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Resource Allocation Techniques for OFDMA-Based Decode-and-Forward Relaying Networks

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Abstract—In this paper, we focus on the design of efficient resource allocation algorithms for a multihop cellular network, with orthogonal frequency division multiple access (OFDMA) as downlink transmission technique and utilizing decode-and-forward cooperation strategy. Our objective is to maximize the total capacity based on the constraint of individual transmission power at each transmitter. We also consider users’ QoS requirements by maximizing the total capacity while giving users proportional fairness weights according to their data requirements. We show that for each data symbol (at each subcarrier) transmitted by base station, the best relaying strategy is to let only one among all the relay stations perform the relaying task. This is true for both objectives of maximizing total capacity and maximizing capacity with proportional fairness constraint. We then propose efficient greedy algorithms for both centralized and distributed resource allocations based on our analysis. Simulation results indicate that our proposed algorithms effectively enhance the total capacity.

I. INTRODUCTION

Multihop relaying is a promising technique to be integrated in the future generation of cellular network. In a multihop relaying network, a source transmits information to a destination with the assistance of one or more than one relay stations. Due to the distributed positions of relay stations, relaying links generally suffer uncorrelated fading paths, and can help mobile users to communicate with base station. Thus they form a virtual Multiple Input Multiple Output (MIMO) or also called Virtual Antenna Arrays (VAA) [1] to support solid connections and enhance throughput as well.

Orthogonal frequency division multiple access (OFDMA) technique is regarded as a leading candidate for the future generation of cellular network [2]. It enables very flexible strategies to do resource allocation, especially for the downlink transmissions. Consequently in this paper 1, we introduce the design of resource allocation algorithms for relaying transmissions in an OFDMA downlink cellular network.

Resource allocation schemes for OFDMA without relaying transmissions have been well studied in literatures [3], [4], [5]. In [5], it is proved that optimization can be achieved when a subcarrier is assigned to only one user who has the best channel gain for that subcarrier, which is also the basic assumption in [3], [4]. For power allocation, it is shown in [4], [5] that equal power allocation among subcarriers has almost the same performance compared to water-filling transmit power adaptation but with less complexity. These algorithms do not consider each user’s data rate requirement.

Relaying for OFDMA systems is considered theoretically in [6]. The paper studies the capacity of the OFDM and OFDMA relaying networks for both amplify-and-forward and decode-and-forward relay schemes, but it does not investigate efficient subcarrier and power allocations for these relaying networks. In [7], the authors investigate the adaptive resource allocation problem in OFDMA relaying system. Their objectives are to maximize capacity with fairness constraint. In their relaying system, only one transmitter transmits data at each subcarrier. Our centralized allocation algorithms assume similar scenarios as [7]. However, we have detailed proof on why such scenarios can be optimal. In [8], the authors impose proportional rate constraints in the OFDMA relaying network to assure that each user can achieve a required data rate. However, all the transmitters including base station and relay stations are limited to one fixed transmitting power, which is not flexible and practical in relaying network. Moreover, all the previous works only consider the scenarios where base station performs centralized resource allocation. They do not consider distributed resource allocation at each relay station.

In this paper, we consider the resource allocation problem in decode-and-forward OFDMA relaying systems, with individual power constraint at each transmitter and users’ fairness considerations. We show that for each data symbol (at each subcarrier) transmitted by base station, the best relaying strategy is to let only one among all the relay stations perform the relaying task. This is true for both objectives of maximizing total capacity and maximizing capacity with proportional fairness. We then propose efficient greedy algorithms for both centralized and distributed resource allocation schemes based on our analysis. Simulation results show that our proposed algorithms effectively enhance the total capacity.

The rest of the paper is organized as follows. Section II describes our relaying network model. In Section III and IV, we investigate both centralized and distributed resource allocation schemes with various objectives and constraints: one is to maximize the overall capacity and the other is to maximize the total capacity under the constraint of the users’ QoS requirements. Section V evaluates the performance of these algorithms in different scenarios. Finally, conclusions

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are drawn in Section VI.

II. SYSTEM MODEL

The system consists of one base station (BS), K mobile users or user equipments (UE) and L relay stations (RS), and each is equipped with a single antenna. As mentioned in previous section, we consider OFDMA downlink transmissions (with N data subcarriers). The relaying scenario works as follows. BS firstly transmits information symbols to RSs and UEs, and then RSs decode the symbols and retransmit them. We assume that all RSs retransmit the symbols in different time slots or frequency bands (free spectrum chunks detected by cognitive radio [9]). This scheme does not require strict synchronization among RSs and can fully utilize the multiuser diversity because different RSs can use different subcarriers for different UEs. Upon receiving information symbols from BS and RSs, optimal combining can be performed at each UE. We further assume that channel gains between BS and RSs are high enough (e.g., selective decode-and-forwarding is performed) so that the probability of decoding error at each relay station is small. In this case, maximal ratio combining (MRC) is a good approximation to maximum likelihood combining [10].

In this paper, we consider both centralized and distributed resource allocation schemes. In centralized resource allocation, we assume that BS knows the entire channel information of source-destination (BS-UE) and relay destination (RS-UE) links. This information can be fed back from UEs and RSs using control channels. After BS performs the subcarrier and power allocations, it will inform the RSs and UEs about the subcarrier allocation results. In distributed resource allocation, each RS has to know all the information about the links from itself to UEs. This can be fed back by UEs or estimated by uplink transmissions if time division duplex (TDD) is used. Each RS performs resource allocation for itself based on the current information it knows. Afterwards, the subcarrier allocation results are sent to UEs. Subcarriers can be grouped to reduce the control overhead.

Throughout this paper, we use \( h_{l,k}^{(n)} \) to represent the channel gain at subcarrier \( n \) from RS \( l \) to UE \( k \), where \( n \in [1,N], l \in [1,L] \) and \( k \in [1,K] \). To simplify the notation, we use \( h_{0,k}^{(l)} \) (when \( l = 0 \)) to represent the channel gain from BS to UE. Similarly, we use \( p_{l,k}^{(n)} \) to represent the power allocated to subcarrier \( n \) at BS or RS \( l \) to UE \( k \). We use \( \Omega_{l,k} \) to represent ordered subcarrier set allocated to user \( k \) at BS or RS \( l \). In our model, we assume that for each data symbol transmitted at BS, there is one subcarrier that relays for it at each RS. Therefore, for all the BS and RSs, same number of subcarriers should be allocated to each UE \( k \), and the length of \( \Omega_{l,k} \) should be same for all \( l \in [0,L] \) and same \( k \). We use \( C_k \) to represent the length. \( \Omega_{l,k}(j) \) is the \( j \)th element in \( \Omega_{l,k} \), which refers to the subcarrier that is used to transmit \( j \)th symbol for UE \( k \) at BS or RS \( l \). We assume that the total bandwidth for \( N \) subcarriers is \( B \), and the bandwidth for each subcarrier is then \( B/N \). Noise at each subcarrier is zero-mean circular symmetric complex Gaussian with variance \( N_0B/N \).

The SNR of \( j \)th symbol from BS or RS \( l \) to UE \( k \) is given by,
\[
\gamma_{l,k}(j) = \frac{\left| p_{l,k}^{(\Omega_{l,k}(j))} \right|^2 \delta_{l,k}(j)}{N_0B/N}.
\]

Assuming maximum ratio combination (MRC) is applied at the UE, the SNR of \( j \)th symbol for user \( k \) will then be,
\[
\Gamma_k(j) = \gamma_{0,k}(j) + \sum_{l=1}^{L} \gamma_{l,k}(j)
\]
for decode-and-forwarding.

The capacity for each user \( k \) will then be,
\[
R_k = \sum_{j=1}^{L} \frac{C_k}{N} \log(1 + \Gamma_k(j)).
\]

III. CENTRALIZED RESOURCE ALLOCATION

In this section, centralized subcarrier and power allocations (\( \Omega \) and \( p \)) are investigated. We will consider both the objectives of maximizing total capacity and of maximizing total capacity with proportional fairness constraint.

A. Maximizing Total Capacity

The problem of maximizing the total capacity can be formulated as,
\[
\max_{\Omega_l,k,\mathbf{p}_{l,k}} \sum_{k=1}^{K} R_k,
\]
subject to,
\[
\Omega_{l_1,k_1} \cap \Omega_{l_2,k_2} = \phi, \text{ for all } l_1 \neq l_2
\]
\[
\bigcup_{l=1}^{K} \Omega_{l,k} = [1,N], \text{ for all } l
\]
\[
\sum_{l=1}^{K} \sum_{n \in \Omega_{l,k}} p_{l,k}^{(n)} = P_l, \text{ for all } l
\]
\[
p_{l,k}^{(n)} \geq 0, \text{ for all } l, k, n
\]

The problem is a mixed integer and continuous variable optimization problem and is generally difficult to solve. Traditional solutions [3], [4], [5] split the problem into two independent problems. Firstly the subcarriers will be allocated for BS and RSs with equal power allocation assumption, and then power will be allocated based on the subcarrier allocation in the first step. In the above optimization problem, even the subcarrier allocation (with equal power assumption) alone can be viewed as a multi-dimensional weighted matching problem and is NP-complete. However, as we will show later, traditional solutions may not work well in this problem and subcarrier allocation cannot be performed efficiently without considering power allocation. In this section, we first give out the theoretical analysis, and then propose a greedy subcarrier and power co-allocation algorithm based on the analysis.

Firstly we consider the power allocation problem when the subcarriers have been allocated to different users, i.e.,
when $\Omega_{l,k}$ has been determined for each $l$ and $k$. The power allocation problem can be reformulated as,

$$\min_{\Omega_{l,k}} \sum_{k=1}^{K} R_k$$

subject to,

$$\sum_{k=1}^{K} \sum_{n \in \Omega_{l,k}} p_{l,k}^{(n)} - P_l = 0, \text{ for all } l$$

$$-\sum_{k=1}^{K} \sum_{n \in \Omega_{l,k}} p_{l,k}^{(n)} \leq 0, \text{ for all } l, k, n$$

Note that since the probability of decoding error at each RS is assumed to be low, it is not a good approach to adjust the power of subcarriers at BS. We can simply assume that all subcarriers use equal power ($P_0/N$) at BS.

The Lagrangian is given by,

$$L = -\sum_{k=1}^{K} R_k - \sum_{l=1}^{L} \sum_{k=1}^{K} C_k \left( \lambda_{l,k,j} p_{l,k}(\Omega_{l,k}(j)) \right)$$

$$+ \sum_{l=1}^{L} \mu_l \left( \sum_{k=1}^{K} C_k \left( \lambda_{l,k,j} p_{l,k}(\Omega_{l,k}(j)) - P_l \right) \right)$$

where $\lambda$ and $\mu$ are Lagrange multipliers.

Applying Karush-Kuhn-Tucker (KKT) conditions [11], the optimal solution exists when,

$$\frac{\partial L}{\partial \left( p_{l,k}(\Omega_{l,k}(j)) \right)} = 0, \lambda_{l,k,j} \geq 0, p_{l,k}(\Omega_{l,k}(j)) \geq 0$$

$$\lambda_{l,k,j} p_{l,k}(\Omega_{l,k}(j)) = 0, \sum_{k=1}^{K} \sum_{n \in \Omega_{l,k}} p_{l,k}^{(n)} - P_l = 0$$

The first two conditions can be simplified to be,

$$\sum_{l'=0}^{L} p_{l',k}(\Omega_{l',k}(j)) \left| p_{l',k}(\Omega_{l',k}(j)) \right|^2 \geq \frac{R \left| h_{l,k} \right|^2}{\mu_l} - \frac{N_0 B}{N}$$

(10)

for all $l \in [1, L]$, $k$ and $j$.

**Lemma 1:** The optimal power of the subcarriers at each RS can be allocated with water-filling algorithm, assuming power allocated to the subcarriers at all other RSs to be constant.

This can be proved easily by treating the power allocated at all other RSs as constant value and moving them to the right hand side of the inequality in Equation 10.

**Lemma 2:** If $p_{l_1,k}(\Omega_{l_1,k}(j)) \neq 0$ and $p_{l_2,k}(\Omega_{l_2,k}(j)) \neq 0$ and $l_1 \neq l_2$, $\mu_{l_1} > \mu_{l_2} = \left| h_{l_1,k} \right|^2 / \left| h_{l_2,k} \right|^2$.

The proofs of Lemma 1 and 2 are omitted here due to space limit.

From Lemma 1 and 2, the following theorem can be derived.

**Theorem 1:** In general situations, i.e., when all $|h|$ are different from each other, for each $k$ and $j$, there is only one RS $l$ among all $L$ RSs that $p_{l,k}(\Omega_{l,k}(j)) \neq 0$.

**Proof:** From Lemma 2, if there are more than one RSs that $p_{l,k}(\Omega_{l,k}(j)) \neq 0$, the value of $\mu_l$ of these RSs is proportional to the value of $|h_{l,k}^{(\Omega_{l,k}(j))}|^2$. Since all the values of $|h|^2$ are different, this is a contradiction. Furthermore, from Lemma 1, the value of $\mu_l$ is determined by water-filling algorithm at the optimal point, which is dependent on the values of all $|h|$, as well as the value of $P_l$. Thus the value $\mu_l$ generally cannot be proportional to the value of $|h_{l,k}^{(\Omega_{l,k}(j))}|^2$.

**Theorem 1** says that, in general situations, for each symbol transmitted from the BS to the UE, only one RS needs to relay for this symbol given the power constraint on each RS. This also suggests that doing subcarrier allocation without considering power allocation may not work well because many subcarriers at each RS may not be allocated power even though they have been chosen.

Combining **Theorem 1** and **Lemma 1** and 2, we have the following corollaries.

**Corollary 1:** $p_{l,k}(\Omega_{l,k}(j)) \neq 0$ only when $\frac{\left| h_{l,k}^{(\Omega_{l,k}(j))} \right|^2}{\mu_l} \geq \frac{\left| h_{l',k}^{(\Omega_{l',k}(j))} \right|^2}{\mu_{l'}}$ for all $l \neq l'$.

**Corollary 2:** Given all the subcarriers such that $p_{l,k}(\Omega_{l,k}(j)) \neq 0$ for RS $l$, the optimal values of $p_{l,k}(\Omega_{l,k}(j))$ can be derived using water-filling algorithm among these subcarriers.

Again, we omit the proof here.

Intuitively, when the water level of the RS $l$ is larger, and the RS-UE channel condition of the subcarrier is better, it is more likely that power is allocated to the subcarrier in RS $l$. Based on this, we propose a greedy subcarrier and power co-allocation algorithm summarized in Figure 1. The algorithm firstly sorts $|h|$ on each antenna $l$ and tries to find the optimal point $i_l$ so that $\frac{|h_{l,i_l}|^2}{\mu_l}$ is as close as possible for different $l$, where $\frac{1}{\mu_l}$ is the water level. Because $|h|$ is sorted, it is easy to show that any $\frac{|h_{l,n}|^2}{\mu_l}$ with $n \in [1, i_l]$ will be greater than $\frac{|h_{l,n}|^2}{\mu_l}$ with $n \in [i_l + 1, N]$ for all $l$. These subcarriers will then be greedily matched to subcarriers in BS, and power will be allocated by water-filling algorithm.
\[ \frac{R_{k}}{c_{k}} - \frac{R_{k}}{c_{k}} = 0, \quad \text{for all } k \in [1, K-1] \quad (11) \]

where \( c_k \) is fairness constant for user \( k \).

The Lagrangian is,

\[
L = - \sum_{k=1}^{K} R_{k} - \sum_{l=0}^{L} \sum_{k=1}^{K} C_{k} \left( \lambda_{l,k,j} p_{l,k}^{(\Omega_{l,k,j})} \right) + \sum_{l=0}^{L} \mu_{l} \left( \sum_{k=1}^{K} \sum_{j=1}^{L} h_{l,k,m}^{(\Omega_{l,k,j})} - P_{l} \right) + \sum_{k=1}^{K-1} \nu_{k} \left( \frac{R_{k}}{c_{k}} - \frac{R_{k}}{c_{k}} \right) \quad (12) \]

where \( \lambda, \mu, \) and \( \nu \) are Lagrange multipliers.

Similarly, applying KKT conditions, besides the conditions in Equation 9, one more condition is shown below,

\[
\frac{R_{k}}{c_{k}} - \frac{R_{k}}{c_{k}} = 0 \quad (13) \]

Equation 10 is also changed to Equation 14,

\[
\sum_{l=0}^{L} \left( p_{l,k}^{(\Omega_{l,k,j})} \right) |h_{l,k,j}^{(\Omega_{l,k,j})}|^2 \geq \begin{cases} \frac{B}{N} - \frac{N_{0}B}{N} \sum_{l=1}^{L} p_{l,k}^{(\Omega_{l,k,j})} |h_{l,k,j}^{(\Omega_{l,k,j})}|^2 & \text{if } k \in [1, K-1] ; \\ \frac{B}{N} - \frac{N_{0}B}{N} \sum_{j=1}^{K} \frac{\nu_{l,k}}{\mu_{l}} & \text{if } k = K. \end{cases} \quad (14) \]

It can be proved that, if \( k \) is fixed, Theorem 1 still holds. Once the subcarriers and total amount of power allocated to UE \( k \) at RS \( l \) is determined, the matching of these subcarriers, as well as the sharing of the power at different subcarriers for each user \( k \) can be determined by maximizing total capacity algorithm shown in Figure 1. Thus, the following algorithm is proposed. The algorithm firstly allocates each user the number of subcarriers that is proportional to its rate constraint at each RS. The total power for each UE at each RS is proportional to the number of subcarriers allocated to it. The resource allocation for each UE \( k \) is then determined by algorithm in Figure 1. Lastly, a local search (to adjust the number of subcarriers and power if \( R_{k}/c_{k} \) is not balanced) can be performed.

**IV. DISTRIBUTED RESOURCE ALLOCATION**

Centralized resource allocation requires the allocation to be performed whenever a RS or UE joins or leaves the system. This may not be scalable to the number of RSs and UEs, for BS will have too much burden when the number of RSs or UEs increases. In a distributed relaying system, each RS makes decision of resource allocation purely on itself. It can join or leave the system at any time without bringing any burden to BS.

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Distributed Resource Allocation for RS \( l \) (Maximizing Total Capacity)

sort the data symbol according to \( \sum_{l'=l}^{L} p_{l',k}^{(\Omega_{l',k,j})} |h_{l',k,j}^{(\Omega_{l',k,j})}|^2 \) in ascending order

for each \( \sum_{l'=l}^{L} p_{l',k}^{(\Omega_{l',k,j})} |h_{l',k,j}^{(\Omega_{l',k,j})}|^2 \)

\[
m = \arg \max_{n} p_{l,k}^{(\Omega_{l,k,j})} \left| h_{l,k,j}^{(\Omega_{l,k,j})} \right|^2 \]

allocate \( m \) to UE \( k \) for its data symbol as \( j \)

let \( h_{l,k,j}^{(m)} = 0 \) perform water-filling algorithm for power allocation

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**V. EVALUATION**

We setup the simulations as follows. In the relaying downlink transmissions, we choose the total bandwidth to be 5MHz, and the number of data subcarriers to be 300, which are
parameters used in the LTE OFDMA downlink system [12]. The wireless channel is modeled as a frequency-selective channel consisting of six independent Rayleigh fading paths. Four algorithms of resource allocation are evaluated and compared in different scenarios. They are centralized resource allocation (CA), distributed resource allocation (DA), centralized resource allocation with fairness constraint (CA-F) and distributed resource allocation with fairness constraint (DA-F). One base case with no resource allocation performed (NA) is also included.

In Figure 3(a), the total capacity versus SNR by applying different algorithms are shown. The SNR here is the average received SNR at UE, which is defined as $\frac{\sum_{k=1}^{K} \sum_{l=1}^{L} E[h_{k,l}^2]}{B N_0}$. In this plot, we set $K = 3$ users and $L = 3$ relay stations. It can be observed that CA achieves the best capacity gain over all range of SNR from 2dB to 20dB. The capacity gain of CA-F is lower than that of CA, and the capacity gain of DA-F is slightly lower than that of DA. This is because they have extra fairness constraint. DA and DA-F also perform worse than CA and CA-F respectively. This is because they cannot optimally let only one RS relay for BS for each data symbol due to the fact that each RS does not know the channel gain information of other RSs in distributed resource allocation.

Figure 3(b) shows the total capacity versus the number of transmitters $(1+L)$. In the plot, $K = 3$ users and SNR=10dB. It is interesting to observe that when the number of relay stations $L$ increases, CA and CA-F tend to have more gain in total capacity than DA and DA-F respectively. This is because when $L$ increases, distributed solutions lose more information comparing to centralized ones. Figure 3(c) shows the fairness index versus the number of transmitters. To compare the fairness level, we use fairness index defined as $\frac{\left(\frac{\sum_{k=1}^{K} R_k}{\sum_{k=1}^{K} (R_k/\bar{R}_k)^2}\right)^2}{\sum_{k=1}^{K} (R_k/\bar{R}_k)^2}$ with the maximum value of 1 to be the greatest fairness case. It is observed that both CA-F and DA-F have a fairness index close to 1, higher than that of NA (which can be considered as a random allocation process). This shows the effectiveness of the two fairness algorithms CA-F and DA-F.

VI. CONCLUSION

In this paper, we systematically investigate the resource allocation problem of OFDMA-based relaying network utilizing decode-and-forward cooperation strategy. We propose efficient greedy algorithms for both centralized and distributed resource allocations based on our analysis. Simulation results show that our proposed algorithms effectively enhance the total capacity, and also satisfy users’ data rate requirements in our proportional fairness constrained algorithms.

REFERENCES