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On the variability of the effective friction angle of Saint Lucian soils: investigations through a laboratory database

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Abstract: For assessment of slope stability in data-scarce regions prone to natural hazards, modelling relies, to a large degree, on estimates of the effective friction angle. Using a database, comprising soil data from Saint Lucia in the Eastern Caribbean, both simple regression and multiple linear regression analysis were performed. These analyses correlate various basic soil parameters with the effective friction angle measurements contained in the database. The developed statistical relationships are then employed for the estimation of the effective friction angle for use in a slope stability simulation scenario. These analyses show the narrowing of the expected range of results for the slope factor of safety when more soil parameters are used in the estimation of effective friction angle.

Keywords: Effective friction angle; Soil plasticity; Uncertainty; Design parameters

1 INTRODUCTION

The construction of informal settlements on slopes in the humid tropics produces a challenge for engineers. As many such slopes are mantled by deep weathered residual soils with low shear strength and relatively high permeability (Wesley 1990), they are frequently susceptible to rainfall-induced landslides (Lumb 1975). These landslides can have significant social and economic cost to communities and developing nations (Petley 2009), hence understanding the localised driving factors, and identifying appropriate preventative measures is valuable for engineers. Landslide hazard risk assessments allow engineers to quantify the stability of slopes in particular regions. Recent studies by Holcombe et al. (2016), Beesley et al. (2017) and Shephard et al. (2018) have simulated the response of a modelled slope to changes in a number of mechanical and geometric properties and showed that relatively small changes in some parameters can create large fluctuations in the calculated slope factor of safety. The slope stability has been shown to rely to a large degree on the estimate of the effective friction angle within the model. However, in many developing nations, such data is often scarce, which may cause unreliability in the results. In these cases, estimation of the appropriate model input parameters is required.

In this paper, a case study of a slope in Saint Lucia is used to investigate methods of estimating the effective friction angle from knowledge of other soil parameters. A database of laboratory test results from the island is analysed to investigate variability in parameter estimations from correlations, as well as their effect on modelled slope stability.

Figure 1. Regression analysis for seven parameters from the Saint Lucia database of geotechnical tests (N.B. ϕ'_{peak} = effective peak friction angle; c' = apparent cohesion; w_L = liquid limit; w_P = plastic limit; PI = plasticity index; SF = silt/clay fraction; w_{nat} = natural water content; r = the correlation coefficient; n = number of points used to generate the regression; SE = standard error and p is the 'p-value' of the correlation)

2 SIMPLE REGRESSION ANALYSIS FOR PARAMETER ESTIMATION

In data-poor regions, engineers must make best use of the available data to develop geotechnical correlations and design rules. As the process for measurement of the effective peak friction angle (ϕ'_{peak}) is fairly involved, correlations with other, simpler to measure parameters may be of use. Fig. 1 shows linear correlations of seven parameters against each other. In this analysis, the silt/clay fraction is used in place of the more commonly quoted 'clay fraction' as the testing facilities on the island do not include the 0.002 mm sieve. Through the correlation coefficient (r) statistic, the most significant predictor of the ϕ'_{peak} is the w_{nat} with $r = -0.6$ (i.e. via a negative correlation). This relationship is inspected in more detail in the lower left of Fig. 1 where it is shown that:

$$\phi'_{peak} = 30.7 - 0.32w_{nat} \quad R^2 = 0.36, p < 0.001, n = 52 \quad (1)$$

The coefficient of determination (R^2) of 0.36 for Eq. (1) suggests that it explains 36% of the variability of the data (see Montgomery et al. 2007 p.294). Despite being the strongest relationship of those calculated (using single predictor variables), this equation still leaves 64% of the variability unexplained, creating significant uncertainty in the results, as is shown in the inset plot in Fig. 1 by the wide bounds, and in the predicted-measured plot in Fig. 2. This latter figure illustrates that about 90% of the data lies within a $\pm 60\%$ bandwidth.

3 MULTIPLE LINEAR REGRESSION ANALYSIS

The wide bounds associated with the expression for estimating the effective friction angle derived in Section 2 suggest significant uncertainties in the results. With a large range of effective friction angles deemed ‘reasonable’ for use in slope stability analysis, a fairly stable slope could be wrongly classed as unsafe, or vice-versa. Multiple Linear Regression (MLR) analysis, where estimations of a certain variable are calculated as a function of two or more other variables (e.g., Ang and Tang 1975, p. 297), is a logical extension to the simple regressions calculated in Section 2. Herein, the multiple linear regression (MLR) analysis is performed using MATLAB (MathWorks 2016).

For the MLR analysis, in this study, six parameters were separated into two groups: ‘predictors’ and ‘dependents’. The trialled ‘predictors’ were w_L , w_P , SF and w_{nat} . The ‘dependents’ were ϕ'_{peak} and c' . The subsequent analysis reported here focuses on the prediction of ϕ'_{peak} .

Figure 2. Predictive power of the simple regression equation (Eq. 1)

The correlations with the highest R^2 for each formulation (linear and the three combinations of logarithmic transforms) were found to all use the same parameters. The best fit equations for two predictors all used w_P and w_{nat} , with $n = 47$; and for three predictors the combination of w_L , w_P and w_{nat} gave the strongest fits, again with $n = 47$. The relationship determined by multiple linear regression with the highest R^2 for the estimation of the angle of friction was for a simple linear relationship of these three variables (Eq. 2). The coefficient of determination for this equation suggests that the relationship explains 56% of the variability of the data. This relationship is illustrated in Fig. 3 using a predicted-measured plot. A $\pm 50\%$ bandwidth encompasses about 90% of the data. Given that w_L and w_P and w_{nat} are all present in Eq. (2), an investigation is prompted into an estimation expression where the parameters are combined, i.e., the liquidity index (LI). Studying the linear formulation, the use of LI as a sole predictor for ϕ'_{peak} gives a lower coefficient of determination of 0.43 (see Vardanega et al. 2018 for the scatter-plot). Therefore, the value in weighting the variables separately (as shown in Eq. 2) is evident.

$$\phi'_{peak} = 19.8 - 0.14w_L + 0.77w_P - 0.46w_{nat} \quad R^2 = 0.56, n = 47, p < 0.001 \quad (2)$$

The low predictive power of Eq. 1 is reasonable for a dataset such as the one used here. Many types of soil are included in the database, thus the range in parameter values is likely to be large. It can be seen that some marginally better estimations of the values and variability of the effective peak angle of

friction using Eq. 2 can be made. However, use of Eq. 2 still presents relatively high scatter, and will likely produce a wide modelled failure envelope.

Figure 3. Predictive power of the three-predictor formula (Eq. 2)

4 SLOPE STABILITY SIMULATION

From the correlation coefficients calculated for the simple and multiple linear regressions in Sections 2 and 3, a drop in the variability in the estimation of ϕ'_{peak} can be observed. This study employs the Combined Hydrology and Stability Model (CHASM) to illustrate an example envelope of the factor of safety (F) for a modelled slope, in which the angle of friction is changed according to the corresponding upper and lower bounds plotted in Figs 2 and 3 to the mean measured friction angle value of 24.2° . The input parameters used in this paper are given in Table 1, and are based on ‘Case B’ from Shepherd et al. (2018).

The CHASM is a physically-based, hydrologically dynamic model that models slope cross sections as two-dimensional matrices of 1m by 1m square cells, for each of which the material type may be specified according to a set of input parameters. Storm events may be applied to the slope in terms of the hourly rainfall. CHASM has been used successfully in previous studies to model the response of specific slopes to vegetation, urbanisation, parameter variation, and rainfall events (e.g., Anderson and Lloyd 1991, Holcombe et al. 2016, Almeida et al. 2017, Shepherd et al. 2018). A full description of the CHASM model and equations can be found in Wilkinson et al. (2002).

The modelled slope is identical to that studied in Shepherd et al. (2018) with a height of 67m, a length of 130m, and an angle of 28° . The uppermost 8m are separated into two 4m deep layers of residual soil (0-4m depth) and decomposed bedrock (4-8m) with properties corresponding to the Hong Kong weathering grades (GEO 1988) (Table 1). In the surface (residual soil) layer, the angle of friction is reduced from an upper value (UV) to a lower value (LV), with values from the simple regression and MLR analyses. The angle of friction is consistent in the two layers below. The stability modelling scenario comprises: 7 dry days to allow the initial estimated hydrological conditions to reach a steady state; then a 24h storm with a 50-year return period (12mm/h intensity) starting at hour 168; and a final 7 dry days to allow soil moisture content, pore pressures and slope stability to respond to the flow of the infiltrated water.

The resulting failure envelopes are illustrated in Fig. 4. The reduced uncertainty from the MLR analysis translates into reduced variability in calculated values of the effective friction angle. The range in F values using the simple regression Eq. (1) is 1.34, whereas the range using the MLR analysis Eq. (2) is 1.11; a reduction of 0.23 in uncertainty.

Table 1. Input soil parameters for the CHASM model (adapted from Shepherd et al. 2018)

Grade	Parameter	Unit	Value	Reference
Grades V-VI	Angle of friction: ϕ'_{peak}	degrees	Simple UV: 38.7	This study
			MLR 33.9	
			LV: 9.7	
	Unit weight: γ_d, γ_{sat}	kN/m ³	17, 19	Irfan (1996)
	Apparent cohesion: c'	kPa	0	-
Grades III-IV	Hydraulic conductivity: K	m/s	1×10^{-5}	Shepherd et al. (2018)
	Angle of friction: ϕ'_{peak}	degrees	45	Rahardjo et al. (2004)
	Unit weight: γ_d, γ_{sat}	kN/m ³	21, 23	Irfan (1996)
	Apparent cohesion: c'	kPa	30	Rahardjo et al. (2004)
Grades I-II	Hydraulic conductivity: K	m/s	1×10^{-5}	Shepherd et al. (2018)
	Angle of friction: ϕ'_{peak}	degrees	50	GCO (1982)
	Unit weight: γ_d, γ_{sat}	kN/m ³	25, 27	Irfan (1996)
	Apparent cohesion: c'	kPa	30	GCO (1982)
	Hydraulic conductivity: K	m/s	1×10^{-6}	Shepherd et al. (2018)

* Grade refers to weathering grades from fresh rock (Grade I) to residual soil (Grade IV) (see GEO 1988, for example).

5 CONCLUSIONS

An analysis of the estimation of the effective peak friction angle from basic soil parameters has been performed, using a case study of the island of Saint Lucia. Initial simple regression analysis yielded very poor parameter correlation relationships, which, when used in conjunction with a slope stability model, gave significantly varied estimations of the factor of safety. The estimation process was then refined with multiple linear regression analysis. This form of proxy relationship gave a stronger correlation, with the strongest fits found for the combination of w_L , w_P and w_{nat} , giving a narrower envelope of estimations of the factor of safety from the CHASM analysis. The use of empirical relationships may be of use to engineers when performing first pass analyses of the stability of a slope in tropical regions.

Figure 4. Comparison of failure envelopes using Eqs (1) and (2)

DATA AVAILABILITY STATEMENT

This research has not generated new experimental data. The authors wish to thank the Government of Saint Lucia Ministry of Infrastructure, Port Services and Transport for supplying the Saint Lucia soils data for in this work.

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