Don’t birth cohorts matter? A commentary and simulation exercise on Reither, Hauser and Yang’s (2009) age-period-cohort study of obesity

Andrew Bell \textsuperscript{a,b} and Kelvyn Jones \textsuperscript{a,b}

\textsuperscript{a}School of Geographical Sciences
University of Bristol
University Road
Bristol
BS8 1SS
UK

\textsuperscript{b}Centre for Multilevel Modelling
University of Bristol
2 Priory Road
Bristol
BS8 1TX
UK

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Corresponding Author:
Andrew Bell
School of Geographical Sciences
University of Bristol
University Road
Bristol
BS8 1SS
UK
Email: andrew.bell@bristol.ac.uk

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Abstract

Reither et al. (2009) use a Hierarchical Age-Period-Cohort model (HAPC - Yang & Land, 2006) to assess changes in obesity in the USA population. Their results suggest that there is only a minimal effect of cohorts, and that it is periods which have driven the increase in obesity over time. We use simulations to show that this result may be incorrect. Using simulated data in which it is cohorts, rather than periods, that are responsible for the rise in obesity, we are able to replicate the period-trending results of Reither et al. In this instance, the HAPC model misses the true cohort trend entirely, erroneously finds a period trend, and underestimates the age trend. Reither et al.’s results may be correct, but because age, period and cohort are confounded there is no way to tell. This is typical of age-period-cohort models, and shows the importance of caution when any APC model is used. We finish with a discussion of ways forward for researchers wishing to model age, period and cohort in a robust and non-arbitrary manner.

Research Highlights

- Reither et al’s HAPC study suggest rises in obesity are caused by period effects
- Simulations suggest that this result may be erroneous
- Identical results were found with data simulated from an entirely different process
- Results show the pitfalls of using APC models without critical forethought.

Keywords

Age-period-cohort models, Obesity, Collinearity, Model identification
1 Introduction

The desire to separate age, period and cohort (APC) effects has been a key feature of both the medical and social sciences for a number of decades (Ryder, 1965). For at least the same period, levels of obesity have been rising at a continuous rate, to the point that in 1997 it was classified by the World Health Organisation as a global epidemic (Caballero, 2007). In 2009, Reither et al. (2009) used the recently developed Hierarchical Age-Period-Cohort (HAPC) model (Yang & Land, 2006) to assess the relative importance of periods and cohorts in the development of the obesity epidemic. Whilst they found some significant cohort effects, the implication of their results was “that period effects were principally responsible for the obesity epidemic” (Reither et al., 2009:1445), and this result was repeated by Yang and Land (2013:215-222).

However, the possibility of separating APC effects is beset by an ‘identification problem’ due to the fact that age, period and cohort when taken together are perfectly collinear. In this paper we show that the HAPC model does not solve this identification problem, and therefore that the results found by Reither et al. should be treated with some scepticism.

The purpose of this paper is twofold. The first substantive contribution is to add to the growing debate in epidemiology regarding the causes of, and therefore possible solutions to, the obesity epidemic. Whether periods or cohorts are responsible for changes in obesity is of profound importance because it should affect how policy interventions are targeted. The second, methodological, contribution is to assess the capabilities of age-period-cohort models, and the dangers of using these models without critical forethought regarding their limits. In this we are building on previous work (Bell & Jones, 2013a, b, c; Glenn, 2005; Luo, 2013; Luo & Hodges, 2013) questioning the capabilities of the HAPC model and other methodological innovations to disentangle APC effects.

We first outline the identification problem and Yang and Land’s proffered solution to it. Second we briefly review the literature on the development of the obesity epidemic. Third we outline our
simulation design which we use to show that the results found by Reither et al. could have been created by a different data generating process (DGP). Finally, we discuss the implications of this both within obesity research and beyond, considering ways forward for researchers wishing to use techniques like the HAPC model to make robust conclusions regarding APC effects.

2 The APC identification problem and Yang and Land’s HAPC Model

The conceptual distinction between age, period and cohort is well known (Bell & Jones, 2013a; Suzuki, 2012). However despite this, there remains the problem of statistically modelling the three effects because of the mathematical dependency between them:

\[
Age = Period - Cohort
\]

(1)

As such, if we know the value of two of the terms, we will always know the value of the third. From an ‘experimental’ standpoint, therefore, it is impossible to hold two of APC constant whilst varying the third. Because of this, each of the following DGPs (and an infinite number more) would produce identical values for a dependent variable \( Y \):

\[
Y = (1 \times Age) + (1 \times Period) + (1 \times Cohort)
\]

(2a)

\[
Y = 2 \times Age + 2 \times Cohort
\]

(2b)

\[
Y = 2 \times Period
\]

(2c)

Given such data, therefore, it would not be possible to tell which DGP actually produced the data. These three instances presented here have very different substantive meanings, yet it would not be
possible to tell which of the three actually produced the data at hand\(^1\). It is for this reason that many see a solution to the identification problem to be a logical impossibility:

“The continued search for a statistical technique that can be mechanically applied always to correctly estimate the effects is one of the most bizarre instances in the history of science of repeated attempts to do the logically impossible.”

(Glenn, 2005:6)

Despite this, numerous supposed solutions to the identification problem have been proposed, each of which imposes some kind of constraint on the model (Mason et al., 1973; Sasaki & Suzuki, 1987; Tu et al., 2011; Yang et al., 2008). The problem arises when these constraints are not clearly stated, are applied arbitrarily on the basis of statistical necessity, and are not grounded in any kind of substantive theory. The models are generally very sensitive to such constraints and as such can provide extremely misleading results when those constraints are not precisely justified and appropriate.

Yang and Land’s proposed solution is to use a cross-classified multilevel model, which treats age as a fixed effect and periods and cohort groups as random effects – contexts in which individuals reside. The model can thus be specified (in the continuous Y case) as:

\[
y_{i(j_1j_2)} = \beta_{0j_1j_2} + \beta_1Age_{i(j_1j_2)} + \beta_2Age^2_{i(j_1j_2)} + e_{i(j_1j_2)}
\]

\[
\beta_{0j_1j_2} = \beta_0 + u_{1j_1} + u_{2j_2}
\]

\[
e_{i(j_1j_2)} \sim N(0, \sigma^2_e), \quad u_{1j_1} \sim N(0, \sigma^2_{u_1}), \quad u_{2j_2} \sim N(0, \sigma^2_{u_2})
\]

(3)

The dependent variable, \(y_{i(j_1j_2)}\) is measured for individuals in period \(j_1\) and cohort \(j_2\). The ‘micro’ model has linear and quadratic age terms, with coefficients \(\beta_1\) and \(\beta_2\) respectively; a constant \((\beta_{0j_1j_2})\) that varies across both periods and cohorts; and a level 1 residual error term \((e_{i(j_1j_2)})\). The

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\(^1\) The technical consequence of this is that a regression with age, period and cohort as linear independent variables will not be estimable (at least with OLS) because the design matrix \(X'X\) cannot be inverted.

5
macro model defines the intercept in the micro model by a non-varying overall intercept $\beta_0$, and a residual term for each of period and cohort. The period, cohort and level-1 residuals are all assumed to follow Normal distributions, each with variances that are estimated.

Putting age in the fixed part and period and cohort in the random part is conceptually attractive; but also, it is argued by Yang and Land that this distinction solves the identification problem:

"An HAPC framework does not incur the identification problem because the three effects are not assumed to be linear and additive at the same level of analysis"

(Yang & Land, 2013:191)

In addition to this, Yang and Land suggest that the inclusion of the quadratic term for age helps to further resolve the identification problem:

the underidentification problem of the classical APC accounting model has been resolved by the specification of the quadratic function for the age effects.

(Yang & Land, 2006:84)

However, it has been shown elsewhere that this methodological advance in fact amounts to another constraint (Luo & Hodges, 2013), and simulation studies have shown that the use of this model, without critical forethought, can lead to misleading results (Bell & Jones, 2013a).

3 The Obesity Epidemic

Historically, obesity was a rare affliction, predominantly affecting those of high socio-economic status (Caballero, 2007). However, levels of obesity increased throughout the twentieth century, particularly amongst those of lower socio-economic status and education levels (Visscher et al., 2010). A number of reasons for this have been proposed, including the more sedentary lifestyle associated with the technological advances of the modern world (Rokholm et al., 2010), and the
greater availability, portion size and fat content of food (Hill & Peters, 1998). However, the question remains as to whether it is via periods or cohorts that these changes occur. If the former, it would suggest that changes in lifestyle have affected all age groups equally, resulting in bad diets and low levels of exercise for all individuals. In contrast, the latter would suggest that these cultural changes particularly affect people in their formative years, and these changes have affected their behaviour and possibly their physiological resistance to obesity throughout their subsequent life-course. In the same vein, interventions to the obesity epidemic should be similarly targeted to the groups most affected. If cohorts are responsible for changes in obesity, then policy interventions should be focused on children in their formative years because interventions targeted at adults are likely to be ineffectual.

Reither et al. argue that the obesity epidemic is the result predominantly of periods, and their results are shown graphically in the third column of figure 1. They argue that “the pattern of predicted probabilities for U.S. adults shows a monotonic increase over time, with no sign of abatement in recent periods of observation” (Reither et al., 2009:1443). Similarly, Allman-Farinelli et al. (2008) find that period effects are the driving force of changes in their APC analysis of obesity in Australia, whilst Rokholm et al. (2010:843) argue that the slight levelling off of the obesity epidemic observed in recent years “occurred at approximately the same time for different age groups”. However, other studies find evidence that cohorts have the greater influence on obesity: for example Olsen et al. (2006) find that non-linearities in cohort trends match for different age groups, but do not match for periods. However, we argue that all of the methods used above have flaws, relying on un-testable assumptions (explaining why the results that have been found are so contradictory). The next section takes the results found by Reither et al (2009) and shows that those results could have been found with a very different data generating process (DGP).
4 Simulation exercise

Reither et al (2009) found a strong, approximately linear trend in periods, and very little in terms of a trend in cohorts (see figure 1). However, we have argued that, because of the identification problem, these results could have arisen from a very different DGP. In order to test this, we ran the model used by Reither et al using the following DGP:

\[
\text{Logit } P(Y = 1) = -2 + (0.1 \times \text{Age}) - (0.001 \times \text{Age}^2) + (0.04 \times \text{Cohort}) + u + u_p
\]

\[ u_c \sim N(0, 0.01) \text{ for cohorts, } u_p \sim N(0, 0.01) \text{ for periods} \]

\[ (4) \]

Here, Y will equal 1 for an individual who is obese, and 0 otherwise. The period and cohort residuals were generated to be Normally distributed, with a variance of 0.01. Crucially, in this DGP we include a linear cohort effect\(^2\) of 0.04, and an age effect that is 0.04 larger than that found by Reither et al (2009). We do not include the period trend found by Reither et al, so this part of the DGP is just random fluctuations from year to year.

This data, generated with this known functional form, was then fitted to a logistic version of Yang and Land’s HAPC model:

\[
\text{Logit } P(Y_{i(j_1 j_2)} = 1) = \beta_{0j_1 j_2} + \beta_1 \text{Age}_{i(j_1 j_2)} + \beta_2 \text{Age}^2_{i(j_1 j_2)}
\]

\[ \beta_{0j_1 j_2} = \beta_0 + u_{j_1} + u_{j_2} \]

\[ u_{j_1} \sim N(0, \sigma^2_{u1}), \quad u_{j_2} \sim N(0, \sigma^2_{u2}) \]

\[ (5) \]

\(^2\) It could be argued that it is unlikely that such a cohort (or period) trend would never be generated in real life in this way, because periods and cohorts are intrinsically random (in contrast to age which has a fixed range and so should be treated as a fixed covariate). However, the model is unable to tell whether a trend is the result of a ‘random’ and fleeting upward fluctuation or a consistent linear trend over all possible time periods/cohorts, since the resulting data sample would be much the same. This is especially the case when the trend found (by Reither et al.) is very much linear in appearance and interpreted as such (“a monotonic increase over time” – Reither et al 2009:1443). As such a linear trend is an appropriate means of generating the data for this situation.
It is implicitly being assumed that any period or cohort trend is appropriately picked up by the period or cohort residuals, since no fixed effect is specified for such trends.

Reither et al. (2009:1442) use 5-year groups to define their cohorts in the models that they fit. This is “conventional in demography”, but Reither et al. argue that a further advantage of this grouping is that they “function as equality constraints” which help to identify the model. In previous work (Bell & Jones, 2013a) it has been shown that the HAPC model is able to correctly estimate trends when groupings exactly match groupings in the data generating process, but not when those groupings are chosen by arbitrary convention as they appear to have been here. It is therefore important to evaluate the possible effect that this grouping has on the question at hand. We therefore examined three grouping scenarios:

- No grouping (i.e. 1 year birth cohorts) in either the DGP or the fitted HAPC model
- No grouping in the DGP, but HAPC model fitted with 5-year birth cohorts
- 7-year birth cohorts in the DGP but HAPC model fitted with 5-year birth cohorts.

Since it is unlikely that we would ever know the exact cohort groups in the DGP (if they are present at all), we do not test a model where the groupings in the DGP and the fitted models match.

The simulations were conducted in a similar way to those in Bell and Jones (2013a), using Bayesian MCMC methods (Browne, 2009) in MLwiN version 2.28 (Rasbash et al., 2013), through Stata using the runmlwin command (Leckie & Charlton, 2013). True values from the DGP were used as starting values as non-informative priors, and the model was run for 20,000 iterations, following a 1000 iteration burn-in. In order to assess convergence of the chains, a sample of parameter trajectories were visually inspected. In addition, a version of the Potential Scale Reduction Factor (Bell & Jones, 2013a; Brooks & Gelman, 1998) and Effective Sample Size were calculated for all parameters. The Stata code for these simulations can be found in the online appendix.
For each grouping scenario 1000 separate simulations were conducted. We have reported the results from the model with the median value for the coefficient associated with age for the scenario where cohorts were grouped in 7-year intervals and modelled with 5-year intervals. We did this, rather than averaging over all 1000 simulations, because the mean results could not have been estimated from a single dataset generated by our DGP (for example, random variation in residuals would be averaged out). The results found are, however, typical of all simulations in all grouping scenarios\(^3\).

The results are shown in the second column of figure 1, alongside the true DGP (column 1) and the results found by Reither et al (column 3). As can be seen, the typical median result does not match the DGP at all. No cohort trend is found, an erroneous period trend is found, and the age effect is underestimated. In fact, the results found very closely resemble those found by Reither et al. The implications of this, of course, is that the same DGP could have generated Reither et al.’s data, and the results that they found could be as misleading as the results found in the simulation.

\[\text{Figure 1 about here}\]

5 Discussion

To be clear: we are not saying that the results found by Reither et al. are necessarily incorrect. However, Reither et al. (2009:1444) argue that their results are “unambiguous” and it is with this that we take issue. There is no reason to think that Reither et al.’s results are the true DGP, rather than the DGP we used to generate our data here. This is important for policy makers considering possible interventions to the obesity epidemic. Whilst Reither et al.’s results suggest that interventions should be targeted to all age groups, the alternative explanation offered by us would suggest that interventions would be better targeted at children in their formative years.

\(^3\) A small minority (~5%) of results for the scenarios with mismatched groupings (between the DGP and the fitted model) produced different results, including results that were correct according to the DGP. However this did not occur when the cohorts were ungrouped. We have not reported these, given that a model that is right less than 10% of the time is not particularly useful.
Reither et al are not alone in finding results that may be misleading using the HAPC model. Dassonneville (2012) finds a period trend in voter turnout volatility, going against the literature which tends to find cohort effects to be most significant. Much like the period trend found by Reither et al, this could be an incorrect finding. Other studies (Plontek et al., 2012; Schwadel, 2010) similarly use the HAPC model to find period and cohort trends which may be over-interpretations of their data.

So what should the researcher of APC effects do? Where there are no trends in the periods or cohorts, the HAPC model works well, meaning that it can be used to assess random variation in periods and cohorts. This assumes that there are not equal and opposite linear period and cohort trends (which, would cancel each other out, with the model estimating a spurious age effect rather than the true period and cohort trends), but this is an assumption that researchers are often willing to make. Furthermore, there remains the possibility that cohort and/or period residuals remain autocorrelated, even when there is no trend; the model can be extended to incorporate this autocorrelation into the model (Stegmueller, 2013).

Where trends do exist, one option would be to make a decision based on theory as to which of periods or cohorts are most likely to have generated the data, and include that term in the HAPC model as a linear fixed effect (Bell & Jones, 2013a). This decision cannot, however, be made on the basis of the data, since a model with period and age fixed linear trends will fit the data as well as one with age and cohort fixed linear trends. This is confirmed by simulations using the same DGP as above, (but with a period or cohort linear term included in the fixed part of the fitted model). The results of these are displayed in Table 1.

[Table 1 about here]

In the case of this study, where the purpose of the research is to examine which of period and cohort are most likely to be the cause of the epidemic, researchers could assess the age trend that is
found and decide whether it seems likely. In the case of the modelled results here, we would argue that the age effect that we generated is more plausible than that found by Reither et al (2009). Whilst we would expect some decline in obesity at older ages – due to physiological reasons and survival bias (Villareal et al., 2005) – we would not expect it to be as large or as early in life as that found by Reither et al. (Villareal et al., 2005; Visscher et al., 2010). Of course it is also possible that the rise of obesity is the result of a mixture of period and cohort effects, in which case the true DGP would be somewhere in between those found in figure 1. Where there are very good theoretical (e.g. physiological) reasons to believe the age trend is known, such belief could be incorporated into the model (e.g. see Tilley & Evans, 2013) potentially in a Bayesian way using strong informative priors (Browne, 2009; Jackman, 2009). However, that would involve a theoretical judgement which, again, cannot be confirmed simply on the basis of the data.

Overall, we hope that this commentary will push researchers towards engaging in more critical forethought regarding APC effects. With appropriate constraints, based on theoretical plausibility rather than statistical necessity, techniques like the HAPC model can be useful in modelling possible APC combinations, so long as those constraints are explicitly stated by the authors. When those constraints are unclear, or not known about in the first place, misleading results may be produced.

References


Figure 1: Age (row 1) cohort (row 2) and period (row 3) effects on obesity, according to the true self-generated DGP\(^a\) (column 1), the median simulation result (column 2) and the result found by Reither et al., 2009 (column 3).

\(^a\) Simulation here is from the scenario where cohorts were grouped in 7-year intervals in the DGP and 5-year intervals in the fitted model. However the results of the other grouping scenarios were substantively similar.
Table 1: Mean fixed effects and model fit criterion (DIC - see Spiegelhalter et al., 2002) for simulation results with a model using (1) age and cohort, and (2) age and period, as fixed linear effects. It can be seen that, if anything, the DIC supports the incorrect specification.

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*a* Cohorts here were grouped by 7-year intervals in the DGP and by 5-year intervals in the model (but the results were substantively similar to the other grouping scenarios tested).