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Analysis and Design of High-Speed Electromagnetic Moving-Iron Actuators

by

Wadi Affane

A thesis submitted to the University of Bristol in accordance with the requirements for the degree of Doctor of Philosophy in the Faculty of Engineering, Electrical and Electronic Engineering Department.

December, 1992
Dedicated to
my parents
Lalmi and Zohra
Abstract

High-speed electromagnetic moving-iron actuators are experimentally investigated and numerically simulated, using digitally-controlled instrumentation techniques, lumped-parameter (magnetic equivalent circuit) networks, and field (finite-element) models. Various actuator topologies, based on the moving-iron principle, that are capable of achieving very high operating speeds, are also investigated.

An optically-based and digitally-controlled instrumentation technique is developed to assess the actuator dynamic performance. A dual voltage (microprocessor-controlled) strategy is also developed to improve actuator speed of response. A lumped-parameter model that accurately simulates, with minimum computation, the dynamic behaviour of the actuator is developed and experimentally verified. This model, whose magnetic parameters are derived from static field results, accounts for magnetic saturation, 3D effects due to width change between iron parts and transverse edge fluxes, and the dynamic coupling of the actuator system variables. A static lumped-parameter model is developed, in parallel, to achieve insight into the underlying actuator design principle, and rapid predictions of the effects of parametric changes. Two-dimensional field models are developed, using a commercial finite-element package, to accurately predict the saturation levels, and to estimate the mmf/flux characteristics of each actuator component (iron and air part) and force characteristics for use in the dynamic lumped-parameter model. The 3D effects are taken into account by incorporating the results of 2D scalar potential models, in typical transverse planes, into the longitudinal (main path) solution using suitable compensation factors. Transient eddy current effects are also investigated.

The study is extended by surveying various topologies of moving-iron devices, and analysing their relative performances. The objective of this investigation is to establish, quantify, and compare the factors limiting the performance, particularly the maximum acceleration rate.
I would like to express my sincere thanks to Prof. B. M. Bird and Dr. C. J. Carpenter, for their continuing encouragement and invaluable guidance throughout the course of this research. Very special thanks are due to Dr. Carpenter for his genuine interest in this research and for the numerous, helpful, and fruitful discussions on the subject of Electromagnetics.

I would like to thank Mr. McMahon for his assistance regarding early use of ANSYS finite-element software on Appolo (DN3000) machines, and later access to DN4000 machine for the use of PE2D package.

Thanks are also due to the technical staff of the Departments of Electrical and Mechanical Engineering.

I am most grateful to the Algerian Government for providing the grant and financial support.

Lastly, but by no means least, my sincere thanks to my parents, my brothers and sisters at home, for their infinite sacrifice, patience and moral support.
Declaration

Unless otherwise acknowledged, the content of this thesis is the original work of the author. No portion of the work in this thesis has been submitted by the author in support of an application for another degree or qualification at this, or any other university, or other institution of learning.

Signed

W. Affane
List of principal symbols

\( R \) Magnetic reluctance
\( \Omega \) Magnetic scalar potential
\( \Phi \) Magnetic flux
\( r \) Iron reluctance
\( \mu_i \) Iron permeability
\( \mu_0 \) Air permeability
\( S \) Area
\( B \) Magnetic flux density
\( H \) Magnetic field strength
\( F \) Magnetomotive force
\( l \) Length
\( w \) width
\( F \) Magnetic force
\( W \) Magnetic co-energy
\( x,y,z \) Cartesian co-ordinates
\( K_g \) Force coefficient
\( N \) Number of turns
\( \alpha_p, \beta_g \) Compensation factors
\( A \) Magnetic vector potential
\( H_c \) Coercive field of permanent magnet
\( J \) Current density
\( \nabla \) Laplacian operator
\( \phi \) Electric potential
\( d_p \) Depth of flux penetration
\( \sigma \) Conductivity
\( m,n \) nodes at extremities of magnetic path
V  Applied voltage  
i  Current  
R  Resistance  
f  Force density  
g  Gap length  
d  Slot depth  
gs  Slot width  
L  Inductance  
ρm  Mass density  
v a  Volume of armature  
Km  Ratio total mass/active mass  
s  Stroke  
δt  time delay  
a  Acceleration  
p  pole pitch  
ρ  Resistivity  
v c  Volume of copper  
a c  Area of copper  
Kd  mechanical damping  
m  mass of armature
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7.1 Summary
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Appendix A Software for the dual voltage drive system
1.1 Project background and objectives

The thesis reports on research into high-speed moving-iron actuators for knitting machine applications and similar high-speed electronically-controlled systems. The first half of the project was undertaken with close co-operation of the departments of Electrical and Electronic Engineering, and of Mechanical Engineering with Bentley Engineering Ltd of Leicester. Half way during the research period there was a change of emphasis toward more generic work because Bentley company was taken over, and subsequently this led to a cessation of co-operative work (despite attempts to establish links with the company that purchased Bentley's business). In the first half of the project [1], the original objectives were to:

(i) Study the dynamics of high-speed electromagnetic actuators for knitting machine applications, and the effects of variations in design and manufacturing parameters on their performance, durability and cost.

(ii) Design instrumentation techniques for production assessment of actuator performance to improve quality control.

(iii) Develop methods for the electronic control of actuating solenoids to achieve good control of actuator dynamics.

(iv) Research the mechanical and electrical design of solenoids for high-speed operation, low cost and long life.
(v) Research novel actuator designs for very high-speed electronically controlled operation.

In the second half of the project, because of the change in the events (cessation of collaboration with Bentley Eng. Ltd), a more generic work was pursued under the two following main objectives:

(i) A study of methods of modelling the static and dynamic characteristics of high-speed electromagnetic moving-iron actuators using lumped-parameter (magnetic equivalent circuit) and field-solving (finite-element) techniques.

(ii) A Survey of alternate high-speed electromagnetic actuator designs, and an investigation of their comparative performance and the criteria that limit this performance.

The remainder of the thesis (Chapters 2 to 6) reflects this division of work between the study of the Bentley bistable permanent magnet actuator, and the associated development of instrumentation techniques, and the generic investigation of performance limitations and comparisons of high-speed moving-iron actuators.

1.2 Thesis structure

Chapter 2 reports on the experimental investigation carried out during the first half of the project. First, three instrumentation techniques [2] for the study of lever motion of the Bentley actuator, are briefly described. The techniques are based, respectively, on the use of a stroboscope, a CCD camera-based system, and an optical probe. These instruments were digitally controlled using a microprocessor for data transmission and processing. An opto-sensor instrument, that is simple and very effective is constructed and used for most tests to measure actuator lever trajectory and operating time. A microprocessor-controlled dual voltage drive strategy is also developed and
applied to the actuator system to improve speed of operation. Armature sticking, and
the change in mechanical construction to eliminate it are also discussed.

The lumped-parameter magnetic equivalent circuit and the finite-element field
models, respectively, in Chapters 3 and 4 are developed simultaneously to,
respectively, achieve insight into the underlying design principle of the Bentley
actuator, and to improve the accuracy of the simulation results. Chapter 4 reports on
an elaborate account on tackling the 3D effects [3] (effects of width change between
iron parts and transverse edge flux), and the effects of transient eddy currents in the
iron parts on actuator performance, using the commercial 2D field-solving (finite-
element) package PE2D [4].

In chapter 5, a dynamic lumped-parameter (magnetic equivalent circuit) model is
developed. The latter has the ability to predict the dynamic actuator behaviour taking
into account magnetic saturation, width change and transverse edge effects, fringing
in the air gaps and dynamic coupling between the electrical, magnetic, and
mechanical actuator system variables. Because the Bentley actuator has an unusual
3D structure (due to width change) that includes a permanent magnet, highly
saturated iron parts and large air gaps (causing substantial flux fringing and corner
effects), detailed static field results obtained from the field model (Chapter 4) are
incorporated into the dynamic lumped-parameter model [5]. The latter gives
sufficiently accurate results that are in good agreement with the measured data.

In Chapter 6 the study is extended by surveying various moving-iron actuator
designs, and analysing their relative performance. The author establishes, quantifies,
and compares the relative merit figures for variable gap-length and variable gap-area
devices (including the Bentley actuator), particularly when used to operate at high
speeds.
Chapter 2

Experimental investigation

2.1 Application of MI actuators in modern industrial knitting machines

The electromagnetic actuators are mainly employed in the circular knitting machines, used primarily for fabric and hose production. In these machines knitting is achieved by sliding needles in slots cut axially around the circumference of a needle cylinder (Fig. 2.1). Cams that rotate with respect to the cylinder act on butts on the needles. These butts project from the cylinder and are raised and lowered to perform the knitting action of the machine (Fig. 2.2). Patterning is achieved by allowing different cam arrangement to act on the needles, or on cam controlled elements known as push-jacks placed below the needles in their slots. Selection of the needle path may use different length of butts on needles, or the deflection or depression of additional needle butts. Many methods have been used for pattern selection. Mechanical techniques include peg cylinder or combs to interact with needle butts. In general, these methods limit the complexity of pattern that may be produced, and pattern change is laborious. The use of electromagnetic actuators under electronic control allows practically unlimited pattern complexity and very rapid pattern change.

The main pattern selection method that has been studied involves the use of electromagnetic actuators for the selective depression of needle or push-jack butts. The actuator flips a lever to one of two positions either to contact or to miss a needle butt (Fig. 2.3 shows the sort of arrangement that might be used). The overall configuration of a single actuator is shown in Fig 2.4. In practice, actuators are stacked, typically into six-high arrangements (Fig. 2.5) and about 1000 actuator
stacks are used on large machines. Needle butts are staggered into chevrons for contact by the actuator stacks (Fig. 2.6).

The reason that actuator stacks have to be used is that a single actuator cannot operate fast enough to interact with all the needles in a cylinder. The limitation in actuator speed also imposes an overall limit on machine speed, particularly on fine gauge machines, in which many, closely-spaced, needles are used. Higher actuator stacks may reduce this limitation, but space constraints limit the extent to which this may be used. Operating speed is a significant constraint on the introduction of electronically controlled machines. A second constraint is the high cost of the large number of actuators required. Improving the actuator speed will assist in both these respects.

2.2 Actuator operation and drive electronics

A substantial part of this research has been the study on the design of the Bentley actuator used on the RTC-E garment-size circular knitting machines. The actuator was manufactured by Bentley Engineering Limited to a design produced by Louis Newmark Ltd. Although full drawings of the design were available, much detail information concerning production techniques and design intent was not available, nor could it be obtained from Louis Newmark. As a consequence, a major part of this research involved characterisation of the design and its performance, and identification of the controls required to achieve satisfactory actuator operation.

The Bentley actuator (Fig. 2.4) is an electromagnetic device that generates forces by controlling the magnetic energies stored in the two main air-gaps between the pole pieces. The device has two separately generated magnetic fluxes. The first is the polarising flux produced by the permanent magnet and channelled through the pole pieces to the air-gaps. The second flux is the control flux generated by the electric coil which surrounds the armature. When an input voltage pulse is applied to the coil,
the control flux interacts with the polarising flux at the air-gaps, thus creating a force on the armature. Reversing the polarity of the voltage pulse, reverses the direction of the control flux across the air-gaps, thus reversing the direction of the force on the armature. This is a bistable operation in which the armature is moved from one pole piece to the other, according to the polarity of the voltage pulse. Normal operation of the Bentley actuator requires an application of a square voltage pulse of 24 volt amplitude and 4 ms duration across the coil. Since this is a bistable operation positive and negative pulses are needed to drive the armature in both directions. The electrical drive circuit used normally by Bentley Eng. was modified to suit a 6809 microprocessor-based control interface and variable input voltages (Fig. 2.7). The microprocessor system controls the duration of the pulses and the delay times between them. Details about the hardware and software control of the pulse widths are reported in Section 2.7. The control pulses are sent through transistor stage amplifiers to instantaneously switch on the power MOSFET transistors and produce positive and negative voltage pulses to be applied to the actuator coil. The top stage amplifier produces an inverted pulse for negative polarity. The two zener diodes determine the voltage between the gate G and the source S necessary to switch on the MOSFETs. In addition to the control of pulse widths and delays, the microprocessor system can be used to investigate different pulse shapes, as reported in Section 2.7.

2.3 Instrumental techniques for measuring actuator lever trajectory

2.3.1 Available measuring techniques

Initially, a number of measuring techniques were investigated to study the dynamic characteristics of actuators. A substantial part of the initial experimental work concentrated on the study of the displacement-time lever trajectories. A number of existing techniques were investigated which included the use of a B&K motion
analyser, high-speed video, miniature accelerometers and eddy-current measuring system for dynamic analysis. Early study indicated that a different approach would be more appropriate because of the following reasons:

(i) The lever size of the Bentley actuator was not appropriate to eddy-current displacement devices, and did not merit the use of accelerometers.

(ii) The B&K motion analyser did not have the desired characteristics.

(iii) The video system available (NAC HSV-200) was specifically designed to record fast occurring events at 200 frames per second (fps) for playback at slower rates, and for inspection and analysis of individual frames. However, to study the lever motion in this way would have demanded equipment with a capability of at least 5000 fps. Useful recordings were nonetheless made by applying the principle used when fast occurring, repeated cycles are observed with a stroboscope, as reported in [6].

Owing to the small size of the actuator components, experimental work concentrated on optical measuring techniques and involved the use of an optical sensor, a fibre-optic probe, a stroboscope control system, and a charge-coupled-device (CCD) camera system. Interface and electronic control of these devices were developed as reported in [2].

2.3.2 Fibre-optic probe

The fibre-optic probe [2] is a fixed transmitter-receiver assembly in which light is transmitted and received along a flexible fibre-optic cable. Electronic control is used for adjusting the gain and sensitivity, as well as the light transmitter receiver. The probe was also linked to a transient data capture equipment and the displacement data together with the actuator applied voltage, on a personal computer. The instrument is
non-contacting and low cost and also, when used in other applications, it can monitor movement in mechanisms that are difficult to reach due to the flexibility of the fibre-optic cable. The problem encountered with this device is that its signal output is weak and needs a large amplification. An additional shortcoming is its high sensitivity to mechanical vibrations, making it difficult to use for measuring actuator lever trajectories.

2.3.3 **Stroboscope and CCD camera systems**

A stroboscope system [7] was used for the visualisation and recordings of repetitive, high-speed motion of actuators or any machinery. The flash rate of a stroboscope may be synchronised with motion frequencies from 15Hz to 10KHz with frequency or phase offset. Software and hardware were developed to control any standard stroboscope with an external trigger input. A signal synchronised to the actuator driving voltage is required by the control system as a reference signal. A delay, either user specified or time varying, is inserted between the start of the reference signal and the strobe pulse. The user specified delay allows the lever undergoing periodic motion to be "frozen" for visual inspection and measurement at any point in its cycle. The position of a target marked on the actuator lever was measured using a travelling microscope for a number of delay times, enabling the lever trajectory to be plotted. The time varying delay made it possible to view the lever in slow motion at a preset speed.

The principal advantage of the stroboscope control system over all the other methods investigated in this study is the possibility of monitoring the actuator lever inside the main air gap (between the pole pieces). The technique was very useful in detecting that there was no observable bounces of the lever tail against the pole piece. The other features include linearity and absence of calibration. However, the technique is very slow and tedious because it relies on reading through the travelling microscope
which also leads to measurement errors. Full details of the hardware and listings of the computer software, may be found in [7].

A more versatile technique [7] involved the use of a CCD camera which can monitor the position of an edge of an opaque object over various displacements (typically greater than 0.5 mm) at a sampling frequency of up to 33 Khz. Output is digital, and the prototype has a spatial resolution of 1 part in 256. It is versatile, non-contact, displacement measurement system, with potential to measure a wide range of displacements, remotely or close to the camera. Although the instrument is non-intrusive, linear, capable of operating over a wide range of sampling frequencies and displacements, and needing no calibration, difficulties were encountered when used for the actuator experimental investigation. The reason for this is that the actual setting up of the apparatus is not straightforward. It is difficult to focus the object onto the camera lens and the CCD is very sensitive to the surrounding light level. This represents a shortcoming when a large number of repetitive experiments are required. The apparatus was used in the early stages of the project and was then abandoned because of the problems mentioned above.

2.4 Optical sensor-based instrument for measuring actuator lever trajectory and operating time

A simple opto-switch instrument (Fig. 2.8) was developed and used to monitor the motion of the actuator lever. The output signal from the optical sensor may be displayed on an oscilloscope, and thus provides a quick and convenient way for investigating the effects of changing input voltages applied to the actuator, for example. The optical sensor consists of a light emitting diode (LED) that transmits a light beam to a reflective surface which sends it back to a phototransistor to generate an electrical signal.
Because of the severe non-linearity of its transfer characteristic, the device must be used in the limited linear region which extends over about 1 millimetre. By positioning the optical component on the appropriate part of the lever where the actual movement is less than 1 mm, the instrument produces the displacement/time characteristic of the actuator. The optical sensor was calibrated using several reflective surfaces in an attempt to obtain maximum amplitude of the linear region. These surfaces comprised tin foil and white paper.

**Calibration of the optical sensor:**

The calibration of the optical sensor was performed by carrying out various tests with respect to angular and linear displacements and to the types of reflective material used on the lever surface. To ensure that the calibration was performed as accurately as possible, the sensor was clamped onto a micrometer. Fig. 2.9 shows diagrams of the angular displacement and linear back-off calibration tests. Fig. 2.10 shows the results of the calibration for angular displacement for tin foil and white paper on the lever surface, respectively. The output signal obtained with the white paper surface varies little when displacing the sensor over an angular range of 18°. In normal actuator operation, the lever angular displacement has an amplitude of 2.8 degrees that is inside the region (-1.5°+1.5°) in Fig. 2.10. In this region, the signal is almost constant with respect to displacement. A better output was obtained with a foil surface, particularly between 0° and +4.5°. Between -1.5° and 0° the signal is almost linear. Since the amplitude of angular displacement of lever in normal actuator operation was about 2.8°, the optical sensor could be used in the range (-1.5° to +1.5°) without causing major errors. In the second test the sensor was calibrated for linear back-off with tin foil on the lever surface (see Fig. 2.12). As the the sensor moves away from the lever (lever and sensor are in contact at zero mm), it starts responding at about 0.6 mm, after which the device enters a region of non-linearity of
about 0.5 mm. The linear region extends over a distance of 1 mm (from 1.1 to 2.1 mm). For correct use of the optical sensor, the lever must be positioned at 1.1 mm away from the sensor and its displacement must not exceed 1 mm amplitude.

After calibration, the optical sensor was mounted, as shown diagrammatically in Fig. 2.11, to measure the lever trajectory. To ensure that the output signal level was in the linear region, the sensor was positioned at the middle of the lever where the amplitude of the linear displacement is about 0.6 mm. The starting position of the lever was at 1.1 mm away from the sensor (inside the linear region). Fig. 2.13 shows the oscilloscope trace produced from the sensor output, and the coil current waveform, after switching the actuator on with a square voltage pulse as shown in Fig. 2.14. The lever displacement in Fig. 2.13 exceeds the expected travel of 0.6 mm due to the presence of oscillations caused by the mechanical contact of the armature with the pole piece which extend the travel to 0.7 mm, well inside the linear region (Fig. 2.12). To derive the linear displacement of the armature end (part between the pole pieces), the measured data were doubled. However, because the materials of the armature and the lever are different and have some elasticity, the point on the lever where the measurement is taken and that of the armature end between the pole pieces may behave mechanically differently causing errors to arise in the measured data.

The advantages of the optical sensor over the devices mentioned in the preceding sections are that it is simple, cheap, non-contacting, and not sensitive to mechanical vibrations. These qualities make the optical sensor more attractive to use than the above devices for measuring the speed of response of the Bentley actuator.

**Measurement of the actuator operating time:**

Because of the bending of the lever at the contact of the armature with the pole piece, and the subsequent result of oscillations on the trajectory, it is difficult to accurately
determine from the displacement curve when the armature hit the pole piece for the first time. The actual operating time is defined as shown in Fig. 2.15. To accurately determine the operating time of the actuator, the optical sensor electronic circuit was developed further as shown in Fig. 2.16. The actuator is switched on from the power MOSFET drive circuit using the microprocessor which triggers a timer. The armature displacement produce the sensor output signal which is sent to the input of a comparator (operational amplifier) to produce a falling edge of a pulse when the low level on the sensor signal is reached. A transistor is also used to convert the op-amp output signal into a TTL compatible pulse which is sent to the input interrupt of the microprocessor to stop the timer, and the operating time is then displayed on the monitor. Since the microprocessor gives only hexadecimal numbers, software development was needed to convert the hexadecimal results in decimal form. A more detailed account on hardware and software control is given in Section 2.7.

2.5 Measurement of magnetostatic force

The awkward position of the armature in the actuator assembly also made the measurement of force difficult to perform. The measurement was carried out using a load cell of a 'NENE' machine. Fig. 2.17 shows a simplified diagram of the measurement rig. An ultra light plastic block, for better contact with the load cell, was glued and positioned at the extremity of the lever, so that the distance between the pivot and the point of measurement is equal to that between the pivot and the main gap where the net static force is produced. The force was measured at various currents and positions. For changing the position, very thin papers (0.1 mm thick) were placed between the pole piece and the armature as shown in Fig. 2.17. After setting the current and position of armature, the 5N load cell was lowered so that it makes contact with the plastic block. The cell was then displaced very slowly downward until the armature was observed to move. The net force value was then
read on the transducer display. Fig. 2.18 shows the measured data of the net force characteristic. The errors that may have emerged during measurement would have the following origins:

(i) The lever and armature have different masses, so that the forces at both ends (on lever and armature) may not be the same.

(ii) Existence of a frictional force on the pivot.

2.6 Dynamic characteristics of the actuator

Two of the main dynamic characteristics that are directly measurable are the lever displacement/time curve and the coil current waveform. They carry information on the dynamic behaviour of the actuator during its operation and are directly linked, respectively, to two important parameters that characterise any actuator, namely the electrical and mechanical time constants. The Bentley actuator is normally operated by application of a standard voltage pulse of 24 V and 4 ms duration (see oscilloscope trace in Fig. 2.14). Notice the negative spike when the voltage is switched off. This is due to the return of energy to the source. In effect, when the voltage is chopped off, the flux decay results in induced emf that is in opposite direction to the supply voltage, and hence, this causes the presence of a negative surge. The actuator lever trajectory that was monitored with the optical sensor in response to the applied voltage is shown in Fig. 2.13. The displacement/time curve shows three stages in the operation of the device:

(i) A delay period (or electrical delay time), during which the flux in the closed main gap is gradually reduced to the point when the net force changes direction, and becomes a pulling rather than a holding force.
(ii) A travel time (or mechanical delay) from pole to pole. This is due to the accelerating pull-off force produced in the closing gap.

(iii) Oscillation of the selector lever after impact with the pole piece. It has to be pointed out that the oscillations occur only on the end lever due to its bending and that stroboscopic examination showed no observable bounce of the lever tail piece (or armature) on the pole piece.

The displacement/time curve was used to study the effect of voltage increase and changes in the magnetic circuit on the delay and travel times. Fig. 2.13 shows the oscilloscope trace of the actuator current response. It is interesting to observe that the curve has two significant dips due to saturation effects. The first dip occurs at the start of motion showing a sudden rise in $\frac{di}{dt}$ caused by a sudden decrease in the circuit inductance. If the armature were held into position the current would continue to rise until it reaches the steady-state value. Allowing armature motion produces the second dip at the end of the travel. Measurement of the travel time, using the opto-sensor instrument, shows that the second dip occurs, precisely, at the first impact of armature on the pole piece. The dynamic lumped-parameter model in Chapter 5 will show that the second dip is caused by further saturation in the armature. After the second dip, the current starts rising again until it reaches the steady-state value of $\frac{V}{R}$. One feature to note is the large $\frac{di}{dt}$ on the current response (Fig. 2.13) just after the voltage was applied. This may be explained using the electrical circuit equation which reads:

$$V = iR + N\frac{d\Phi}{dt}$$

(2.1)

where $V$ is the applied voltage, $i$ the current, $R$ the coil resistance, $N$ the number of turns and $\frac{d\Phi}{dt}$ the coil flux time derivative. At the initial time ($t=0$), the current is equal to zero and $\frac{d\Phi}{dt}$ takes a large value equal to $\frac{V}{N}$ which results in a large $\frac{di}{dt}$. 
Under normal operating conditions, the electrical time delay and the mechanical time delay (or travel time) are of the same order of magnitude (see Fig. 2.13). The actuator operating time could be substantially reduced by either reducing the electrical time constant \( (L/R) \) or reducing the mechanical time constant (proportional to armature mass/total accelerating force) or both. The first strategy would require only modification of the electrical drive circuit and does not affect the actuator design, and, hence, it is the first choice in an attempt to reduce the actuator response time (Section 2.7). The second strategy would require significant design changes of the magnetic circuit such as the shape of the pole pieces, the armature and the size of the air gaps. This would need substantial calculation to predict actuator performance (see Chapters 3 to 5).

### 2.7 Dual voltage strategy to increase actuator speed

A dual voltage strategy was employed to reduce the actuator operating time. The technique consists of applying a square dual voltage as illustrated in Fig. 2.19. The effect of the short pulse \((V_1, t_1)\) is to force the flux up and drives the current up as quickly as possible. In effect, by applying as high a voltage as possible, maximum \(di/dt\) (and hence maximum \(d\Phi/dt\)) can be achieved to reduce the flux in one of the main air gaps to zero as quickly as possible, and obtain maximum acceleration force as early as possible during operation time. The second pulse \((V_2, t_2)\) provides enough voltage for \(iR\), since \(d\Phi/dt\) is zero, and the latch on then holds the armature. The voltages \(V_1\) and \(V_2\) were set using variable power supplies, whilst the times \(t_1\) and \(t_2\) were controlled by a 6809 microprocessor-based system.

**System hardware:**

To implement the dual voltage strategy, a microprocessor-controlled drive system was developed. The overall system hardware is shown in diagrammatic form in Fig.
2.20. Fig. 2.21 shows a simplified block diagram of the microprocessor (MC6809) board. The latter has a CPU (Central Processor unit) that communicates with its peripherals via a 8-bit data bus and a 16-bit address bus at clock cycles of 1 μsecs. The timer uses a 4MHz clock as its time base for counting. The timer (MC6840) was used to generate the interrupt request signal (IRQ) and operates as a monostable. The asynchronous communications interface adapter (ACIA MC6850) provides the control and formatting of data transmission and reception between the CPU and the keyboard/monitor unit. The software program described in the next sub-section was stored in a 4 Kbyte ROM (MC2716). The system also uses a 8 Kbytes RAM (MC6264) for temporary data storage. Both ROM and RAM have an access time of 150 nsecs.

The dual voltage drive circuit (Fig. 2.22) was realised by linking two single-drive circuits in parallel. In one of the circuits, the MOSFETS are switched on to high (positive and negative) voltages (V₁), and in the other, they are switched on to low voltages (V₂). To switch on the MOSFETs in each single circuit, pulses of durations t₁ and t₂ are sent from the microprocessor at predetermined times controlled by software. Let's consider the upper single-drive circuit. Pulse A(t₁) is sent to MPSA42 (NPN) transistor whose collector provides sufficient current to open the gate G of the MOSFET IRF621 which, instantaneously switches on to the positive voltage +V₁. The MOSFET output D (drain) produces the high positive voltage pulse. The signal B(t₁) sent through transistors BC337 and MPSA92 opens the MOSFET IRF9631 which produces the high negative voltage pulse. The signal C(t₂) and D(t₂) switch on the MOSFETs in the lower drive circuit to generate the low (positive and negative) voltage pulses.
System software:

An Assembly language program was written and fed into the microprocessor to generate the 4 pulses $A(t_1)$, $B(t_1)$, $C(t_2)$, and $D(t_2)$ with controllable widths at chosen instants of time, and to determine the actuator operating time at the reception of an interrupt signal sent from the opto-sensor instrument (Section 2.4). Full listing of the Assembly language program is given in Appendix A. The program flowchart is shown in simplified block diagram in Fig. 2.23. The latter helps to explain and understand clearly how the assembly language program listed in Appendix A works. Before the main program starts, each memory location (address) that will be used for data storage and retrieval must be reset and identified with variable names supplied by the user. These addresses are of the form $11XXh$, $12XXh$, and $XXh$ (see Appendix A). Special addresses for use of the timers, the universal transmitter receiver (urt), the interrupt, and the escape key must be enabled and initialised. The main program starts at subroutine START1 which, at a hit of a key on the keyboard, will identify, via the subroutine KEY the ASCII characters that have been typed. The entered characters will be stored temporarily at addresses 'xloc' in CONV1 subroutine. When the return key (CR) is hit, all entered characters will be converted from ASCII to hexadecimal numbers through the subroutine CONVD. The pulses $A(t_1)$, $B(t_1)$, $C(t_2)$, and $D(t_2)$ were generated using the subroutine WAVE1 which uses the loop DELAY to set the durations of $t_1$, $t_2$, and of the rest time $t_r$ (delay time between pulses), with the ASCII numbers entered. The program was made user friendly by displaying instruction-like messages which appear in the last part of the program listing.

Constant energy strategy and test results:

The voltages $V_1$ and $V_2$ set on the variable supplies and the times $t_1$ and $t_2$ to be supplied to the microprocessor system, were determined using a constant energy strategy. These parameters were determined so that the maximum electric input
energy is the same as for the single standard voltage pulse ($V_3=24\text{V}$, $t_3=4\text{ms}$). The following condition was then set:

$$E_1 + E_2 = E_3$$ (2.2)

where $E_1$, $E_2$, and $E_3$ are maximum energies during the times $t_1$, $t_2$, and $t_3$, respectively. Transformation of equ. 2.2 yields:

$$R_1 I_1^2 t_1 + R_2 I_2^2 t_2 = R_3 I_3^2 t_3$$ (2.3a)

and, results in:

$$V_1^2 t_1 + V_2^2 t_2 = V_3^2 t_3$$ (2.3b)

The combinations of $V_1$, $t_1$, $V_2$, and $t_2$ that verify equ. 2.3b and the following condition

$$t_1 + t_2 = t_3$$ (2.4)

are unlimited. Various combinations of $V_1$, $t_1$, $V_2$, and $t_2$ were computed using a simple algorithm to solve equs. 2.3 and 2.4. $V_1$ was allowed to change between 70 and 120 V (maximum voltage supply available) with increment of 10. For each value of $V_1$, $t_1$ was allowed to vary from 0.1 to 0.5 ms by increments of 0.1 ms. A set of tables giving 5 values of $t_1$, $t_2$ and $V_2$ for each value of $V_1$ were obtained, and tested to drive the actuator. $V_1$ and $V_2$ were set from the variable power supplies and $t_1$ and $t_2$ were entered using the keyboard and the monitor attached to the microprocessor board. Table 2.1 shows the results of the tests. For each value of $V_1$, only one set of values of $t_1$, $V_2$, and $t_2$ that produced minimum operating time is given.
Table 2.1 Results of dual voltage

<table>
<thead>
<tr>
<th>$V_1$(volts)</th>
<th>$t_1$(ms)</th>
<th>$V_2$(volts)</th>
<th>$t_2$(ms)</th>
<th>Oper.Time(ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>70</td>
<td>0.1</td>
<td>23.4</td>
<td>3.9</td>
<td>2.8</td>
</tr>
<tr>
<td>80</td>
<td>0.1</td>
<td>19.6</td>
<td>3.9</td>
<td>2.5</td>
</tr>
<tr>
<td>90</td>
<td>0.24</td>
<td>9.6</td>
<td>3.76</td>
<td>2.2</td>
</tr>
<tr>
<td>100</td>
<td>0.2</td>
<td>9.5</td>
<td>3.8</td>
<td>2.2</td>
</tr>
<tr>
<td>120</td>
<td>0.1</td>
<td>9.3</td>
<td>3.9</td>
<td>2.2</td>
</tr>
</tbody>
</table>

As can be observed in Table 2.1, the lowest operating time of 2.2 ms was achieved for $V_1$ values between 90 and 120 volts. Fig. 2.25 shows the oscilloscope trace of a dual-voltage pulse of 90 V/0.24 ms and 9.6 V/3.76 ms that was applied to the Bentley actuator. The corresponding displacement/time and coil current responses are illustrated in Fig. 2.26. As can be observed the effect of the dual voltage was to reduce the delay time by 50 % and the overall time by 27 %. This is accounted for by the increase in the rate-of-change $di/dt$. However, the subsequent travel time remained unchanged because the attractive force is limited by the saturation in the armature. It was also observed that provided the armature has started to move, the voltage pulse can be chopped off and the actuator still operates correctly (Fig. 2.27) with slightly longer operation time which suggests that during the travel time the energy is provided by the permanent magnet only and this could lead to significant energy savings. The improvement in actuator speed has been realised at the expense of electronics cost since twice as many components were used as compared to a single-voltage drive. Thus, the proper shaping of the pulse is an effective way of reducing the response time of the actuator. This technique can improve the performance of any high-speed actuator, provided that its ratio electrical time constant/mechanical time constant is not small. The work on modification to the drive strategy was not pursued further because Bentley were unwilling to consider the use
of drive voltages above 56 volts, or modifications to the hybrid electrical drive circuit used by the company.

2.8 Changes in mechanical construction

One of the problems that was given attention by Bentley Engineering Ltd and the Bristol University research team was that the actuator had a tendency to stick (and thus mis-select stitches, or even damage the machine) after a period of inactivity. In addition actuator sticking made the existing design very expensive, with much of the cost arising in machining, and plating of the soft iron parts. At least 11 stages were involved in the manufacture of a pole piece, and these include carburising, induction hardening and chromium plating of the end of the component. The problem or risk of sticking caused Bentley Engineering Ltd to adopt an electrical drive strategy for the actuators such that a voltage pulse was applied to an actuator even if it was already in the correct position. By pulsing the actuators every time, the risk of sticking was reduced significantly, and this meant that the actuators had a much higher electrical duty than the average pattern demands, which limited the DC voltage that could be applied, and thus the speed of operation.

In the past, it was believed that sticking was caused by a mixture of knitting oil and lint gumming up the actuator lever and pole pieces, and that this was compounded by low drive voltages, and low ambient temperatures, in some installations. To investigate this, a number of 'used' actuators were tested. These were driven with square voltage pulses of between 16 and 25 volts, and with varying degrees of knitting oil ingress, and the actuator response measured. The overall finding was that actuator operation time was not significantly affected by temperature (actuator temperature in the range 0-20°C), or by bearing clearance on worn actuators. Oil ingress had the effect of increasing operation time, but did not lead to sticking at the voltages tested.
In the reference documentation [8] on Remko soft iron it is suggested that, to achieve satisfactory magnetic properties, the iron had to undergo an annealing treatment. If this were not carried out, there was a risk of high remanent flux leading to solenoid sticking which was not observed during the experimental investigation. Bentley Eng. Ltd did not carry out this anneal phase, because of their requirement to case-harden the actuator components to withstand the mechanical impacts for a 100 million operation life and then plate them to reduce residual flux and minimise the risk of sticking.

The main work in this area involved collaboration with Bentley Eng. Ltd on the design of an experimental actuator (Fig. 2.28) with polymer stops to control air-gap thickness and to provide a 'seat' against which the actuator lever tail piece would impact. With these stops it would not be necessary to harden or plate the pole piece or lever, which could be blanked from stock material, and these components could also be annealed to achieve optimum magnetic properties. The prototype actuator performed satisfactorily on test machines since no sticking was observed. The polymer used in trials is Nylatron, a molybdenum disulphide filled nylon. Alternate materials were also investigated. It was noted that the durability of the material under repeated impacts may be crucial to the reliability of the actuators. Unfortunately, progress with the development of the experimental actuator was interrupted by the sale of the Bentley Engineering company.
Fig. 2.1 Knitting-machine cylinder

Fig. 2.2 Close view of needle butts
Fig. 2.3 Diagrammatic representation of lever/needle interaction

Fig. 2.4 Configuration of a single actuator
Fig. 2.5 Six-high actuator arrangement

Fig. 2.6 Chevron arrangement of butts
**Fig. 2.7** Microprocessor-controlled single drive system

**Fig. 2.8** Schematic diagram of the optical sensor

**Fig. 2.9** Linear back-off and angular displacement for calibration tests
Fig. 2.10 Calibration curves of opto-sensor for angular displacement
(a) white paper surface, (b) tin foil surface.
Fig. 2.11 Positioning of the optical sensor

Fig. 2.12 Calibration curve for linear back off with tin foil on level surface

Fig. 2.13 Oscilloscope traces: The lever trajectory (top) coil current response (bottom). 0.25 mm/division 0.2 amp/division, 1ms/division
Fig. 2.14 Input voltage pulse: 10Volts/div, 1ms/div

Fig. 2.15 Displacement time showing three stages during actuator operation
Fig. 2.17 Test rig for force measurement

Fig. 2.18 Measured force characteristics
Fig. 2.19 Dual voltage pulse

Fig. 2.20 Microprocessor-controlled dual voltage drive system

Fig. 2.21 Simplified diagram of microprocessor board and peripherals
Fig. 2.22 Dual-voltage circuit diagram
(interface with microprocessor)
Fig. 2.23 Program flowchart for control of pulse widths

Fig. 2.24 Pulse sequence for generating dual voltage pulse

t1: duration of high voltage pulse

t2: duration of low voltage pulse

tr: duration of rest period
Fig. 2.25 Oscilloscope trace of dual voltage pulse: 
20 Volts/division, 1ms/division

Fig. 2.26 Displacement/time (top) and current response (bottom) 
to dual voltage: 0.25 mm/div, 0.2 amps/div, 1ms/div
Fig. 2.27 Displacement/time curve (top) and current response (bottom)
Dual voltage chopped off at start of armature travel.
0.25 mm/div, 0.2 amp/div, 1ms/div

Fig. 2.28 Modified actuator design to eliminate sticking
(no contact between armature and pole piece)
Chapter 3

Static lumped-parameter magnetic circuit analysis

3.1 Objectives

The present chapter will focus more on understanding of the working principle of the Bentley actuator than on achieving accuracy of the magnetic modelling which will be the subject of the next chapter. Important information on how the device works can be obtained by studying the magnetic circuit under static conditions, using a lumped-parameter approach which will be described in the next sections. However, this is only an initial analysis which is based on the major assumption that the magnetic fields are essentially static, that is, no induced currents flow in any part of the device. The effects of eddy currents in the conductive iron parts will be investigated in Chap. 4. Although the device looks simple in its overall structure, its mathematical modelling is relatively difficult because of the various complex features which will be examined in this section. Another aspect of this chapter is to discuss how these complexities are tackled by the applied prediction technique.

Fig. 3.1a shows a 3-dimensional diagram of the actuator. For the sake of clarity only magnetic parts are shown (for device operation see Chap. 2). The actuator is characterised by long air gaps and, for mechanical reasons, the cross-section of the moving armature is much smaller than those of the other parts. The actuator is also characterised by the armature protrusion inside the main air gap beyond the pole tips which results in two air gaps of complex structure. The main flux path seems to be confined to the longitudinal (x,y) plane as shown in Fig. 3.1b. However, the transverse (y,z) cross-sections in Figs. 3.1c, 3.1d show a considerable change in width
between the different iron parts which suggests that some flux paths will lie mainly in transverse planes.

The actuator presents a complex three-mesh magnetic circuit as shown in Fig. 3.2. It operates by interaction of flux $\Phi_m$ with $\Phi_1$ and $\Phi_2$. The role of the coil is to switch the flux from gap $G_a$ to gap $G_b$ (or vice versa) in order to create the accelerating force that drives the armature from one pole piece to the other. The aim of this study is to understand how the three-mesh circuit works using a simple lumped-parameter approach. In addition, the determination of how the fluxes vary with current will give a quantitative assessment of the circuit inductance which is related to the electrical time constant, one of the key parameters in the design of high-speed actuators.

One of the characteristic features is the uneven shapes of the main air gaps $G_a$ and $G_b$ between the pole pieces and the armature as illustrated in Fig. 3.3 which shows the latter at the starting position against the top pole piece. Note that the armature protrudes beyond the pole tips which is a characteristic feature of the actuator. The small initial air gap between the pole and the armature is the thickness of the plating. The hold-on force $F_{ga}$ prevents the armature from moving, whereas, the pull-off force $F_{gb}$ drives the armature from pole to pole and determines the armature acceleration. Another aspect of the problem is the effect that the fringing flux (Fig. 3.3) may have on the initial pull-off force. This effect does vary with armature position. All these aspects will be considered in the development of the magnetic circuit model.

The pole pieces and armature undergo heavy saturation due to their small cross-sections, so that this will have an effect on the pull-off force and the speed of response of the actuator. The pole pieces and armature are made of Swedish Remko B, a non retentive soft iron of high permeability, low remanent flux density and maximum flux density $B_s$ of about 2 T. Since Remko B does not have major
hysteresis loops, for practical purposes its B/H characteristic can be modelled using a single-valued approximation curve.

The actuator includes Alnico 9 permanent magnet which operates on minor and major hysteresis loops. A full description of the hysteretic characteristics of the magnet material, including both the major and minor loops is not presently realistic because it requires knowledge of the past history of the magnet. However, since the permanent magnet operates over a limited range, it is sufficient to model it using a linearised approximation curve as described in Section 3.6.

Because of the existence of a pivot at one end of the armature (Fig. 3.1), the motion produced is rotary although it is a variable gap length device working by attraction force principle (devices working under this principle are often called linear actuators). Most rotary actuators work under a different mode of operation principle based on varying the gap area (see Chap. 6). For the purpose of the electromagnetic analysis, and because of a small angular displacement (about 3° maximum), the armature displacement was assumed linear.

One other important feature in the actuator is the simultaneous change of current and armature position during operation. This variation occurs in a manner that can only be determined by a time-stepping method which requires the coupling of electrical, magnetic, and mechanical actuator system variables (see Chap. 5).

3.2 Magnetic circuit lumping and other considerations

The actuator geometry was subdivided into rectangular iron and air blocks (Fig. 3.4) whose length $l_i$ and thickness $t_i$ were estimated from early finite-element analysis of the actuator flux distribution. Fig. 3.5 shows the lumped magnetic equivalent circuit model in which the magnetic saturation in the iron parts is taken into account (see Section 3.4). The iron components are represented by non-linear reluctance elements
(r), whereas the air gaps are represented by linear reluctances \((R)\). The magnetic sources are, respectively, a coil source \((NI)\) where \(N\) is the number of turns, and a permanent magnet mmf source. Due to the relative transverse dimensions (width change) and the corner effects, finite-element results were necessary to accurately determine the reluctances of the air gaps.

The displacement of the armature is angular and bidirectional, so that the static behaviour in both directions is the same when reversing the current. Therefore, the study of the effect of displacement (represented by the variable \(y\)) in one direction only is needed. Since the distance between the pivot and the contact point armature/pole piece is very large compared with the gap dimension \(l_g\) (see Fig. 3.4), the motion in the main gap is assumed to be linear for magnetic analysis purposes.

### 3.3 Calculation of air gap reluctances

The effects of width change between the iron components (Fig. 3.1) and the sharp edges of their corners, particularly those of the poles in the main air gap region (Fig. 3.3) make the reluctances of the air gap geometries difficult to determine. Solutions for specific air gap geometries such as those given by Roters [9] do not fit the actuator configuration under study, and thus cannot be applied here. Because early performance analysis of the actuator was carried out using the commercial (finite-element) field-solving package PE2D [4] and at the same time the lumped-parameter model was being developed, the author used the finite element results to calculate the air gap reluctances since these are difficult to accurately determine by any classical method due to the 3D effects. Development of a PE2D model and detailed analysis of the field distribution is reported in Chapter 4. Because the use of a 3D field-solving package such as TOSCA [10] was cumbersome and time consuming, the 3D effects were dealt with by performing 2D field calculations in longitudinal and transverse planes as discussed in Chapter 4. Due to symmetry, reluctance calculations for air
gaps $G_a$, $G_1$, and $G_2$ only were needed. Detailed account of how these reluctances were calculated is reported in Chapter 5 (for the simplified dynamic model) and only the results are given here. The total reluctance of any air gap at any specified current in the coil and position of armature is the ratio of the gap mmf $\Omega_g$ (as defined in Chapter 5) to the total air gap flux $\Phi_g$:

$$R_{ga} = \frac{\Omega_{ga}}{\Phi_{ga}}$$ (3.1)

Figs. 5.6 in Chapter 5 show linear $\Omega_{ga}/\Phi_{ga}$ curves for the main air gap $G_a$. The effect of armature displacement on the main air gap reluctance is as shown in Fig. 3.6. In the lumped model, this curve was simply fitted with a two-segment piecewise linearised curve. The reluctance of gap $G_2$ (Fig. 5.7) was little affected by armature displacement since the latter is small compared to the gap length. A reluctance change of less than 10% was observed for gap $G_2$, and thus for the purpose of this analysis, its reluctance was assumed constant. The reluctance of $G_1$, however, remains constant since there is no geometric change in its vicinity.

### 3.4 Non linear elements

The elements $r_{s1}$, $r_{s2}$, $r_{p1}$, $r_{p2}$, and $r_a$ in the 3-mesh circuit of Fig. 3.5 are non-linear variable reluctances that characterise the behaviour of the shelves, pole pieces, and armature, respectively. Each element $r_i$ was evaluated as

$$r_i = \frac{l_i}{\mu_i A_i}$$ (3.2)

where $l_i$, $\mu_i$, and $A_i$ are the mean path length, the permeability, and the cross-sectional area of the corresponding iron part. The permeability $\mu_i$ is the value of $B/H$ at the relevant operating point on the Remko BH curve. The common approach is to search for the value of the absolute permeability

$$\mu_{i(abs)} = \frac{B}{H}$$ (3.3)
during an iteration process. This requires an analytical function that can approximate the B/H curve which is difficult to obtain. As B varies between 0 and the saturation limit, $\mu_i(\text{abs})$ may vary continuously over a wide range which slows down the convergence of the iteration process. The technique used here is simple and easy to implement. It consists of modelling the Remko BH curve using a piecewise linearised approximation as shown in Fig. 3.7. The approximate BH curve is made up of 8 segments whose linear analytical expressions are known. The following expressions are those of 3 segments selected from Fig. 3.7:

- $B < B_1$ \quad $H = (1/\mu_1) B$ \quad ($\mu_1$ is the initial permeability)
- $B_1 < B < B_2$ \quad $H = (1/\mu_2) B + b_2$
- $B > B_s (B_s=2T)$ \quad $H = (1/\mu_s) B + b_s$ \quad ($\mu_s=\mu_o$ is the permeability at saturation)

where $1/\mu_1$ is the constant reluctance for the corresponding linear segment and is equal to $\Delta B/\Delta H$. It was assumed that for $B > 2T$, the slope of the BH curve becomes equal to $\mu_o$.

### 3.5 Modelling of permanent magnet

The Alnico 9 permanent magnet is one part of the actuator structure that is under influence of changing magnetic fields and geometry (armature displacement). It operates on recoil lines within the major hysteresis loop and it is necessary to determine the actual operating recoil line in order to model it. The actuator magnet can be represented either as a flux source in parallel with a reluctance or an mmf source in series with a reluctance [11]. The former representation of equivalent circuit is preferable only for magnets that approach the ideal behaviour of constant flux and very high equivalent reluctance. Since this is not the case here, the magnet was
modelled using the equivalent mmf representation. Fig. 3.9 shows the relation between the magnetic flux $\Phi_m$ and the mmf $F_m$ of the block of the magnet material. This curve was obtained by rescaling the demagnetisation portion of the BH curve of Fig 3.8. If the magnitude of the mmf applied in the negative direction does not exceed $F_a$, the magnet operates along the locus a-b. This locus may be closely approximated by a straight line of slope $1/R_o$ denoted by the expression

$$F_m = -F_o + R_o \Phi_m$$

(3.4)

This equation describes the equivalent magnetic circuit of Fig. 3.10. The part of the system external to the magnet is simply the equivalent reluctance seen by the magnet. The magnet is represented as a source of mmf $F_o$ in series with a reluctance $R_o$. The latter is defined as

$$R_o = \mu_m l_m / A_m$$

(3.5)

where $\mu_m$ is the actual slope of the recoil line in Fig. 3.9, and $l_m$ and $A_m$ are, respectively, the length and the cross-sectional area of the block of permanent magnet. Once $R_{eq}$ is known, the operating point can be determined by finding the coordinates of the intersection point c of the two straight lines as shown in Fig. 3.9.

### 3.6 Magnetic force calculation

As shown in Fig. 3.4 two magnetic forces $F_{ga}$ and $F_{gb}$ act simultaneously on the armature. These forces, referred to as the holding force and the pulling force, respectively, were calculated from an energy-balance analysis [12]. Generally, a magnetic force $F_g$ in a single mesh magnetic circuit can be expressed as

$$F_g = \Sigma (-\partial W_{ij}/\partial y)_i=\text{constant}$$

(3.6)
where \( W_i = \int R_i \Phi \, d\Phi \) is the energy stored in each circuit element. Since only variable air gaps contribute to the summation of equ. 3.6 the force expression becomes

\[
F_g = -\frac{\partial}{\partial y} \int R_i \Phi \, d\Phi = -0.5 \Phi^2 \frac{\partial R_g}{\partial y}
\]  

(3.7)

In the case of the actuator under study, the magnetic forces acting on the armature are expressed as follows:

\[
F_{ga} = 0.5 \Phi_{ga}^2 \frac{\partial R_{ga}}{\partial y}
\]

(3.8a)

\[
F_{gb} = 0.5 \Phi_{gb}^2 \frac{\partial R_{gb}}{\partial y}
\]

(3.8b)

where \( \Phi_{ga} = \Phi_1 + \Phi_m \) and \( \Phi_{gb} = \Phi_2 + \Phi_m \) are the fluxes in the main air gaps \( G_a \) and \( G_b \), respectively, and the net magnetic force is simply

\[
F_{net} = F_{gb} - F_{ga}
\]

(3.9)

The gap reluctances \( R_{ga} \) and \( R_{gb} \) were modelled using a 2-segment piecewise linearised curve (Fig. 3.6). Their derivatives with respect to displacement \( y \) take only two different values depending in which half of the travel the armature is. The net force can be expressed

\[
F_{net} = K_{gb} \Phi_{gb}^2 - K_{ga} \Phi_{ga}^2
\]

(3.10)

where \( K_{ga} \) and \( K_{gb} \) depend on \( y \).

3.7 Model equations and solution technique

The corresponding system of equations describing the magnetic equivalent circuit model is given below.
Analytic solution of this system is not possible because of the nonlinear nature of many of the magnetic-circuit elements, and so an iterative numerical solution technique was employed [13]. The technique is illustrated by the program flowchart shown in Fig. 3.11. Initially, the armature is assumed to be positioned against the top pole piece and the three fluxes \( \Phi_1, \Phi_2, \) and \( \Phi_m \) were assumed to have initial values to find trial flux densities in each iron parts. The above equation system was solved for \( \Phi_1, \Phi_2, \) and \( \Phi_m \) for a range of armature positions and currents in the coil. The current was allowed to vary from zero to \( V/R \) (\( V \) is the applied voltage, \( R \) the coil resistance) at small constant \( \delta_i \) steps. The value of \( \delta_i \) was chosen sufficiently small to avoid oscillations during the iteration process. Some other precautions were taken to ensure the convergence of the iteration process. Trouble arises from the severe nonlinearity of the magnetisation curve of Remko B soft iron. This is because relatively small differences in the flux density between successive iterations caused large differences in the permeability and hence in the coefficients of equ. 3.11 which gave rise to oscillations and divergence. This was corrected by keeping the changes in the coefficients below a certain limit using an acceleration (or underrelaxation) factor less than unity. The 3-mesh flux calculation and force computation were performed using a Fortran program developed by the author and run on IRIX (Unix System) mainframe. The lumped-parameter model took only 25 seconds to solve for one static position of armature which is very rapid compared to the field solution running time.
of 4 hours. These running times are not meaningful since two different machines were used, but they do show the difference between the 2 models in terms of size.

3.8 Static performance results

To be able to understand how the 3-mesh circuit works, the flux and force characteristics, with respect to current in the coil and displacement of the armature, were plotted (Fig. 3.12-3.19). For the sake of simplicity and clarification of the performance analysis, three positions of armature were given for each graph. The most interesting cases to examine are the variations of flux and force, with respect to current, at the starting position, since at this position, the initial pull-off force is developed to produce the initial acceleration of the armature.

The main parameters to examine are the main gap fluxes $\Phi_{ga} (=\Phi_m-\Phi_1)$, $\Phi_{gb}$ ($=\Phi_m+\Phi_2$), the hold-on force $F_{ga}$, and the pull-off force $F_{gb}$ acting on the armature. A clear understanding of the actuator magnetic design was obtained by plotting $\Phi_{ga}$ and $F_{ga}$ ($\Phi_{gb}$ and $F_{gb}$) together. Fig. 3.16 shows that at zero current (actuator excited by permanent magnet only), $F_{ga}$ falls sharply when the armature is displaced from starting to middle position, particularly at zero current. When increasing the current $\Phi_{ga}$ (Fig. 3.15) at starting position decreases at a higher rate-of-change $d\Phi_{ga}/di$ than in the middle and final positions, due to the large decrease of $\Phi_m$ (see Fig. 3.14). This results, as shown in Fig. 3.16, in a large rate-of-change $dF_{ga}/di$. Notice in Fig. 3.16 that $F_{ga}$ starts increasing again after reaching the zero level at high currents because $\Phi_{ga}$ reverses. This is a negative effect since it will reduce the net pulling force. As shown in Figs. 17-18, the flux $\Phi_{gb}$ and particularly, $F_{gb}$ remain at a much lower level at starting and middle positions. $F_{gb}$ increases substantially only at the final position when most of the armature travel has been completed. Fig. 3.19 shows the resulting net magnetic force $F_{net}$ which is mainly dominated by the change of $F_{ga}$ at the starting position, whereas at the final position, the dominating force is $F_{gb}$.
which seems to be affected by moderate saturation as shown in Fig. 3.17. The $F_{\text{net}}$ curve, at starting position, also shows that when the mmf reaches the value of 330 At the forces $F_{ga}$ and $F_{gb}$ become equal, point at which the armature is pulled away under the action $F_{gb}$. The smaller is this value, the smaller is the required mmf to drive the actuator. Fig. 3.19 also shows reasonably good agreement between theoretical and experimental results, particularly at low current levels at the three positions. However at high currents ($N\geq600$ At), particularly at the final position, the results diverge. These discrepancies originate principally from the simple approximation of the BH curve which does not follow exactly the manufacturer's BH characteristic, especially in the saturated region.

3.9 Design analysis

The above results show that the Bentley actuator design is based on the requirement that the hold-on force must be sufficiently high (much higher than the pull-off force) to secure the latch before switching on the device. This explains the existence of the protrusion of armature inside the main air gap. In effect, as shown in Fig. 3.3, the extended part of armature (at starting position) does not affect the hold-on force since most of the flux passes through the short gap and only a negligible amount crosses the parallel gap (the 2 gaps form $G_a$). However, the protrusion weakens the flux density in the lower main gap ($G_b$) because the flux crosses a wider gap ($G_b$), and hence giving a much lower force than in $G_a$ as shown in Fig. 3.18. Therefore, a large net latch force is achieved by merely extending the armature. However, this results in an increase in mechanical inertia due to the extra mass, and hence reduce armature acceleration. To counter this negative effect, the pulling force must be increased. It is suggested this may be achieved by using magnetic shunts across the secondary air gaps $G_2$ and $G_4$. 
One of the questions that remain to be answered at this stage is: why was the actuator designed with large secondary air gaps particularly G2 and G4? In effect, these absorb much of the applied mmf in the coil. The lumped-parameter model was used to investigate the effects of magnetic shunts across G2 and G4 (see Fig. 4.16). Figs. 3.20-3.21 show their effects on the hold-on force, the pull-off force and the net force, at starting position. There is a substantial improvement in the force characteristics. The comparison with the original design shows an increase in the hold-on force, at zero current (before switching the device), by almost 40%. F_{ga} decreases more sharply with current and displacement. However, at high currents, F_{ga} starts rising to almost half its maximum value, which may represent a drawback in the actuator performance. However, the inclusion of shunts has two contrasting implications: the circuit mesh reluctances have decreased, which has resulted in increase in circuit inductance L, and hence, increase in the electrical time constant L/R. This represents a disadvantage for a high-speed device. Hence, it explains the existence of large air gaps and a large coil to supply the large input mmf needed. On the other hand, the positive result of increase in force shown by Fig. 3.20-21, will produce a higher acceleration of the moving mass, and hence a smaller mechanical time constant. The effects of shunts on actuator dynamic response is quantified using the simplified dynamic model developed in Chapter 5.

3.10 Conclusions

The lumped-parameter model has predicted the performance of the actuator with a reasonable degree of accuracy (less than 20% maximum error in predicting the net force). It has given an insight into the working principle of the device. The model has shown that the basic requirement in the actuator design is that the hold-on force F_{ga} must be as high as possible to secure the latch and meet the load requirement, and must be reduced to a zero level at maximum rate-of-changes \( \frac{dF_{ga}}{di} \) and \( \frac{dF_{ga}}{dy} \).
The model has also shown that this is achieved at the expense of a much lower pull-off force $F_{gb}$ that increases substantially only at the end of the armature travel.

Because of the important levels of saturation in the actuator and the subsequent effects of the shunts in increasing these levels, there was a need for a more refined prediction using a field-solving package such as PE2D. The latter was also used to check the validity of the lumped-parameter model.
Fig. 3.1 3D and 2D schematic diagrams of the Bentley actuator
Fig. 3.2 Schematic 3-mesh flux distribution

Fig. 3.3 Magnified view of the main air gaps
Fig. 3.4 Dimensions of magnetic circuit lumps

Fig. 3.5 Three-mesh magnetic equivalent circuit
Fig. 3.6 Main gap reluctance versus displacement

Fig. 3.7 Piecewise linearised approximation for BH curve of Remko B
Fig. 3.8 BH demagnetisation curve of Alnico 9 magnet

Fig. 3.9 Linearised approximation for magnet flux/mmf characteristic

Fig. 3.10 Magnet equivalent circuit
I read data

set lump dimensions and BH curves

set constants

set armature position

set coil current

calculate circuit reluctances

calculate approximate B from BH curve

check sum of mmfs around meshes and adjust B

convergence?

no

yes

calculate loop fluxes and the main gap fluxes

Calculate net magnetic force

range completed?

no

yes

position range completed?

no

yes

plot results

stop

Fig. 3.11 Program flowchart for static lumped-parameter model
Fig. 3.12 Flux versus mmf in mesh 1

Fig. 3.13 Flux versus mmf in mesh 2

Fig. 3.14 Flux versus mmf in magnet mesh
Fig. 3.15 Flux-current-displacement characteristics of upper main air gap Ga

Fig. 3.16 Hold-on force-current-displacement characteristics
Fig. 3.17 Flux-current-displacement characteristics of lower main air gap Gb

Fig. 3.18 Pull-off force-current-displacement characteristics
Fig. 3.19 Computed and measured net force characteristics
Fig. 3.20 Effects of shunts on hold-on and pull-off force characteristics

Fig. 3.21 Effects of shunts on net force characteristic
Chapter 4

Magnetic field analysis

4.1 Objectives

Because of the high saturation levels in the iron parts, particularly in the armature, the large air gaps (inherently three-dimensional) and the varying flux distribution caused by current change which results in eddy currents in the conductive parts, it is necessary to investigate more closely the actuator magnetic circuit using more accurate methods of solution than that used in Chapter 3. In effect, the static lumped-parameter model did not predict fluxes and forces, at high currents, with sufficient accuracy, due to the use of a simple approximation to model the iron BH curve. A field analysis is carried out to deal with the highly nonlinear permeability of the iron parts, and to estimate the effects of transient eddy currents whose intensity varies with space and time, and depends on several other factors such as the shape of the magnetic circuit, the electrical conductivity and the magnetic permeability. These problems were treated using a commercial 2D field-solving (finite-element) package [3]. The actuator problem is, however, three-dimensional and of dynamic nature, which requires cumbersome and time-consuming solutions because of the large amount of computation involved.

In the absence of a field-solving package that can deal with 3D dynamic problems with eddy currents and mechanical motion, as is the case here, the commercially available 2D finite-element package PE2D [3] was used. The latter has the ability to deal with static and transient nonlinear magnetic fields including eddy currents, but limited to problems not involving motion. The full dynamic modelling of the actuator taking account of the mechanical motion is the subject of the next chapter. PE2D was
used in a manner that accounts for the width change of the actuator components and
the edge effects by solving for the magnetic vector potential and scalar potential in
longitudinal and transverse directions, respectively. This technique referred as 'quasi-
3D' analysis combines the 2D vector potential \( A \) solution in the longitudinal plane
with the scalar potential \( \Omega \) solutions in the transverse planes, so that only one
component of \( A \) is needed instead of three in a 3D problem. This technique which has
required less computation and cpu time than a full 3D analysis gave sufficiently
accurate results.

The major part of this analysis involves investigation of the magnetostatic fields and
the magnetic forces for a series of current levels and armature positions with the
assumption that eddy currents do not exist in the iron parts. The static field results
were obtained with a minimum of computation time. The magnetostatic field analysis
allowed the study of the influence of changes of various magnetic and geometric
parameters that affect the actuator static performance. In the last part (Section 4.9) of
the analysis transient fields in the actuator were investigated to assess the effects of
eddy currents on actuator performance, particularly the effect on the net magnetic
force. This involved the computation of time-varying potentials using the PE2D
transient program.

4.2 Initial analysis using a 3D field-solving package

As was shown in Fig. 3.1a (Chapter 3), the actuator under study has, inherently, 3D
geometric features. The change in width of the different iron parts makes the
magnetic field problem essentially 3-dimensional, hence the use of a 3D field-solving
(finite-element) package such as TOSCA [10] may be appropriate. The latter was
used at the initial stage of this project and run on the Prime computer. The package
was found to be cumbersome in the pre- and post-processing of data, and
computation time were relatively long. In addition, the maximum number of nodes
allowed could not exceed 6,000 which was not sufficient to describe the model geometry in moderate detail. A simple TOSCA model for the actuator was constructed with about 6,000 nodes and required 20 hours of cpu times on the Prime machine. The limitation in the number of nodes (or mesh density) achievable and the long computation times made the 3D solution less practical than its 2D counterparts, particularly for this actuator configuration which requires a high mesh density to deal with the important levels of saturation and also a series of magnetic field solutions for each level of current in the coil and each position of the armature. In addition, TOSCA is purely a 3D static solver and cannot be used to carry out eddy-current calculations necessary for the actuator transient problem. Another aspect of the 3D computation is the difficulty in visualising and interpreting 3D fields. All these considerations created strong incentives to use 2D approximations.

4.3 **Quasi-3D magnetostatic analysis**

As pointed out in Chapter 3 the main flux path around the various parts of the actuator is assumed to be confined to the longitudinal (x,y) plane (Fig. 4.1). However, straightforward 2D analysis using the longitudinal cross-section as the field problem region would not give correct results because the iron components do not have the same depth (into the page) along the z-axis. The change in width from one iron part to another is significant (60 % to 160 %) and must be taken into account as is shown by the transverse (y,z) cross-sections in Figs. 4.2a,b,c. The flux density in these parts can be corrected for by way of adjustment of the iron permeability \( \mu_i \). This aspect of the problem will be examined in great detail in the next section. Although the correction for the width change, which is the major compensation, produced sufficient results for the purpose of the analysis, the effects of the transverse fluxes that must exist near the edges of the air gaps, were also investigated. These fluxes are
most important in the largest air gaps such as the ones between the shelves and the armature (gaps G₂ and G₄ in Fig. 3.1d).

In order to deal with the effects of the width change and the transverse fields in the air gaps 2D field calculations were carried out in two directions (longitudinal and transverse) leading to a quasi-3D analysis. The latter consists in incorporating the results obtained from transverse scalar potential solutions into the longitudinal vector potential solution.

4.4 Width change effects

The differences in the widths of the various parts greatly affect the saturation levels and also cause transverse fluxes which cannot be represented in the longitudinal A model. The 2D approximation assumes that there is no variation of \( B_x \) in the transverse z-direction, and that the flux is transferred only between adjacent parts, whereas some passes around the narrower parts. The most important effect is the width change which was allowed for by adjusting the permeability of the materials in pole pieces and shelves in the longitudinal model, a method commonly applied to ventilating duct problems in electrical machines [15,16]. However, the difference in width is, there, relatively small. In the actuator iron parts the flux distribution is complex, but is of little interest unless the flux density is high, when the component \( B_x \) predominates, and tends to become uniform.

Let's illustrate the effects that the width change has on the longitudinal component \( B_x \) with the use of the diagram in Fig. 4.1. Assuming that the thickness is the same for all the iron parts, any particular amount of flux passing through them will produce lower flux density \( B_x \) in the pole pieces and the shelves than in the armature. Subsequently, the longitudinal 2-D model with a depth equal to the base width \( w_a \) will give correct flux density \( B_x \) in the armature, but incorrect \( B_x \) in the pole pieces.
and the shelves. One simple approach to this problem is through modification of the
B/H curves of the pole pieces and the shelves. Another alternative is to change the
thickness of the iron parts to adjust the flux density $B_x$. However, changing the
thickness will alter the overall geometry and can then affect the overall flux
distribution. Hence, the B/H curve compensation method was chosen.

The approach in [15] is usually used for only small corrections such as the problem of
ventilating ducts in machines. Here, the corrections needed are much larger because
of the large ratios between iron parts. The method of compensation consists in
rescaling the BH curve of the soft magnetic material in the pole pieces and the
shelves, whilst that of the armature stands as it is. It is based on the following
argument. For a given magnetomotive force, the actual pole piece and shelf carry
more flux than would a pole piece and shelf of width $w_a$. Provided the flux density
lies in the longitudinal plane everywhere in the pole piece and shelf, their iron may be
replaced by homogeneous, fictitious materials whose permeabilities are higher than
that of the real iron. The magnetic field $H$ in the fictitious materials is the same as it
is in the actual pole pieces and shelves, respectively, because $H$ does not depend on
the geometry of the iron parts. On the other hand, the flux densities $B'_p$ and $B'_s$ in the
fictitious materials must be such as to produce the correct total flux in the pole pieces
and the shelves. The fictitious flux densities will therefore be related to the actual flux
densities $B_p$ and $B_s$ by

$$B'_p = \alpha_p B_p \quad (\alpha_p = w_p/w_a) \tag{4.1a}$$

$$B'_s = \alpha_s B_s \quad (\alpha_s = w_s/w_a) \tag{4.1b}$$

where $\alpha_p$ and $\alpha_s$ represent the compensation factors. The analysis of the actuator
longitudinal (A) model should therefore employ magnetisation curves which have
magnetic field $H'$ identical to that of the actual iron ($H_1=H'$), but a different $B'$ as
shown in Fig. 4.3. The B/H curves of the fictitious materials were obtained through the following transformations:

\[
\begin{bmatrix}
H_p \\
B_p
\end{bmatrix} \times \begin{bmatrix}
0 \\
\alpha_p
\end{bmatrix} = \begin{bmatrix}
H'_p \\
B'_p
\end{bmatrix}
\]  

(4.2a)

for the pole pieces, and,

\[
\begin{bmatrix}
H_s \\
B_s
\end{bmatrix} \times \begin{bmatrix}
0 \\
\alpha_s
\end{bmatrix} = \begin{bmatrix}
H'_s \\
B'_s
\end{bmatrix}
\]

(4.2b)

for the shelves.

These adjustments of the B/H curves will produce the correct total flux in the pole piece and the shelf, but local flux densities in these iron parts (not in the armature) will require inverse scaling. This technique was applied to the 2D vector potential model of the actuator which is described in the next section.

4.5 Vector potential (A) model

4.5.1 The PE2D software package

A 2D vector potential model was developed using the PE2D static analysis (finite-element) program which deals with highly non-linear magnetic materials such the ones found in the actuator (Remko B and Alnico 9). PE2D was used to solve for the magnetic vector potential using the following equation [17,18]

\[
\nabla \times \left( \frac{1}{\mu} \nabla \times A - H_c \right) = J
\]

(4.3)

where A, H_c, and J are respectively, the magnetic vector potential, the coercive field of the permanent magnet and the current density of the coil winding. This equation (derived from Maxwell's equation: \( \nabla \times H = J \)) describes the static magnetic field
taking into account the properties of the magnetic materials. In order to determine the vector potential distribution inside the actuator the following assumptions must be made:

(i) The vector potential distribution inside the actuator is constant along the z direction. The length of the model in this direction is assumed to be equal to the armature width and the technique outlined in the last section is applied.

(ii) The magnetic materials of the actuator are isotropic. The B/H curves of Remko B iron and Alnico 9 permanent magnet are modelled using their magnetisation curves which are single-valued (minor hysteresis loops are ignored).

(iii) The magnetic vector potential \( A \) and the current density vector \( J \) have only z-directed components.

4.5.2 Finite-element mesh

Since there is no magnetic symmetry in the actuator (except when the armature is in the middle position and no current flows in the coil), the entire device must be modelled. To build the finite-element mesh, the actuator geometry (magnetic components, coil and surrounding air) was broken down into quadrilateral regions (Fig. 4.4) which were assigned codes to define the material type and the current source density. The regions were then subdivided into triangular elements by specifying the number of elements desired in the two local directions of each region. Adjacent regions were subdivided so that all elements along the common boundary are compatible. Fig. 4.5 illustrates a mesh of 3600 elements which was used to model the actuator with the armature at the initial position. The material boundaries are highlighted to show the actuator components. An important requirement is the need for a fine mesh in the main air gaps where magnetic forces are produced, and in the
armature in which high levels of saturation occur. In the initial analysis a coarser mesh of 900 nodes gave rapid solutions but produced large local and global errors in the fields (RMS error over whole problem of about 20 %), whereas the 3600 element model lead to a global error of 3.5% after refining the mesh in the regions of large variations of the field.

PE2D offers the choice of using either first-order or second-order elements. An investigation was done at the initial stage to find out which type will be appropriate for the actuator analysis. Results showed that when using a fine mesh there is no significant difference between 1st and 2nd-order finite-element results (1 to 2 % difference, on force and mmfs), however, the overall solution error was reduced by 50% (the global error was brought down from 3.5% to 1.8% when using 2nd-order elements. In terms of computation time there is an advantage in using 1st-order elements since the running time is 6 times shorter than with 2nd-order type, which makes significant time savings. When high solution errors were obtained these were reduced by refining the mesh in the regions where large variations of the flux density occurred. Under these considerations the 1st-order element type was chosen for the actuator analysis.

4.5.3 Modelling of armature displacement

An additionnal requirement in this analysis is the construction of several meshes (or finite-element models) to study the effect of armature displacement. The successive meshes were derived from only one mesh (armature in the middle) by changing the coordinate of the armature region corners. To avoid squashing and stretching of the elements at either side of the armature (in the main air gap) after displacement, the number of elements was reduced in the gap which is closing and increased in the one which is opening. To ensure a minimum of discrepancy between the local errors in the fields of the various solutions, the number and the distribution of the elements
was kept constant. Also, the shape of the triangular elements, particularly in the main air gaps and armature, was made as regular as possible so as to minimise the solution errors.

For the purpose of this analysis 5 geometric models with the same number of finite-elements were constructed. Each model corresponds to a particular position of armature. As for the lumped model, the same set of positions was used.

4.5.4 Current change

For each fixed position a number of cases were studied to reflect the change of current through the coil. For a given position the current density $J_z$ over the coil area is set to 1 A/cm² and a range of current density values is specified. In this way only one run of the PE2D solver is needed during which $J_z$ is multiplied by each one of these values to produce a series of solutions. The current density was allowed to vary between 0 (actuator excited by the permanent magnet only) and a maximum value that correspond to the peak current ($I_{peak} = \text{applied voltage/coin resistance}$). For this study 7 values of current were specified at equal intervals for each of the 5 positions. Hence, a total of 35 field solutions were obtained for a full investigation.

4.5.5 B/H curves

Soft iron and permanent magnet used in the Bentley actuator were modelled by specifying their B-H and demagnetisation curves, respectively. The procedure used in the PE2D program consists in converting the graphical data of the manufacturer's B-H curve into numeric data. This procedure allows for direct entry of experimental points without any need for interpolation. Remko B iron was modelled using 20 points as shown in Fig. 4.6. PE2D provides an internal routine that convert the experimental data into mathematical form using interpolation techniques. The program then calculates the reluctivity and its derivatives. The system does not use
the B/H curve in its classical form $H = H(B)$, but rather the reluctivity as a function of flux density squared to avoid mathematical complications such as square root operation. Automatic extrapolation take place beyond the point corresponding to maximum $B$ and $H$.

To take account of the effect of width change, the pole pieces and the shelves were assigned BH curves different from the actual BH curve of Remko B. These fictitious curves (Fig. 4.6) were derived from the actual curve as explained in Section 4.4.

The manufacturer's B/H curve of the alnico 9 permanent magnet (Fig. 4.7) was approximated using the recoil line in Fig. 3.9.

4.5.6 Boundary conditions

The homogeneous Dirichlet boundary condition was applied to the outer surface of the mesh. This consists in forcing the nodes on the outer boundary line to have the vector potential $A$ equal to zero (Fig. 4.8). The outside surfaces of the shelves were taken as boundaries for the whole mesh since the field in the air outside the shelves is negligible. The tangential component of the magnetic field intensity $H$ and the normal component of the magnetic flux density vector $B$ are continuous along the iron-air boundary of the actuator. To allow for fringing effects the vertical outer boundary lines of the mesh were drawn sufficiently distant from the actual device.

4.5.7 Computer considerations

The PE2D data and solutions for the 35 cases (armature and current change) were developed using, respectively, the PE2D preprocessor and static solver on a Appolo DN4000 (Unix Sun 5). The average cpu time for a single solution was about 20 minutes for a non-linear problem of 3600 elements (1900 nodes). All the 35 cases
were run overnight by batch processing (execution commands assembled into a file which was read by the PE2D solver at a specified time).

4.6 **Results and comparisons with lumped-parameter model and experimental data**

4.6.1 **Analysis of flux and mmf distributions**

After solving for the vector potential $A$ at every node for a specified current level and armature position, the flux distribution was obtained by plotting lines of constant potential. For the sake of comparison between the field model and the lumped-parameter model, it was sufficient to give the results for 3 armature positions. Figs 4.9-4.11 show flux plots for the starting, the middle, and the final position of armature, respectively. For each specified position the mmf is allowed to change from zero to a maximum level at equal intervals. As discussed in the last chapter, the most interesting case to examine is that of the armature at initial position (Fig 4.9). At this position and zero mmf in the coil (actuator excited by permanent magnet only), the field plot shows clearly the 3-mesh flux pattern of the device as was modelled in Chapter 3. As the coil mmf increases, the flux in the upper main gap is reduced to almost zero and that of the lower main gap remains almost constant as was determined from the lumped-parameter model.

Although the upper shelf seems to be saturated at 804 At, its maximum flux density was found to have a low value (well into the linear region of the actual BH curve) after inverse scaling using the correction factor as discussed in Section 4.4. Whereas, the readjustment of $B$ in the bottom pole piece shows that it is well saturated. The armature does not need inverse scaling since it was modelled with the actual BH curve. The advantage of the field model over the 3-mesh circuit model is that it shows the saturation occurring locally in the iron parts, whereas in the circuit model the
saturation level is the same everywhere in the iron block. The flux density calculation have shown that the armature and the bottom pole piece become heavily saturated at high currents. Because of the existence of large air gaps with large mmfs, saturation has no great effect on actuator performance unless the mmf drops in the iron parts are important. To investigate the mmf distribution in the actuator, the mmfs in the iron and air parts were calculated from the field solutions (including the scaling factors) at the specified current and position. The mmf computation is discussed in Chapter 5, and for the purpose of this analysis only the results are presented. Table 4.1 shows the mmf levels in the armature, the bottom pole piece, and the air gaps that correspond to each field solution in Figs. 4.9-4.11, respectively. By reason of symmetry, it is sufficient to investigate the mmf levels in air gaps $G_1$, $G_2$, and $G_a$ only. As can be observed in Table 4.1, and although the bottom pole piece becomes heavily saturated at high currents, its mmf drop remains at negligibly low level as compared to those in the air gaps. However, the mmf in the armature reaches important levels at high current due to heavy saturation. This important result shows the need to model the armature as accurately as possible. In this respect, the lumped-parameter model in Chapter 3 did not handle saturation sufficiently. To check the validity of the lumped model, its force-current-displacement characteristics were compared to those of the field model as is reported in Section 4.6.3.

4.6.2 Computation of the net magnetic force

The vector potential solution was also used to calculate two of the most important single factors that determine actuator performance: the hold-on and the pull-off forces acting on the armature. These were calculated for the various currents and armature positions to derive a force characteristic diagram that will be indicative of the sort of accelerations achieved in dynamic operation.
For each field solution (at specified current and armature position), the magnetic forces were calculated using the Maxwell stress tensor method [19]. To determine both forces the following integral must be computed:

$$F_g = \int \int_{S} T \, ds$$  \hspace{1cm} (4.4)

where $T$ is the stress tensor (a function of the flux density in the air on the armature surface), and $S$ a surface surrounding the armature upon which the force is exerted. Equ. 5.2 is exact even if the field is not perpendicular to the air/iron boundary surface (saturation taken into account). The contour of integration does not have to enclose the whole armature since most forces act on the surface sections inside the main air gap region between the pole pieces $G_a$ and $G_b$. The forces acting on the remaining sections of the armature were found to be negligibly small.

To avoid calculation errors that may arise when integrating the stresses, the following steps were taken:

(i) A large number of elements surrounding the armature was used, particularly in the regions of the main air gaps.

(ii) The contour of integration was specified so that it goes through the centroid of the elements.

(iii) The force was evaluated using three contours of integration to check any deviation between the results. Calculations showed that this deviation is less than 10 % which was acceptable for analysis purposes.

Since the motion is angular (with 3 degrees amplitude) the torque was also derived by multiplying the force by the distance between the action point (pivot) and the point of impact (pole piece/armature).
4.6.3 **Force-current-displacement characteristic**

Figs. 4.12-4.14 show, respectively, the variations of the hold-on force $F_{ga}$, the pull-off force $F_{gb}$, and the net force $F_{net}$ with respect to the coil mmf at various armature positions as obtained from both the field model and the lumped-parameter model. For the sake of comparisons between the two models, the results are given for 3 armature positions. As can be clearly seen in Fig. 4.14a, the field model and the lumped-parameter model seem to be in good agreement only at the middle position. The major differences are observed at both ends of the armature travel. At the starting position the discrepancy between the two curves is due to large errors, particularly at high currents, produced from the calculation of $F_{ga}$ with the lumped-parameter model. At this position, any errors produced in predicting the flux $\Phi_{ga}$ (Fig. 3.15) lead to larger errors in the calculation of $F_{ga}$. The other consideration is that $F_{ga}$ starts increasing at high currents in the lumped-parameter model. The field model shows that this does not happen and that $F_{ga}$ remains at zero level because heavy saturation prevents increase of flux $\Phi_{ga}$. At the other end of armature travel (final position), the errors obtained in the prediction of $F_{net}$ with the lumped-parameter model, particularly at high currents, originate mainly from the use of a simple approximation of the BH curve as reported in Section 3.8. At this position, the field model show strong effects of saturation in the armature and the bottom pole piece (Fig. 4.11). The error produced from the field model to predict the net magnetic force was found to be less than 5% which was satisfactory for the actuator study as can be observed in Fig. 4.14b.

4.7 **Transverse edge effects**

The flux entering the armature at its transverse edges contributes to the saturation levels and needs to be considered. This flux which takes partly transverse and partly vertical directions cannot be represented by the longitudinal A model. The edge flux
was found to have a substantial effect on the armature saturation. To compensate for the edge effect either of the two following techniques could be used. The first one suggests another correction applied to the BH characteristic, however, this is not valid because the saturation effects are in different cross-sections from the plane in which the fringing is calculated. The proportion of fringe flux relative to gap flux is constant in the x-direction in some regions but not in others. A second more direct method is to change the permeability of the air between any two iron parts in the A model to compensate for the local transverse field conditions. Although the amount of flux that passes directly from shelf to shelf (or pole to pole) is important it is found to have no great consequence on the saturation levels in these parts because of their greater widths. Hence, the problem was simplified to the study of armature edge flux only since this is what affects saturation in the armature, and the mmf distribution in the actuator.

The edge fluxes are taken into account by making another adjustment in the A model. The permeability of air between iron parts is increased using a correction factor β as follows:

$$
\mu_g = \beta_g \mu_o
$$

$$
\beta_g = (\Phi_e + \Phi_u) / \Phi_u
$$

where $\Phi_e$ represents the edge flux and $\Phi_u$ is the flux that would cross the gap if the field were uniform. In a similar way the permeability $\mu_m$ of the magnet is adjusted to compensate for the transverse fluxes that exist at its edges. This is a local adjustment since the edge fluxes affect only the local operating conditions of the magnet. The new permeability is obtained as follows

$$
\mu_m' = \lambda \mu_m
$$
where $\lambda$ is the magnet correction factor determined from calculation of edge flux. The correction factors $\beta_e$ and $\lambda$ were evaluated from flux plot postprocessing.

The transverse problems were solved assuming the iron part surfaces to be equipotentials. This assumption is justified because saturation is due primarily to $B_x$, which does not cause $\Omega$ potential variations in the transverse (y-z) plane. Scalar potential values are extracted from the longitudinal solution and used as boundary conditions for the transverse problem regions. The transverse models were terminated by assuming flux line boundaries to exist at some distance from the edges of the iron parts. In the main air gaps the boundary conditions of the transverse model were set as follows:

- $\Omega = \Omega_{p1}$ on the upper pole piece surface,
- $\Omega = \Omega_{p2}$ on the lower pole piece surface, and,
- $\Omega = \Omega_{a}$ on the armature surface.
- $d\Omega/dn = 0$ along the flux line boundary.

where $\Omega_{p1}$, $\Omega_{p2}$ and $\Omega_{a}$ take values which are extracted from the longitudinal vector potential solution. Since these values vary with current and armature position, a set of scalar potential problems must be solved. Fig 4.15a,b,c show, respectively, the transverse scalar potential distribution in the main air gap region, the shelf/armature gap region, and the magnet region. To obtain a transverse flux distribution in these regions, equipotential lines orthogonal to the scalar potential lines must be drawn. Since the PE2D program does not produce orthogonal flux lines when the problem is solved for $\Omega$, the dotted lines in Fig 4.15 were drawn manually to illustrate the important amounts of edge flux, particularly in gaps G4 and G4. To determine the
edge flux in each air gap, the flux density was integrated along horizontal lines (x-axis).

The prediction of the fluxes in the gaps is most important because of the effect on gap permeance and armature reluctance. The relatively large gap length/gap area ratios make the corner effects non negligible, particularly in the regions of air gaps G2 and G4. However, the effect in the main air gap region (Ga and Gb) is not severe because the armature protrusion inside the main gap helps to reduce it, and hence, the variation of the transverse flux proportion in the z-direction. The proportional effect of the transverse flux is largest at maximum gap length.

As shown in Table 4.2 the correction factor $\beta$ for the main air gaps (Ga and Gb) depends on the coil mmf and armature position. It is found that this factor varies more with armature displacement than current. However, for the gaps G2 and G4 between shelf and armature it is almost constant (about 1.6) with current and displacement. In effect, in these regions the displacement of armature is negligibly small compared with the lengths of G2 and G4. For the magnet region, $b$ was found equal to 1.1.

<table>
<thead>
<tr>
<th>Table 4.2 Variation of $\beta_{ga}$ with current and position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil mmf (At)</td>
</tr>
<tr>
<td>Starting position</td>
</tr>
<tr>
<td>Position 2</td>
</tr>
<tr>
<td>Mid-position</td>
</tr>
<tr>
<td>Position 4</td>
</tr>
<tr>
<td>Final position</td>
</tr>
</tbody>
</table>

After determining the correction factors, the 5 finite-element models that correspond to the 5 positions of armature were modified. In all of these models, the elements inside the secondary air gaps G1, G2, G3, and G4 were assigned permeabilities greater than that of the air as follows:

For G1 (and G3) $\mu_{g1} = 1.1 \mu_0$
For $G_2$ (and $G_4$) \[ \mu_{g2} = 1.6 \mu_0 \]

In the main air gaps $G_a$ and $G_b$, the permeabilities were modified according to the position of the armature. In each of the 5 models, the air elements inside $G_a$ (and $G_b$) were assigned permeability values that were obtained by multiplying the factors in Table 4.2 by $\mu_0$:

\[ \mu_{ga} = \beta_{ga} \mu_0 \]

As $\beta_{ga}$ varies only slightly with current (Table 4.2), it was assumed to have one fixed value equal to the average of the values corresponding to the specified armature position. After assigning all the air gaps new permeabilities, the 5 models were solved using the magnetostatic field solver of the PE2D package, for the same set of applied coil mmfs.

The compensations for edge fluxes will correct the air gap permeances and bring the gap fluxes to the right total level. The major interest from the results of these compensations was to investigate the effects of edge flux on the actuator mmf distribution. Table 4.3 shows the mmfs in the various components of the actuator. These values were compared to those of Table 4.1. Notice that the mmf in the armature has increased considerably at high currents. At 806 At in the coil, the armature mmf has more than doubled. Subsequently, this has caused a reduction in the mmfs in the large secondary air gaps, particularly in $G_2$ where it has decreased by 20%. Although this effect is most significant only at maximum current, it must be taken into account in the dynamic model (Chapter 5) since it may affect the electrical time constant.
4.8 Effects of magnetic shunts

A preliminary investigation of the effects of shunts across $G_2$ and $G_4$ in Chapter 3 showed that these improve the actuator static performance as was shown in Figs. 3.20-21. However, the hold-on force $F_{ga}$ is increased beyond the value giving zero force to almost 3 N at 800 At. To fully investigate the effects of the shunts the field model was modified by assigning some elements of the air gaps $G_2$ and $G_4$ the permeability of Remko B iron, leaving two small air gaps between the shunts and the armature to allow the armature free to move. Fig. 4.16 shows a close view of the field distribution at the starting position of armature, for 4 values of applied mmf. When comparing these plots with those of Fig. 4.9, it is interesting to observe that the flux in the upper main gap is higher at 0 At and that it decreases at a higher rate to zero as can be seen in the cases of 536 At and 806 At, whereas in the original design (Fig. 4.9), there is still some flux at high currents. Notice the difference between the 2 results in the case of 536 At. These changes in the main air gap flux has affected more significantly the hold-on force, as can be seen in Fig. 4.17. $F_{ga}$ has increased by 40 % at 0 At and decreases at a much higher rate with respect to current and displacement. Notice that $F_{ga}$ does not rise again at high current as was shown from the results of Chapter 3. There is, however, a negligibly small (almost zero) increase of $F_{ga}$ at 806 At. Fig. 4.18. shows an increase in the pull-off force with respect to current and displacement. Notice the large increase of $F_{gb}$ at the final position and also the stronger effect of saturation. The net force characteristics have been significantly improved (up to 40 % max) as shown in Fig. 4.19. An interesting remark from observing the $F_{net}$ variation at the starting position, is that the current at which $F_{gb}$ becomes greater than $F_{ga}$ is 30 % lower which shows less mmf is required to drive the armature. The results here have confirmed those obtained with the lumped model, that the inclusion of shunts across the large secondary air gaps improves the magnetostatic performance of the actuator. However, because of the reduction in
circuit mesh reluctance their effect on dynamic performance result in an increase in the time constant $L/R$ due to an increase in $L$ (see Chapter 5).

4.9 Transient eddy-current analysis

One physical phenomenon that can affect the performance of the actuator is the effect of eddy currents that were neglected in the field analysis. The eddy currents arise in the conductive (non-driving) parts of the actuator from the time-varying fields that result from a sudden application of a constant voltage across the coil terminals. The aim of this analysis is to evaluate their effects on several magnetic parameters, particularly the mmf along the conducting parts and also the net magnetic force acting on the armature.

In electrical machine problems the effects of eddy currents can be reduced by using laminations in the magnetic circuit. However, the iron parts of high-speed devices such as the Bentley actuator (particularly the armature and pole pieces) cannot be laminated because of the need to use solid iron pieces that can withstand repetitive impacts, and hence, their behaviour is influenced by eddy current effects. The dynamic behaviour of the Bentley actuator depends greatly on the rate at which the device can be operated, or in other words how quickly the flux penetrates into the iron parts. To understand the details of the field penetration process, it should first be studied for a simple case such as the problem of a semi-infinite space, in which the field penetration is one-dimensional. Aldefeld [20,21] performed one-dimensional calculation of this kind which he applied to a simple cylindrical configuration in which the penetration depth (skin depth) was small compared with the other dimensions. This investigation is relevant to the Bentley actuator example because a material with a BH characteristic similar to that of Remko B iron was used. The results of this investigation are shown in Fig. 4.22 which illustrates the flux-density distributions under excitation of a step pulse (a magnetic field parallel to the surface
of the material is suddenly switched on and remains constant) and also under excitation of a triangular pulse (with the surface field strength rising linearly to its peak value). It is interesting to notice that for a given instant of time the depth of penetration is greater in the case of the step pulse (best case) than in the case of a triangular pulse (worst case). The thin armature of the Bentley actuator (made of the same material as above) undergoes heavy saturation. It has a thickness varying from 1.2 mm to 1.5 mm (half the thickness is 0.6 to 0.75 mm since the flux has to travel to the middle of the armature) and is subject to an exponential external field (exponential current pulse applied to the coil) with a time constant of about 1.2 ms. On the basis of the results by Aldefeld it can be suggested that the flux will penetrate into the armature in a period of time well below 1.2 ms. To obtain a measure of the penetration depth in the armature for the case of a step pulse the following formula can be used [20]:

\[
d_p = \sqrt{2 \frac{t}{\sigma \mu}} \quad (\mu = B_s/H_s)
\]

where \(d_p\) is the penetration depth, \(\sigma\) the conductivity of the material, \(\mu\) its permeability. For \(t=1.2\) ms, and \(B_s=2\)T corresponding to \(H_s=28000\) A/m, \(d_p\) is found equal to 2.4 mm, which is well above the actual half-thickness of the armature. These unconfirmed results seem to be suggesting that the effects of eddy currents induced in the actuator iron components will not significantly affect the device performance. To confirm or disprove the above implication, a 2D transient field analysis of the actuator was needed. The transient field distribution of the Bentley actuator was studied using the PE2D transient program. The results were obtained for a case where the coil was excited by a current pulse as shown in Fig. 4.21. This means that to calculate the eddy currents the driving coil current must be known in advance because the available PE2D version does not solve the coupled problem (transient magnetic/electric circuit), where usually the voltage is applied (which is the case...
here) and it is the coil current that is computed step by step. The prescribed time-varying coil current $i_s(t)$ is the actual current waveform that was obtained from switching the actuator coil with a voltage step.

The PE2D transient program solves the following 2D vector potential diffusion equation (derived from the well known Maxwell's equations $\text{curl } H = J$ and $\text{curl } E = -\partial B/\partial t$) with the vector potential as the unknown variable:

$$\nabla \frac{1}{\mu} \nabla A_z - (\nabla \times H_c)_z = J_s - \sigma (\partial A_z/\partial t - \nabla \phi)$$

(4.9)

with the assumption that the current density over the cross-section of any iron part is equal to zero as expressed by the constraint equation:

$$\int_S (-\sigma \partial A_z/\partial t - \sigma \nabla \phi) \, ds = 0$$

(4.10)

$A_z$ and $\phi$ are the z-component of the magnetic vector potential and the electric scalar potential, respectively, and $S$ is the area of any considered conducting iron part. $J_s$ is the magnetising current density and $H_c$ represents the coercive force of the permanent magnet. The electrical conductivity was assumed to be independent of the electric field. Eddy currents due to motion and present in the armature only ($-\sigma \nabla \times B$) were neglected. The actuator possesses 5 conducting iron parts (shelves, pole pieces, armature), electrically isolated and for which equ. 4.3 must be satisfied [22]. The transient PE2D program was used to solve the above equation system (equ. 4.2-4.3) taking into account the nonlinearity of the iron parts and the permanent magnet.

The actuator finite-element meshes used for this study were set up in such a way that the lengths of the triangular elements in the iron parts were smaller in the main directions of the field penetration. Under the assumption that the penetration depth is of the same order of magnitude as that of the iron part thicknesses, the regions were
subdivided by 8 elements along the penetration direction which was satisfactory. The
equation system was also discretised in time, with the time derivative of the vector
potential approximated by the first-order expression

$$\partial A_z / \partial t = (A_z(t+1) - A_z(t)) / \delta t$$

(4.11)

where $\delta t$ is the time step. After applying a current-time function drive at $t=0$, the
program starts with a given time step $\delta t$ (e.g. 0.2 ms). After the first step, the time step
is automatically adjusted to achieve time-stepping errors of less than the supplied
tolerance. To verify the efficiency of the method, the number of iterations that were
required during the numerical calculation, was checked. For this study, 20 iterations
per time step, on average, were needed to achieve sufficient accuracy and acceptable
computer time. This number is also dependent on the mesh density and on the shape
of the BH curve. Initial conditions are set by the permanent magnet which provide the
DC background field. The latter was determined using the PE2D static solver with the
driving current set to zero. This static solution (Fig. 4.9a) was used as the initial
condition for the transient solution at $t=0$.

As discussed previously, the most important case to examine is when the armature is
held at the starting position for which the initial force and, hence, the initial
acceleration of the armature can be determined. The time taken to develop this force,
just after switching on the actuator coil is directly dependant on the build up of flux in
the iron parts. The transient solution for the initial position case has been obtained to
examine the effects of eddy currents on the net magnetic force and on the mmf in the
armature. To be able to assess these effects, the transient solutions were determined
for two cases. One in which the conductivity $\sigma$ of the iron parts was set to zero to
represent an ideal actuator without eddy currents, and in another case, $\sigma$ was set to the
actual value of the RemkoB iron material. Figs. 4.24-4.25 illustrate, respectively, the
flux distribution and the direction and intensity of eddy currents in the conducting
iron parts at 4 instants of time selected from the transient solution. Fig. 4.24 shows that although the flux distribution is distorted in the iron parts, the flux patterns in the air gaps appear to be less affected by the eddy currents which may suggest that eddy currents have no serious effects on the initial magnetic force.

To evaluate the effects on the net magnetic force, the two force/time curves corresponding to both ideal and non-ideal cases were plotted together as shown in Fig. 4.24. Assuming no other forces are exerted on the armature, the motion would start when the net force is equal to zero ($F_{ga} = F_{gb}$) which corresponds to a delay time of about 0.87 ms in the ideal case, and 1.05 ms when the eddy currents are present. Therefore, the existence of eddy currents cause an increase in the delay time by 20%. The effect on the mmf distribution, particularly in the armature, were also examined. Fig 4.25 show the armature mmf/time curves for both cases. The results show that the difference between the two curves is important only between 0.4 ms and 1 ms, however, it is not crucial since the mmf levels are very low compared to those of the air gaps in this initial period. These results show that eddy currents are important only to some extent, and can be neglected for dynamic analysis purposes, provided the results of dynamic simulation do not greatly diverge from experimental data.

4.10 Conclusions

The amount of computation has been greatly reduced, by making two-dimensional field calculation using only one component of A and a scalar $\Omega$. The interdependence requires adjustment of the permeability of both the air region and the iron parts. The proportionate amount of edge flux is nearly uniform over extended regions, hence only few cross-sections are needed, and this further reduces the amount of computation considerably. The accuracy achieved is comparable to that of a very extensive 3-dimensional model. Comparison of results obtained with and without the
\( \beta \) corrections for the transverse flux effects shows a 50% maximum change in mmf in the armature, which has a dominant effect in the main flux path.

The static lumped-parameter developed in Chapter 3 has been validated by comparing its results with those of the finite-element model. The two models were shown to be in a reasonably good agreement. However, there were major discrepancies between the two models at high currents, mainly due to the use of a simple approximation to model saturation in iron.

The analysis of the effects of magnetic shunts across the large secondary air gaps has shown that the actuator static force characteristic can be improved. However, the shunts reduce the circuit reluctance which results in an increase in circuit inductance \( L \), and hence, an increase in electrical time constant \( L/R \). The effect of the shunts on dynamic response is quantified in Chapter 5.

The analysis of transient eddy currents in the actuator has shown that their effects are not negligible since the thickness of the iron parts is comparable to the depth of penetration. However, the magnitude of these effects are not as important as those of the width change and the transverse edge flux that were examined in this chapter. For analysis purposes, they were ignored in the dynamic model developed in Chapter 5, although they can be taken into account using additional couplings.
Fig. 4.1 Longitudinal and main-section of Bently actuator

Fig. 4.2 Typical transverse cross-sections
(a) Coil region
(b) Magnet region
(c) Main air gap region
Fig. 4.3 Rescaling of BH curve to account for width change.
Fig. 4.4 Model subdivision into quadrilateral regions

Fig. 4.5 2D finite-element mesh
Fig. 4.6 BH demagnetisation curve for Alnico 9

Fig. 4.7 Nineteen-segment approximation curves for real and fictive Remko B BH curves
Fig. 4.8 Dirichlet boundary conditions on the outer surface of mesh
Fig. 4.9 Static flux distribution inside actuator with armature at starting position and applied coil mmfs:
(a) 0 At  (b) 268 At  (c) 536 At  (d) 806 At
Fig. 4.10 Static flux distribution inside actuator with armature at mid-position and applied coil mmfs:
(a) 0 At  (b) 268 At  (c) 536 At  (d) 806 At
Fig. 4.11 Static flux distribution inside actuator with armature at final position and applied coil mmfs:
(a) 0 At  (b) 268 At  (c) 536 At  (d) 806 At
### Table 4.1 MMF values (in A) in the actuator components at specified current and position

<table>
<thead>
<tr>
<th></th>
<th>(0 At)</th>
<th>(403 At)</th>
<th>(806 At)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Armature</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>0.67</td>
<td>2.1</td>
<td>44</td>
</tr>
<tr>
<td>P2</td>
<td>0</td>
<td>3</td>
<td>96.34</td>
</tr>
<tr>
<td>P3</td>
<td>0.23</td>
<td>70.7</td>
<td>320</td>
</tr>
<tr>
<td><strong>Bottom pole piece</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>1.31</td>
<td>1.9</td>
<td>2.2</td>
</tr>
<tr>
<td>P2</td>
<td>1.79</td>
<td>2.4</td>
<td>2.9</td>
</tr>
<tr>
<td>P3</td>
<td>3.1</td>
<td>11.9</td>
<td>22</td>
</tr>
<tr>
<td><strong>Gap Ga</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>67</td>
<td>33.3</td>
<td>2.47</td>
</tr>
<tr>
<td>P2</td>
<td>196.2</td>
<td>123.4</td>
<td>73.2</td>
</tr>
<tr>
<td>P3</td>
<td>318.3</td>
<td>279.8</td>
<td>267.6</td>
</tr>
<tr>
<td><strong>Gap G1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>29.5</td>
<td>86.6</td>
<td>136.6</td>
</tr>
<tr>
<td>P2</td>
<td>62.3</td>
<td>109</td>
<td>142</td>
</tr>
<tr>
<td>P3</td>
<td>91.5</td>
<td>134.2</td>
<td>147.2</td>
</tr>
<tr>
<td><strong>Gap G2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P1</td>
<td>43.3</td>
<td>338.5</td>
<td>614.4</td>
</tr>
<tr>
<td>P2</td>
<td>131.7</td>
<td>407.2</td>
<td>626.1</td>
</tr>
<tr>
<td>P3</td>
<td>218.4</td>
<td>475</td>
<td>626.3</td>
</tr>
</tbody>
</table>

P1: Starting position  
P2: Middle position  
P3: Final position
Fig. 4.12 Hold-on force characteristics computed with PE2D and lumped-parameter models

Fig. 4.13 Pull-off force characteristics computed with PE2D and lumped-parameter models
Fig. 4.14 Computed and measured net force characteristics
(a) PE2D and lumped-parameter results
(b) PE2D and measured results
Fig. 4.15 Transverse flux distribution in typical cross-sections
(a) main air gap region
(b) coil region
(c) magnet region
### Table 4.3 MMF values (in A) in the actuator components

<table>
<thead>
<tr>
<th>Component</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armature</td>
<td>1.9</td>
<td>2.1</td>
<td>117.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td>218.5</td>
</tr>
<tr>
<td></td>
<td>1.9</td>
<td>107.5</td>
<td>354.6</td>
</tr>
<tr>
<td>Bottom pole piece</td>
<td>5.4</td>
<td>6.5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>5.8</td>
<td>10.5</td>
<td>12.7</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>36.5</td>
<td>45.6</td>
</tr>
<tr>
<td>Gap Ga</td>
<td>74.8</td>
<td>35.8</td>
<td>10.1</td>
</tr>
<tr>
<td></td>
<td>193.2</td>
<td>115</td>
<td>85.2</td>
</tr>
<tr>
<td></td>
<td>302.2</td>
<td>252.4</td>
<td>248.7</td>
</tr>
<tr>
<td>Gap G1</td>
<td>35.8</td>
<td>100</td>
<td>141</td>
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<tr>
<td></td>
<td>68.4</td>
<td>123</td>
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<tr>
<td></td>
<td>98.8</td>
<td>135.7</td>
<td>144.5</td>
</tr>
<tr>
<td>Gap G2</td>
<td>46.8</td>
<td>323</td>
<td>524</td>
</tr>
<tr>
<td></td>
<td>117</td>
<td>373.1</td>
<td>524.3</td>
</tr>
<tr>
<td></td>
<td>186.8</td>
<td>397</td>
<td>524.3</td>
</tr>
</tbody>
</table>

P1: Starting position  
P2: Middle position  
P3: Final position

*Table 4.3 MMF values (in A) in the actuator components after correction for edge fluxes*
Fig. 4.16 Static flux distribution in modified design with shunts. Armature at starting position and applied mmfs: (a) 0 At (b) 403 At (c) 806 At
Fig. 4.17 Hold-on force characteristics in original and modified designs (with shunts). Computed with PE2D

Fig. 4.18 Pull-off force characteristics in original and modified designs (with shunts). Computed with PE2D
Fig. 4.19 Net force characteristics in original and modified designs (with shunts).
Computed with PE2D
Flux-density distributions under excitation of a step pulse

Magnetic vector-potential distributions under excitation of a step pulse

Fig. 4.20 Flux penetration into a soft magnetic material of similar BH characteristic to that of Remko B (calculated by Aldelfeld [20])
Fig. 4.21 Drive pulse used for PE2D transient eddy current calculation
Fig. 4.22 Transient flux distribution (armature at start pos.)
(a) $t=0.5$ ms  (b) $t=1$ ms  (c) $t=1.5$ ms  (d) $t=2$ ms
Fig. 4.23 Direction and intensity of eddy currents
(size of + and - in iron parts indicate their magnitude)
(a) $t=0.5$ ms  (b) $t=1$ ms  (c) $t=1.5$ ms  (d) $t=2$ ms
Fig. 4.24 Effect of eddy currents on net magnetic force (armature at starting position)

Fig. 4.25 Effect of eddy currents on armature mmf (armature at starting position)
Chapter 5

Dynamic model

5.1 The dynamic problem

The aim of developing a dynamic model is to predict the actuator armature movement in response to an application of a constant voltage across the coil. This problem is represented by a set of coupled equations describing the electric circuit, the magnetic field, and the mechanical motion. This system of equations is highly non-linear because of saturation in the iron parts. Because the PE213 package does not include mechanical motion it was not possible to use it to determine the dynamic performance parameters of the actuator. An alternative method which combines the accuracy of PE2D and the simplicity of a lumped-parameter circuit similar to that developed in Chapter 3, was used. The method consists of incorporating vector potential solutions into a simplified scalar potential model.

In recent years, the approach to solve the dynamic problem has been to use a method in which the electrical equivalent circuit and mechanical equations are directly coupled to the electromagnetic finite element equation and solved simultaneously with them in an iterative manner, taking into account eddy currents [23,24,25]. The armature motion requires remeshing after each time step. The advantage of this method is its ability to model motion and to link, directly, the finite element solution to the electrical and mechanical equations. However, this technique is expensive due to the considerable amount of computation and cpu time required, the large number of meshes needed and the many iterations through each time step due to saturation. In the last section of Chapter 4 it was shown that the computation of eddy currents, including saturation and ignoring armature motion (one position only), required 30
hours of cpu time on Appolo DN4000, a reasonably fast machine, which gives an idea on the size of the problem. Because of the unavailability of a commercial finite-element package that deals with full dynamic modelling, the author turned to two other methods. These are less costly in computation than the above method and consists of decoupling the electromagnetic finite element equations from the electrical and mechanical equations.

The first of these two methods has been used as an alternative [26,27] to the fully integrated finite-element method. It is based on predetermining the magnetic force and inductance for a range of currents and displacement values, using a field-solving package, such as PE2D, and storing the values in the form of look-up tables. These static results are then integrated in time to solve, simultaneously, the electrical circuit, and the mechanical motion equations. This method is limited only to single-mesh circuits, and has the defect of using the inductance, which is inappropriate in a non-linear problem. Another drawback is the use of much data which is not subsequently used.

The method developed by the author, and described in this chapter offers a more direct approach than the force/inductance method by using a magnetic equivalent network model with few elements or component which was derived from the vector potential model (allowing for transverse effects) developed in Chapter 4. This method is particularly suitable to multiple mesh circuits and, gives information on flux-time variations in each iron and air gap components and force-time patterns for both main air gaps. The coupled magnetic equivalent network/electric circuit model is easy to solve and requires much less computation than a direct finite-element/electrical circuit problem. An additional advantage of the presented method over the force/inductance method is the possibility of modelling eddy currents by lumping the magnetic circuit reluctances together with eddy inductance elements that can be
estimated using simple theoretically-based techniques usually applied to simple geometries such as rectangular bars [28]. In section 4.8 the effects of eddy currents on the delay time before armature motion starts, were evaluated and although not negligible, they were found to have a limited effect on performance. Thus, in general, it is important to be able to take them into account, although they were neglected here, so that the circuit model components were simple magnetic reluctances.

5.2 Simplified scalar potential model

In Chapter 4 a static vector potential model was developed using about 3500 elements. The basic problem here is to reduce this number in order to derive a simplified network model consisting of a considerably low number of elements or components, for the purpose of dynamic modelling. This depends on the identification of flux paths and potential nodes so that the field problem can be transformed from a description in terms of the magnetic vector potential A to one expressed in terms of a scalar potential $\Omega$ [29].

It was shown in Chapter 4 that the pole pieces and the shelves produced sufficiently low mmf drops compared to those of the armature and air gaps, so that for the purpose of the dynamic analysis, they need not be modelled. Fig. 5.1 shows the derived 3-mesh magnetic network model (Chapter 3) where the only path reluctances considered are those of the armature and the air gaps. The method consists in determining the mmf/flux characteristic of each iron and air part of the actuator from the various solutions of the vector potential model at specified currents and armature positions. As shown in a typical vector potential solution in Fig. 5.2, the flux distribution in most parts of the actuator is severely non-uniform. Therefore, the reluctance of any non-uniform path cannot be expressed in terms of the flux density $B$ and the magnetic field $H$, but rather in terms of the mmf drop $\Omega$ across the component and the flux $\Phi$. The latter is defined as the flux in the region of highest
flux density, since this makes the major contribution to the mmf, while the former is determined by integrating as follows:

$$\Omega_{mn}(\Phi) = \int_{m}^{n} H \, dl$$

(5.1)

where \(m\) and \(n\) represent the nodes at each extremity of the contour line of any considered non-uniform flux path.

As the flux paths in the actuator do not coincide with the x-y coordinate directions, it is necessary to resolve the vector \(H\) into components tangential to the path (parallel to the direction of the line segment \(dl\)) and normal to it. Once the contour is known and the component of \(H\) tangential to the contour is found at all points of the contour, the following integration is then performed:

$$\Omega_{mn}(\Phi) = \int_{m}^{n} H_t \, dl$$

(5.2)

5.3 Modelling of armature

Figs. 5.2 to 5.4 show magnified flux plots at various levels of coil excitation and positions of armature. As can be observed the field in the armature is highly non-uniform at low excitation levels and its pattern changes significantly with current and displacement. As the coil current rises, the flux distribution becomes less influenced by displacement. Using the various PE2D vector potential solutions, the mmf and flux in the armature were calculated in the manner stated above.

A series of \(\Omega_a/\Phi_a\) curves were plotted for the armature at different positions by determining the \((\Omega, \Phi)\) solution points as shown in Fig. 5.5. As can be seen, there are small differences between the curves in the unsaturated region (low flux levels) due to the change of field pattern inside the armature caused by displacement and change
of current. In effect, the field pattern changes in such a way that for a given flux, the mmf, as defined earlier, varies with displacement. However at high flux levels the influence of displacement is negligible, and as a result all the curves coincide with each other. Since in the unsaturated region of the $\Omega/\Phi$ curves, the mmf levels are so low compared with those of the air gaps, the accurate determination of the curves in this region does not matter, and the armature can be modelled using one piecewise linearised curve that fits into the PE2D ($\Omega, \Phi$) points.

5.4 Modelling of air gaps and magnetic sources

As was shown in Chapter 3, the mmfs produced in the large air gaps greatly influence the actuator overall performance. Thus, the calculation of these mmfs must be as accurate as possible to produce a reliable dynamic model. The magnetic field in the main air gaps ($G_a$ and $G_b$) has a non-uniform pattern due to the pole piece corner effects and the relatively large gap length/gap area ratios as can be clearly seen in Figs. 5.2 to 5.4. When the armature is at initial position, $G_a$ is at minimum length, and has a uniform field along its width, except near the pole step where the field is slightly deformed. The uniformity of the field is due to the small gap length/gap area ratio. However, the opposite main air gap $G_b$ is at maximum length and has a non-uniform field pattern due to a much larger gap length/gap area ratio. Due to symmetry, the $\Omega_g/\Phi_g$ characteristics of only one main air gap is needed. At the starting position $G_a$ appear to be two air gaps in parallel. The mmfs in both air gaps were determined by integrating $H_t$ along a line contour (equ. 5.2) in the middle of each gap where $H$ is maximum. The results showed, as expected, almost equal mmfs in both gaps. The mmf for the whole gap was determined as the average of these two values. As the armature moves away from the top pole piece, the effect of the pole step diminishes and the whole air gap appears as one. For each specified armature position (from pole to pole) and each current level, the gap mmf was calculated along
several vertical contours and the average value was determined. To check the variation in $\Omega$, the ratio $\Delta \Omega/\Omega_{\text{ave}}$ was determined and was found not to exceed 5%. As a result of this, the accurate determination of the positions of the nodes at both ends of the main gap is not necessary as long as the nodes are on the boundaries pole/air and armature/air. Fig. 5.6 shows linear curves $\Omega_{ga}/\Phi_{ga}$ at various armature positions. The absolute ratios $\Omega_{ga}/\Phi_{ga}$ for each position are the actual gap reluctances at the corresponding positions. Fig. 3.6 (Chapter 3) shows the effect of displacement of these reluctances. This curve is a 2-segment piecewise linearised curve that is used in the dynamic simulation. Although few positions were taken to model the whole motion, this simple 2-segment linearised curve was found sufficient for analysis purposes.

In the secondary air gaps $G_2$ (or $G_4$), the field distribution is quite different. The magnetic field component $H_y$ in this gap increases from zero in the z-direction to a maximum value and then drops to zero. $H_y$ does not vary significantly with z in the region around the right-hand side edge of the coil. The mmf was calculated by integrating $H_{t\text{max}}$ along the corresponding contour line. Fig. 5.7 shows linear curves $\Omega_{g2}/\Phi_{g2}$ at various armature positions. As can be observed, the effect of changing armature position is not as significant as in the case of the main air gaps because the armature displacement in this region is negligibly small compared to the length of $G_2$. For dynamic analysis purposes, this effect was ignored and the gap $G_2$ was modelled using one linear curve. The determination of mmf in gap $G_1$ (or $G_3$) is more simple since $\Omega_{g1}$ does not vary in the z-direction, and the mmf was calculated by integrating in the middle of the gap. The corresponding $\Omega_{g1}/\Phi_{g1}$ curve is shown in Fig. 5.8.

To complete the simplified model, the permanent magnet and the coil sources were considered. The permanent magnet was modelled as an mmf source for the purpose
of the method of calculation since it is more convenient here than using a flux source magnet model. Using the PE2D solutions, the operating point of the permanent magnet was found to vary along a very small portion of its recoil line. As is shown in Fig. 5.9, \( \Omega \) varies along a straight line with respect to \( \Phi \) for all positions, which makes the magnet modelling straightforward using only one approximation curve.

### 5.5 Main Air Gap Forces

As was shown in Chapter 3, two magnetic forces, namely \( F_{ga} \) and \( F_{gb} \) act simultaneously on the armature in each main air gap. Using the energy balance analysis, \( F_{ga} \) and \( F_{gb} \) are expressed as follows

\[
F_{ga} = K_{ga} \Phi_{ga}^2 \quad (5.3)
\]

\[
F_{gb} = K_{gb} \Phi_{gb}^2 \quad (5.4)
\]

Since the forces \( F_{ga} \) and \( F_{gb} \) and the fluxes \( \Phi_{ga} \) and \( \Phi_{gb} \) were obtained from the PE2D solutions at the specified currents and armature positions, \( K_{ga} \) and \( K_{gb} \) were easily derived. These were found to be independent of current and to vary with displacement as shown in Fig. 5.10. \( K_{ga} \) and \( K_{gb} \) have similar variations due to symmetry between the two gaps (when \( K_{ga} \) is minimum \( K_{gb} \) is maximum). The net magnetic force acting on the armature can be written as

\[
F_{net} = K_{gb}(y) \Phi_{gb}(y)^2 - K_{ga}(y) \Phi_{ga}(y)^2 \quad (5.5)
\]

Piecewise linearised approximations were fitted into the PE2D values of \( K_{ga} \) and \( K_{gb} \), and incorporated in the simplified dynamic model.

### 5.6 Actuator System Equations

The dynamic behaviour of the actuator system can be represented by three sets of coupled equations which must be solved simultaneously. The 3-mesh magnetic
network equations were established using the equivalent of Kirchhoff's voltage law as follows:

\[ \Omega_{g1} \Phi_1 + \Omega_{g2} \Phi_1 + \Omega_a \Phi_a + \Omega_{ga} \Phi_{ga} = N_i \]
\[ \Omega_{g3} \Phi_2 + \Omega_{g4} \Phi_2 + \Omega_a \Phi_a + \Omega_{gb} \Phi_{gb} = N_i \]
\[ \Omega_{ga} \Phi_{ga} + \Omega_{gb} \Phi_{gb} = \Omega_m \Phi_m \]  \hspace{1cm} (5.6)

where \( \Phi_a=\Phi_1+\Phi_2, \Phi_{ga}=\Phi_1-\Phi_m, \Phi_{gb}=\Phi_2+\Phi_m \) are, respectively, the fluxes in the armature and in the upper and lower main air gaps. Each term \( \Omega_i \Phi_i \), representing a magnetic component, is then expressed in terms of the loop fluxes \( \Phi_1, \Phi_2, \) and \( \Phi_m \). When solving the system, these terms must be updated at each value of displacement \( y \) and current \( i \). To start the solution of the above equation system, initial conditions were assumed for the three fluxes \( \Phi_1, \Phi_2, \) and \( \Phi_m \). Their initial values were determined from the field model solution at the initial position of armature and current \( i \) equal to zero. This solution corresponds to the case where the actuator is excited by the permanent magnet only.

The armature flux \( \Phi_a \) and coil current \( i \), in eqn. 5.6, are linked to the electrical subsystem equation

\[ V = i R + N \frac{d \Phi_a}{dt} \]  \hspace{1cm} (5.7)

where \( R \) is the resistance of the coil, \( V \) is the voltage across the coil terminals, and \( N \) the number of turns. Eqn. 5.7 is expressed directly in terms of the flux \( \Phi_a \) instead of the current \( i \) which is derived from eqn. 5.6, thus avoiding the various terms in incremental inductance which appear when current is used as a parameter [27].
The third coupled equation is the mechanical motion equation which is obtained by summing all the forces acting on the armature. There is no significant damping, so that acceleration is given by

\[ m \frac{d^2 y}{dt^2} = F_{\text{net}}(i, y) - F_f \]  

(5.8)

where \( F_f \) is a small (constant) friction force and \( m \) is the equivalent mass of the equivalent linear system. The equivalent mass was determined by equating the energies in both linear and angular systems. The net magnetic force \( F_{\text{net}}(i, y) \) may be computed from eqn. 5.5.

5.7 Time-stepping procedure used in the dynamic simulation

In order to determine the dynamic performance parameters of the actuator eqns. 5.6, 5.7 and 5.8, describing the magnetic, electric and mechanical subsystems, respectively, were solved simultaneously using a Fortran computer program developed by the author and run on IRIX (Unix System) mainframe. Conceptually, the system of equations may be abstracted in the block diagram form shown in Fig. 5.11.

\( \Phi_a \) was predicted at a particular time from the corresponding \( \Phi_a \) and \( d\Phi_a/dt \) at an earlier time, thus

\[ \Phi_a(t+\delta t) = \Phi_a(t) + \delta t \left( \frac{d\Phi_a}{dt} \right)_t \]  

(5.9)

The time-stepping procedure is as follows. At \( t=0 \), when voltage is applied, current is equal to zero and \( \Phi_a \) has the initial value \( \Phi_{a0} \) (obtained from the field solution with armature at initial position and current equal to zero), and \( d\Phi_a/dt \) is unknown. An initial estimate of current at the end of the first time-step is obtained using one of the magnetic system equations with mesh fluxes taking initial values. Using eqn. 5.7, the first iteration gives the first estimate of flux \( \Phi_a \) and, hence, \( d\Phi_a/dt \)’s. Thus, the
induced voltage in the coil is available at the start of the second iteration and a better estimate of current can be made. The procedure is repeated until changes in $\Phi_a$ are below a specified limit. The time-stepping procedure starts with the armature at initial position. Time-stepping is then continued until armature travel has been completed. Initial calculations showed non-convergence of the magnetic parameters due to the small size of the time-step $\delta t$ needed for the subsequent estimate of current. The non-convergence problem was overcome by adopting double-precision variables. This did not increase greatly the computation time which was about 45 seconds.

5.8 Model verification and dynamic analysis

In order to check the validity of the dynamic model, the computed results were compared to the experimental data measured under normal actuator operation.

The results presented in Figs. 5.12 and 5.13 give the predicted transient response of the actuator under static mechanical conditions. Fig. 5.12 shows the coil current waveforms at the starting, middle, and final positions of armature and an applied voltage of 24 volts. Notice the strong saturation effect in the current trace at the final position (lower main gap closed), and at the starting position, the armature just reached saturation. As pointed out in Chapter 2, one important feature to note here is the large $\frac{dV}{dt}$ at $t=0$ (just after applying the voltage $V$). Let's consider equ. 5.7. At $t=0$, the current is equal to zero and, thus, $\frac{d\Phi_a}{dt}$ takes a large value $V/N$ which results in a large $\frac{dI}{dt}$. This abrupt initial rise of current was predicted fairly well for the case of starting position, however, the model did not predicted it for the middle and final position cases. Fig 5.13 shows the corresponding predicted armature fluxes at the same fixed positions. Notice that $\Phi_a$ does not start from zero at the starting and final positions, because of the presence of permanent magnet. At the starting position $\Phi_a$ starts from a negative value and rises to a zero level at about 0.2 ms. Using the $\Phi_a$
vs time curve, at starting position, the electrical time constant was computed and found equal to 1.15 ms which agreed well with the experimental value of 1.2 ms.

Model verification was carried out by comparing the predicted transient electrical, magnetic, and mechanical behaviour to experimental results. The experimental and predicted armature displacement versus time curves are shown in Fig. 5.14. The most appreciable error in the predicted results appears to be the shorter (by approximately 0.25 ms) time delay that the model predicts before armature motion begins. The total operating time is in error by less than 4%. This includes the error produced from the tests since the armature trajectory was measured by monitoring its motion outside the actuator assembly (see Chapter 2). Another origin of errors between experimental and predicted results is the neglect of eddy currents in the model. For the purpose of this analysis, the oscillations on the experimental displacement/time curve in Fig. 2.13 (Chapter 2), were not predicted. These oscillations were found to be caused by the bending of the lever and no bounces were detected at the impact pole/armature. The experimental and theoretical results for the coil current versus time are shown in Fig. 5.15. As discussed earlier, the experimental current rises very quickly initially (large $\frac{di}{dt}$) due to a large $\frac{d\Phi_a}{dt}$. As the armature starts to move (after 1.6 ms on the experimental waveform), the rapid increase in the armature flux induces a negative voltage in the coil reducing the current change $\frac{di}{dt}$ temporarily (hence the first dip). When the armature hits the pole piece at the other end, the armature flux increases rapidly again causing further saturation, and hence a second dip on the current waveform. Once the armature is heavily saturated, the current rises quickly to its steady-state value $\frac{V}{R}$. Figs. 5.14-5.15 taken as a whole, clearly show the dynamic coupling between the actuator systems variables as mechanical motion affects the magnetic flux which, in turn, affects the electric current.
The Lumped-parameter and field models in Chapters 3 and 4 showed that the use of magnetic shunts (Fig. 4.16) across the large air gaps G2 and G4 improved the actuator force characteristics. However, the concern was that the shunts increase the inductance L in the magnetic circuit. The dynamic model was used to quantify their effects on dynamic performance. To evaluate these effects the current/time and displacement/time curves for both original and modified designs, were plotted together as shown in Figs. 5.16 and 5.17. Notice in Fig. 5.16 that the shunts reduce the rate-of-change di/dt because of the increase in inductance, in turn due to the reduction in circuit reluctance. This is reflected on the displacement/time curve in Fig. 5.17 which shows a longer delay time (by 0.2 ms). The model showed no change in the travel time.

5.9 Conclusions

The simplified dynamic model has proved to be an effective means of predicting the dynamic characteristics of the actuator with less computation than with a fully integrated dynamic finite-element model. An important additional advantage of the simplified model is the insight that it gives into the underlying design problem. It can also be used to predict the dynamic performance of other high-speed moving-iron actuators, provided that its parameters are derived from field studies.

Although the magnetic shunts have slightly increased the operating time (as was shown theoretically and experimentally), they are advisable. Further work in this area would also look at the change on current feed and reduction in the number of coil turns as other ways to reduce actuator response time.

The static and dynamic analysis (Chapters 2 to 5) of the Bentley actuator has shown that it is electromagnetically, well designed. This has created some incentives for the
author to extend the study, to investigate various topologies of moving-iron actuators for achieving high operating speeds, and compare their relative performances, including that of the Bentley actuator (see Chapter 6).
Fig. 5.1 Simplified dynamic model
Fig. 5.2 Magnified view of flux paths in armature and air gaps.
Armature at starting position and applied mmfs:
(a) 0 At  (b) 403 At  (c) 806 At
Fig. 5.3 Magnified view of flux paths in armature and air gaps.
Armature at mid-position and applied mmfs:
(a) 0 At  (b) 403 At  (c) 806 At
Fig. 5.4 Magnified view of flux paths in armature and air gaps.
Armature at final position and applied mmfs:
(a) 0 At  (b) 403 At  (c) 806 At
Fig. 5.5 mmf/flux characteristics of armature

Fig. 5.6 mmf/flux characteristics of upper main air gap
Fig. 5.7 mmf/flux characteristics of secondary air gap G2

Fig. 5.8 mmf/flux characteristics of secondary air gap G1
Fig. 5.9 Magnet mmf/flux characteristics

Fig. 5.10 Effect of armature displacement on force coefficient
INPUT
applied voltage V
resistance R
circuit number of turns N
equivalent mass
friction force Ff

Set time step \( \delta t \)
set initial conditions:
initial values for current i
mesh fluxes \( \Phi_1, \Phi_2 \) and \( \Phi_m \)
armature flux \( \Phi_a \)
armature starting position

calculate \( \Phi_a \) from electrical equation
\[ V = iR + N\Phi_a/dt \]

solve for mesh fluxes \( \Phi_1, \Phi_2 \) and \( \Phi_m \)
and current i

calculate fluxes in the main air gaps

use \( K_{ga}(y) \) and \( K_{gb}(y) \) to calculate the magnetic forces

integrate mechanical equation
\[ m \frac{d^2y}{dt^2} = F_{net} - F_f \]
to obtain armature position

armature travel completed?

is \( F_{ga} \neq F_{gb} \)?

YES

plot results

STOP

NO

Fig. 5.11 Computer block diagram of dynamic model
Fig. 5.12 Coil current versus time, armature held at starting, middle and final positions

Fig. 5.13 Armature flux versus time at starting middle and final positions
Fig. 5.14 Armature displacement versus time. Computed and measured results

Fig. 5.15 Coil current response. Computed and measured results
Fig. 5.16 Effect of shunts on current response

Fig. 5.17 Effects of shunts on actuator time response
Chapter 6

Performance limitations and comparisons of various high-speed moving-iron actuators

6.1 Objectives

The work on the Bentley actuator is extended by establishing, quantifying, and comparing the factors limiting the performance of high-speed moving-iron actuators, particularly the maximum acceleration rate. One objective is to attempt to answer the questions: can the Bentley actuator be significantly improved? more specifically, can its mechanical time constant be substantially reduced, and how does the actuator compare with the relative performances of the various devices analysed in this chapter?

In most moving-iron actuators, the required reluctance changes are produced by varying the length of an air gap, in contrast with most rotating reluctance machines, in which the effective gap area changes. The chapter describes the results of a comparative study of the different forms of linear moving-iron actuators, when used in applications in which the speed of response is the most important of the performance criteria.

6.2 The two modes of MI operations and their fundamental differences

The MI actuators form one of the largest classes of electromagnetic device applications after rotating machinery. They are generally used to provide forces or motions to do work in machines or to position parts or close switches. The simplest forms of these devices are the linear motion configurations illustrated in Figs. 6.1 and
6.2. Fig. 6.1 shows an actuator in which the operating principle depends on a change in the length of the working air gaps, while Fig. 6.2 represents an actuator in which the gap overlap area changes. The two operating modes can also be used to provide rotational motion.

The variable gap area (VGA) actuator (Fig. 6.2), generally designed for long stroke applications, can be made to have a linear relationship between current and force. This is a desirable feature for servo positioning applications in which it is also necessary to have the force independent of armature position. The VGA actuator is generally regarded as a device of low force delivery and small force-per-volume ratio which make it, in principle, not suitable for high-speed applications. Although this is the general belief, VGA actuators in rotary form such as the stepper motors have been designed to produce fast response. They are used when rotary motion and/or discrete positioning is desired. With an appropriate gear, they can be used for linear motion. A typical application is their use in computer peripherals for which they are preferred to the DC servo, synchronous or induction motors because of their compatibility with digital electronics. The use of rare-earth permanent magnet in the stepper motors has extended their high-speed capability, high specific torque and torque/inertia ratio. This leads to suggests that the VGA type deserve examination with regard to high-speed application. Other examples of VGA actuators include the Law's relay actuator [30].

The variable gap length (VGL) type, shown in Fig. 6.1, is most useful for simple point to point movement and produces a larger force than that of the VGA type at the expense of linearity and length of stroke, assuming constant current. The VGL actuator is most popular for applications requiring high speeds and high forces over short strokes. There are a wide variety of configurations of this type, many of which employ permanent magnets such as the Bentley actuator. Other examples of the VGL
type include the Philips actuator used in dot matrix printers [31], and the General Motors solenoid actuators for automotive control applications [32,33].

The characteristic feature of a VGA type is a progressive increase in the volume of the working air gap during the stroke, and a corresponding increase in the magnetic stored energy which is needed to maintain a constant force, whereas the VGL device demands maximum energy initially to meet the same requirement. This affects the dynamic performance of high-speed actuators, since the rate at which the magnetic energy can be supplied determines the initial force and acceleration rate.

It follows that the most suitable way of driving a VGL actuator effectively, is to maintain the maximum possible force (as limited by saturation). To do this, in a VGL actuator, $\Phi$ must be kept constant to maintain constant flux (by short-circuiting the drive coil in a zero-resistance circuit). This requires maximum energy initially, very rapidly, which is a disadvantage since it cannot be supplied. Hence, it may be slower than a VGA device. Note that the VGL actuator is inherently a constant $\Phi$ device. If the flux were constant from $t=0$, then it would work ideally (with zero delay), but the constant-current source has to supply a lot of energy very quickly. In contrast, in the VGA devices a linear rate-of-change of flux requires a constant supply voltage to maintain constant current and force. This difference in the dynamics suggests that the usual method of excitation by switching on a DC source is generally more appropriate to the second type of device. It is capable of developing the maximum force more quickly, and maintaining it more uniformly, as well as providing low-speed characteristics better suited to some mechanical load requirements. If necessary, the electrical and mechanical time-constants could be matched in a design in which a uniform velocity is maintained for a substantial part of the stroke. Because the VGA device has obvious advantages when relatively long strokes, and small forces, are needed, and it is then necessarily slow in operation, it tends to be regarded
as essentially low-speed. Thus one of the questions to be answered is whether or not it is capable of operating at speeds comparable to those of VGL devices. To be able to compare the high capability of both types, there is a need to establish and quantify the factors that limits the performance of MI devices.

6.3 Force characteristics

The usual view in the design of moving-iron actuator is that the increase in force as required by the load requirement is not realised without reducing the operating speed of the device, as it can be shown with the aid of illustration in Fig. 6.3. For the sake of simplification the slot side is chosen equal to the pole width \( w \). After specifying the stroke \( s \), \( w \) is chosen to obtain the required force. If, for instance, the force is doubled by doubling \( w \), the armature volume would be 4 times greater so that the ratio force per unit volume of armature \( (F/v_a) \) is halved and so is the acceleration. However, the following argument demonstrates that it is not necessary to use a large pole pair (or unit 'cell') to obtain the required force, but use a multiplicity of cells joined side by side or by increasing the length of the 'cell' as shown in Fig. 6.4. In this case the force and the volume of armature increase in the same proportion with the linear size of the actuator and the ratio force per unit volume of armature remains constant. However, for practical reasons the number of cells or the length of the cell cannot be increased linearly infinitely to achieve the maximum force possible. Other configurations must be sought to obtain a more solid and compact design such as the helical forms (the Helenoid and Colenoid actuators) devised by H. Seilly [34,35], the cylindrical type developed by Ford [36], and the disc-shaped (Disole) solenoid actuator by T. Kushida [37].

For comparison purposes, the 2-pole 'cell' or 'unit' arranged in the rectilinear form is considered (Fig. 6.1), with length \( l_z \) in the remaining dimension. Assuming that the
poles have infinite permeability and that the field \( H_g \) is uniform in the air gap and zero field outside gives the usual expression for force

\[
F_1 = \mu_0 H_g^2 w l_z
\]

\[
= \left(1/\mu_0\right) B_g^2 w l_z
\]

by integrating the Maxwell stress \((1/2\mu_0) B_g^2\) in the two gaps. The VGA device, shown in Fig. 6.2, gives a field \( H_s \), in the slot before the armature is inserted. Integrating the Maxwell stress over the dotted surface in Fig. 6.5 shows that, by assuming that the poles have infinite permeability and that the field is uniform in the small air gap \( g \) and in the large air gap \( g_s \) [19],

\[
F_2 = \mu_0 \left(g_2 H_g^2 - g_s H_s^2/2\right) l_z
\]

\[
= \mu_0 H_g^2 \left(1 - 2g_2/g_s\right) g l_z
\]

and in terms of flux density \( B \),

\[
F_2 = \left(1/\mu_0\right) B_g^2 \left(1 - 2g_2/g_s\right) g l_z
\]

where \( H_s \) is the field in the large air gap, and \( g_s/g \) must be sufficiently large to prevent an undue reduction in \( F_2 \). In both devices the mechanical work, \( F \) times stroke, gives the total gap energies,

\[
W_1 = \left(1/\mu_0\right) B_g^2 w l_z s
\]

\[
W_2 = \left(1/\mu_0\right) B_g^2 \left(1 - 2g_2/g_s\right) g l_z s
\]

but the factors determining force and stroke are interchanged. If the dimensions of both were chosen so that the width of each gap were the same as its length, then the strokes would be the same, and so also would be the forces, were it not for the \( 2g_2/g_s \) term. Thus, in the limit of practical air gap dimensions, the two outputs are directly
comparable, and increasing the width of the gap increases either the force or the stroke, depending on which device is chosen.

In actuators designed for a high operating speed an important figure of merit is the force density, $F/v_a$, where $v_a$ is the volume of the armature. In Fig. 6.1

$$f_1 = F_1/v_a = (1/\mu_0) B_g^2 w / t p$$

(6.5)

in terms of the pole pitch

$$p = 2w + g_{s1}$$

which must be as small as is consistent with an acceptable $g_s/g$ ratio.

whereas in Fig. 6.2

$$f_2 = F_2/v_a = (1/\mu_0) B_g^2 (1 - 2g_2/g_s) g_2 l_z/(g_s-2g_2) s l_z$$

(6.6a)

$$= (1/\mu_0) B_g^2 g_2 / s g_s$$

(6.6b)

If the stroke, $s$, were continued beyond the constant-force region to the point at which $F_2$ falls to zero and the armature no longer protrudes. Assuming $g_s=4g_2$ in Fig. 6.2, reduces the multiplying factors to

$1/4w$ and $1/4s$

respectively, and illustrates the conditions required to give similar accelerations from both devices, if the total accelerated mass is in some constant proportion to $v_a$. As a basis of comparison, we observe that the arrangement shown in Fig. 6.6 gives for a disc shaped armature (assuming the flux densities gaps 1 and 2 are the same)

$$F = (1/2\mu_0) B_g^2 \pi d_1^2/4$$

(6.7)

$$= (1/2\mu_0) B_g^2 \pi d_2 t$$

(6.8)
and a force per unit (volume \( v_a = \pi t d_2^2 / 4 \)),

\[
F/v_a = (2/\mu_0) B_g^2 / d_2
\]  \hspace{1cm} (6.9)

For a stroke \( s \), the work per unit volume is

\[
w = s F/v_a = (2/\mu_0) B_g^2 s / d_2
\]  \hspace{1cm} (6.10)

If \( s_{\text{max}} = 0.5 d_2 \),

\[
w = (1/\mu_0) B_g^2
\]  \hspace{1cm} (6.11)

which is twice the energy density in the gap (= 0.5 H B). Equs. 6.9 and 6.11 provide data for assessing the figures of merit for any moving-iron actuator.

The dimensions shown in Figs. 6.1 and 6.2 define the 'electromagnetically active' part of the entire armature, whose practical design must necessarily include additional structural material. The effect of the consequent increase in mass, and loss of net force density, is shown by a factor \( K \) defined as total mass/active mass by which equs. 6.5 and 6.6, have to be multiplied to give the actual acceleration, and operating time. Analysis of commercially-available high-speed variable-gap actuators show \( K \) values in the range between 1.25 to 3.

The response time of both devices increases in proportion to the linear dimensions, because the force is a surface effect, acting on the larger side of the armature in Fig. 6.1. It acts on the end-surface in Fig. 6.2, and is proportional to the area, whilst the stroke \( s \) determines the amount of armature material, so that increasing \( s \) causes a proportional reduction in the force density. The latter is increased by keeping \( g_s / g_2 \) small, with a limiting value of 2 as the armature thickness tends to zero. However, \( g_s / g_2 \) is constrained by the need to maximise \( F_2 \), from a given pole structure, so that
the practical limit of $g_s / g_2$ is about 4, and the energy density (product of force-density and stroke),

$$f_2 s = \frac{1}{\mu_0} B_g^2 g_2 / g_s$$

is largely determined $B_g^2$. Thus specifying the stroke limits the maximum acceleration rate, regardless of the other dimensions of the device, or the force which it is required to produce. Although an increase in $F$ requires more armature material, this does not necessarily cause any reduction in force density if the ratio of surface/volume is maintained, as can be done either by increasing the length, $l_2$, or the number of poles (by combining different geometric arrangements of the 2-pole unit). It is the stroke, rather than the force, or any other measure of size, which directly affects the maximum acceleration.

Similar considerations apply to VGL actuators, even though they do not appear so explicitly. Since the length, $l_2$, and the number of poles, may be chosen to meet the force specification, the unit dimensions (particularly the pole pitch) can be kept as small as possible, to give maximum acceleration rate. The least size depends directly on the stroke. More specifically, the ratio $g_s / g$, and $w/g$, where $w$ is the pole width, must be sufficient to maintain the surface force density, which depends on $B_g^2$. Thus the two types of actuator may give similar amount of energy conversion, per unit gap volume, similar forces, when appropriately compared, and similar maximum acceleration rates, suggesting that the VGA device may be more attractive than its relative neglect suggests. Modern experience with rotating machines also suggest high specific outputs from variable-area (stepper-type) devices when compared with wound-rotor machines, which are the rotary equivalent of MC actuators.
6.4 Force/stroke ratio

Although both types of actuator shown in Figs. 6.1 and 6.2 can be designed to produce performances which do not differ greatly over a range of forces and strokes, the VGL device is obviously better suited to applications in which the force/stroke ratio is large, and, conversely, the VGA device to those characterised by a large stroke/force. This can be quantified by writing equ. 6.1 in the form

\[
F_1/g_1 = \left(\frac{1}{\mu_0}\right) B_g^2 \left(\frac{w}{g_1}\right) l_z \tag{6.13}
\]

and equ. 6.2

\[
F_2/s = \left(\frac{1}{\mu_0}\right) B_g^2 \left(1 - 2\frac{g_2}{g_s}\right) \left(\frac{g_2}{s}\right) l_z \tag{6.14}
\]

giving the force/stroke ratios in terms of the aspect ratios, \(w/g_1\) and \(g_2/s\), of the two air gaps. A gap flux density \(B_g\) of slightly more than 1 Tesla and a gap aspect ratio of 1 \((w = g)\) gives

\[
F_1/l_z g_1 = \left(\frac{1}{\mu_0}\right) B_g^2 = 10^6 \text{ N/m}^2
\]

or 10 N per cm length, if \(g_1\) is in mm. Thus, if the stroke is limited to 1 mm, a force of more than 10 N per cm length suggests a VGL device, and this force increases in proportion to \(g_1\), so that a 10 mm stroke suggests the same choice if the specified force exceeds 100 N per cm length. Conversely, a stroke of more than 10 mm suggests a VGA device if the force does not exceed 100 N per cm.

These figures depend on \(B_g^2\), which will be increased in a high-performance device, and ignore the \(2g/g_s\) factor, but they provide a useful measure of the operating range to which each device is most appropriate. Exceeding the specified stroke/force ratios, in a VGL device, will require a gap length greater than the pole width \((g/w > 1)\), and similar consequences in a VGA device in which the force/stroke ratio is excessive.
6.5 **Influence of gap aspect ratios**

Equ. 6.5 can be expressed in terms of the ratio $w/g_s$ as

$$\frac{F_1}{v_a} = \left(\frac{1}{\mu_0}\right) \frac{B_g^2}{\left(2w/g_s+1\right)} g_s$$

(6.15)

In practice, specifying the stroke $g$ in a VGL actuator (Fig. 6.1), specifies the slot width $g_s$ needed to obtain the flux in the gap and armature. That is $g/g_s$ is approximately fixed. The pole width $w$ is chosen to produce the specified force $F$, but as $w$ increases $F/v_a$ diminishes, so that the smaller $w$ the better. In the limit as $w$ tends to zero $F/v_a$ reaches a maximum value of $\left(\frac{1}{\mu_0}\right) \frac{B_g^2}{g_s}$, assuming the flux density $B_g$ is uniform for the distance $w$ and equal to zero outside. In practice, there is a limit $w_o$ for which the armature will carry the minimum flux $\Phi_o$. As $w$ decreases towards this limit, the relative concentration of mmf gradient near to the pole (Fig. 6.7) progressively reduces $B_g$ on the flat surface, and hence the force $F$. The uniform field region which is needed to keep large $B_g^2$ dissappears. Field calculations (using PE2D), taking account of saturation in the iron core, showed that the ratio $w/g_1$ should be between 0.5 and 3. Below 0.5, the force starts dropping sharply because of the loss of uniformity of $B_g$ and above 3, $F/v_a$ decreases rapidly because of the greater effect of $v_a$. By keeping $w/g_1$ in this range, it is possible to achieve maximum values of $F/v_a$, and hence, maximum acceleration rates. The required force $F$ can be obtained by simply increasing the length $l_z$ or increase the number of 'units' without changing $F/v_a$. The commercial high-speed VGL devices [34-37] have been designed with $w/g$ ratios well above the theoretical limit of 3 (usually between 5 and 10), thus working with low values of $F/v_a$. Fig. 6.8 shows an example of a commercial high-speed VGL actuator.

In the VGA type, the corresponding aspect ratio of interest is $s/g_2$. Fig. 6.9 shows the design of a VGA actuator for a stroke of 5 cm, and an air gap length of 1 cm, giving a
force of 500 N for a length $l_z$ of 10 cm. The constant-current force-displacement curve, calculated from PE2D, shows that starting the stroke from the $x=0$ position shown in Fig. 6.10, with the armature outside the gap, gives an initial force little less than the maximum value.

The total flux in any given armature position can be expressed in the form

$$\Phi = \Phi_0 + \Phi_1 \frac{x}{s}$$  \hspace{1cm} (6.16)

where the flux $\Phi_0$ which is established in the $x=0$ position remains approximately constant for a large part of the stroke. It is this which causes the initial force. Reducing the stroke, $s$, and hence the armature mass has little effect on the initial force, so that, if there were no other mass, the maximum acceleration rate would be obtained from an armature whose stroke dimension is less than $g$ ($s/g < 1$).

### 6.6 Effects of saturation and slot geometry

If high accelerations are to be obtained, the pole and armature immediately adjacent to the air gap must be heavily saturated. At the same time, the slot must be kept small ($g_s$ small). If all other considerations are ignored, the problem of maximising $F/v_a$ is that of achieving the maximum $F$ from the arrangement shown in Fig. 6.11 in which:

(i) The parameter $g_s$ is chosen according to $g$ ($g_s/g$ specified). This is the basic size effect.

(ii) The pole profile maximises $B$ at the gap surface.

(iii) The armature is shaped for uniform $B$ since the maximum $B$ in it is less than that in the pole, so the additional mmf loss is small.

(iv) The ratio $t/g_s$ should be sufficiently small to keep the armature volume down.
Saturation effects are the most important in the armature when they dominate the problem of keeping the volume down. There is no obvious advantages in saturating the rest of the poles, and increasing the angle $\theta$ (Fig. 6.11) has the effect of increasing $F$. Thus, it is reasonable to assume an unsaturated pole surface to examine the problem of the best $t/g_s$ and $g_s/g$ proportions. Reducing $g_s$ will eventually reduce the force, as well as increasing the slot flux, the inductance, and the pole saturation. The basic problem is keeping $g_s$ as small as possible, consistent with achieving the flux in the armature.

Alongside with the ratios $g/g_s$ and $t/g_s$, the slot aspect ratio $d/g_s$ must be considered. In a simple rectangular slot (Fig. 6.1), the area is proportional to the depth $d$:

$$B_g = J \mu_0 g_s d / 2 g$$  (6.17)

Hence, if $g_s/g$ is kept constant, the smaller the depth $d$, the more current density is needed to maintain $B_g$. This will increase saturation in all parts of the actuator and reduces the flux in the gap (because of leakage), and hence, the force $F$. On the other hand the larger the ratio $d/g_s$, the lower the current density $J$ needed, but the slot flux increases. What is needed here is the absolute size of the device, which gives reasonable $d/g_s$ ratios. PE2D calculations were carried out for a 2-pole VGL unit, keeping $g_s/g$ constant and varying the depth $d$ and current density $J$. The results showed that for maximum $F/v_a$ the ratios $d/g_s$ must be in the range 1 to 3.

### 6.7 Mechanical time delay

The $F/v_a$ ratio controls the acceleration rate, but the most important single measure of high-speed performance is the operating time, which is directly influenced by the length of the stroke. The VGA device gives a constant force, if the current is assumed to be constant. Then the acceleration, $a$, is also constant, if the only retardation is inertial, so that
\[ F = \rho_m v_a K_m a \] \hspace{1cm} (6.18)

where \( \rho_m \) is the mass density and \( K_m \) the ratio of the total armature mass to the 'active' mass \( \rho_m v_a \). The time, \( \delta_t \), to complete the stroke, \( s \), is given by

\[ (\delta t)^2 = 2s / a \] \hspace{1cm} (6.19)

Substituting in equ. 6.6,

\[ \delta t = (s / B_g) \sqrt{\left(2 \rho_m K_m \mu_0 / g_s^2\right)} \] \hspace{1cm} (6.20)

and is proportional to \( s \).

Of the parameters, only \( B_g \) is subject to much variation, and we can obtain a measure of the performance by substituting the values \( B_g = 1 \), \( \rho_m = 7,600 \text{ Kg/m}^3 \) and \( g_s / g = 4 \), giving

\[ \delta t = 0.28 \text{ s} \] \hspace{1cm} (6.21)

If \( K_m \) is unity. This is therefore a minimum value, in the sense that the additional mass will increase \( \delta t \) according to \( \sqrt{K_m} \). It ignores the delay due to the electrical time-constant, but the result is otherwise comparable with performance data given graphically by Seilly [34] for the high-speed VGL Helenoid design. This shows a linear relationship between operating time and stroke, with a slope of 0.8 ms/mm. For the VGL type, and substituting from equ. 6.5 instead of equ. 6.6 replaces \( s \) by \( \sqrt{s} p \), in equ. 6.20, and removes the \( \sqrt{g_s / g} \) factor, giving similar results.

**6.8 Comparison with the Bentley actuator and proposed design to reduce the mechanical time constant**

The Bentley actuator has a mechanical time delay of 1.4 ms for a stroke of 1.2 mm, giving a ratio \( \delta t / s \) of 1.16, 4 times larger than the above figure (equ. 6.21). The
relatively large mechanical delay in the Bentley device results from the need to use a long armature which is inserted inside a large coil needed to supply the large mmf required by the large air gaps. Note that in the Bentley device a large part of the armature is used to carry the flux and only a small part (where the main forces are exerted) is active, which results in a poor force per-unit-volume of armature. In effect, the equivalent linear mass (half the total mass) is much larger than the active mass, resulting in a small w/p ratio equal to 0.13 (equ. 6.5). This results in $F/v_a$ far below the maximum merit figure given by equ. 6.9. The relatively large mechanical delay in the Bentley device is the price to pay in achieving a small electrical time constant. In effect, the increase in reluctance using large air gaps, reduce the inductance ($N^2$/reluctance), and hence $L/R$. In the Bentley design, the accelerating force (or pulling force) is produced in one air gap only, since the other gap produces the hold-on force (in opposition). Whereas, in a simple 2-pole unit the 2 forces in the gaps are added. This a second price to pay when a bistable operation is required using a permanent magnet.

It is proposed that the design shown in Fig. 6.12 will substantially reduce the mechanical time constant since it uses less moving mass (hence larger $F/v_a$) and less inertia without affecting the electrical time constant $L/R$. The coil can be split into 2 coils in series (same total number of turns as in the original design) which allows insertion of the non-magnetic spindle in the freed space, and use of a smaller (active) armature (section 1 in Fig. 6.12). The other section (2) is necessary for the return of the flux path. This can either be free to move by attaching it to the spindle or static, leaving a small gap between it and the spindle. The lever can be attached to the spindle and inserted through a hole in the middle of the shelf. With this proposed configuration, the single actuators have to be stacked side-by-side instead of on the top of each other as shown by the actual stack arrangement in Fig. 2.5 (Chapter 2), which may be a problem for practical reason of the Bentley application.
6.9 **Electrical time-constant**

The operating time depends on both the \(v_a/F\) ratio, and on the electrical time constant \(L/R\). The inductance \(L\) of an unsaturated device with uniform field \(B_g\) in similar air gaps, is given by

\[
L_i^2/2 = (1/\mu_0) B_g^2 v_g / 2
\]

in terms of the total volume, \(v_g\), of the air gaps. This ignores the flux in the excitation slot, as well as fringe effects adding to \(L\), giving a minimum possible

\[
L/R = (1/\mu_0) B_g^2 v_g / i^2 R
\]

and the \(i^2R\) loss depends on the current density, \(J\), resistivity \(\rho\), and total volume, \(v_c\), of copper. Substituting,

\[
L/R = (1/\mu_0) B_g^2 v_g / j^2 \rho v_c
\]

(and is otherwise independent of the number of turns in the winding), where the copper volume depends on the current density, since

\[
B_g = \mu_0 J a_c / 2 g
\]

in Figs. 6.1 and 6.2, in terms of the area of copper, \(a_c\), in the excitation slot. Denoting the mean length of turn \(K_{t*}z\),

\[
L_1/R = B_g w / \rho J K_t
\]

in Fig. 6.1, in terms of the mean length of turn \(K_{t*}z\). In Fig. 6.2 the volume of the gap \(g\) increases with displacement, \(x\), whilst the volume of the gap \(g_s\) diminishes, giving

\[
L_2/R = (B_g / \rho J K_t) [(2g/g_s) s + (1 - 2g/g_s) x]
\]
with a maximum value when \( x = s \), reducing the square bracket to \( s \). Thus, if the air
gap flux density, and current densities, are both kept constant, the maximum
electrical time constant in Fig. 6.2, like the \( v_g / F_2 \) ratio (equ. 6.6) and \( \delta t \) (equ. 6.20)
increases in proportion to the stroke, but does not otherwise vary with size or with
air-gap length. In practice the current density increases in smaller devices because of
the increased ratio of surface area to volume of copper, and the method of cooling has
to be taken into account, following the general rule that \( L/R \) can always be reduced
by reducing the copper section. However, the assumption that \( J \) is constant greatly
simplifies and clarifies the description.

Comparing equs. 6.5 and 6.6 shows that the \( v_a / F_2 \) ratio in Fig. 6.2 includes a factor,
\( g_s \), but otherwise the electrical and mechanical responses both depend on the
dimension \( w \), and hence on the specified force, \( F \) (equ. 6.1). In general, the
mechanical and electrical delays vary in the same way, regardless of the mode of
operation. Substituting typical values shows that the \( L/R \) delay is the greater of the
two in both devices if the structural (non-active) part of the armature mass is ignored,
although the practical comparison depend on the various design factors. The \( L_2/R \) and
\( L_1/R \) ratios are equal when the two devices have the same stroke, and a pole with/gap
ratio (\( w/g \)) of unity, and the other factors are the same, but this refers to \( L_2 \) at the end
of the stroke, and \( L_1 \) at the beginning, if \( B_g \) and \( J \) are to take their design values. The
characteristic feature of the variable-area device is the reduction of \( L_2/R \) to its
minimum value at \( x = 0 \), giving a more rapid rise in current and force at the start of the
stroke, when they matter most. However, equ. 6.27 shows that this advantage is
limited, since the minimum inductance \( L_0 \) is related to the maximum, \( L_s \), by

\[
L_0/L_s = 2g/g_s
\]

(6.28)
6.10 **Equivalent circuit**

A computer solution of the field usually gives the vector $\mathbf{A}$. The equivalent circuit equation is then

$$v_s = iR + \frac{d}{dt} \mathbf{A} \cdot d\mathbf{l} \tag{6.29}$$

If we define the average value

$$A_a = \frac{1}{l_w} \left( \int A \, dl \right) \tag{6.30}$$

for a coil wire of length $l_w$, then

$$v_s = iR + l_w \left( \frac{\partial A_a}{\partial i} \right) \frac{di}{dt} + l_w \left( \frac{\partial A_a}{\partial x} \right) \frac{dx}{dt} \tag{6.31}$$

where $v_s$ is the supply voltage, $R$ the resistance, and $x$ the armature position. The vector $\mathbf{A}$ can be interpreted in terms of flux linkage, or, more directly, as the momentum density of the moving conduction electrons [38] (viewed in these terms, the actuator is a device from converting electrical to mechanical momentum). The corresponding energy input in time $\delta t$ is

$$\int v_s \delta t = i^2 R \delta t + i l_w \left[ \left( \frac{\partial A_a}{\partial i} \right) di + \left( \frac{\partial A_a}{\partial x} \right) dx \right] \tag{6.32}$$

It is convenient to represent the terms by the equivalent circuit shown in Fig. 6.13, in which the $\frac{\partial A_a}{\partial i}$ term is represented by a coil, in the usual way, and the $\frac{\partial A_a}{\partial x}$ term is split into two, one representing the conversion to mechanical work, and the other the change in electrical energy with position (some of which is also converted by hysteresis). To clarify the discussion we assume ideal, linear, components, although saturation is important in a detailed study. Then $l_w \frac{\partial A_a}{\partial i}$ is independent of $i$, defining the inductance, $L$, the electrical energy represented by this term is
\[ E_s = \frac{1}{2}L \dot{i}^2 \]  \hspace{1cm} (6.33)

and is stored (i.e. 'magnetically', or as the kinetic energy of the moving electrons [38]). The displacement \( \delta x \) causes equal changes in stored energy and mechanical work.

\[ F \delta x = \frac{1}{2} \dot{i}^2 \delta L/2 \]  \hspace{1cm} (6.34)

giving equal voltages \( v' \) and \( v'' \) in Fig. 6.13

\[ v' = v'' = 0.5 i (\partial L / \partial x) u = 0.5 i \omega (\partial A_a / \partial x) u \]  \hspace{1cm} (6.35)

where \( u = dx/dt \) is the velocity, and \( iv'' \) accounts for the mechanical work rate, \( Fu \). \( F \) is given by

\[ F = 0.5 i^2 (\partial L / \partial x) = 0.5 \omega (\partial A_a / \partial x) i \]  \hspace{1cm} (6.36)

and equ. 6.34 and 6.35 describe the momentum exchange. Assuming a mechanical damping \( K_d \) and mass \( m \), i.e.

\[ F = K_d u + m \dot{u} + F_L(x,u) \]  \hspace{1cm} (6.37)

where \( F_L(x,u) \) is a specified mechanical load characteristic, completes the equivalent circuit. The effects of eddy currents, when significant, are easily added by suitable couplings and impedances.

The circuit model helps to illustrate the dynamic behaviour in diagramatic form. At the start of the stroke \( v' \) and \( v'' \) are zero, and the current increases to \( v_g / R \) at a rate determined by \( L_0 / R \), where \( L_0 \) is the initial inductance. This tends to zero in an ideal variable-area device when the gap area is zero, although it is convenient to assume a sufficient initial overlap to give a uniform-field region. The force rapidly increases with \( i \) to its maximum value, given by equ. 6.36, where \( \partial A_a / \partial x \) depends on the flux.
density $B_g$ in the gap, and the device is designed so that this is at the saturation level.

Equ. 6.36 can also be expressed in the form

$$F = 0.5 B_g N_i$$

(6.38)

and this provide a direct performance comparison with a moving-coil actuator, since the same maximum force would be exerted on a winding whose ampere-turns, $N_i$, are sufficient to 'terminate' the air-gap field, reducing it from $B_g$ on one side to zero on the other. Thus we obtain the same force on the armature, in Fig. 6.2, as would act on a pair of moving coils placed in the two air gaps if the two coil together had the same ampere-turns as the main exciting winding. This illustrates in simple terms one of the performance advantages of the moving-iron devices, since the same ampere-turns cannot be accommodated in the gap without a considerable increase in the electrical loading, as defined by $J^2$, where $J$ is the current density.

Keeping the armature stationary defines the first 'steady-state' solution, given by

$$i_1 = v_s / R$$

(6.39)

When freed the armature accelerates at maximum rate, in response to the corresponding force, $F$, and the voltages $v'$ and $v''$ appear in the equivalent circuit, one representing the increase in stored energy, and the other the useful work. These reduce the current towards a second 'steady-state' condition, defined by constant velocity $u$, giving constant $v'$ and $v''$, and hence constant $i$. The $L \partial i / \partial t$ voltage falls to zero, and the current is controlled by velocity, not inertial, effects to a value

$$i_2 = v_s / (R + K_u u)$$

(6.40)

where

$$K_u = l_w \frac{\partial A_a}{\partial x} = i \frac{\partial L}{\partial x}$$

(6.41)
and is independent of x if the gap area varies in proportion to x. The value of the velocity, \( u \), in equ. 6.40, will depend on the mechanical load characteristics, and whether or not the 'steady-state' will be achieved will depend on the relative importance of the mass and kinetic-damping terms.

The same equivalent circuit describes the behaviour of VGL devices, in which \( L \) would increase to very high values as the gap tends to zero, were it not for the limiting effect of saturation, together with any additional air gaps in the magnetic circuit (as in the Bentley actuator). The change in inductance due to the non-linearity causes a further induced-voltage term due to \( \partial L/\partial i \), which opposes that due to \( \partial L/\partial t \), but can be avoided by using the more fundamental form in equ. 6.29 when computing the current/time curve. The initial inductance, and hence time-constant, \( L/R \), is determined by the initial gap energy, which is similar to the final gap energy, and thus \( L/R \), in a VGA device having the same mechanical work capability. If the current were established with the armature stationary, and a short-circuit applied through zero resistance, the velocity terms would remove the initial energy by reversing the sign of \( di/dt \). In practice the supply voltage, \( v_s \), overcomes the iR drop, but also causes a rise in current which merely tends to increase the saturation level, rather than the force, if the designed \( B_g \) is achieved early in the closing period.

6.11 Conclusions

The study shows that it is the stroke which determines the operating speed of a well-designed moving-iron actuator, and it controls the mechanical and electrical delays, both varying linearly with stroke in an idealised device. The force, on the other hand, can be increased without necessarily slowing the operation down. This is true of each of the method of operation by changing either the length, or the area, of the air gap. The choice between two modes of operation depends partly on the specified ratio of force to stroke, but the variable area device also offers possibilities in increased
operating speed by allowing a stroke in excess of the air-gap length. This allows a reduction in pole-pitch, provided that the number of poles, or length, can be increased to meet the total force requirements.

The variable-area device also has other potential advantages, giving less variation of force with time, and a smaller electrical time-constant at the start of the stroke. It is generally well-matched to the usual excitation method by switching a DC voltage source, in contrast with the variable-length which need a constant-current source that has to supply a lot of energy very quickly. The design principles of the VGA device have been examined, and suggest that, although it is inherently well suited to long-stroke applications, it deserves consideration when this is not an obvious requirement. Although only single-acting devices have been considered without permanent magnets, the VGA principle can be applied to double-acting and polarised actuators.

It was also shown that the mechanical time delay of the Bentley actuator is 4 times larger than the minimum limit figure. Its mechanical time constant can substantially be reduced by making appropriate design changes without affecting its electrical time constant.
Fig. 6.1 Variable gap-length actuator

Fig. 6.2 Variable gap-area actuator
Fig. 6.3 Size effect on force per unit volume of armature

Fig. 6.4 The two basic modes of multicell arrangements to achieve large forces without reducing the acceleration of armature
Fig. 6.5 Dotted line shows contour of integration of Maxwell stress tensor.

Fig. 6.6 Arrangement giving maximum force per unit volume of armature.
Fig. 6.7 Effect of reducing pole width $w$ on $B^z$

Fig. 6.8 Example of a high-speed VGL actuator: the Helenold
Fig. 6.9 VGA actuator design for a stroke of 5 cm and gap length of 1 cm giving a force of 500N for lz=10cm

Fig. 6.10 Constant-current force-displacement curve computed with PE2D
Fig. 6.11 Armature and pole geometry for maximum B

Fig. 6.12 Proposed modified design of the Bentley actuator to reduce mechanical time delay

Fig. 6.13 Actuator equivalent circuit
Chapter 7

Summary, conclusions and recommendations

7.1 Summary

In this study, high-speed electromagnetic moving-iron actuators were experimentally investigated using digitally controlled instrumentation techniques to assess actuator dynamics, and numerically simulated with a dynamic lumped-parameter (magnetic equivalent circuit) whose magnetic parameters were derived from field (finite-element) studies. A dual voltage strategy was developed to improve actuator speed of response. This consisted in applying a high voltage dual pulse whose widths were changed to obtain the desired shape. The pulse widths were controlled using appropriate hardware and software.

The lumped-parameter model was shown to accurately predict the dynamic response of a high-speed moving-iron actuator. This lumped model accounts for magnetic saturation, effects of width change between the iron components, effects of transverse edge fluxes, and the dynamic coupling of actuator system variables. The dynamic lumped model was obtained by deriving its magnetic parameters from finite-element results with the aid of a commercial field-solving package. In parallel, a static lumped model was developed and shown to simulate the actuator static performance with reasonable accuracy, and to give an insight into the underlying actuator design principle. The static lumped model improved the understanding of the working principle of the device under study, and determined the key parameters of actuator performance. Two-dimensional finite-element models were developed to accurately predict the saturation levels, and to assess the 3D effects (due to width change between the iron parts and to transverse edge fluxes). The 3D effects were evaluated
by incorporating the results of 2D scalar potential models in typical transverse cross-sections into the vector potential solution in the longitudinal (main flux path) cross-section. The 2D finite-element models were used to determine the air gap reluctances for the static lumped model, and to estimate the mmf/flux and force characteristics for use in the dynamic lumped model. Transient eddy currents under static mechanical condition were also determined from finite-element results, and were found to have limited effects on actuator performance due to thin dimensions of the iron parts.

The study was extended by surveying various moving-iron actuator designs, and analysing their relative figures of merit for high-speed operation. The factors limiting the performance of both operating moving-iron modes, particularly the maximum possible acceleration rates were established, quantified, and compared.

7.2 Conclusions

In general, this research has shown the feasibility of using a dual voltage drive strategy to improve the dynamic performance of practical, high-speed moving-iron actuator systems. It has also shown the capability of combined lumped-parameter/finite element techniques to provide a great deal of useful design information. The extended study has shown that it is the stroke which determines the operating speed of a moving-iron actuator, and it controls the mechanical and electrical delays, both varying linearly with stroke in an idealised device. More specifically, it has been shown that:

(i) The operating time of the Bentley actuator could be reduced by 27% using the dual voltage strategy. This is accounted for by reducing the electrical time delay by 50 %, and hence, the electrical time constant L/R. Thus, the proper shaping of the pulse is an effective way of reducing the actuator response time.
The hybrid actuator model which combines the simplicity of a lumped parameter technique with the accuracy of finite-element (field-solving) methods, proved to be a very efficient and inexpensive tool for improving understanding of the Bentley device. The model was used to investigate the effect of parametric design changes. Static and dynamic analyses showed that the use of magnetic shunts improve the static force characteristics, but increase the actuator operating time. It was concluded that the Bentley device is, electromagnetically, well designed.

The operating time of a high-speed actuator, as determined by both the mechanical and electrical time-constants is, not fundamentally, determined by the force capability, but only by the required stroke. The force in both variable-length and variable-area devices, on the other hand, can be increased without necessarily slowing the operation down. The variable-area device also offers possibilities in increased operating speed by allowing a stroke in excess of the air gap length. It has advantages in the stored-energy changes when switched on to a DC voltage supply. It is also shown that the maximum force densities, and hence operating speeds, in both modes of operation, do not differ greatly.

7.3 Recommendations

There are several specific areas in which the instrumentation techniques, the dual voltage strategy, the numerical simulation methods, and the results of this research could be extended:

(i) The dual-voltage technique should be used to improve the performance of any high-speed actuator, provided that its ratio electrical time constant/mechanical time constant is not small.
(ii) The actuator modelling techniques developed should be used to improve the performance of other types of moving-iron actuators (other than bistable permanent magnet devices). These should include both variable gap-length and variable-area actuators. The model developed for dynamic simulation should be used to predict the dynamic performance of other high-speed moving-iron actuators, provided that its parameters are derived from field studies.

(iii) The results of the performance limitations and comparisons of high-speed moving-iron actuators should be verified experimentally, particularly those suggesting high potential of the variable-area device for high-speed applications, and those suggesting a reduction of the mechanical time constant in the proposed modified design of the Bentley actuator. The maximum merit figures should also be compared to those of other high-speed devices, such as the moving-coil and the moving-magnet actuators.
References


Publications


Appendix A

Software for the dual voltage drive system

Program written in Assembly Language for a 6809 microprocessor

```
.processor m6809
urt1st .equ 22h ;universal receiver transmitter
urt1dt .equ 23h
vara .equ 1100h ;duration of the pulse
varb .equ 1102h ;duration of the rest time
varc .equ 1104
temp4 .equ 1108h
temp3 .equ 1109h
temp2 .equ 110ah
temp1 .equ 110bh
temp  .equ 110ch
xloc  .equ 1110h
yloc1 .equ 1112h
yloc2 .equ 1114h
temp5 .equ 1116h
temp6 .equ 1118h
temp7 .equ 111ah
tempc .equ 111ch
tim1c .equ 0010h ;address of timer1 control register
tim2c .equ 0011h ;address of timer2 control register
tim1d .equ 0012h ;address of timer1 buffer register
int   .equ 0048h ;interrupt address
temp1 .equ 1204h ;16bits
xmem  .equ 1206h
ymem  .equ 1208h
count .equ 120ch
t1    .equ 120eh
t2    .equ 120fh
t3    .equ 1210h
t4    .equ 1211h
t5    .equ 1212h
t6    .equ 1213h
p12   .equ 1215h
table .equ 1220h
table2 .equ 1260h
esc   .equ 01bh ;escape key
pdata .equ 0f00ch;
pclrf .equ 0f00eh
k1    .equ 1402h
k2    .equ 1403h
k3    .equ 1404h
k4    .equ 1405h
k5    .equ 1406h
```
k6 .equ 1407h
k7 .equ 1408h
k8 .equ 1409h
l1 .equ 140ch
l2 .equ 140eh
l3 .equ 1410h
test .equ 1412h
mem1 .equ 1414h

zero:
.org zero+1800h
ldd £table
std xmem
ldd £00h
std ymem
lda £01h
sta tim2c ; set timer2 control register to 1
lda £0a2h
sta tim1c ; initialise timer1 in continuous mode
lda £03h ; uart
sta urt1st
lda £52h ; uart
sta urt1st
ldd £32 ; vara and varb set with this value
std vara
ldd £495
std varb
ldd £9999
std varc
lbr mes4

read:
clr temp1
clr temp2
clr temp3
clr temp4
ldd £temp4
std xloc

start:
lda urt1st
anda £01h
lbne key

wavel:
lda £0ffh
sta 2
sta 4
lbr ptm
ldd £32
lbsr  delay
lda  £00
sta  2
ldd  £495
lbsr  delay
lda  £00
sta  4
ldd  £2000
lbsr  delay
lda  £2000
sta  3
sta  5
ldd  £32
lbsr  delay
lda  £00
sta  3
ldd  £495
lbsr  delay
lda  £00
sta  5
ldd  £2000
lbsr  delay
lbra  start1

********************************************************************

ptm:

ldd  £0ffff
std  tim1d
rts

********************************************************************

delay:

subd  £1
bne  delay
rts

********************************************************************

key:

lda  urt1dt
cmpa  £61h
lbeq  adj a
cmpa  £62h
lbeq  adj b
cmpa  £63h
lbeq  adj c
cmpa  £30h
lbeq  conv 1
cmpa  £31h
lbeq  conv 1
cmpa  £32h
lbeq  conv 1
cmpa  £33h
lbeq  conv 1
cmpa  £34h
lbeq  conv 1
cmpa E35h
lbeq convl
cmpa E36h
lbeq convl
cmpa E37h
lbeq convl
cmpa E38h
lbeq convl
cmpa E39h
lbeq convl
cmpa £39h
lbeq convl
cmpa £39h
lbeq convl
cmpa £39h
lbeq convl
cmpa £39h
lbeq mes2
lbeq £0dh;CR
lbne mavel
lbsr convd
lbra wave1

*********************************************************************************

convl: 
lbsr output
ldx xloc
sta ,x+
stx xloc
ldd xloc
cmpd £temp
lbeq mes1
lbra start1

*********************************************************************************

mes1: 
ldx £m3
jsr [pdata]
lbra start1

mem2: 
jsr [pcrlf]
ldx £m4
jsr [pdata]
jsr [pcrlf]
lbra read

mem3: 
jsr [pcrlf]
jsr [pcrlf]
ldx £m5
jsr [pdata]
jsr [pcrlf]
rts

mem4: 
jsr [pcrlf]
ldx £m6
jsr [pdata]
jsr [pcrlf]
rts
adja: ldd tempc std vara lbra wave1

adjb: ldd tempc std varb lbra wave1

adjc: ldd tempc std varc lbra wave1

convd: ldb temp4
       andb £0fh
       stb temp4
       ldb temp3
       andb £0fh
       stb temp4
       ldb temp3
       andb £0fh
       stb temp3
       lda temp4
       ldb £0fah
       mul std temp5
       addd temp5
       addd temp5
       std temp5
       lda temp3
       ldb £64h
       mul std temp6
       lda temp2
       ldb £0ah
       mul std temp7
       addd temp6
       addd temp5
       addb temp1
       std tempc
       lbsr mes3
       rts
;convert decimal number into ascii character

scrn:  adda  £30h
      jsr     output
      rts

;clear screen

clear: lda  £esc
       jsr     output
       lda  £';'
       jsr     output
       rts

;transmit to the screen

output: pshu  a

ready:  lda     urt1st
        anda  £02h
        beq     ready
        pulu  a
        sta     urt1dt
        rts

******************************************************************

m1: .text "(press any key to continue)"
    .byte 00h

m2: .text "The following values are the time intervals taken"
    .byte 0ah,0dh
    .text "by the armature to move across the gap"
    .byte 0ah,0dh
    .text "Time interval = delay time + travel time"
    .byte 0ah,0dh
    .text "The values are in microseconds"
    .byte 0ah,0dh
    .text "(press any key to continue)"
    .byte 00h

m3: .text "(press return to store the number or 'z' to delete it)"
    .byte 00h

m4: .text "type a new number"
    .byte 00h

m5: .text "the number is stored, press 'a','b'or(and)'c' to change"
    .byte 0ah,0dh
    .text "a, b or(and) c,"
    .byte 0ah,0dh
    .text "press 'z' before typing a new number,"
    .byte 00h
This is a two-level voltage waveform. The period of this signal is set with initial values a, b and c:
a: duration of high voltage pulse, b: duration of low voltage pulse and c: rest time
Initial values are:
a--0.24 ms, b=3.76 ms and c=70 ms,
period T=2(a+b+c).
The durations a, b and c of the signal can be changed by typing a number of 4 digits.
This number will be multiplied by 1.085 * 7 microseconds to give the actual durations a, b, c desired.
Type a number of 4 digits