to the Least Mean Squares (LMS) in a fast fading mobile channel, although it has a higher implementation cost. A complex iterative algorithm using RLS update was profiled on the Texas Instruments TMS320C6701 floating-point DSP with all data held in on-chip memory. The C code was compiler optimized (using the highest level) in which loops are unrolled and software is pipelined. A single iteration for sixteen optimized (using the highest level) in which loops are unrolled and independent 16-bit multipliers. The cycle estimates also assume the DSP is otherwise idle and hence will represent a lower-bound.

An initial overhead is required in fetching the data from memory and in setting up of loops, though pipelining means that successive fetches and operations are often executed in successive processor cycles. For example, a real reciprocal takes \( 8x+14 \) cycles [9], where \( x \) is the number of consecutive divisions to be computed. When \( x \) is small the cycle count per single reciprocal operation is larger than when \( x \) is large.

The calculation of \( F^\dagger H^{-1}y(n) \) necessitates \( 16*16*8 \) complex multiplications and requires 2063 cycles. The total complexity for a single antenna basic ZF scheme is then 11,413 cycles, when the matrix inversion steps are taken into account.

The complexity for a \( M=2 \) system with STBC involves inversion of a 32x32 channel matrix. We have calculated that this procedure takes 39569 cycles using the same RLS based iterative algorithm.

B. FFT based ZF scheme with four iterations

The inversion of the complex diagonal matrix \( \Lambda^{-1} \), can be efficiently computed using the expressions below, where \( \lambda_i=a+bi \) is an arbitrary diagonal element:

\[
\Lambda^{-1} = \left( \text{diag}(\lambda_1,...,\lambda_N) \right)^{-1} = \text{diag}\left(\lambda_1^{-1},...,\lambda_N^{-1}\right)
\]  

To compute the complex reciprocal for \( \lambda_i \) we can use,

\[
\lambda_i^{-1} = \frac{1}{a+bi} = \frac{a-bi}{a^2+b^2}
\]  

First the denominator is computed, which requires two real multiplications and one addition. The real reciprocal is then computed, which requires 142 cycles for 16 successive computations. Two further multiplications are required to compute the numerator and reciprocal product. The total count is 142+(x*4) = 206 cycles.

To compute four iterations of (3) requires 1780 cycles plus a single matrix inversion. The product \( (W^H F) \) is pre-computed and therefore doesn’t feature in the cycle count. The total complexity for a \( M=1 \) system is therefore calculated at 1986 cycles.

We next compute the complexity of the iterative ZF scheme for the case \( M=2 \) transmit antennas. Starting from the 32x1 column vector in (8) and pre-multiplying the other three matrices require 1613 cycles. The complexity over four iterations is 6452 cycles, plus 237-cycles to compute the inverse of the diagonal matrix \( \Lambda \).

C. Code orthogonality based selection scheme

This method requires the computation of the code orthogonality from the SIR, in addition to the FFT based ZF iterative algorithm above.

To compute the signal power, \( S \) requires 19 cycles for \( R=3 \) significant paths in the Rake receiver. The computation of the interference powers \( I_n \) and \( I_s \) requires 41 cycles and 42 cycles
respectively assuming 6 impulses in the channel delay profile. A single real division requires 23 cycles and hence computing the SIR requires 125 cycles. The total complexity for this scheme (including the calculations in the previous section) is hence estimated at 2111 cycles.

The complexity of computing $I_m$ and $I_s$ each increases by a factor of four with two transmit antennas, as shown by (10) and (11). The signal power in (9) requires twice the computational effort. With a single real division, the SIR requires 446 cycles.

D. Summary of results

The complexities of the three ZF methods are summarized in Table 1. The basic ZF method has the largest cycle count despite being considerably less complex than computing the inverse through Gaussian elimination based methods. The FFT based iterative ZF scheme takes advantage of converting the channel matrix to a circulant structure. We have shown through (12) and (13) that the inversion of a diagonal matrix requires relatively very few clock cycles. With a 1ns cycle time, the FFT based iterative ZF with code orthogonality selection can be computed with relatively little extra overhead and well within the channel coherence time. For both $M=1$ and $M=2$ schemes, this additional overhead is around 6%. The final clock cycle estimates presented below in Table 1.

<table>
<thead>
<tr>
<th>Transmit Antennas ($M$)</th>
<th>Basic ZF (RLS)</th>
<th>FFT based ZF</th>
<th>Selection w.r.t. Code Orthogonality</th>
<th>Orthogonality overhead (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9365</td>
<td>1986</td>
<td>2111</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>39569</td>
<td>6689</td>
<td>7135</td>
<td>6.7</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

In this paper we have introduced a selection diversity scheme based on code orthogonality of the radio channel, which improves the performance of an iterative ZF equalization scheme. The simulated BER results indicate that at high SNR, the above selection metric outperforms the traditional received signal power based selection. The complexities of these schemes were evaluated in terms of required clock cycles in a latest DSP chip. The FFT based iterative ZF scheme substantially reduces the complexity of basic ZF equalization. The proposed selection diversity through code orthogonality adds very little overhead to this iterative ZF scheme, but offers substantial gains at high SNR.

REFERENCES


