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Linear Equalizers for Quasi-synchronous Block Spreading CDMA Systems

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Abstract— Recently, a block spreading code division multiple access (BS-CDMA) technique was presented whereby user-specific precoding along with orthogonal spreading codes are used to achieve multi-user interference (MUI) free when all users arrive at the base station simultaneously. In practice however, imperfect synchronization destroys the orthogonality among users and MUI occurs. This paper investigates the design of linear frequency domain equalizers to reduce the MUI in a quasi-synchronous BS-CDMA system. An optimal frequency domain linear minimum-mean squared error (LMMSE) equalizer is derived. Further simplification leads to a novel sub-optimal equalizer with reduced computational complexity. It is shown through simulation that the proposed equalizers effectively suppress the error floor due to quasi-synchronous reception when channel coding is applied.

I. INTRODUCTION

Code division multiple access (CDMA) is a popular multiple access technique that is used to support multiple users simultaneously in a network. Recently, under the assumption of perfect synchronization among users, a novel block spreading CDMA (BS-CDMA) [2] framework was presented whereby user-specific, channel-independent precoding along with special spreading codes are used to ensure orthogonality among users and multi-user interference (MUI) free reception. In practice, however, perfect synchronization for signals from different users is usually hard to achieve due to different signal propagation delays [1], resulting in MUI. Consequently, conventional equalizers designed for synchronous BS-CDMA fail to work in asynchronous systems.

In this paper, we study the design of the equalizers for BS-CDMA systems where the received signals from different users are not perfectly synchronized at the base station. In particular, we consider the realistic case where the timing control at the base station is good enough such that signals from different users are approximately synchronized to within a few chips, thus a quasi-synchronous BS-CDMA system [1]. We focus our study on the impact of quasi-synchronous reception and assume that the channel state information and the delays among the users are perfectly known at the base station. Furthermore, we assume chip-level synchronization, where the delays among users are integer multiples of one chip duration.

We present a general system model for quasi-synchronous BS-CDMA, extending on the work of [2]. MUI due to quasi-synchronous reception is derived, based on which an optimal frequency domain linear minimum-mean squared error (LMMSE) equalizer is proposed. Having noticed that the optimal equalizer requires a matrix inverse operation which could lead to high computational complexity when the block length or the number of users is large, we propose a sub-optimal equalizer to reduce the complexity. It is shown through simulation that both equalizers can effectively reduce the MUI due to quasi-synchronous reception. Moreover, benefiting from a reduced computational complexity, the sub-optimal equalizer is shown to provide a performance close to that of a similar system using the optimal equalizer.

The rest of the paper is organized as follows. We derive the system model for quasi-synchronous BS-CDMA in Section II. The equalizer designs are presented in Section III. Performance of the proposed equalizers is given in IV. Conclusions are drawn in Section V.

Notation: We use blackboard bold capital letters and lowercase boldface letters to denote vectors, and uppercase boldface letters and calligraphic letters to denote matrices; $\mathbf{A}^H$ and $\text{Tr}\{\mathbf{A}\}$ to denote the Hermitian transpose and the trace of the matrix $\mathbf{A}$, respectively. Moreover, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix, $\mathbf{F}$ denotes the $P \times P$ Fourier transform matrix with its (m,n)th entry given by $F(m,n) = 1/\sqrt{P}\exp(-j2\pi(m-1)(n-1)/P)$, $\mathbb{R}^{m \times n}$ and $\mathbb{C}^{m \times n}$ denote the fields of real and complex numbers in the $m \times n$-dimensional space, respectively, and $\mathbb{E}$ denotes the expectation operation. Finally, $\otimes$ denotes the Kronecker product.

II. SYSTEM MODEL

Fig. 1 shows the block diagram of the quasi-synchronous BS-CDMA system. Assume a BS-CDMA system with a total M users. At the transmitter of the μth user, information bits are encoded, interleaved, and mapped to constellation symbols, which are then arranged into blocks of P symbols, with the ith block of signals for the μth user given by a length-P column vector $\mathbf{s}_\mu^i$. Assume that the symbols within and across blocks are independent, each block having a mean of
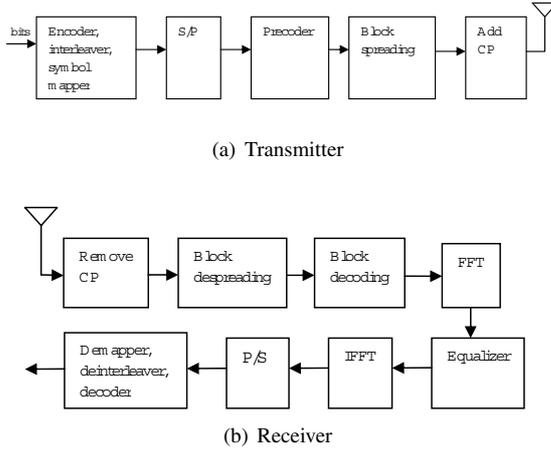
(a) Transmitter

(b) Receiver

Fig. 1. Transceiver structure of a BS-CDMA system

zero and a variance of $\sigma_{\mu,i}^2$. Each block is then precoded with a $P \times P$ user-specific precoding matrix $\mathbf{\Lambda}_\mu$, and subsequently block spread by a length-M spreading code $\mathbf{c}_\mu$. The resulting signal of the μth user in the ith block is therefore given by $\mathbf{x}_\mu^i = (\mathbf{c}_\mu \otimes \mathbf{\Lambda}_\mu)\mathbf{s}_\mu^i$, where $\mathbf{x}_\mu^i$ contains MP chips.

A cyclic prefix (CP) of length L_{CP}, at least equal to the memory order of the channel impulse response (CIR), is then added at the beginning of $\mathbf{x}_\mu^i$.

Cyclically extended signals of each user then go through the channel. We consider slow time-varying channel where the CIRs in different blocks within one frame of the transmitted data are the same. For simplicity, we also assume that the CIRs for different users are of the same length L. Denote the CIR of the μth user as $\mathbf{h}_\mu = [h_\mu(0), \cdots, h_\mu(L-1)]^T$. The noiseless ith received block of the μth user is given by [2]

$$\mathbf{y}_\mu^i = \mathbf{H}_\mu^L \mathbf{T} \mathbf{x}_\mu^i + \mathbf{H}_\mu^U \mathbf{T} \mathbf{x}_\mu^{i-1} \qquad (1)$$

where $\mathbf{T} \in \mathbb{R}^{N \times MP}$ denotes the CP insertion matrix given in, e.g., [3]; $\mathbf{H}_\mu^L \in \mathbb{C}^{N \times N}$ is a lower triangular Toeplitz matrix with its first column being $\mathbf{h}_\mu$ zero padded to length N, and $\mathbf{H}_\mu^U \in \mathbb{C}^{N \times N}$ is an upper triangular Toeplitz matrix with its first row being $[0, \cdots, 0, h_\mu(L-1), \cdots, h_\mu(1)]$.

Assume the base station is synchronized to one particular user (e.g., the mth user), which we refer to as the reference user. Users that are not synchronized to the reference user are referred to as the interference users. Let τ_μ denote the delay between the μth ($\mu \neq m$) user and the reference user. We consider two application scenarios where the μth user arrives τ_μ chips later or earlier than the reference user. In either case, we assume that the delays between the interference users and the reference user are reasonably small such that $\max\{\tau_\mu\} \leq L - 1 \leq L_{CP} << MP$.

Now consider the ith received block of the reference user and the ath user that arrives τ_a chips later than the reference user. The interference block from the ath user to the reference user contains the last τ_a chips from the $(i-1)$th received block of the ath user as its first τ_a chips, and the first $(N - \tau_a)$ chips from the ith received block of the ath user as its last $(N - \tau_a)$ chips. The interference from the ath user to the ith received block of the reference user is therefore given by

$$\mathbb{S}_a^i = \mathbf{P}_{N-\tau_a}^r \mathbf{y}_a^{i-1} + \mathbf{P}_{\tau_a}^d \mathbf{y}_a^i \qquad (2)$$

where $\mathbf{P}_{N-\tau_a}^r$ and $\mathbf{P}_{\tau_a}^d$ are the $N \times N$ permutation matrices obtained by shifting $\mathbf{I}_N$ right by $(N - \tau_a)$, and down by τ_a, respectively.

Similarly, for the bth user that arrives τ_b chips earlier than the mth user, the interference from the bth user contains the last $(N - \tau_b)$ chips of its ith block as the first $(N - \tau_b)$ chips and the first τ_b chips of its $(i+1)$th block as the last τ_b chips. The interference from the bth user to the ith received block of the reference user is therefore given by

$$\mathbb{S}_b^i = \mathbf{P}_{\tau_b}^r \mathbf{y}_b^i + \mathbf{P}_{N-\tau_b}^d \mathbf{y}_b^{i+1}. \qquad (3)$$

At the receiver, CP is removed from the beginning of the received signal of the mth user. It is known that the noiseless ith received block of the reference user after CP removal is given by $\mathbb{S}_m^i = \tilde{\mathbf{H}}_m \mathbf{x}_m^i$ [2], where $\tilde{\mathbf{H}}_m \in \mathbb{C}^{MP \times MP}$ is a circulant channel matrix with its first column being $[h_m(0), \cdots, h_m(L-1), 0, \cdots, 0]^T$. The composite received ith block of the signal after CP removal is therefore given by

$$\mathbf{r}_i = \mathbb{S}_m^i + \mathbf{R}\left(\sum_{a=1}^{N_a} \mathbb{S}_a^i + \sum_{b=1}^{N_b} \mathbb{S}_b^i\right) + \mathbf{v}_i \qquad (4)$$

where N_a and N_b are the number of users that arrive after and before the mth user, respectively, $\mathbf{R} \in \mathbb{R}^{MP \times N}$ accounts for CP removal [3], and $\mathbf{v}_i$ is the Gaussian noise with a mean of zero and a variance of σ_v^2. A block decoding and despreading matrix $\mathbf{D}_m = \mathbf{c}_m \otimes \mathbf{\Gamma}_m$ is then employed to recover the signals for the mth user, where $\mathbf{\Gamma}_m \in \mathbb{C}^{P \times P}$ is the decoding matrix. The ith received block of the mth user after block despreading and decoding is given by

$$\mathbf{z}_m^i = \mathbf{D}_m^H \mathbf{r}_i, \qquad \mathbf{z}_m^i \in \mathbb{C}^{P \times 1} \qquad (5)$$

It was shown in [2] that when $\mathbf{c}_m$, $m = 1, \cdots, M$, are the discrete Fourier transform (DFT) spreading codes[1], and the precoding and decoding matrices are given by $\mathbf{\Lambda}_m = \mathbf{\Gamma}_m = \text{diag}\{e^{j\theta_{m,0}}, \cdots, e^{j\theta_{m,P-1}}\}$ where $\theta_{m,p} =$

[1]Note that other spreading codes can be used as long as the code is mutually shift orthogonal.

$-2\pi p(m-1)/MP$, we have

$$\mathbf{D}_\mu^H \tilde{\mathbf{H}}_m \mathbf{x}_m^i = \begin{cases} M\tilde{\tilde{\mathbf{H}}}_m \mathbf{s}_m^i, & \text{for } \mu = m \\ 0, & \text{for } \mu \neq m \end{cases} \quad (6)$$

where $\tilde{\tilde{\mathbf{H}}}_m$ is a $P \times P$ kernel circulant channel matrix given in [2]. We can therefore rewrite (5) as

$$\mathbf{z}_m^i = M\tilde{\tilde{\mathbf{H}}}_m \mathbf{s}_m^i + \mathbb{I} + \mathbf{D}_m^H \mathbf{v}_i \quad (7)$$

where

$$\mathbb{I} = \mathbf{D}_m^H \mathbf{R} \left(\sum_{a=1}^{N_a} \mathbb{S}_a^i + \sum_{b=1}^{N_b} \mathbb{S}_b^i \right) \quad (8)$$

accounts for the MUI due to quasi-synchronous reception.

A fast Fourier transform (FFT) operation is then applied to allow a frequency domain equalizer for each user, followed by an inverse FFT (IFFT) to convert the signals back into the time domain. Our target is to design frequency domain equalizers to reduce the MUI due to quasi-synchronous reception. Denote the equalizer to be designed as a $P \times P$ matrix $\mathbf{G}$. The signal after FFT, frequency domain equalization, and IFFT is given by

$$\mathbf{w}_m^i = \mathbf{F}^H \mathbf{G} \mathbf{z}_m^i. \quad (9)$$

The equalized time domain signals are then demapped, deinterleaved, and decoded to recover the transmitted bits of the desired user.

III. FREQUENCY DOMAIN EQUALIZERS

We first simplify the MUI term in (8), and then propose two types of equalizers to reduce the MUI. Let $\mathbb{I}_a = \mathbf{D}_m^H \mathbf{R} \mathbb{S}_a^i$. Following (1) and (2), we have

$$\mathbb{I}_a = \mathbf{D}_m^H \mathbf{R} \big[\mathbf{P}_{N-\tau_a}^r \left(\mathbf{H}_a^L \mathbf{T} \mathbf{x}_a^{i-1} + \mathbf{H}_a^U \mathbf{T} \mathbf{x}_a^{i-2} \right) \\ + \mathbf{P}_{\tau_a}^d \left(\mathbf{H}_a^L \mathbf{T} \mathbf{x}_a^i + \mathbf{H}_a^U \mathbf{T} \mathbf{x}_a^{i-1} \right) \big].$$

Since $\mathbf{P}_{N-\tau_a}^r$ contains non-zero entries only on its first τ_a rows, and left multiplying $\mathbf{R}$ with $\mathbf{P}_{N-\tau_a}^r$ removes the first L_{CP} rows of $\mathbf{P}_{N-\tau_a}^r$, $\mathbf{R}\mathbf{P}_{N-\tau_a}^r$ results in a zero matrix due to the assumption that $\tau_a \leq L_{CP}$. Furthermore, define a $MP \times MP$ matrix $\mathbf{\Delta}_a^U = \mathbf{R}\mathbf{P}_{\tau_a}^d \mathbf{H}_a^U \mathbf{T}$. It can be shown that $\mathbf{\Delta}_a^U$ is an upper triangular Toeplitz matrix with its first row being $[\mathbf{0}_{1\times(MP-l_a)}, h_a(L-1), \cdots, h_a(L-l_a)]$, where $l_a = L - L_{CP} + \tau_a - 1$ when $L_{CP} < L + \tau_a - 1$. For $L_{CP} \geq L + \tau_a - 1$, $\mathbf{\Delta}_a^U$ is a zero matrix. It will become clear later that in this case, there is no interference from a user that arrives later than the reference user. Through matrix derivations we have

$$\mathbf{R}\mathbf{P}_{\tau_a}^d \mathbf{H}_a^L \mathbf{T} = \mathfrak{C}_{\tau_a}^d \tilde{\mathbf{H}}_a - \mathbf{\Delta}_a^U \mathfrak{C}_{L_{CP}}^d \quad (10)$$

where $\mathfrak{C}_x^d \in \mathbb{C}^{MP \times MP}$ is a circulant matrix obtained by circulantly shifting $\mathbf{I}_{MP}$ down by x. Therefore

$$\mathbb{I}_a = \mathbf{D}_m^H \left(\mathfrak{C}_{\tau_a}^d \tilde{\mathbf{H}}_a \mathbf{x}_a^i - \mathbf{\Delta}_a^U \mathfrak{C}_{L_{CP}}^d \mathbf{x}_a^i + \mathbf{\Delta}_a^U \mathbf{x}_a^{i-1} \right) \\ = \mathbf{D}_m^H \left(\mathbf{\Delta}_a^U \mathbf{x}_a^{i-1} - \mathbf{\Delta}_a^U \mathfrak{C}_{L_{CP}}^d \mathbf{x}_a^i \right) \quad (11)$$

where the last equality is due to the mutually shift orthogonal property of the DFT spreading codes. For the bth user that arrives earlier than the reference user, denoting $\mathbb{I}_b = \mathbf{D}_m^H \mathbf{R} \mathbb{S}_b^i$, we have

$$\mathbb{I}_b = \mathbf{D}_m^H \mathbf{R} \big[\mathbf{P}_{\tau_b}^r \left(\mathbf{H}_b^L \mathbf{T} \mathbf{x}_b^i + \mathbf{H}_b^U \mathbf{T} \mathbf{x}_b^{i-1} \right) \\ + \mathbf{P}_{N-\tau_b}^d \left(\mathbf{H}_b^L \mathbf{T} \mathbf{x}_b^{i+1} + \mathbf{H}_b^U \mathbf{T} \mathbf{x}_b^i \right) \big].$$

It can be shown that $\mathbf{R}\mathbf{P}_{\tau_b}^r \mathbf{H}_b^U \mathbf{T}$ is a zero matrix, and

$$\mathbf{R} \left(\mathbf{P}_{\tau_b}^r \mathbf{H}_b^L + \mathbf{P}_{N-\tau_b}^d \mathbf{H}_b^U \right) \mathbf{T} = \mathfrak{C}_{\tau_b}^u \tilde{\mathbf{H}}_b + \mathbf{\Delta}_b^L$$

where $\mathfrak{C}_x^u$ is a circulant matrix obtained by circulantly shifting $\mathbf{I}_{MP}$ up by x, $\mathbf{\Delta}_b^L$ is a lower triangular Toeplitz matrix with the first column being $[\mathbf{0}_{1\times(MP-\tau_b)}, -h_b(0), \cdots, -h_b(\tau_b - 1)]^T$, and

$$\mathbf{R}\mathbf{P}_{N-\tau_b}^d \mathbf{H}_b^L \mathbf{T} = -\mathbf{\Delta}_b^L \mathfrak{C}_{L_{CP}}^d.$$

The interference from the bth user that arrives earlier than the reference user can therefore be rewritten as

$$\mathbb{I}_b = \mathbf{D}_m^H \left(\mathfrak{C}_{\tau_b}^u \tilde{\mathbf{H}}_b \mathbf{x}_b^i + \mathbf{\Delta}_b^L \mathbf{x}_b^i - \mathbf{\Delta}_b^L \mathfrak{C}_{L_{CP}}^d \mathbf{x}_b^{i+1} \right) \\ = \mathbf{D}_m^H \left(\mathbf{\Delta}_b^L \mathbf{x}_b^i - \mathbf{\Delta}_b^L \mathfrak{C}_{L_{CP}}^d \mathbf{x}_b^{i+1} \right) \quad (12)$$

and MUI is given by

$$\mathbb{I} = \sum_{a=1}^{N_a} \mathbb{I}_a + \sum_{b=1}^{N_b} \mathbb{I}_b. \quad (13)$$

Note that $\mathbf{\Delta}_a^U$ and $\mathbf{\Delta}_b^L$ contain the last l_a taps and the first τ_b taps of the channel, respectively. It is known that in realistic channels such as an exponential decay channel, the energy of the first few taps dominates. A user that arrives earlier than the reference user therefore contributes majority of the interference power while the performance degradation due to the interference from a user that arrives later than the reference user is negligible. This phenomenon can also be verified through simulation.

Having obtained the MUI in quasi-synchronous BS-CDMA, we propose in the following two frequency domain equalizers to mitigate the interference.

A. Optimal LMMSE equalizer

An optimal LMMSE equalizer minimizes the mean squared error (MSE) between the transmitted signal and the signal after equalization in the time domain (see [4] and the reference therein). The optimal LMMSE equalizer, denoted as $\mathbf{G}_{opt}^*$, can be obtained by solving $\partial \epsilon_m^i / \partial \mathbf{G}_{opt}^* = 0$, where ϵ_m^i is the MSE, given by

$$\epsilon_m^i = \text{Tr} \left\{ \mathbb{E} \left[(\mathbf{s}_m^i - \mathbf{w}_m^i)(\mathbf{s}_m^i - \mathbf{w}_m^i)^H \right] \right\}. \quad (14)$$

Following (9), (7) and (13), the key step of computing the MSE is to derive $\mathbb{E} \left[\mathbb{I}_a \mathbb{I}_a^H \right]$ and $\mathbb{E} \left[\mathbb{I}_b \mathbb{I}_b^H \right]$, where $\mathbb{I}_a$ and $\mathbb{I}_b$ are given in (11) and (12), respectively. Based

on the assumptions of the independence of the transmitted symbols, taking expectation over the transmitted symbols, we have

$$\mathbb{E}\left[\mathbb{I}_a \mathbb{I}_a^H\right] = \eta_a^2 \mathbf{D}_m^H \mathbf{\Delta}_a^U (\mathbf{c}_a \mathbf{c}_a^H \otimes \mathbf{I}_P)(\mathbf{\Delta}_a^U)^H \mathbf{D}_m \quad (15)$$

where $\eta_a^2 = \sigma_{a,i-1}^2 + \sigma_{a,i}^2$. Eq. (15) is due to the mixed product property of the Kronecker product, i.e., $(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AB} \otimes \mathbf{CD}$, and the circularity of $\mathbf{c}_a \mathbf{c}_a^H \otimes \mathbf{I}_P$ when the DFT spreading codes are used, i.e.,

$$\mathcal{C}_{L_{CP}}^d (\mathbf{c}_a \mathbf{c}_a^H \otimes \mathbf{I}_P)(\mathcal{C}_{L_{CP}}^d)^H = \mathbf{c}_a \mathbf{c}_a^H \otimes \mathbf{I}_P. \quad (16)$$

Similar results can be obtained for $\mathbb{E}\left[\mathbb{I}_b \mathbb{I}_b^H\right]$. The optimal LMMSE equalizer $\mathbf{G}_{opt}^*$ is therefore given by

$$\mathbf{G}_{opt}^* = M \sigma_{m,i}^2 \mathbf{\Xi}_m^H \left[\mathbf{\Pi} + \mathbf{F}\mathbf{\Omega}_{opt}\mathbf{F}^H\right]^{-1} \quad (17)$$

where $\mathbf{\Xi}_m = \mathbf{F}\tilde{\bar{\mathbf{H}}}_m \mathbf{F}^H$ is a $P \times P$ diagonal matrix with its kth diagonal entry being the kth frequency domain channel coefficient of the CIR $\mathbf{h}_m$ [3]

$$\mathbf{\Pi} = M^2 \sigma_{m,i}^2 \mathbf{\Xi}_m \mathbf{\Xi}_m^H + M \sigma_v^2 \mathbf{I}_P \quad (18)$$

and

$$
\begin{aligned}
\mathbf{\Omega}_{opt} &= \mathbf{D}_m^H \left[\sum_{a=1}^{N_a} \eta_a^2 \left(\mathbf{\Delta}_a^U (\mathbf{c}_a \mathbf{c}_a^H \otimes \mathbf{I}_P)(\mathbf{\Delta}_a^U)^H\right)\right. \\
&\quad \left. + \sum_{b=1}^{N_b} \eta_b^2 \left(\mathbf{\Delta}_b^L (\mathbf{c}_b \mathbf{c}_b^H \otimes \mathbf{I}_P)(\mathbf{\Delta}_b^L)^H\right)\right] \mathbf{D}_m
\end{aligned}
$$

where $\eta_b^2 = \sigma_{b,i}^2 + \sigma_{b,i+1}^2$. Details in deriving the optimal LMMSE equalizer are omitted for brevity. Note that when $\mathbf{\Delta}_a^U$ and $\mathbf{\Delta}_b^L$ are zero matrices, i.e., when there is no MUI, $\mathbf{\Omega}_{opt}$ is a zero matrix, and (17) becomes the conventional frequency domain LMMSE equalizer for synchronous BS-CDMA with MUI free reception, as is given in [4].

Calculating the optimal MMSE equalizer requires matrix multiplication and inverse, which can be computationally complex especially when the block size and the number of users is large. Reduced complexity can be achieved when the expectation operation in computing the MSE is also taken over the channel. Next, we derive a low complexity sub-optimal LMMSE equalizer.

B. Sub-optimal LMMSE equalizer

When $L_{CP} < L + \tau_a - 1$, $\mathbf{\Delta}_a^U$ can be decomposed into a Kronecker product

$$\mathbf{\Delta}_a^U = \mathbf{J} \otimes \mathbf{\Theta}_a^U \quad (19)$$

where $\mathbf{J} \in \mathbb{R}^{M \times M}$ is obtained by shifting $\mathbf{I}_M$ to the right by $M - 1$, and $\mathbf{\Theta}_a^U \in \mathbb{C}^{P \times P}$ is an upper triangular Toeplitz matrix with the first row being $[\mathbf{0}_{1 \times (P-l_a)}, h_a(L-1), \cdots, h_a(L-l_a)]$. Similarly, $\mathbf{\Delta}_b^L$ can be decomposed as

$$\mathbf{\Delta}_b^L = \mathbf{J}^H \otimes \mathbf{\Theta}_b^L \quad (20)$$

where $\mathbf{\Theta}_b^L \in \mathbb{C}^{P \times P}$ is a lower triangular Toeplitz matrix with the first column being $[\mathbf{0}_{1 \times (P-\tau_b)}, -h_b(0), \cdots, -h_b(\tau_b - 1)]^T$.

Assuming that the channel taps for each user are independent, when the expectation operation is also taken over the channel, (15) becomes

$$
\begin{aligned}
\mathbb{E}\left[\mathbb{I}_a \mathbb{I}_a^H\right] &= \eta_a^2 \left[(\mathbf{c}_m^H \mathbf{J} \mathbf{c}_a \mathbf{c}_a^H \mathbf{J}^H \mathbf{c}_m) \otimes (\mathbf{\Gamma}_m^H \mathbf{\Omega}_a^U \mathbf{\Gamma}_m)\right] \\
&= \eta_a^2 \mathbf{\Omega}_a^U \quad (21)
\end{aligned}
$$

where $\mathbf{\Omega}_a^U = \mathbb{E}\left[\mathbf{\Theta}_a^U (\mathbf{\Theta}_a^U)^H\right]$ is a diagonal matrix with its first l_a diagonal entry being $\alpha_h^2(k) = \sum_{l=0}^{l_a-k-1} \mathbb{E}|h_a(L-l-1)|^2$, for $k = 0, \cdots, l_a - 1$. The second equality in (21) holds due to the fact that $\mathbf{c}_m^H \mathbf{J} \mathbf{c}_a \mathbf{c}_a^H \mathbf{J}^H \mathbf{c}_m = 1$ when the DFT spreading codes are used, and that $\mathbf{\Gamma}_m$ is a diagonal matrix with $\mathbf{\Gamma}_m^H \mathbf{\Gamma}_m = \mathbf{I}_P$. Similarly,

$$\mathbb{E}\left[\mathbb{I}_b \mathbb{I}_b^H\right] = \eta_b^2 \mathbf{\Omega}_b^L \quad (22)$$

where $\mathbf{\Omega}_b^L = \mathbb{E}\left[\mathbf{\Theta}_b^L (\mathbf{\Theta}_b^L)^H\right]$ is a diagonal matrix with its last τ_b diagonal entry being $\beta_h^2(k) = \sum_{l=0}^{k} \mathbb{E}|h_b(l)|^2$, for $k = 0, \cdots, \tau_b - 1$. Following the same approach as in Section III-A, defining the $P \times P$ diagonal matrix as

$$\mathbf{\Omega}_{subopt} = \sum_{a=1}^{N_a} \eta_a^2 \mathbf{\Omega}_a^U + \sum_{b=1}^{N_b} \eta_b^2 \mathbf{\Omega}_b^L \quad (23)$$

the sub-optimal LMMSE equalizer is given by

$$\mathbf{G}_{subopt}^* = M \sigma_{m,i}^2 \mathbf{\Xi}_m^H \left[\mathbf{\Pi} + \mathbf{F}\mathbf{\Omega}_{subopt}\mathbf{F}^H\right]^{-1}. \quad (24)$$

Taking advantage of the diagonal structure of the matrices, the matrix inverse in (24) can be computed recursively with a reduced complexity. Details on the computation and the complexity of the matrix inverse are given in Appendix I. It will be shown that, with a reduced order of complexity from $\mathcal{O}(P^3)$ to $\mathcal{O}(P^2)$, the sub-optimal equalizer can provide a performance close to that of a similar quasi-synchronous BS-CDMA system using the proposed optimal equalizer.

IV. SIMULATION RESULTS

Performance of a quasi-synchronous BS-CDMA system using the proposed optimal and the sub-optimal equalizer is simulated and compared to that using the conventional LMMSE equalizer. Performance of BS-CDMA with synchronous reception is also simulated. In all the simulations, we used the system model given in Section II with QPSK modulation, a block length of $P = 16$, a CP length of $L_{CP} = 8$, and a channel length of 9. We used Rayleigh fading channels with exponential decay. The decay factor is approximately 0.86, which leads to the difference in power between the first and the last channel taps being about 30 dB. As it was shown in Section III that MUI is dominated by the interference from the users that arrive earlier than
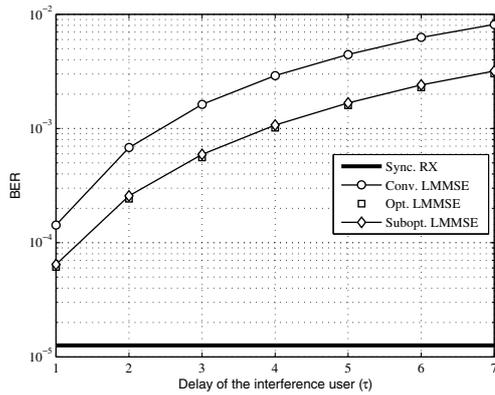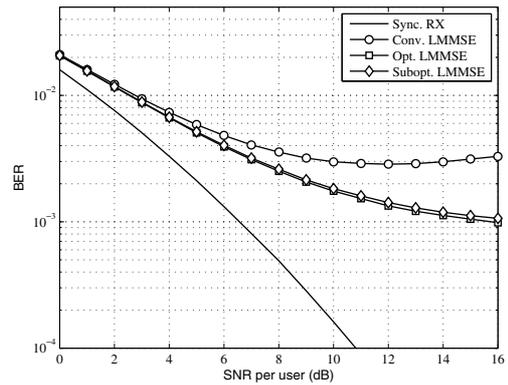
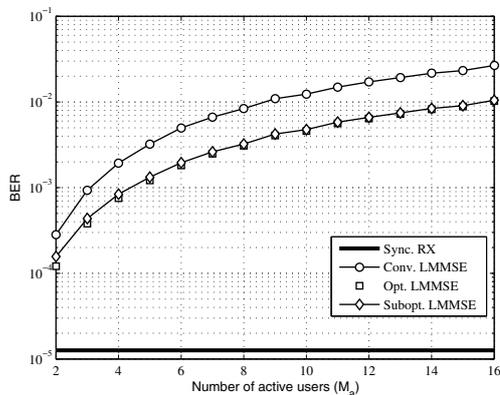Fig. 2. Effects of the delays.

Fig. 4. Uncoded performance.

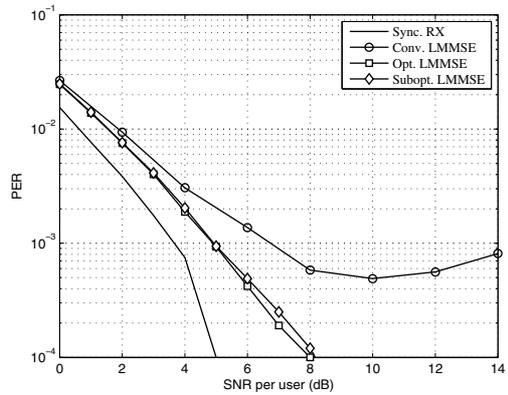Fig. 3. Effects of the number of active users.

Fig. 5. Coded performance.

the reference user, and the interference user with the largest delay contributes the most to the MUI, in the following, we focus our simulations on the case where MUI is caused by the only interference user that arrives the earliest among all the active users.

Effects of system parameters on the performance of the aforementioned four systems are presented in Figs. 2 and 3. Fig. 2 presents the effects of different delays from an interference user, where it was assumed that among the 8 active users, all users are synchronized to the reference user except one. In Fig. 3, we show the effects of different number of active users, where it was assumed that all users are synchronized to the reference user except one, who has a delay of 7. In both figures, the bit error rates (BERs) were obtained at a signal-to-noise ratio (SNR) of 14 dB. It is observed that systems using the proposed optimal and sub-optimal LMMSE equalizers achieve very close performance. Compared to the BER of a system using the conventional LMMSE equalizer, systems using the proposed equalizers provide improved BERs by a factor of approximately $1/3$ and

$1/2$ in Figs. 2 and 3, respectively. Moreover, although the performance of a system with synchronous reception does not change with longer delays or larger number of active users, that of systems with quasi-synchronous reception degrades due to the increased MUI.

Fig. 4 presents the performance of the four systems without channel coding. We assume 8 active users in the simulation, where all the users are synchronized to the reference user except one, who has a delay of 7. Note that we considered a relatively pessimistic scenario where the delay is close to the channel memory order, therefore severe MUI occurs. It is again observed that the proposed sub-optimal equalizer achieves a close performance to that of a quasi-synchronous system using the optimal equalizer, both outperform the system using the conventional LMMSE equalizer where an obvious error floor occurs due to MUI. Compared to the ideal system where synchronous reception can be achieved, performance degradation due to quasi-synchronous reception is observed. This degradation can be mitigated by using effective channel coding. As is shown in Fig. 5, when

a rate $1/2$ convolutional code with constraint length 7 is applied, the performance degradation due to quasi-synchronous reception is only about 1.2 dB at a packet error rate (PER) of 10^{-3} when the proposed optimal and sub-optimal equalizers are used. The packet length for the coded case is 1024 bits.

V. Conclusions

Designs of frequency domain equalizers to reduce the MUI in a quasi-synchronous BS-CDMA system were investigated. An optimal LMMSE equalizer was derived, based on which a sub-optimal equalizer with reduced complexity was proposed. It was shown through simulation that both equalizers effectively reduce the MUI due to quasi-synchronous reception, and the sub-optimal equalizer provides a performance close to a similar system using the optimal equalizer.

Acknowledgment

Appendix I

Let $\mathbf{Q} = \left[\mathbf{\Pi} + \mathbf{F}\mathbf{\Omega}_{subopt}\mathbf{F}^H\right]^{-1}$. Assuming $Q = \max\{l_a\} + \max\{\tau_b\} < P$, there are Q non-zero diagonal entries in $\mathbf{Q}$. Define $m_{l_a} = \max\{l_a\}$, $m_{\tau_b} = \max\{\tau_b\}$, and $\mathbf{\Omega}_F = \mathbf{F}\mathbf{\Omega}_{subopt}\mathbf{F}^H$. We can decompose $\mathbf{\Omega}_F$ into the following summations:

$$
\mathbf{\Omega}_F = \mathbf{F}\left[\mathrm{diag}\left\{\sum_{a=1}^{N_a}\eta_a^2\alpha_h^2(0), 0, \cdots, 0\right\} + \cdots \right.
$$
$$
+\mathrm{diag}\left\{0, \cdots, 0, \sum_{a=1}^{N_a}\eta_a^2\alpha_h^2(m_{l_a}-1), 0, \cdots, 0\right\}
$$
$$
+\mathrm{diag}\left\{0, \cdots, 0, \sum_{b=1}^{N_b}\eta_b^2\beta_h^2(\tau_b - m_{\tau_b}), 0, \cdots, 0\right\}
$$
$$
\left. + \cdots + \mathrm{diag}\left\{0, \cdots, 0, \sum_{b=1}^{N_b}\eta_b^2\beta_h^2(\tau_b - 1)\right\}\right]\mathbf{F}^H
$$
$$
= \sum_{a=1}^{N_a}\eta_a^2\alpha_h^2(0)\mathbf{f}_0\mathbf{f}_0^H + \cdots
$$
$$
+\sum_{a=1}^{N_a}\eta_a^2\alpha_h^2(m_{l_a}-1)\mathbf{f}_{m_{l_a}-1}\mathbf{f}_{m_{l_a}-1}^H
$$
$$
+\sum_{b=1}^{N_b}\eta_b^2\beta_h^2(\tau_b - m_{\tau_b})\mathbf{f}_{P-m_{\tau_b}}\mathbf{f}_{P-m_{\tau_b}}^H + \cdots
$$
$$
+\sum_{b=1}^{N_b}\eta_b^2\beta_h^2(\tau_b - 1)\mathbf{f}_{P-1}\mathbf{f}_{P-1}^H
$$

where $\mathbf{f}_p$ denotes the pth column of the $P \times P$ matrix $\mathbf{F}$ for $p = 0, \cdots, P-1$. Matrix $\mathbf{Q}$ can be rewritten as

$\mathbf{Q} = [\mathbf{\Pi} + \sum_{q=0}^{Q-1}\rho_q\mathbf{u}_q\mathbf{u}_q^H]^{-1}$ where

$$
\rho_q = \begin{cases} \sum_{a=1}^{N_a}\eta_a^2\alpha_h^2(q), & q = 0, \cdots, m_{l_a}-1 \\ \sum_{b=1}^{N_b}\eta_b^2\beta_h^2(q - m_{l_a} + \tau_b - m_{\tau_b}), & q = m_{l_a}, \cdots, Q-1 \end{cases}
$$

$$(25)$$

and

$$
\mathbf{u}_q = \begin{cases} \mathbf{f}_q, & q = 0, \cdots, m_{l_a}-1 \\ \mathbf{f}_{P-m_{\tau_b}-m_{l_a}+q}, & q = m_{l_a}, \cdots, Q-1 \end{cases}.
$$

$$(26)$$

The matrix inverse lemma states that if a matrix $\mathbf{C}$ is defined as

$$\mathbf{C} = \mathbf{A} + \mathbf{x}\mathbf{y}^H \tag{27}$$

the inverse of matrix $\mathbf{C}$ can be computed as

$$\mathbf{C}^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{x}\mathbf{y}^H\mathbf{A}^{-1}}{1 + \mathbf{y}^H\mathbf{A}^{-1}\mathbf{x}}. \tag{28}$$

Applying the matrix inverse lemma, a recursive approach can be used to compute the inverse of $\mathbf{Q}$ by decomposing $\sum_{q=0}^{Q-1}\rho_q\mathbf{u}_q\mathbf{u}_q^H$ into Q additions, and applying (28) Q times. The pseudocode of computing the inverse of $\mathbf{Q}$ is given in Table I.

Since matrix $\mathbf{\Pi}$ is diagonal, the inverse on $\mathbf{\Pi}$ can be easily obtained, and the rest of the computation involves Q multiplications of size $P \times P$ diagonal matrices, with a complexity of $\mathcal{O}(P^2)$, as opposed to the complexity of computing the matrix inverse of a $P \times P$ matrix in the optimal LMMSE equalizer, which is $\mathcal{O}(P^3)$. The computational complexity for the sub-optimal LMMSE equalizer is therefore greatly reduced, especially with a large matrix dimension P.

TABLE I

PSEUDOCODE OF COMPUTING $\mathbf{Q}^{-1}$

Initialization: $\mathbf{Q}_0 = \mathbf{\Pi}$.
For $q = 1 : Q-1$

Compute $\mathbf{Q}_q^{-1} = \mathbf{Q}_{q-1}^{-1} - \frac{\rho_q\mathbf{Q}_{q-1}^{-1}\mathbf{u}_q\mathbf{u}_q^H\mathbf{Q}_{q-1}^{-1}}{1+\rho_q\mathbf{u}_q^H\mathbf{Q}_{q-1}^{-1}\mathbf{u}_q}$

end for

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