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Optimal Monetary Interventions in Credit Markets^{*}

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Abstract

We characterize optimal credit market interventions with respect to two fundamental frictions—limited commitment and limited monitoring. We consider two classes of interventions: an inflationary policy which uses inflation tax to forgive private debt, and a deflationary policy which uses credit tax to increase the real rate of return on money. We show that both money and debt are essential and that intervention is generically optimal. The nature of the optimal intervention depends on the fundamentals of the economy and we provide conditions under which the optimal intervention is inflationary and under which it is deflationary. Our results hold irrespective of whether we use a bargaining protocol or optimal trading mechanisms.

JEL Codes: E50; E51; E52

Keywords: money; credit; limited commitment; optimal monetary policy; mechanism design; inflation

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1 Introduction

The use of both money and credit as means-of-payments is widespread. However, the effect of monetary interventions in economies where money and credit coexist remains largely an open issue. While there is a sizable literature on the impact of interventions in the monetary sector (see, for example, Lagos, Rocheteau, and Wright (2017) for recent developments), most literature on the impact of interventions in the credit sector consider economies without money (see, for example, Gertler and Kiyotaki (2010)). In this paper, we aim at bridging this gap by considering monetary interventions in the credit market.

In our view, a model where money and credit coexist and where monetary interventions in the credit market can be meaningfully analyzed, requires two elements. First, one needs a friction which renders credit imperfect, thus making liquidity (or its lack thereof) important. In pure credit economies, Kehoe and Levine (1993) and Alvarez and Jermann (2000) have shown that the inability of individuals to commit introduces an endogenous borrowing constraint which limits the possibility of implementing the first-best allocation. Second, one needs a friction which renders money essential. This requires some degree of imperfect monitoring, as pointed out by Wallace (2010, 2014). It turns out to be a demanding task to combine limitations in commitment and imperfections in monitoring in ways that make both money and credit essential. In particular, Gu, Mattesini, and Wright (2016) show that money and credit cannot be both relevant in a large class of economies based on Lagos and Wright (2005) (LW henceforth), an environment which is extensively used to study interventions in the monetary sector.

In this paper, we first propose a model which builds on LW but that combines limited commitment and imperfect monitoring in a way that makes both money and credit essential as means of payment. We then use the model to study optimal monetary interventions in the credit market among policies that are consistent with the underlying frictions of the environment. For instance, we do not allow taxation schemes, such as lump-sum taxes, that abscond resources from activities that are not monitored. Moreover, we only consider interventions that respect agents' voluntary participation. We identify two novel classes of monetary policies that can improve welfare. The first class, labeled inflationary policies, uses inflation tax to forgive private debt; the second class, labeled deflationary policies, uses taxes on the credit sector to increase the real rate of return on money. The first class of policies

tightens liquidity in the monetary sector by reducing the real rate of return on money but it increases liquidity in the credit sector by relaxing the borrowing constraint, while the second class does the opposite.

Thus, the determination of the optimal policy in our environment depends on the liquidity needs across the two sectors, a trade-off which does not exist in economies without money and which differs greatly from the usual approach of modeling monetary injections through lump-sum transfers. In particular, since these policies include taxes and subsidies to the credit market, their implementation requires the joint cooperation of both monetary and fiscal authorities.¹ We label these policies “monetary” because they are aimed at correcting a specific inefficiencies inherited in the liquidity services provided by money and debt as means-of-payments, and not at redistributing income across agents.

Our model modifies LW in two ways. First, we introduce debt by assuming an imperfect monitoring technology which records promises-to-pay from buyers and through which sellers can access past records in a fraction of meetings. Second, we assume that agents interact in three rounds in each period: the first two rounds correspond to the decentralized market (DM) in LW, and the last round corresponds to the centralized market (CM). This last modification is motivated by findings in Gu, Mattesini, and Wright (2016). As said above, they show that money and credit cannot both play meaningful transactional roles in versions of the LW model, in particular versions with permanent buyer and seller types and in which agents alternate between one round of DM trade and one round of CM trade. We focus on the model with two DM rounds for simplicity, but most results generalize to models with more than two DM rounds.

We characterize implementable allocations in an economy with limited monitoring, in which only sellers participating in one of the DM rounds have access to the monitoring technology. In order to provide useful benchmarks against which our results can be interpreted, we also characterize implementable allocations in two extreme scenarios: an economy without money and with unlimited monitoring, i.e., the monitoring technology is available in both DM rounds; and an economy with a constant money supply and no monitoring, i.e., neither DM round is monitored.

¹Credit subsidies are common in many modern economies. For a partial list and the monetary size of federal credit programs in the US, see Bickley (2012). Similarly, the Japanese government is well known for its credit subsidies program, FILP; for details see Doi and Hoshi (2003).

We begin with the results for the extreme scenarios. In the unlimited monitoring economy, implementability requires that buyers are willing to repay the debt accumulated in the two DM rounds, given the expected gains from trade in all future periods. This constraint endogenously determines the debt limit that buyers face and hence the level of production in the DM rounds. In the no monitoring economy with a constant money supply, trades depend on the trade-off between the cost of holding money across periods and the expected gains from trade. This trade-off imposes more stringent restrictions than the participation constraint in the unlimited monitoring economy, and the optimal allocation under unlimited monitoring delivers a higher welfare.

Consider, now, the limited monitoring economy. If the money supply is constant, the optimal allocation exhibits the highest debt limit that is consistent with the repayment constraint. Under this allocation, agents do not want to use money in monitored meetings, and trades in non-monitored meetings resemble those in the no monitoring economy. This gives rise to a welfare ranking among the three economies: the unlimited monitoring economy has the highest welfare and the no monitoring economy has the lowest. Our main result then shows that active intervention by the monetary authority is optimal in the limited monitoring economy. The nature of the optimal intervention, however, depends on the details of the economy and on the bargaining protocol. For most of our analysis we focus on take it or leave it (TIOLI) offers but we also consider optimal trading mechanisms. We obtain that, if agents are very patient, it is feasible to implement a deflationary policy that corresponds to the Friedman rule (through taxes from credit trades) and achieves the first-best under TIOLI offers. By continuity, deflationary policies are also optimal under TIOLI offers for a range of lower discount factors. The first best can also be achieved with a constant money supply if agents are patient and use the optimal trading mechanism.

However, if agents are relatively impatient, intervention is generically optimal, but it can be either inflationary or deflationary, irrespective of whether agents use TIOLI offers or the optimal trading mechanism. Under TIOLI offers, we provide examples to illustrate the optimality of inflation. Under the optimal trading mechanism, we provide an explicit characterization of the region of parameters under which inflation is optimal and under which deflation is optimal. In particular, deflationary policy is optimal when liquidity in the non-monitored round is “tight” relative to the no-intervention benchmark, in the sense that

it would be better to redistribute liquidity away from the monitored to the non-monitored round. Such redistribution is incentive infeasible under *laissez faire*, but feasible under deflationary policy. Such policy taxes credit trades and hence hurts production in monitored meetings, but it redistributes liquidity from those meetings to non-monitored ones by way of higher rate-of-return to money and increases production in the latter. Inflationary policy does exactly the opposite and is optimal when monitored round has tight liquidity in a similar sense. Interestingly, we also find that, thanks to its redistributive role, inflationary interventions can achieve welfare that is strictly higher than the welfare under unlimited monitoring.

The result that intervention is generically optimal is reminiscent of the conjecture in Wallace (2014) in the context of pure currency economies. In both cases no lump-sum taxes are allowed and the monetary authority is required to balance the budget. A key difference, however, is that the intervention in our framework makes use of the monitoring technology, which is absent in Wallace (2014), to tax credit trades or to forgive private debts. Moreover, our results suggest that a fully anticipated debt purchase intervention by the monetary authority can mitigate limited commitment in the credit market and increase lending in the private sector. Because of this benefit, the optimal long-term inflation rate can be positive, and its precise level depends on the liquidity needs in the money and the credit sectors.

Finally we review related literature. Kehoe and Levine (1993) and Alvarez and Jermann (2001) introduce endogenous borrowing constraints in pure credit economies, and this concept was applied to the LW setting by a few papers to obtain coexistence of money and credit. Sanches and Williamson (2010) obtain coexistence between money and credit by introducing an exogenous cost of using money, while Liu, Wang, and Wright (2015) assume perfect enforcement of debts but introduce an exogenous cost of using credit. Gomis-Porqueras and Sanches (2013) and Lotz and Zhang (2013) obtain coexistence under limited monitoring, but both consider only lump-sum transfers.² None of these papers consider the two classes of interventions considered here. There are other papers that also study the LW setting with two DM rounds. Guerrieri and Lorenzoni (2009) study the amplification mechanism and Telyukova and Wright (2008) explain the credit card debt puzzle. In contrast to ours, both

²Berentsen, Camera and Waller (2007) and Ferraris and Watanabe (2008) assume that debts need to be settled in cash, which also leads to the coexistence of money and credit. This coexistence, though, is driven by their complementarity instead of substitutability, as considered here and in the papers mentioned above.

papers assume perfect enforcement and hence have no endogenous borrowing constraints.³

2 Model

2.1 Environment

Time is discrete and the horizon is infinite. The economy is populated by three types of unit measure agents, labeled buyers, type-1 sellers, and type-2 sellers. Each period is divided into three rounds. Buyers randomly meet type- j sellers in round j , and the probability of a successful meeting is σ_j , where $j = 1, 2$. There are three goods, one for each round. At round $j = 1, 2$, a type- j seller can produce y_j units of good j for a buyer at a linear cost $\rho_j y_j$ and the buyer's utility is $u_j(y_j)$. We assume that both u_j 's are strictly concave and increasing, continuously differentiable, and satisfy the Inada conditions: $u_j'(0) = \infty$, $u_j'(\infty) = 0$. Let y_j^* be the solution to $u_j'(y) = \rho_j$, the quantity that maximizes the surplus. In the last round, agents meet in a centralized market. In this market, they can all consume and produce, and the (dis)utility is linear. Agents maximize their life-time expected utility with discount factor β . We let $r = \frac{1-\beta}{\beta}$.

There exists a technology which keeps track of buyers' trading histories and is available to a fraction of sellers. We call a meeting a *monitored meeting* if the seller has access to this technology, and a *non-monitored meeting* otherwise. The technology works as follows. For each buyer b , a recorded history at period t is a triple, $h = (h_1, h_2, h_3) \in H$, such that for $j = 1, 2$, $h_j = (b, s(j), d_j)$ keeps track of the buyer's round j DM promise to the seller, where b is the identity of the buyer, $s(j)$ is the identity of the seller, and d_j is the promise-to-pay in terms of CM good (debt); and $h_3 \in \mathbb{R}_+$ keeps track of the total repayment. For $j = 1, 2$, if the buyer does not meet a seller in round- j , or if the meeting is non-monitored, h_j is empty.

This monitoring technology is accompanied by a record-keeping technology for a given *debt limit* D . In our analysis D is endogenously determined subject to incentive compatibility constraints, and the choice of D is formulated as a mechanism design problem to maximize the social welfare. The set of records is given by $R = \{G, B\}$, which is updated by the function $\omega : \{G, B\} \times H \rightarrow \{G, B\}$ described below. The records are only accessible to sellers in monitored meetings. If the monitoring technology is used in both rounds, in the

³Some papers use two DM rounds to study pure currency economies, including Berentsen, Camera, and Waller (2005) on the short-run neutrality of money, and Ennis (2009) on the hot potato effect.

second DM round the seller also observes the buyer's debt d_1 . We restrict attention to an updating function ω in which all buyers start with a record G (good), and a buyer keeps this record if and only if he pays his debt up to D at the end of the period; and acquires a record B (bad) otherwise. Precisely, $\omega(r, \emptyset) = r$ for $r \in \{G, B\}$, $\omega(B, h) = B$ for all $h \in H$, and $\omega(G, h) = G$ iff $h_3 \geq \min\{D, d_1 + d_2\}$. Finally, we assume that only buyers with a good record can use the monitoring technology to issue debts. This record-updating technology is with no loss of generality because any equilibrium outcome can be characterized by such a record-updating rule. Intuitively, under this updating rule buyers are excluded from future credit trades if he fails to repay loans up to the debt limit but not for defaults on bigger loans. This would in turn deter sellers from extending loans larger than the debt limit. Bethune, Hu, and Rocheteau (2018) have proved this in a pure-credit economy and their results can be easily generalized to our setting.

Lastly, there is an intrinsically useless, divisible, and storable object, called money. The money supply at the end of period t is denoted by M_t . We assume that agents' money holdings are observable in a match. The price of money in the CM is given by ϕ_t . In what follows, we focus on symmetric stationary equilibria where real balances $Z_t = \phi_t M_t$ do not change over time.

2.2 Strategies and equilibrium

In every meeting in a DM round, terms of trade are determined by take it or leave it (TIOLI) offers. In a non-monitored meeting, the buyer makes the offer. In a monitored meeting, the buyer makes the offer if he has a good record and he chooses to use the monitoring technology. Otherwise, the seller makes the offer.⁴ Implicitly we assume that the surplus of the buyer depends on his record in monitored meetings. In Section 4 we show that this is a feature of the optimal mechanism (see Remark 1).

We denote by s_b the strategy of a buyer. In each DM meeting, s_b maps the buyer's history up to that meeting, his real balance, and his record, to his choice of using the technology (if the choice is available), and to his offer (in case he makes the offer) or to his response to the seller's offer (otherwise). Note that a buyer's trading history is his private information,

⁴The buyer can use both money and debt in monitored meetings. Thus, absent intervention by the monetary authority, it is a weakly dominant strategy of a buyer with a good record to use the technology. However, it might not be so when intervention involves a tax on using the technology.

and his matched partner can only observe his real balance and, if available, his record and current debt. In the CM round s_b maps the buyer's trading history up to that round to his repayment decisions and to his final money holdings when leaving the CM. In turn, we denote by s_j the strategy of a type- j seller, where $j \in \{1, 2\}$. If the j^{th} DM round is monitored, the strategy s_j maps the seller's private history as well as the observables (buyer's real balance and, if available, his record and current debt) to her trading behavior: if the buyer uses the technology, the seller's response; otherwise, the seller's offer. If round- j DM round is not monitored, the strategy s_j maps the seller's private history as well as the observables (buyer's real balance and buyer's offer) to her response. We assume, without loss of generality, that sellers do not carry money across periods.

We use stationary symmetric Perfect Bayesian Equilibrium as our solution concept, and denote a strategy profile by $s = (s_b, s_1, s_2)$, where s_b is the buyer strategy for all buyers, and s_j is the strategy for all sellers. Throughout the paper we restrict attention to equilibria where trade always takes place in the DM rounds, buyers with good record always repay their debts up to the debt limit D and they always choose to use the technology in monitored meetings, and the initial distribution of money across buyers is degenerate. We call such equilibria simple equilibria.

In what follows, to simplify the presentation and to convey our main insights, we restrict attention to the case where $\sigma_1 = 1$, and let $\sigma_2 \equiv \sigma$. Given this assumption, an equilibrium allocation can be characterized by the DM trades, (y_1, y_2) , where y_j is the round- j DM consumption for buyers. The social welfare associated with such an allocation is given by

$$\mathcal{W}(y_1, y_2) = u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]. \quad (1)$$

As we discuss below, the equilibrium allocation is determined by the debt limit we choose, and typically there is a continuum of debt limits that satisfy the incentive compatibility constraints for repayment. We deal with this multiplicity by choosing the constrained optimal debt limit and hence the constrained optimal allocation, i.e., the allocation that maximizes (1) among all allocations that are incentive compatible.

We finish this section by characterizing the constrained optimal allocation in two extreme economies: an economy with unlimited monitoring, where there is no money and all sellers have access to the monitoring technology; and an economy with no monitoring, where there is a constant money supply and no sellers have access to the monitoring technology.

These scenarios serve as benchmark to the economy where monitoring is limited and there is intervention by the monetary authority, which will be analyzed in the next section.

2.3 Unlimited monitoring

In a simple equilibrium, the buyer keeps a good record if he repays his debt up to the limit D . Thus, a seller will not extend a loan to a buyer whose total obligation will exceed D in the coming CM. This implies that $d_1 + d_2 \leq D$, where d_j denotes the amount of debt the buyer issues in round- j DM. Now, the buyer issues debt to acquire goods and TIOLI offers imply that d_j is such that $\rho_j y_j = d_j$, i.e., the seller is indifferent between producing and not producing y_j . Thus, we can summarize the objects in a simple equilibrium by the triple (D, y_1, y_2) .

Given D , the pair (y_1, y_2) is uniquely determined by the solution to the problem of the buyer with a good record, given by

$$\max_{y_1 \geq 0, y_2 \geq 0} \{u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]\} \quad (2)$$

$$\text{s.t. } \rho_1 y_1 + \rho_2 y_2 \leq D. \quad (3)$$

In turn, given (y_1, y_2) , repaying D is incentive compatible if and only if

$$-D + \frac{\beta}{1 - \beta} \{u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]\} \geq 0,$$

that is, if and only if the temptation to renege, given by $-D$ is overcome by the discounted sum of all expected future gains from trade, $u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2]$. This can be reduced to

$$-rD + u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2] \geq 0. \quad (4)$$

A triple (D, y_1, y_2) constitutes a simple equilibrium if and only if it satisfies (2)-(4).

The constrained optimal allocation is then given by the triple (D, y_1, y_2) that maximizes (1) subject to (2)-(4). We first claim that, for any given (y_1, y_2) , there exists D such that (D, y_1, y_2) satisfies (2)-(4) if and only if $y_1 \leq y_1^*$, $y_2 \leq y_2^*$,

$$\frac{u'_1(y_1)}{\rho_1} - 1 = \sigma \left[\frac{u'_2(y_2)}{\rho_2} - 1 \right] \quad (5)$$

$$-r(\rho_1 y_1 + \rho_2 y_2) + u_1(y_1) - \rho_1 y_1 + \sigma[u_2(y_2) - \rho_2 y_2] \geq 0. \quad (6)$$

This implies that we can restrict attention to allocations (y_1, y_2) satisfying (5)-(6). We omit a complete proof of this claim, but we provide a sketch by considering two separate cases. First, let

$$r \leq r_c \equiv \frac{u_1(y_1^*) - \rho_1 y_1^* + \sigma[u_2(y_2^*) - \rho_2 y_2^*]}{\rho_1 y_1^* + \rho_2 y_2^*}. \quad (7)$$

In this case, (y_1^*, y_2^*) satisfies (5)-(6), and the solution to (2)-(3) is equal to (y_1^*, y_2^*) for any D such that $D \geq D^* \equiv \rho_1 y_1^* + \rho_2 y_2^*$. Now, let $D < D^*$. In this case, the pair (y_1, y_2) that solves (2)-(3) satisfies (5) and (6). The latter is true because the pair (y_1, y_2) that solves (2)-(3) satisfies (3) with equality, which implies that (4) is equivalent to (6). Conversely, for any pair (y_1, y_2) that satisfies (5)-(6), if we set $D = \rho_1 y_1 + \rho_2 y_2$, the triple (D, y_1, y_2) satisfies (2)-(4). The proof for the case where $r > r_c$ is similar. The following lemma fully characterizes the optimal allocation.

Lemma 1 *The optimal allocation maximizes (1) subject to the incentive compatibility condition (6). If $r \leq r_c$, the first-best is the optimal allocation. In this case, $D \geq D^* \equiv \rho_1 y_1^* + \rho_2 y_2^*$. If, instead, $r > r_c$, the optimal allocation is given by the unique solution, denoted by (y_1^c, y_2^c) , to (5) and (6) at equality. In this case, $D = \rho_1 y_1^c + \rho_2 y_2^c$.*

The optimal allocation maximizes (1) subject to (5) and (6). Lemma 1 shows that we can ignore (5). The problem then becomes a standard maximization problem: the objective function (1) is strictly concave, and the constraint (6) gives a convex set, and hence it has a unique solution characterized by first-order conditions, which turn out to coincide with (5).

One can show that (6) is a necessary condition for any bargaining protocol (see the Supplemental Material for formal details). Thus, Lemma 1 also shows that TIOLI offers is an optimal trading mechanism under unlimited monitoring. This makes it transparent that the only friction that prevents achieving the first-best under unlimited monitoring is the presence of limited commitment, as captured by (6). Lastly, note that, while we assume that money does not circulate in the economy, under the optimal debt limit agents would optimally choose not to hold any money even if they were allowed to do so.

2.4 No monitoring

In the no monitoring economy, the only way to sustain trades is to use money. As standard in the LW framework, if $W(z)$ is the value function of a buyer with z real balances at the

beginning of the CM round, then $W(z) = z + W_0$, where W_0 is a constant independent of z . Standard arguments also show that the buyer will simply bring enough real balances to finance his DM trades. Finally, TIOLI offers imply that all the surplus in each DM round goes to the buyer. Moreover, since $\sigma_1 = 1$, the buyer does not have to contemplate on the contingency where he does not meet a seller at round-1 DM. Thus, the buyer's problem in the CM can be written as

$$\max_{z_1 \geq 0, z_2 \geq 0} \left\{ -r(z_1 + z_2) + u_1 \left(\frac{z_1}{\rho_1} \right) - z_1 + \sigma \left[u_2 \left(\frac{z_2}{\rho_2} \right) - z_2 \right] \right\}, \quad (8)$$

where z_j is the amount of real balances the buyer plans to spend in round- j . Under TIOLI offers, these real balances will allow him to consume $\frac{z_j}{\rho_j}$ units of round- j DM goods, $j \in \{1, 2\}$. The first term in (8) is the opportunity cost of carrying $z_1 + z_2$ real balances across periods, while the second and the third terms capture the surplus in each DM round. Equivalently, one may rewrite the problem in terms of planned consumptions (y_1, y_2) in the two DM rounds, with $y_j = \frac{z_j}{\rho_j}$. This gives rise to a characterization of the equilibrium allocation under no monitoring.

Lemma 2 *The unique equilibrium allocation under no monitoring, given by (y_1^m, y_2^m) , solves*

$$\frac{u'_1(y_1) - \rho_1}{\rho_1} = \frac{\sigma [u'_2(y_2) - \rho_2]}{\rho_2} = r. \quad (9)$$

Lemma 2 restricts attention to a constant money supply. More generally, we can consider monetary interventions implemented by lump-sum transfers to all agents, and which respect agents' incentive constraints. In this case, any positive growth rate of money supply τ is also feasible. In turn, for any given τ , the equilibrium allocation solves a modified version of equation (9) in which the real interest rate r is replaced with the nominal interest rate i .⁵ Note that, since $\tau \geq 0$ implies $i \geq r$, welfare is maximized under no intervention.

3 Limited monitoring

We now consider the limited monitoring economy. Throughout, we deal with the case where the second DM round is monitored. We briefly comment on the case where the first DM

⁵We use the Fisher equation $1 + i = (1 + r)(1 + \tau)$ to replace the real interest rate r with the nominal interest rate i . Precisely, the Fisher equation gives i as the nominal interest rate of an illiquid bond that is issued in one centralized round and matures in the next centralized round.

round is monitored at the end of this section, but we defer a formal analysis of this scenario to the Appendix B1.

3.1 No intervention

As a benchmark, let us first assume a constant money supply. Irrespective of his record, the problem of the buyer in the non-monitored meeting is exactly as in the no monitoring economy. Instead, in the monitored round, a buyer with a good record can access the monitoring technology and use both money and debt. The decision on how much money to use in the monitored round is similar to the one in the no monitoring economy. The main difference is that the buyer has additional liquidity since he can issue debt up to the limit D .⁶ Assuming that he repays his debt D , the choice of real balances by the buyer in the CM is then determined by

$$\max_{z_1 \geq 0, z_2 \geq 0} \left\{ -r(z_1 + z_2) + u_1 \left(\frac{z_1}{\rho_1} \right) - z_1 + \sigma \left[u_2 \left(\frac{z_2 + D}{\rho_2} \right) - z_2 - D \right] \right\}, \quad (10)$$

where TIOLI offers imply that $\frac{z_1}{\rho_1}$ is the consumption of the buyer if he brings z_1 real balances to use in the first DM round, and $\frac{z_2 + D}{\rho_2}$ is the consumption of the buyer if he issues an amount of debt equal to D and brings z_2 real balances to use in the second DM round. Note that it is never optimal for the buyer to replace debt with money, i.e., he always uses all the debt and he may choose to bring additional money if he wants to consume more than what the debt limit allows. This is so because debt is cheaper than money for every $r > 0$: debt can be issued at the spot, while money must be carried across periods.

As in the no monitoring economy, we may rewrite the choice of real balances with the choice of DM consumptions by letting $y_1 = \frac{z_1}{\rho_1}$ and $y_2 = \frac{z_2 + D}{\rho_2}$. This also implies that we can use (D, y_1, y_2) as the equilibrium objects, and the real balances (z_1, z_2) can be inferred from those objects. The following FOCs determine the solution to the buyer's problem:

$$\frac{u'_1(y_1) - \rho_1}{\rho_1} = r, \quad (11)$$

and

$$\begin{cases} \rho_2 y_2 = D & \text{if } \sigma \frac{u'_2(D/\rho_2) - \rho_2}{\rho_2} < r, \\ \sigma \frac{u'_2(y_2) - \rho_2}{\rho_2} = r & \text{otherwise.} \end{cases} \quad (12)$$

⁶We assume that $D \leq \rho_2 y_2^*$ so the buyer has enough debt limit to acquire the efficient quantity but not more than that. This assumption is without loss of generality as it is never optimal for the buyer to consume more than y_2^* in the second DM round.

According to (11), $y_1 = y_1^m$, and round-1 DM consumption is identical to that in the no monitoring economy. According to (12), if D is relatively high and the expected marginal surplus using debt alone is lower than the real interest rate, the buyer has no incentive to bring money to use in round-2 DM. In this case, he consumes up to the available debt limit and $y_2 > y_2^m$. Otherwise, the buyer supplements the debt with money so that the expected marginal surplus is equal to the real interest rate, and $y_2 = y_2^m$. In this case, the amount D is irrelevant for the equilibrium allocation; a lower D will simply be substituted by higher real balances. This result is related to Gu, Mattesini and Wright (2016), who obtain that when the debt limit is too low, then it becomes irrelevant for the equilibrium allocation for a wide class of bargaining protocols.

While (11)-(12) determine (y_1, y_2) from D , we need to check the incentive of buyers to repay D . Here we claim that a triple (D, y_1, y_2) constitutes a simple equilibrium if and only if (y_1, y_2) solves (11)-(12) under D and

$$-r\rho_2 y_2 + \sigma [u_2(y_2) - \rho_2 y_2] \geq 0. \quad (13)$$

To prove this claim, we need to show that (13) is necessary and sufficient for buyers to repay the debt limit. Since the only punishment for not repaying the debt is to give the buyer a bad record, we first discuss the optimal strategy for a buyer with a bad record. If a buyer has bad record, he cannot use the monitoring technology and the seller makes a TIOLI offer in the second DM round. This implies that, although his surplus from round-1 DM does not depend on his record, he receives no surplus from round-2 DM. As a result, a buyer with a bad record only brings money to finance his round-1 DM trade. Thus, repayment is better than default if and only if

$$-r(D + z_2) + \sigma [u_2(y_2) - D - z_2] \geq 0, \quad (14)$$

where y_2 is given by (12) and $z_2 = \rho_2 y_2 - D$. Note that we did not include the cost of holding money and the trade surpluses from round-1 DM on both sides as they are not affected by the bad record. Note also that the presence of z_2 reflects the possibility that the buyer also uses money in the monitored round. The constraint (13) follows from (14) by simple algebra. Lemma 3 fully characterizes the optimal allocation, denoted by $\mathcal{L}^{cm} = (y_1^{cm}, y_2^{cm})$.

Lemma 3 *Let*

$$r_{cm} \equiv \frac{\sigma[u_2(y_2^*) - \rho_2 y_2^*]}{\rho_2 y_2^*}. \quad (15)$$

If $r \leq r_{cm}$, the optimal allocation without intervention is given by $(y_1^{cm}, y_2^{cm}) = (y_1^m, y_2^)$. Otherwise, the optimal allocation is given by $y_1^{cm} = y_1^m$, and y_2^{cm} is the unique positive solution to (13) at equality. In both cases, buyers do not use money in the monitored round.*

If $r \leq r_{cm}$ then y_2^* satisfies (13), and hence $(D, y_1, y_2) = (\rho_2 y_2^*, y_1^m, y_2^*)$ constitutes a simple equilibrium. Consider now $r > r_{cm}$. Note that the choice of D does not affect equilibrium in the first DM round, which always equals y_1^m . Moreover, for D small such that the second case in (12) holds, equilibrium in the second DM round is also not affected by D —it always equals y_2^m . But at y_2^m the constraint (13) is slack, and the surplus $u_2(y_2) - \rho_2 y_2$ is maximized by increasing y_2 until it hits y_2^{cm} , which is the unique optimal allocation.⁷

Our first main result shows that the optimal allocation in the limited monitoring economy without intervention is strictly better than the optimal allocation in the no monitoring economy, but worse than the optimal allocation in the unlimited monitoring economy.

Theorem 1 *Under no intervention by the monetary authority, we have the following ranking of welfare:*

$$\mathcal{W}(y_1^m, y_2^m) < \mathcal{W}(y_1^{cm}, y_2^{cm}) < \mathcal{W}(y_1^c, y_2^c). \quad (16)$$

Theorem 1 shows that limited monitoring delivers a strictly higher welfare than no monitoring, which implies that the use of debt is essential. This is so because the optimal allocation achieved with credit cannot be replicated by a constant money supply under TIOLI offers, as shown by Lemma 3. The use of money is also essential because credit is not available in the non-monitored round. Lastly, it is worth pointing out that the key friction delivering the above ranking of welfare is the presence of limited monitoring and not the presence of limited commitment, i.e., this ranking would still exist under perfect enforcement of debt; see Appendix B2 for details.

⁷Lemma 3 shows that, if agents are not patient enough, limited commitment prevents achieving the efficient allocation in the monitored round. To make the role of limited commitment transparent, in the Appendix we show that, absent limited commitment, the efficient quantity always obtains in the monitored round.

3.2 Intervention

We consider two classes of interventions: either the monetary authority uses inflation tax to forgive private debt or uses credit tax to increase the real rate of return on money.⁸ We label interventions in the former category *inflationary policies* and interventions in the latter category *deflationary policies*. These policies are quite different from lump-sum interventions usually considered in the literature, and the analysis of their implications is the main contribution of this paper.

We first characterize the optimal allocation for any given deflationary policy and the optimal allocation for any given inflationary policy.⁹ We then allow the monetary authority to choose among all policies, including the passive policy of no intervention, and we show that intervention is always optimal. In particular, we show that the optimal intervention can be inflationary.

3.2.1 Deflationary policies

In a deflationary policy, the monetary authority taxes credit trades and uses these taxes to purchase money and deflate. In order to be consistent with the underlying frictions of the environment, only buyers with a good record who want to have future access to the monitoring technology are required to pay the credit tax in the end of the CM each period. It further implies that the only punishment in case the buyer refuses to pay the credit tax is the assignment of a bad record and the exclusion of future access to the monitoring technology.

Let χ denote the credit tax (in terms of the CM good). When solving his problem, each buyer takes as given the nominal interest rate i , the debt limit D , and the credit tax χ . As in the case of no intervention, the buyer always issues D units of debt in the monitored meeting and he may use money to finance some additional consumption.¹⁰ Assuming that

⁸The literature also considers interventions which use the inflation tax to increase the real rate of return on money (Andolfatto (2010), Wallace (2014)). These policies require the observability of individual money holdings by the monetary authority, which we do not assume here. Besides, we show that these policies are not useful in our environment when the optimal trading mechanism is used. In contrast, the optimality of the interventions considered here is robust to the optimal trading mechanism.

⁹In what follows, in many instances we use the Fisher equation to characterize interventions in terms of the nominal interest rate i , instead of the growth rate of money supply τ . See footnote 6.

¹⁰As in the case of no intervention, we are assuming $D \leq \rho_2 y_2^*$ so the buyer has enough debt limit to acquire the efficient quantity, and he wants to use all the debt limit because debt is cheaper than money for every $i > 0$. We break the tie in favor of debt if $i = 0$.

the buyer repays his debt D , the problem on how much money to bring into the DM rounds is given by

$$\max_{z_1 \geq 0, z_2 \geq 0} \left\{ -i(z_1 + z_2) + u_1 \left(\frac{z_1}{\rho_1} \right) - z_1 + \sigma \left[u_2 \left(\frac{z_2 + D}{\rho_2} \right) - z_2 - D \right] \right\}, \quad (17)$$

and the corresponding FOCs are (where we let $y_1 = \frac{z_1}{\rho_1}$ and $y_2 = \frac{z_2 + D}{\rho_2}$)

$$\frac{u'_1(y_1) - \rho_1}{\rho_1} = i, \quad (18)$$

and

$$\begin{cases} \rho_2 y_2 = D & \text{if } \sigma \frac{u'_2(D/\rho_2) - \rho_2}{\rho_2} < i, \\ \sigma \frac{u'_2(y_2) - \rho_2}{\rho_2} = i & \text{otherwise.} \end{cases} \quad (19)$$

The key difference with the case of a constant money supply is the opportunity cost of money, now given by i . Note also that, while y_1 only depends on i , y_2 depends both on i and D . To emphasize this dependence, we use $[y_1(i), y_2(i, D)]$ to denote the solution to (18)-(19).

Each buyer takes as given the credit tax χ . However, the equilibrium value of χ depends on the growth rate of the money supply and on the real balances that buyers carry out of the CM. For instance, if real balances are given by z , and the monetary authority wants to increase the real rate of return on money by taking a fraction τ of these balances out of circulation, it needs to raise τz in credit taxes. In our case, since equilibrium real balances are given by $\rho_1 y_1(i) + \rho_2 y_2(i, D) - D$, the tax required from each buyer with a good record is given by

$$\chi(i, D) = -\tau [\rho_1 y_1(i) + \rho_2 y_2(i, D) - D] = (r - i) [\rho_1 y_1(i) + \rho_2 y_2(i, D) - D] / (1 + r). \quad (20)$$

Note that under a deflationary policy we have $\tau < 0$ and hence $i < r$.

Henceforth, the pair (i, D) denotes the policy of the monetary authority. We now turn to the buyer's incentive to keep a good record and thus participate in monitored meetings in future periods. It is sufficient to consider the case of a buyer who participated in a monitored meeting in the current period, since, in addition to paying the credit tax and possibly acquiring enough real balances to participate in the monitored meeting, this buyer has an outstanding debt equal to D , which must be paid if he wants to keep the good record. As a result, it is optimal for a buyer with a good record to repay his debt if and only if

$$-(1 - \tau)z_2 - D - \chi(i, D) + \frac{\beta}{1 - \beta} \{ \sigma [u(y_2(i, D)) - \rho_2 y_2(i, D)] + \tau z_2 - \chi(i, D) \} \geq 0,$$

where $z_2 = \rho_2 y_2(i, D) - D$, and this can be reduced to

$$-r [D + \chi(i, D)] - i[\rho_2 y_2(i, D) - D] + \sigma \{u_2[y_2(i, D)] - \rho_2 y_2(i, D)\} - \chi(i, D) \geq 0. \quad (21)$$

The right hand side of (21) is equal to zero because the buyer with a bad record cannot access the monitoring technology, in which case the seller makes the TIOLI offer and appropriates all the surplus. Note that, since the buyer may carry real balances to use in the monitored round, we included the cost of carrying those balances across periods. Note also that, for i small relative to r , there may not be any D for which (21) is satisfied. Intuitively, if i is too small, the amount of credit taxes required to implement it may be too large to induce the participation of buyers in credit trades. Thus, there is a limit on how much taxation can be imposed on credit trades. Lemma 4 characterizes the optimal allocation for any feasible deflationary policy.

Lemma 4 *There exists $\underline{i} < r$ such that for all $i \in [\underline{i}, r]$, the set of debt limits satisfying (21) is nonempty. Moreover, for such i 's, the optimal allocation is characterized as follows:*

$$\chi(i) = \frac{(r - i) \rho_1 y_1(i)}{(1 + r)}, \quad (22)$$

$y_2(i)$ is the largest $y_2 \leq y_2^*$ such that

$$-r [\rho_2 y_2 + \chi(i)] + \sigma [u_2(y_2) - \rho_2 y_2] - \chi(i) \geq 0. \quad (23)$$

In particular, under the optimal allocation, the buyer does not use money in the monitored round.

We obtain (23) from (21) by using the fact that money is not used in the monitored round and the fact that the constrained optimal debt limit is equal to $\rho_2 y_2(i)$. Note that deflationary policies include as a particular case the scenario of no intervention, where $i = r$. In this case, there is no credit tax.

3.2.2 Inflationary policies

Under an inflationary policy, the monetary authority can pay a maximum amount of private debt, denoted by η (in terms of CM goods), for each buyer. If the monetary authority chooses to pay this maximum amount, a buyer with debt d_2 entering the CM only needs to pay $\max\{0, d_2 - \eta\}$ to keep his good record. Apart from the fact that nominal interest rates

are higher under inflationary policies, the problem of the buyer on how much real balances to bring out of the CM is set up in exactly the same way as in the case of deflationary policies. Precisely, the buyer solves (17) and the corresponding FOCs are given by (18) and (19).

As in the case of deflationary policies, the equilibrium value of η depends on the growth rate of the money supply and on the real balances that buyers carry out of the CM. For instance, if real balances are given by z , the monetary authority needs to increase these balances by $\tau z = \sigma \eta$ in order to pay for an amount η of private debt, and this gives a lower bound on the money-creation rate, τ . Note that, unlike in the case of deflationary policies, the fraction σ of buyers participating in monitored meetings matter because only these buyers will have incurred some debt. It turns out, as it will be shown below, that this feature of inflationary policies can make them quite efficient when σ is small.

In our case, since equilibrium real balances are given by $\rho_1 y_1(i) + \rho_2 y_2(i, D) - D$, the maximum amount of debt that the monetary authority can pay is given by¹¹

$$\eta(i, D) = (i - r) [\rho_1 y_1(i) + \rho_2 y_2(i, D) - D] / \sigma(1 + r). \quad (24)$$

It remains to check the incentive of a buyer with a good record to participate in monitored meetings. As in the case of deflationary policies, it suffices to consider the case of a buyer with an outstanding debt equal to D . If this buyer chooses not to pay his part of the debt, he gets a bad record and receives zero surplus in all future monitored meetings. Thus, he chooses to keep the good record if and only if

$$-r [D - \eta(i, D)] - i[\rho_2 y_2(i, D) - D] + \sigma \{u_2[y_2(i, D)] - \rho_2 y_2(i, D) + \eta(i, D)\} \geq 0. \quad (25)$$

Note that the monetary authority can always pay some of the debt incurred by the buyer. This is because $u'_1(0) = \infty$ implies $y_1(i) > 0$ for any $i > r$, and hence $\eta(i, D) > 0$ for any D . Lemma 5 characterizes the optimal allocation for any inflationary policy.

Lemma 5 *For any given inflationary policy with $i \geq r$, the set of debt limits satisfying (25) is nonempty, and, under the optimal allocation, the buyer does not use money in the*

¹¹We are implicitly assuming that the monetary authority never pays for more debt than what the buyer actually incurred. This is without loss of generality because such policy only hurts non-monitored trades with no benefit to monitored trades.

monitored round. The optimal allocation is characterized as follows:

$$\eta(i) = \frac{(i - r) \rho_1 y_1(i)}{(1 + r)\sigma}, \quad (26)$$

$y_2(i)$ is the largest $y_2 \leq y_2^*$ such that

$$-r [\rho_2 y_2 - \eta(i)] + \sigma \{u_2(y_2) - \rho_2 y_2 + \eta(i)\} \geq 0. \quad (27)$$

Note that inflationary policies include as a particular case the scenario of no intervention, where $i = r$. In this case, there is no inflation tax and the monetary authority does not fund any portion of the buyer's private debt. However, when $i > r$, (26) implies that the maximum amount of debt that the monetary authority can pay varies non-monotonically with i ; for i small it is increasing, but it will eventually be decreasing since the tax base for the inflationary tax, $y_1(i)$, approaches zero as i becomes very large.

3.2.3 Optimal policies

The monetary authority solves

$$\max_{D,i} \mathcal{W}[y_1(i), y_2(D, i)]$$

choosing among the set of all incentive feasible deflationary and inflationary allocations, which amounts to the set that satisfies (21) in the case of deflationary policies ($i \leq r$) and the set that satisfies (25) in the case of inflationary policies ($i \geq r$). The monetary authority can also choose not to intervene by setting the nominal interest rate equal to r . Note that, in the latter case, the allocation is the same, irrespective of whether we use Lemma 4 or Lemma 5.¹²

Theorem 2 *The optimal policy (i, D) is such that $i \neq r$. Thus, intervention is always optimal.*

The proof is given by the following two lemmas.

Lemma 6 *If $r \leq r_{cm}$, then the optimal intervention is always deflationary. In particular, for r sufficiently small, the first-best is implementable.*

¹²There always exist a solution to the maximization problem because the welfare function is continuous on i and D , and the set of incentive feasible deflationary policies and inflationary policies is compact.

The proof of Lemma 6 is based on the following observation. When $r < r_{cm}$ and when $i = r$, the constraint (23) is slack for $y_2 = y_2^*$ (as $\chi(r) = 0$ by (22)). Since $y_1(r) = y_1^m < y_1^*$, a small decrease in i will keep y_2 at y_2^* and increase y_1 , and this improves welfare. In this sense, we may say that round-2 DM is relatively slack in terms of liquidity, while round-1 is relatively “tight.” Deflationary policy is optimal as it redistributes liquidity from the slack DM round to the tight round: it tightens the borrowing constraint in monitored meetings but increases the rate-of-return on money. When agents are very patient, it is in fact feasible to set the nominal interest rate to zero, i.e., the Friedman rule, while keeping $y_2 = y_2^*$, and this implements the first-best. For less patient buyers, although the first-best is not feasible, marginal substitution between y_1 and y_2 in the welfare (1) dictates that such liquidity redistribution performed by deflationary policy is optimal as long as $r \leq r_{cm}$.

Now let $[y_1(i), y_2(i)]$ be given by Lemma 4 (for $i \in [\underline{i}, r]$) and by Lemma 5 (for $i \geq r$), and let

$$\bar{\mathcal{W}}(i) \equiv \mathcal{W}[y_1(i), y_2(i)].$$

Lemma 7 *Suppose that $r > r_{cm}$. Then, $\bar{\mathcal{W}}(i)$ is continuously differentiable for all $i \in [\underline{i}, \infty) - \{r\}$, and either*

$$\lim_{i \downarrow r} \frac{d}{di} \bar{\mathcal{W}}(i) > 0 \text{ or } \lim_{i \uparrow r} \frac{d}{di} \bar{\mathcal{W}}(i) < 0.$$

Lemma 6 shows that we only need to consider $r > r_{cm}$. Lemma 4 and 5 show that it is with no loss of generality to consider $\bar{\mathcal{W}}(i)$, at least when we look for optimum around $i = r$. According to Lemma 7, in this case, although there may be a kink, $\bar{\mathcal{W}}(r)$ cannot be a local maximum. This proves Theorem 2. Note that the possibility of the kink at $\bar{\mathcal{W}}(r)$ is natural since deflationary policies are qualitatively different from inflationary policies. That is, although there is continuity near i , there is no reason to think that these policies should have the same marginal effects.

In the next section, we will check the robustness of our results by replacing TIOLI offers with optimal trading mechanisms. It turns out that an additional benefit of introducing optimal trading mechanisms is that it allow us to provide a precise characterization of the region of parameters under which deflation is optimal and under which inflation is optimal, as well as the actual value of the optimal deflation rate and of the optimal inflation rate. For

Figure 1: Welfare Difference between Optimal Inflationary and Deflationary Policy

this reason, we defer a careful interpretation of our result on the optimality of interventions to the next section. Here, for the sake of illustration, we will present two figures.

Figure 1 shows the welfare difference between the optimal inflationary and the optimal deflationary policy, as a function of σ . For each $j = 1, 2$, we let $u_j(y_j) = y_j^{1-\theta_j}/(1-\theta_j)$. We use $r = 0.08$, $\theta_1 = 0.5$, $\theta_2 = 0.25$, and $\rho_1 = \rho_2 = 1$. We obtain that deflation is optimal if σ is relatively large, while inflation is optimal if σ is relatively small. Interestingly, note that, at $\sigma = 0.203$, where deflation overtakes inflation as the optimal policy, the optimal deflation rate and the optimal inflation rate are both nonzero.

Figure 2 looks at the difference between welfare under optimal intervention with limited monitoring and welfare in the optimal allocation under unlimited monitoring, as a function of σ . We use $r = 0.2$, $u_1(y_1) = \sigma \frac{y_1^{0.65}}{0.65}$, $u_2(y_2) = \frac{y_2^{0.4}}{0.4}$, $\rho_1 = \sigma$ and $\rho_2 = 1$. Interestingly, note that interventions by the monetary authority can upset the welfare ranking given by Theorem 1, which says that the optimal allocation in the unlimited monitoring economy always dominates the one in the limited monitoring economy. A rough intuition for this result is that, when σ is relatively small, a given inflation tax allows the monetary authority to pay for a large amount of debt to each buyer participating in a monitored trade. As said above, we defer a more precise intuition to the next section.

We finish this section by briefly considering the scenario where the first DM round is monitored. It turns out that the key difference is that, although intervention is generically optimal and it can be inflationary, it is no longer the case that interventions can lead to

Figure 2: Welfare Difference between Optimal Welfare under Intervention and Unlimited Monitoring

allocations that are superior to the one achieved under unlimited monitoring. The reason is that, since all buyers participate in monitored meetings when round-1 DM is monitored, they all benefit from the debt forgiveness. In contrast, when round-2 DM is monitored and $\sigma < 1$, only those who participate in monitored meeting benefit from debt forgiveness. Thus, in the latter case, the inflation tax required by the monetary authority to pay for the private debt is relatively small. A natural conjecture is that, if we allow the meeting probability in round-1 DM to be smaller than one, inflationary policies can also achieve a strictly higher welfare than unlimited monitoring.

4 Mechanism Design

In this section, we allow for arbitrary trading mechanisms that are incentive compatible subject to the frictions in the environment. We characterize the set of incentive feasible allocations and then look for the allocation that maximizes social welfare within this set. We show that, in the economy with limited monitoring, interventions can still improve welfare. An allocation is denoted by

$$\mathcal{L} = [(y_1, y_2), (z_1, z_2)],$$

where y_j denotes a buyer's consumption in round- j DM and z_j denotes the CM consumption of a round- j seller, $j \in \{1, 2\}$. We restrict our attention to allocations that satisfy $z_1 \leq u_1(y_1) \leq u_1(y_1^*)$ and $z_2 \leq u_2(y_2) \leq u_2(y_2^*)$. This restriction is without loss of generality as

far as constrained efficient allocations are concerned.

We start with the unlimited monitoring economy as a benchmark. In this case, it is straightforward to verify that an allocation $[(y_1, y_2), (z_1, z_2)]$ is consistent with the repayment constraint of buyers and the participation constraint of sellers if and only if

$$-r(z_1 + z_2) + u_1(y_1) - z_1 + \sigma[u_2(y_2) - z_2] \geq 0, \quad (28)$$

$$\rho_1 y_1 \leq z_1, \quad \rho_2 y_2 \leq z_2. \quad (29)$$

It is also straightforward to verify that Lemma 1 holds. Precisely, if $r \leq r_c$, the first-best is implementable, while if $r > r_c$, $\mathcal{L}^c = [(y_1^c, y_2^c), (\rho_1 y_1^c, \rho_2 y_2^c)]$ maximizes (1) subject to (28)-(29). For later purposes, recall that $\mathcal{L}^c$ is determined by (28) and (29) at equality, and by the FOC

$$\underbrace{[u'_1(y_1) - \rho_1]}_{(a)} - \underbrace{\frac{-r\rho_1 + [u'_1(y_1) - \rho_1]}{-r\rho_2 + \sigma[u'_2(y_2) - \rho_2]}}_{(b)} \underbrace{\sigma[u'_2(y_2) - \rho_2]}_{(c)} = 0. \quad (30)$$

This FOC also has a simple interpretation: term (a) is the marginal benefit of increasing y_1 , while term (c) is the marginal benefit of increasing y_2 . In turn, term (b) is the “shadow price” implied by (28) (with $z_j = \rho_j y_j$, $j = 1, 2$) at which we can convert y_2 into y_1 . If the left side of (30) is positive it is beneficial to increase y_1 and when it is negative it is beneficial to increase y_2 . The need to satisfy (28) at equality also implies that, with the exception of a knife-edge case, either

$$-r\rho_1 y_1^c + [u_1(y_1^c) - \rho_1 y_1^c] < 0, \quad (31)$$

and round-1 DM has a “tight” liquidity constraint, or

$$-r\rho_2 y_2^c + \sigma[u_2(y_2^c) - \rho_2 y_2^c] < 0, \quad (32)$$

and round-2 DM has a “tight” liquidity constraint. Intuitively, liquidity in round- j DM is tight if the buyer uses more liquidity in that round than the one sustainable by the threat of exclusion from that round alone, and, as we shall see later, this notion of tightness has implications on how to relocate liquidity optimally through interventions.

We now turn to the limited monitoring economy. As in Section 2.1, the choice of the debt limit D is formulated as a mechanism design problem to maximize social welfare subject to incentive compatibility constraints. We also maintain the same set of records $R = \{G, B\}$, and the same function $\omega : \{G, B\} \times H \rightarrow \{G, B\}$. The key difference is that terms of trades

are now determined by the mechanism designer. Precisely, we have a pair of functions (o_1, o_2) defined as follows:

$$o_1(m) = (y_1, z_{1,m}) \text{ and } o_2(m, r, d) = (y_2, z_{2,c}, z_{2,m}),$$

where m is the buyer's real balance holdings, r is his record, d is his debt limit, $(y_1, z_{1,m})$ is the proposed trade in a non-monitored meeting and $(y_2, z_{2,c}, z_{2,m})$ is the proposed trade in a monitored meeting – $z_{j,m}$ is the transfer of real balances in round j , and $z_{2,c}$ is the promise of the buyer in the monitored round. Proposals are implemented as follows. First, both the buyer and the seller respond with *yes* or *no* to the corresponding proposed trade. If both respond with *yes* then they move to the next stage; otherwise, the meeting is autarkic. If they move to the next stage, the buyer makes a TIOLI offer, which is implemented if the seller responds with *yes* while the originally proposed trade by the mechanism is carried out otherwise. This trading protocol is in the spirit of a direct mechanism. In particular, it allows arbitrary ways to split the trading surpluses only subject to individual rationality and coalition-proofness.¹³ Finally, in the CM, each buyer chooses how much to repay of his promises, and agents trade competitively against ϕ_t to rebalance their money holdings.

We restrict attention to symmetric equilibria where real balances $Z_t = \phi_t M_t$ do not change over time, and proposals are stationary. Strategies are as in Section 2.2, with the difference that we replace TIOLI offers by the proposal and the trading protocol described above. The equilibrium concept is symmetric Perfect Bayesian Equilibrium, and we restrict attention to simple equilibria: buyers with good record always repay their debts up to the debt limit D and they always choose to use the technology in monitored meetings, buyers and sellers respond with *yes* in all DM meetings and buyers always offer the trades proposed by the mechanism. Finally, the initial distribution of money across buyers is degenerate.

We will prove that interventions are generically optimal by providing an upper bound on the welfare that can be achieved without intervention, and a lower bound on the welfare that can be achieved with intervention; and then showing that the latter is strictly larger than the former. We start by giving necessary and sufficient conditions for implementation without intervention. The former defines an upper bound on the set of allocations can be

¹³This trading mechanism generalizes the trading protocols considered in Zhu (2008) and Hu, Kennan, and Wallace (2009) to our setting with monitored meetings. As in those papers, the first stage ensures that the mechanism satisfies individual rationality, and the second stage ensures that it satisfies the pairwise core requirement and hence is coalition-proof.

achieved without intervention, while the latter will help us identify the nature of the optimal intervention.

Lemma 8 (i) *An allocation $\mathcal{L} = [(y_1, y_2), (z_1, z_2)]$ is implementable under a constant money supply only if it satisfies (28)-(29) and*

$$-rz_1 + u_1(y_1) - z_1 \geq 0. \quad (33)$$

(ii) *An allocation $\mathcal{L} = [(y_1, y_2), (z_1, z_2)]$ is implementable under a constant money supply if it satisfies (29), (33), and*

$$-z_2 + \frac{1}{r}\sigma[u_2(y_2) - z_2] \geq 0. \quad (34)$$

Lemma 8 (i) provides necessary conditions for implementation and hence can be used to derive a upper bound on welfare achievable without intervention; (ii) provides sufficient conditions and hence a lower bound can be derived. Note that (33) and (34) together imply (28), and hence the sufficient conditions are stronger than the necessary. The need for (28) and (29) in (i) is straightforward. The constraint given by (28) is based on the buyer's incentive to deviate in the CM: the term $-r(z_1 + z_2)$ reflects the total cost of liquidity, including both money and credit, while the remaining terms reflect the total DM surplus to the buyer per period. If (28) does not hold, the buyer is better off holding no money and not paying back his debt in the CM. In turn, (29) ensures that sellers want to participate in the DM trade.¹⁴ Lemma 8 (i) highlights that one more condition, (33), is necessary. This condition states that the cost rz_1 of bringing real balances to use in the non-monitored round cannot be larger than the benefit of those balances in that round, given by $u(y_1) - z_1$. Intuitively, since round-1 DM is not monitored, buyers with different private histories who enter round-2 DM with the same money holdings must be treated equally. Thus, a buyer can always deviate by choosing not to participate in that round with no impact on his continuation payoff. The presence of (33) ensures the voluntary participation of the buyer in round-1 DM trade.

Sufficiency requires another condition, (34), which, ensures that a buyer is willing to repay the equilibrium debt, z_2 , for round-2 DM trade; failure to do so would result in losing all future round-2 DM surplus, $\frac{1}{r}\sigma[u_2(y_2) - z_2]$ and hence the condition. Let $\hat{\mathcal{L}}^{cm} =$

¹⁴Similar constraints also appear in Hu, Kennan, and Wallace (2009), who consider the baseline Lagos-Wright environment with no monitoring and with one DM round.

$[(\hat{y}_1^{cm}, \hat{y}_2^{cm}), (\hat{z}_1^{cm}, \hat{z}_2^{cm})]$ be the allocation that maximizes welfare subject to (29), (33), and (34).¹⁵ Although this allocation only sets a lower bound on what can be accomplished without intervention, as said above, it will help us identify the nature of the optimal intervention. We have the following lemma.

Lemma 9 *Assume that*

$$r > \hat{r}_{cm} = \min \left\{ \frac{u_1(y_1^*) - \rho_1 y_1^*}{\rho_1 y_1^*}, r_c \right\}.$$

(i) *If (31) holds, then*

$$[u'_1(\hat{y}_1^{cm}) - \rho_1] - \frac{-r\rho_1 + [u'_1(\hat{y}_1^{cm}) - \rho_1]}{-r\rho_2 + \sigma[u'_2(\hat{y}_2^{cm}) - \rho_2]} \sigma[u'_2(\hat{y}_2^{cm}) - \rho_2] > 0. \quad (35)$$

(ii) *If (32) holds, then*

$$[u'_1(\hat{y}_1^{cm}) - \rho_1] - \frac{-r\rho_1 + [u'_1(\hat{y}_1^{cm}) - \rho_1]}{-r\rho_2 + \sigma[u'_2(\hat{y}_2^{cm}) - \rho_2]} \sigma[u'_2(\hat{y}_2^{cm}) - \rho_2] < 0. \quad (36)$$

The restriction $r > \hat{r}_{cm}$ in Lemma 9 ensures that $W(\hat{y}_1^{cm}, \hat{y}_2^{cm}) < W(y_1^c, y_2^c)$. In particular, it ensures that $\hat{\mathcal{L}}^{cm}$ does not implement the first-best in both rounds. This is necessary if intervention is ever going to be optimal.

It is easier to interpret Lemma 9 if we take as a benchmark the case where monitoring is unlimited and the optimal allocation $\mathcal{L}^c$ is achieved under (28)-(29). Under $\mathcal{L}^c$, liquidity can be moved from one DM round to another by adjusting the quantities y_1 and y_2 in accordance to (28), with the relative “shadow price” for doing so given by the term (b) in (30). Now, under $\hat{\mathcal{L}}^{cm}$, if we consider the same shadow price, then (33) or (34) present an additional constraint in adjusting liquidity. Two possibilities arise. If (31) holds and round-1 DM is tight in liquidity, (35) suggests that it would be optimal to transfer liquidity from the monitored to the non-monitored sector. We will show below that a deflationary intervention can implement this transfer of liquidity and improve welfare. Similarly, if (32) holds and round-2 DM is tight in liquidity, (36) suggests the opposite. In this case, we will show that an inflationary intervention can implement a transfer of liquidity from the non-monitored to the monitored sector and improve welfare. A critical component of the transfer implemented by our inflationary intervention is that it occurs at a shadow price that is lower than the

¹⁵Note that, without intervention, the optimal equilibrium allocation under TIOLI offers satisfies these sufficient conditions, with (33) slack. This implies that TIOLI offers constitutes a sub-optimal trading mechanism.

one under unlimited monitoring. As a result, it allows to achieve allocations that are better than $\mathcal{L}^c$.

Deflationary policy

We first characterize the set of implementable allocations using a deflationary policy that taxes credit trades and purchases money. Note that if the average real balance per buyer is given by z_1 in equilibrium, then budget balance requires a tax on credit trade with $\chi = -\tau z_1$, where $\tau < 0$ is the corresponding deflation rate. The following theorem provides an upper bound and a lower bound on implementable allocations under deflationary policies.

Theorem 3 (i) *An allocation, $\mathcal{L} = [(y_1, y_2), (z_1, z_2)]$, is implementable with a deflationary policy only if it satisfies (28) and (29).*

(ii) *An allocation, $\mathcal{L} = [(y_1, y_2), (\rho_1 y_1, \rho_2 y_2)]$, is implementable with a deflationary policy with rate $\tau \in (-\frac{r}{1+r}, 0)$ if it satisfies*

$$-(1+r)\tau\rho_1 y_1 - r\rho_1 y_1 + [u_1(y_1) - \rho_1 y_1] \geq 0, \quad (37)$$

$$-(-\tau\rho_1 y_1 + \rho_2 y_2) + \frac{1}{r}\sigma[u_2(y_2) - \rho_2 y_2 + \tau\rho_1 y_1] \geq 0. \quad (38)$$

Theorem 3 (i) establishes that (28)-(29) is necessary for any deflationary policy, which implies that the optimal allocation under deflation cannot be better than $\mathcal{L}^c$. The condition (28) is based on the same deviation in the CM as that for Lemma 8; the two conditions coincide because the increase in the liquidity available in the non-monitored round enabled by the increase in the real rate of return on money is exactly offset by the decrease in the liquidity available in the monitored round caused by the taxation of credit trades.

Theorem 3 (ii) gives two sufficient conditions for implementability under a given deflationary policy. The condition (37) ensures that the buyer is willing to carry $z_1 = \rho_1 y_1$ units of real balances across the CM's. Note that, since $\tau < 0$, the first term adds the increase in the rate-of-return on money relative to what we have in (33).¹⁶ In turn, (38) ensures that the buyer is willing to repay his equilibrium debt ($z_2 = \rho_2 y_2$) as well as the credit tax ($\chi = -\tau\rho_1 y_1$) in order to continue participating in credit trades. Note also that credit tax also decreases per period surplus. If $\tau = 0$, (37)-(38) coincide with (33)-(34).

¹⁶The proof is based on the mechanism constructed in Hu, Kennan, and Wallace (2009), which can effectively discipline buyers to hold any amount of real balances as long as the cost of holding it across the CM's is compensated by the surplus in DM trades.

We can now use Theorem 3 (ii) to identify a lower bound on the welfare that can be achieved under a deflationary intervention. Precisely, let $[y_1(\tau), y_2(\tau)]$ be the allocation that maximizes welfare under (37)-(38), and define

$$\mathcal{W}^{def}(\tau) = \mathcal{W}[y_1(\tau), y_2(\tau)] \text{ for all } \tau \in \left(-\frac{r}{1+r}, 0\right).$$

Note that $[y_1(0), y_2(0)] = (\hat{y}_1^{cm}, \hat{y}_2^{cm})$. We have

$$\begin{aligned} \frac{\partial \mathcal{W}^{def}(\tau)}{\partial \tau} &= C_0(\tau) \cdot C_{def}(\tau), \\ \text{where } C_0(\tau) &= \frac{(1+r)\rho_1 y_1}{-(1+r)\tau\rho_1 - r\rho_1 + [u'_1(y_1(\tau)) - \rho_1]} < 0, \\ \text{and} \\ C_{def}(\tau) &= [u'_1(y_1(\tau)) - \rho_1] - \frac{-r\rho_1 + [u'_1(y_1(\tau)) - \rho_1]}{-r\rho_2 + \sigma[u'_2(y_2(\tau)) - \rho_2]} \sigma[u'_2(y_2(\tau)) - \rho_2]. \end{aligned} \tag{39}$$

The impact on the welfare of a change in the deflation rate τ has two components. The first component, $C_0(\tau)$, simply captures the fact that a decrease in τ leads to an increase in y_1 . The second term, $C_{def}(\tau)$, shows that the shadow price at which we can convert y_2 into y_1 is exactly the same shadow price observed under unlimited monitoring. Indeed, $C_{def}(\tau)$ is equal to left side of (30).

We are now ready to provide the conditions under a deflationary intervention is optimal. For this, consider the region of parameters where (31) holds. In this case, since $[y_1(0), y_2(0)] = (\hat{y}_1^{cm}, \hat{y}_2^{cm})$, Lemma 9 (i) implies $C_{def}(0) > 0$. Together with $C_0(\tau) < 0$, we have that, starting from the point of no intervention, a small deflation improves welfare relative to $\hat{\mathcal{L}}^{cm}$. In fact, deflation can lead us all the way to $\mathcal{L}^c$, i.e., it is straightforward to verify that $\mathcal{L}^c = (y_1^c, y_2^c)$ satisfies the sufficient conditions (37)-(38) with $\tau = \tau^*$ given by

$$\tau^* = -1 + \frac{u_1(y_1^c)}{(1+r)\rho_1 y_1^c} \in \left(-\frac{r}{1+r}, 0\right).$$

Theorem 3 (i) implies that τ^* is the optimal deflation rate. This is so because a deflationary intervention must satisfy (28)-(29), and thus can do no better than $\mathcal{L}^c$. Lastly, if (31) holds, Lemma 8 (i) implies that L^c cannot be implemented under no intervention. As a result, it must be the case a deflationary policy is necessary to achieve optimality. We have the following corollary.

Corollary 4 *Suppose that $r > \min\{r_c, \hat{r}_{cm}\}$ and that (31) holds. The optimal intervention under limited monitoring is deflationary.*

Remark 1 *In the proof of Theorem 3 (ii), the proposed trade uses debt alone in round-2 DM. By doing so the pairwise core requirement is satisfied, and it will not be true in general without using debt. In that sense debt is essential for implementation. We also remark here that while money is allowed in round-2 DM, the optimal mechanism has the feature that a buyer with a bad record is punished by the optimal mechanism by receiving zero surpluses in round-2 DM.*

Inflationary policy

We now characterize the set of implementable allocations using an inflationary policy that uses the revenue from the inflation tax to purchase private debt. If the average real balance holding per buyer is given by z_1 in equilibrium, then budget balance requires that the maximum feasible amount of credit subsidies, η , satisfy $\sigma\eta = \tau z_1$, where $\tau > 0$ is the corresponding inflation rate. The following theorem provides an upper bound and a lower bound on implementable allocations under inflationary policies.

Theorem 5 (i) *An allocation, $\mathcal{L} = [(y_1, y_2), (z_1, z_2)]$, is implementable with an inflationary policy only if it satisfies (29), (33), and*

$$-r(z_1 + z_2) + u_1(y_1) - z_1 + \sigma[u_2(y_2) - z_2] \geq \frac{r(1 - \sigma)}{r + \sigma} \{-rz_2 + \sigma[u_2(y_2) - z_2]\}. \quad (40)$$

(ii) *An allocation, $\mathcal{L} = [(y_1, y_2), (\rho_1 y_1, \rho_2 y_2)]$, is implementable with an inflationary policy with rate $\tau > 0$ if it satisfies*

$$-(1 + r)\tau\rho_1 y_1 - r\rho_1 y_1 + [u_1(y_1) - \rho_1 y_1] \geq 0, \quad (41)$$

$$-[\rho_2 y_2 - \tau\rho_1 y_1/\sigma] + \frac{1}{r}\sigma[u_2(y_2) - \rho_2 y_2 + \tau\rho_1 y_1/\sigma] \geq 0. \quad (42)$$

The condition (29) in Theorem 5 (i) ensures that sellers want to participate in DM trade, while (33) ensures that buyers want to participate in the non-monitored trade. However, condition (40) replaces (28) as the overall participation constraint, and its necessity is proved with the same argument used to prove the necessity of (28) in Theorem 3, except that now we have credit subsidy and inflation tax. Note that the only difference between (40) and (28) lies in the right side of (40). Importantly, this difference suggests that the corresponding shadow price in (40) differs from that in (28), as we shall see below.

The condition (40) is obtained from an appropriate weighted sum of the two sufficient conditions, (41) and (42), so that τ cancels out. The condition (41) ensures that the buyer is willing to carry $z_1 = \rho_1 y_1$ units of real balances across the CM's, but now the cost also includes the inflation tax, $(1+r)\tau z_1$. In turn, (42) ensures that the buyer is willing to repay his equilibrium debt to finance his round-2 DM trade, $z_2 = \rho_2 y_2$, but now there is a credit subsidy, $\eta = \tau z_1 / \sigma$, as reflected in the first term. In turn, the second term is the discounted sum of all future round-2 DM surpluses. A key difference with a deflationary policy is that, while all agents receive the benefit of credit taxes, here only agents that participate in monitored meetings receives the benefit of the credit subsidy. This difference explains the right side of (40).

Similarly to the case of deflationary interventions, we can now use Theorem 5 (ii) to identify a lower bound on the welfare that can be achieved under an inflationary intervention. Precisely, let $[y_1(\tau), y_2(\tau)]$ be the optimal allocation under constraints (41)-(42), and define

$$\mathcal{W}^{inf}(\tau) = \mathcal{W}[y_1(\tau), y_2(\tau)] \text{ for all } \tau \in (0, \infty).$$

We then have

$$\frac{\partial \mathcal{W}^{inf}(\tau)}{\partial \tau} = C_0(\tau) \cdot C_{inf}(\tau), \text{ where} \quad (43)$$

$$C_{inf}(\tau) = [u'_1(y_1(\tau)) - \rho_1] - \frac{\sigma + r}{\sigma(1+r)} \frac{-r\rho_1 + [u'_1(y_1(\tau)) - \rho_1]}{-r\rho_2 + \sigma[u'_2(y_2(\tau)) - \rho_2]} \sigma[u'_2(y_1(\tau)) - \rho_2],$$

and $C_0(\tau) < 0$ is the same as before. The difference is that, as inflation increases, $y_1(\tau)$ decreases. However, the shadow price in $C_{inf}(\tau)$ at which we can convert y_2 into y_1 differs from the one in $C_{def}(\tau)$. Precisely, since $\sigma < 1$, it is cheaper to increase y_2 at the expense of a decrease in y_1 . This reflects the fact that the inflation tax is redistributed only to the σ fraction of buyers with a successful round-2 DM meeting.

We can now provide the conditions under which an inflationary intervention is optimal. For this, consider the region of parameters where (32) holds. Since $\widehat{\mathcal{L}}^{cm} = [y_1(0), y_2(0)]$ and $\sigma < 1$, Lemma 9 (ii) implies that $\frac{\partial \mathcal{W}^{inf}(0)}{\partial \tau} > 0$ and a small increase in the inflation rate improves welfare relative to $\widehat{\mathcal{L}}^{cm}$. It turns out that, unlike in the case of a deflationary intervention, we can do better than $\mathcal{L}^c$. To see this, observe that, if (32) holds, $\mathcal{L}^c$ satisfies conditions (41) and (42) with

$$\hat{\tau} = \frac{1}{(1+r)\rho_1 y_1^c} [-(1+r)\rho_1 y_1^c + u_1(y_1^c)] > 0.$$

In fact, since at $\hat{\tau}$, $\mathcal{L}^c$ satisfies conditions (41) at equality but satisfies (42) with strict inequality, it follows that $y_1(\hat{\tau}) = y_1^c$ and $y_2(\hat{\tau}) > y_2^c$, and hence $[y_1(\hat{\tau}), y_2(\hat{\tau})]$ is a strictly better allocation than $\mathcal{L}^c$. The following corollary then directly follows from Lemma 8 (i). (By the necessary conditions in Theorems 3 (i) and 5 (i), it is straightforward to verify that under (35) inflation does not help while under (36) deflation does not help.)

Corollary 6 *Suppose that $r > r_c$ and that (32) holds. The optimal intervention under limited monitoring is inflationary.*

Corollaries 4 and 6 show that, unless the first-best allocation is implementable with a constant money supply, intervention is generically optimal. This result then extends the result in Theorem 2 to optimal mechanisms and shows that the need for intervention is not driven by assuming a suboptimal trading mechanism.¹⁷ These theorems also give a precise condition for the optimal policy to call for inflation or deflation, according to the liquidity needs across the DM rounds.¹⁸ Finally, these findings also help provide intuition on the findings in Figures 1 and 2 under TIOLI. Since as the shadow price in $C_{inf}(\tau)$ gets cheaper for increasing y_2 as σ decreases, inflationary policy is more likely to be optimal when σ is relatively small.

5 Concluding remarks

In this paper we propose a model of monetary interventions in credit markets in which both money and debt play essential liquidity roles. The interventions are monetary in the sense that money is the only instrument through which the monetary authority intervenes. Our model features heterogeneous liquidity needs in monetary and credit sectors, and the optimal monetary intervention balances these needs by redistributing liquidity across these two sectors. Specifically, inflation is optimal when the credit sector has a tight borrowing constraint. In contrast, when agents are not willing to hold enough real balances and the credit sector

¹⁷Bjaj et al. (2016) show that one can use inflation and the scheme considered in Andoffatto (2010) under TIOLI offers to obtain the constrained efficient allocation that is obtained in Hu, Kennan, and Wallace (2009) in the LW model with one DM round. Such schemes could help in our previous section with TIOLI offers, but not under the optimal trading mechanisms considered here.

¹⁸It is worth pointing out that some of our result on optimality of interventions survive if we assume that limited commitment is not a problem and debts are perfectly enforced, as long as lump-sum tax is not available. However, in this case, since the efficient quantity always obtains in the monitored round, an inflationary intervention is never optimal. The formal analysis of this scenario is in the Appendix B2.

has a relatively loose borrowing constraint, deflation is optimal. Since the liquidity needs in the two sectors are endogenously determined, the precise shape of intervention depends on the details of the economy, such as the discount factor, the matching probability, and the curvature of the utilities.

We made a few special assumptions to obtain our results. The linear cost of production in the DM can be easily generalized. We assume that round-1 DM has no matching uncertainty and there are only two DM rounds at each period. Under the optimal trading mechanism, we can extend Theorems 3-5 and Corollaries 4-6 to more general trading patterns, including more than two rounds of DM trades and matching uncertainty in all rounds, as long as we assume that a buyer can have a successful meeting in a later round only if he had a successful meeting in an earlier round. This excludes heterogeneous trading histories for active buyers. Otherwise, there would be an endogenous distribution of different debt limits and money holdings. While we suspect that in such a case both our credit market intervention and the interventions considered in Wallace (2014) would have nontrivial and interesting interactions, the analysis of such interventions is beyond the scope of this paper.

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Appendix A: proofs of Lemmas and Theorems

Proof of Lemma 1: The Lagrangian of maximizing (1) subject to (6) is given by

$$\begin{aligned} u'_1(y_1) - \rho_1 + \nu[-r\rho_1 + u'_1(y_1) - \rho_1] &= 0, \\ \sigma[u'_2(y_2) - \rho_2] + \nu\{-r\rho_1 + \sigma[u'_2(y_2) - \rho_2]\} &= 0, \end{aligned}$$

where $\nu \geq 0$ is the Lagrange multiplier. When $r \leq r_c$, the solution is (y_1^*, y_2^*) , and (5) is satisfied. When $r > r_c$ the constraint (6) is binding and hence $\nu > 0$. This implies that

$$\frac{u'_1(y_1)}{\rho_1} - 1 = \frac{r\nu}{1 + \nu} = \sigma \left[\frac{u'_2(y_2)}{\rho_2} - 1 \right],$$

which implies (5).

Proof of Lemma 2: Let $V_j(z)$ be the value function of a buyer with z real balances at the beginning of the j^{th} DM round, where $j \in \{1, 2\}$. We start with V_2 :

$$V_2(z_2) = z_2 + W_0 + \sigma \max_{0 \leq \rho_2 y_2 \leq z_2} \{u_2(y_2) - \rho_2 y_2\},$$

where TIOLI offers implies that his consumption is equal to y_2 if he spend $\rho_2 y_2$ real balances. If $z_2 \geq \rho_2 y_2^*$, then $y_2 = y_2^*$; otherwise, $y_2 = z_2/\rho_2$. Now consider V_1 :

$$V_1(z) = \max_{0 \leq y_1 \leq \frac{z}{\rho_1}} [u_1(y_1) + V_2(z - \rho_1 y_1)]. \quad (44)$$

The Inada conditions imply $y_1 > 0$ and $y_2 > 0$. There are two cases to consider. If $z \geq \rho_1 y_1^* + \rho_2 y_2^*$, then the optimal choice is $y_1 = y_1^*$ and $y_2 = y_2^*$; otherwise, the buyer anticipates that he will spend $z - \rho_1 y_1$ in the second DM round, and hence (44) becomes

$$V_1(z) = (1 - \sigma)z + \max_{0 \leq y_1 \leq \frac{z}{\rho_1}} \left[u_1(y_1) + \sigma u_2 \left(\frac{z - \rho_1 y_1}{\rho_2} \right) - (1 - \sigma)\rho_1 y_1 \right]. \quad (45)$$

The first-order condition is

$$u'_1(y_1) = (1 - \sigma)\rho_1 + \sigma u'_2 \left(\frac{z - \rho_1 y_1}{\rho_2} \right) \frac{\rho_1}{\rho_2}. \quad (46)$$

The buyer equates the marginal benefit of spending one unit of real balances in the first DM round, with the marginal benefit of bringing one unit of real balances into the second DM round. Thus, the buyer's CM problem is

$$\max_{z \geq 0} [-(1 + r)z + V_1(z)], \quad (47)$$

and standard arguments show that it is never optimal to choose more real balances than the necessary to purchase the first-best in both DM rounds. Using the Envelope Theorem, the FOC for (47) becomes

$$(1 - \sigma) + \sigma \frac{u'_2(y_2)}{\rho_2} = 1 + r. \quad (48)$$

where $\rho_2 y_2 = z - \rho_1 y_1$. Combining (46) and (48), we obtain (9).

Proof of Lemma 3: If $r \leq r_{cm}$, the optimal allocation is (y_1^m, y_2^*) , which can be implemented by $D = \rho y_2^*$. Suppose now that $r > r_{cm}$. If D is so small that $\sigma[u'_2(D/\rho_2)/\rho_2 - 1] < r$, then in equilibrium y_2 solves

$$\sigma \frac{u'_2(y_2) - \rho_2}{\rho_2} = r,$$

which implies that $y_2 = y_2^m$. Now, since for y_2^m (13) is satisfied with a strict inequality due to the concavity of u_2 , we can get a strictly higher y_2 by increasing D to be above $\rho_2 y_2^m$ without violating (13), which delivers a strictly better welfare. In fact, in equilibrium $y_2 = D/\rho_2$, and there exists a largest $y_2 > y_2^m$ such that (13) holds (at which it holds with equality); this is our y_2^{cm} that delivers optimal allocation. It can be implemented with $D^{cm} = \rho_2 y_2^{cm}$.

Proof of Theorem 1: First we show that $\mathcal{W}(y_1^c, y_2^c) > \mathcal{W}(y_1^{cm}, y_2^{cm})$. We consider two cases.

(1.a) Suppose that $r \leq r_c$. Then, $(y_1^c, y_2^c) = (y_1^*, y_2^*)$, and $y_1^{cm} = y_1^m < y_1^*$, $y_2^{cm} \leq y_2^*$. Thus, $\mathcal{W}(y_1^c, y_2^c) > \mathcal{W}(y_1^{cm}, y_2^{cm})$.

(1.b) Suppose that $r > r_c$. Then, at (y_1^c, y_2^c) , (6) holds at equality. The strict concavity of u_1 implies that

$$-r\rho_1 y_1^{cm} + [u_1(y_1^{cm}) - \rho_1 y_1^{cm}] > 0,$$

and Lemma 2 implies that

$$-r\rho_2 y_2^{cm} + \sigma[u_2(y_2^{cm}) - \rho_2 y_2^{cm}] \geq 0.$$

Thus, at (y_1^{cm}, y_2^{cm}) , (6) holds at strict inequality, and hence it is feasible when maximizing (1) subject to (6). This implies that $\mathcal{W}(y_1^c, y_2^c) > \mathcal{W}(y_1^{cm}, y_2^{cm})$.

Now we show that $\mathcal{W}(y_1^{cm}, y_2^{cm}) > \mathcal{W}(y_1^m, y_2^m)$. First note that $y_1^{cm} = y_1^m$, and hence we only need to compare y_2^{cm} against y_2^m . By Lemma 3, either y_2^{cm} satisfies (13) at equality, or $y_2^{cm} = y_2^*$. However, strict concavity of u_2 implies that $y_2^m < y_2^{cm}$. Thus, $\mathcal{W}(y_1^{cm}, y_2^{cm}) > \mathcal{W}(y_1^m, y_2^m)$.

Proof of Lemma 4: We may assume that $i < r$. First we show that, there exists $\underline{i} < r$ such that for all $i \geq \underline{i}$, it is never optimal to set a debt limit so low that buyers use money in the monitored meeting.

Let $y_1(i)$ be the solution to (18) and let $\chi(i) = \frac{(r-i)\rho_1 y_1(i)}{(1+r)}$. Recall that by Lemma 3, y_2^{cm} is the largest y_2 for which (13) holds (and it holds with equality). Since $\chi(r) = 0$ and χ is continuously differentiable, by the Implicit Function Theorem, there exist $\underline{i}' < r$ such that for each $i \in [\underline{i}', r]$, $y_2(i)$ exists such that

$$-\rho_2 y_2 - \chi(i) + \frac{\beta}{1-\beta} \sigma [u_2(y_2) - \rho_2 y_2] - \chi(i) = 0, \quad (49)$$

with $y_2(r) = y_2^{cm}$. Moreover, since we have

$$\sigma \frac{u_2'(y_2(r)) - \rho_2}{\rho_2} < r,$$

there exists $\underline{i} < r$ such that for all $i \in [\underline{i}, r]$,

$$\sigma \frac{u_2'(y_2(i)) - \rho_2 y_2(i)}{\rho_2} \leq i,$$

and hence buyers do not spend money in round-2 DM. Moreover, for such i 's $[y_1(i), y_2(i)]$ is better than any other allocation in which the buyer uses money in round-2 DM.

Proof of Lemma 5: We may assume that $i > r$. Let y_2 solve

$$\sigma \frac{u_2'(y_2) - \rho_2}{\rho_2} = i.$$

Now, since $i > r$, $y_2 < y_2^{cm}$. Since y_2^{cm} is feasible at $D = D_{cm} = \rho_2 y_2^{cm}$ and it achieves a strictly higher welfare than y_2 , there is no loss in generality in considering $D > D_{cm}$. Otherwise, the debt limit would be so low that it would be better to let the buyer bring real balances to trade in the second DM-round. If $D > D_{cm}$, in equilibrium we have $y_2 = D/\rho_2$, and (25) can be rewritten as

$$-r [\rho_2 y_2 + \eta(i)] + \sigma \{u_2(y_2) - [\rho_2 y_2 + \eta(i)]\} \geq 0, \quad (50)$$

where $\eta(i) = -\tau \rho_1 y_1(i)/\sigma$ and it is independent of D . In this case, there exists a largest $y_2 > y_2^{cm}$ such that (50) holds (at which it holds with equality), denoted by $y_2(i)$. Then, $[y_1(i), y_2(i)]$ is the optimal allocation and the debt limit to implement it is $D(i) = \rho_2 y_2(i)$.

Proof of Lemma 6: If $r \leq r_{cm}$, the first-best y_2^* is implementable in the second DM round without any intervention, that is, $y_2^{cm} = y_2^*$. Since $y_1^{cm} = y_1^m < y_1^*$, it is optimal to increase y_1 and to decrease y_2 . A deflationary policy will achieve that and hence will increase welfare relative to no intervention. A inflationary policy cannot increase y_1 .

Proof of Lemma 7: First we give some derivatives. For $y_1(i)$, it does not depend on whether $i > r$ or $i < r$ and

$$y_1'(i) = \frac{\rho_1}{u_1''[y_1(i)]}. \quad (51)$$

For $y_2(i)$, we need to distinguish the two cases:

$$y_2'(i) = \frac{\sigma + r}{\sigma(1+r)} \frac{\rho_1 y_1(i) + (i-r)\rho_1 y_1'(i)}{(r+\sigma)\rho_2 - \sigma u_2'[y_2(i)]} \text{ if } i > r. \quad (52)$$

$$y_2'(i) = \frac{\rho_1 y_1(i) + (i-r)\rho_1 y_1'(i)}{(r+\sigma)\rho_2 - \sigma u_2'[y_2(i)]} \text{ if } i < r; \quad (53)$$

Suppose, by contradiction, that

$$\lim_{i \downarrow r} \frac{d}{di} \bar{\mathcal{W}}(i) \leq 0 \text{ and } \lim_{i \uparrow r} \frac{d}{di} \bar{\mathcal{W}}(i) \geq 0.$$

Then, we have

$$\begin{aligned} \frac{\rho_1[u_1'(y_1^{cm}) - \rho_1]}{u_1''(y_1^{cm})} + \frac{\rho_1 y_1^{cm}[u_2'(y_2^{cm}) - \rho_2]}{(r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm})} &\geq 0, \\ \frac{\rho_1[u_1'(y_1^{cm}) - \rho_1]}{u_1''(y_1^{cm})} + \frac{\sigma + r}{\sigma(1+r)} \frac{\rho_1 y_1^{cm}[u_2'(y_2^{cm}) - \rho_2]}{(r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm})} &\leq 0. \end{aligned}$$

This then implies that

$$-\frac{(1-\sigma)r}{\sigma(1+r)} \frac{[u_2'(y_2^{cm}) - \rho_2]}{(r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm})} \geq 0,$$

but, since $0 < y_2^{cm} < y^*$ and $(r+\sigma)\rho_2 y_2^{cm} - \sigma u_2(y_2^{cm}) = 0$,

$$u_2'(y_2^{cm}) - \rho_2 > 0 \text{ and } (r+\sigma)\rho_2 - \sigma u_2'(y_2^{cm}) > 0,$$

a contradiction.

Proof of Lemma 8: (i) It is straightforward to verify that (29) is necessary and sufficient to ensure participation of sellers. Here we show that (28) is necessary to ensure participation of buyers. In equilibrium, the continuation value upon entering the DM at any given period is

$$V = \frac{1}{1-\beta} \{[u_1(x_1) - z_1] + \sigma[u_2(x_2) - z_2]\}.$$

Consider the CM decision of a buyer with the largest amount of debt and the smallest amount of real balances along the equilibrium path. To follow the equilibrium behavior, he needs to produce $z_1 + z_2$ in order to repay his debt and rebalance his money holdings. However, he can deviate by producing zero in the CM and not participating in any future trade afterwards. This deviation is not profitable only if

$$-(z_1 + z_2) + \beta V \geq 0,$$

which is equivalent to (28).

To show the necessity of (33), note that the buyer can choose to bring just sufficient real balances for his round-2 DM trade (which can be, of course, zero). To deter this deviation, it must be the case that

$$-z_1 + \beta \{u_1(y_1) + z_1\} \geq 0,$$

that is,

$$-(1 - \beta)z_1 + \beta[u_1(y_1) - z_1] \geq 0,$$

which is equivalent to (33) and hence (33) is necessary. We omit the proof of (ii) here, since it is a corollary of Theorem 5 (ii) with $\tau = 0$.

Proof of Lemma 9: (i) Since $r > \hat{r}_{cm}$, the first-best cannot be implemented under no intervention and it must be the case that (33) and (34) bind at the optimal allocation. Now, if (31) holds and (33) and (34) bind, it must be that $\hat{y}_1^{cm} < y_1^c$ and $\hat{y}_2^{cm} > y_2^c$. Combining this result with the fact that (y_1^c, y_2^c) satisfies (30), we obtain (35).

(ii) If, instead (32) holds and (33) and (34) bind, it must be that $\hat{y}_1^{cm} > y_1^c$ and $\hat{y}_2^{cm} < y_2^c$. Again, combining this result with the fact that (y_1^c, y_2^c) satisfies (30), we obtain (36).

Proof of Theorem 3 (i) Assume that the allocation can be implemented with a deflation $\tau < 0$. To finance consumption in the first round the buyer has to bring at least z_1 units of real balances. Note that budget-balancedness requires (as we charge fees at the end of CM)

$$\chi = -\tau z, \tag{54}$$

where $z \geq z_1$ is the equilibrium money holding and the buyer uses $z - z_1$ units of cash to finance his round-2 DM trade (with debt $z_2 - (z - z_1)$). Consider a buyer with $z_2 - (z - z_1)$ units of debt in the CM. To follow the equilibrium behavior, he needs to repay $z_2 - (z - z_1) + \chi$

and buy $(1 + \tau)z$ units of real balances. The buyer can deviate and choose not to repay the debt and taxes and not to hold any cash, thus forgoing all future DM trades. Thus, for him to follow the equilibrium behavior, it must be the case that

$$-(1 + \tau)z - [z_2 - (z - z_1) + \chi] + \frac{\beta}{1 - \beta} \{ [u_1(y_1) - z_1] + \sigma[u_2(y_2) - z_2] - \chi - \tau z \} \geq 0.$$

which, plugging (54), implies (28). As usual, (29) is necessary, otherwise sellers would not be willing to participate.

(ii) We now prove sufficiency. Given $\tau \in (-\frac{r}{1+r}, 0]$, set

$$\chi = -\tau z_1, \tag{55}$$

and set $z_j = \rho_j y_j$, $j = 1, 2$. The debt limit is $D = z_2$ if the buyer has a record G , and is equal to $D = 0$ if the buyer has a record B . In equilibrium the buyer always remains with record G . Standard arguments in the LW environment show that the continuation value at the beginning of the CM is linear in money holdings; we use $W_G(m, x_c)$ to denote the continuation value for a buyer entering the CM with record G , with m units of real balances and with x_c units of debt, and use $W_B(m)$ to denote the continuation value for a buyer entering the CM with record B and with m units of real balances. We thus have

$$W_G(m, x_c) = m - \min\{D, x_c\} + W_G(0, 0), \tag{56}$$

$$W_B(m) = m + W_B(0). \tag{57}$$

We now specify the proposed trades $o_1(m)$ and $o_2(m, R, D)$ for all $m \geq 0$ and $R \in \{G, B\}$. We start with $o_2(m, R, D)$. Let $o_2(m, G, D)$ be a solution to

$$\begin{aligned} \max_{(y, x_c, x_m) \in \mathbb{R}_+ \times [0, D] \times [0, m]} & -\rho_2 y + x_c + x_m \\ \text{s.t.} & u_2(y) - x_c - x_m \geq u_2(y_2) - z_2, \end{aligned} \tag{58}$$

and let $o_2(m, B, 0)$ be a solution to

$$\begin{aligned} \max_{(y, x_m) \in \mathbb{R}_+ \times [0, m]} & -\rho_2 y + x_m \\ \text{s.t.} & u_2(y) - x_m \geq 0. \end{aligned} \tag{59}$$

The solutions to (58) and (59) exist and are unique, with the constraints binding at the optimum. Moreover, it can be verified that

$$o_2(0, G, D) = (y_2, z_2, 0).$$

It is also straightforward to see that, regardless of the buyer's record, his surplus in round-2 DM is independent of m .

We now move to $o_1(m)$. Since the first DM round is non-monitored, the payoff of the buyer in this round cannot depend on his record or on the debt limit. Let $\zeta(m) = u_1(y_1) - z_1$ if $m \geq z_1$ and let $\zeta(m) = 0$ otherwise. In turn, let $o_1(m)$ be a solution to

$$\begin{aligned} \max_{(y,x) \in \mathbb{R}_+ \times [0,m]} \quad & -\rho_1 y + x \\ \text{s.t.} \quad & u_1(y) - x \geq \zeta(m). \end{aligned} \tag{60}$$

The solution to (60) exists and is unique with the constraints binding at the optimum.

Given the proposed trades, we can now write down the Bellman equations:

$$\begin{aligned} W_G(0,0) &= \max_{m,R} \{-\mathbf{1}_{R=G}\chi - m + \beta V_R(m)\}, \\ W_B(0) &= \max_m \{-m + \beta V_B(m)\}, \end{aligned}$$

where

$$\begin{aligned} V_G(m) &= \zeta(m) + m + \sigma[u_2(y_2) - z_2] + W_G(0,0), \\ V_B(m) &= \zeta(m) + m + W_B(0), \end{aligned}$$

give the continuation value of a buyer leaving the CM with m units of real balances and a record R , for $R = G, B$.

We now show that the following strategies form a simple equilibrium. All agents respond with *yes* to the proposed trades and buyers offer the proposed trades, on and off the equilibrium path. Buyers with record G always repay their debts up to D plus the fee χ , and buyers with record B never repay their debt. All buyers leave the CM with z_1 units of real balances, on and off the equilibrium path.

We first show that the buyer carry z_1 real balances to the DM, irrespective of his record $R \in \{G, B\}$. By linearity of W_R , the CM decision on real balances is given by

$$\begin{aligned} & \max_{m \geq 0} -(1 + \tau)m + \beta V_R(m) \\ &= \beta \{-[r + (1 + r)\tau]m + \zeta(m) + \mathbf{1}_{R=G}\{\sigma[u_2(y_2) - z_2]\} + W_R(0,0)\}. \end{aligned}$$

Note that $\zeta(m)$ is constant for all $m \geq z_1$ and is constant for all $m \in [0, z_1)$. Thus, we only need to show that bringing z_1 is better than zero, which will be the case if and only if

$$-[r + (1 + r)\tau]z_1 + u_1(y_1) - z_1 \geq 0,$$

and this holds by (37) with $z_1 = \rho_1 y_1$.

We now show that buyers are willing to repay their debts, together with the fee for accessing the monitoring technology, χ . Following the equilibrium behavior,

$$\begin{aligned} W_G(0, z_2) &= -[(1 + \tau)z_1 + z_2] - \eta + \frac{\beta}{1 - \beta} \{[u_1(y_1) - z_1] + \sigma[u_2(y_2) - z_2] - \tau z_1 - \chi\}, \\ W_B(0) &= -(1 + \tau)z_1 + \frac{\beta}{1 - \beta} \{[u_1(y_1) - z_1] - \tau z_1\}. \end{aligned}$$

We need to show that the buyer wants to keep the record G . He has incentive to repay $z_2 + \chi$ in order to keep this record if and only if

$$-(z_2 + \chi) + \beta V_G(z_1) \geq \beta V_B(z_1),$$

and this is equivalent to

$$-(z_2 + \chi) + \beta \{\sigma[u_2(y_2) - z_2] + W_G(0, 0) - W_B(0)\} \geq 0,$$

and can also be rewritten as

$$-(z_2 + \chi) + \frac{\beta}{1 - \beta} \{\sigma[u_2(y_2) - z_2] - \chi\} \geq 0,$$

which holds with equality by (55) and (38) with $z_2 = \rho_2 y_2$. Lastly, since $y_1 \leq y_1^*$ and $y_2 \leq y_2^*$, as far as constrained efficient allocations are concerned, it is always optimal to set $z_1 = \rho_1 y_1$ and $z_2 = \rho_2 y_2$.

Proof of Corollary 4: Since by Lemma 8 (i) and by (31) (y_1^c, y_2^c) is not implementable with a constant money supply but it is implementable with a deflationary policy, deflationary policy is strictly better than no intervention. Moreover, by Theorem 5 (i) (y_1^c, y_2^c) is not implementable with an inflationary policy either. In fact, it is straightforward to verify that by Lemma 9) we have (35), which implies that the optimal allocation that maximizes welfare subject to (33), (34) and (29) is indeed $(\hat{y}_1^{cm}, \hat{y}_2^{cm})$, and hence inflationary policy cannot do better than (y_1^c, y_2^c) by Theorem 5 (i).

Proof of Theorem 5: (i) First, we prove necessity. Clearly, (29) is necessary, otherwise sellers would not be willing to participate. Consider now the incentives of the buyer to participate in the first DM-round. Since the first DM round is not monitored, the buyer can deviate and still keep a good record, i.e., if the buyer only brings real balances (if any) to

participate in the second DM-round, the punishment from bringing zero real balances to use in the first DM-round is just exclusion from trade in that round. As a result, if the inflation rate is τ , it is necessary that

$$-(1+r)\tau z_1 - rz_1 + [u_1(y_1) - z_1] \geq 0. \quad (61)$$

To prove the necessity of (40), consider an arbitrary inflationary policy (η, τ) with $\eta > 0$. Consider a buyer with z_2 units of debts in the CM. To follow the equilibrium behavior, this buyer needs to repay $\min\{z_2 - \eta, 0\}$ and buy at least $(1+\tau)z_1$ units of real balances. Alternatively, he can deviate by not repaying his debt and bringing zero real balances into the DM rounds. Thus, for not deviation to take place, it must be the case that

$$-rz_1 - rz_2 + [u_1(y_1) - z_1] + \sigma[u_2(y_2) - z_2] - (1+r)\tau z_1 + (r+\sigma)\eta \geq 0.$$

Since budget-balancedness implies $\sigma\eta = \tau z_1$, the above inequality, together with (61), implies (40).

(ii) We now prove sufficiency. For a given $\tau > 0$, we set

$$\eta = \frac{\tau}{\sigma} z_1, \quad (62)$$

and set $z_j = \rho_j y_j$, $j = 1, 2$. The debt limit is $D = z_2$ if the buyer has a record G , and is equal to $D = 0$ if the buyer has a record B . In equilibrium the buyers always keep the record G . Standard arguments in the LW environment show that the continuation value for a buyer with record G entering the CM with m units of real balances and with x_c units of debt is given by

$$W_G(m, x_c) = m - \min\{D - \eta, \max\{x_c - \eta, 0\}\} + W_G(0, 0). \quad (63)$$

It remains to specify the proposed trades $o_1(m)$ and $o_2(m, R, D)$ for all $m \geq 0$ and $R \in \{G, B\}$. We start with $o_2(m, R, D)$. Let $o_2(m, G, D)$ be a solution to

$$\begin{aligned} \max_{(y, x_c, x_m) \in \mathbb{R}_+ \times [0, D] \times [0, m]} & \quad -\rho_2 y + x_c + x_m \\ \text{s.t.} & \quad u_2(y) - \max(x_c - \eta, 0) - x_m \geq u_2(y_2) - (z_2 - \eta), \end{aligned} \quad (64)$$

and let $o_2(m, B, 0)$ be a solution to

$$\begin{aligned} \max_{(y, x_m) \in \mathbb{R}_+ \times [0, m]} & \quad -\rho_2 y + x_m \\ \text{s.t.} & \quad u_2(y) - x_m \geq 0. \end{aligned} \quad (65)$$

The solutions to (64) and (65) exist and are unique, with the constraints binding at the optimum. Moreover, it can be verified that

$$o_2(0, G, D) = (y_2, z_2, 0).$$

It is also straightforward to see that, regardless of the buyer's record, his surplus in round-2 DM is independent of m .

We now move to $o_1(m)$. Since the first DM round is non-monitored, the payoff of the buyer in this round cannot depend on his record or on the debt limit. Let $\zeta(m) = u_1(y_1) - z_1$ if $m \geq z_1$ and let $\zeta(m) = 0$ otherwise. In turn, let $o_1(m)$ be a solution to

$$\begin{aligned} \max_{(y,x) \in \mathbb{R}_+ \times [0,m]} \quad & -\rho_1 y + x \\ \text{s.t.} \quad & u_1(y) - x \geq \zeta(m). \end{aligned}$$

The solution to (60) exist and is unique with the constraints binding at the optimum. Hence, for $o_1(m) = (y, x)$ and for a buyer with a record G and m units of real balances upon entering the DM, his continuation value by following the equilibrium is given by

$$\zeta(m) + m + \sigma_2[u_2(y_2) - (z_2 - \eta)] + W_G(0, 0),$$

which can be rewritten as

$$V_G(m) = \zeta(m) + m + \sigma[u_2(y_2) - (z_2 - \eta)] + W_G(0, 0). \quad (66)$$

Similarly, for $o_1(m) = (x, y)$ and for a buyer with a record B and m units of real balances upon entering the DM, his continuation value by following the equilibrium is given by

$$u_1(y) + (m - x) + W_B(0) = \zeta(m) + m + W_B(0),$$

that is,

$$V_B(m) = \zeta(m) + m + W_B(0). \quad (67)$$

Note also that the construction of $o_1(m)$ ensures that there is no profitable deviating offer in the first DM. Now we show that $o_1(z_1) = (y_1, z_1)$. If this is not the case, there exists $(y, x) \neq (y_1, z_1)$, with $x < z_1$, which provide a higher surplus to the seller without violating the constraint. Hence

$$u_1(y) - x \geq u_1(y_1) - z_1 \text{ and } -\rho_1 y + x \geq -\rho_1 y_1 + z_1,$$

which implies

$$u_1(y_1) - \rho_1 y_1 \geq u_1(y_1) - \rho_1 y_1,$$

and $y > y_1$. Since $-\rho_1 y + x \geq -\rho_1 y_1 + z_1$, we have $x > z_1$, a contradiction.

We now show that the following strategies form a simple equilibrium. All agents respond with *yes* to the proposed trades and buyers offer the proposed trades, both on and off the equilibrium path. Buyers with record G always repay their debts up to D , and buyers with record B never repay their debt. All buyers leave the CM with z_1 units of real balances, both on and off the equilibrium path.

First we show that the buyer carry z_1 real balances to the DM, irrespective of his record $R \in \{G, B\}$. By linearity of W_R , the CM problem is given by

$$\max_{m \geq 0} \{-(1 + \tau)m + \beta V_R(m)\},$$

that is,

$$\max_{m \geq 0} \{\beta \{ -[r + (1 + r)\tau]m + \zeta(m) + \mathbf{1}_{R=G} \{ \sigma[u_2(y_2) - (z_2 - \eta)] \} + W_R(0, 0) \} \}.$$

Note that $\zeta(m)$ is constant for all $m \geq z_1$ and is constant for all $m \in [0, z_1]$. Thus, we only need to show that bringing z_1 is better than zero, which will be the case if and only if

$$-[r + (1 + r)\tau]z_1 + u_1(y_1) - z_1 \geq 0.$$

Now, replacing $z_1 = \rho_1 y_1$, this is (41).

Now we show that buyers are willing to repay their debts. Note that, according to the equilibrium behavior,

$$W_G(0, z_2) = -[(1 + \tau)z_1 + (z_2 - \eta)] + \beta \{ \zeta(z_1) + \sigma[u_2(y_2) - (z_2 - \eta)] + z_1 + W_G(0, z_2) \},$$

and hence

$$W_G(0, z_2) = -[(1 + \tau)z_1 + (z_2 - \eta)] + \frac{\beta}{1 - \beta} \{ [u_1(y_1) - z_1] + \sigma[u_2(y_2) - (z_2 - \eta)] - \tau z_1 \}.$$

Similarly,

$$W_B(0) = -(1 + \tau)z_1 + \frac{\beta}{1 - \beta} \{ [u_1(y_1) - z_1] - \tau z_1 \}.$$

We need to show that the buyer wants to keep the record G . He has incentive to repay $z_2 - \eta$ in order to keep the record if and only if

$$-(z_2 - \eta) + \beta V_G(z_1) \geq \beta V_B(z_1),$$

and by (66) and (67), this is equivalent to

$$-(z_2 - \eta) + \beta \{ \sigma[u_2(y_2) - (z_2 - \eta)] + W_G(0, 0) - W_B(0) \} \geq 0,$$

that is,

$$-(z_2 - \eta) + \frac{\beta}{1 - \beta} \{ \sigma[u_2(y_2) - (z_2 - \eta)] \} \geq 0.$$

which holds by (62) and (42) with $z_2 = \rho_2 y_2$.

Proof of Corollary 6: We have shown that inflation can do better than (y_1^c, y_2^c) , which is an upper bound for no intervention by Lemma 8 (i). To show that deflation cannot do better, note that Theorem 3 (i) implies that deflationary policy cannot achieve a higher welfare than (y_1^c, y_2^c) .

Appendix B: Supplemental Material

B1. Monitored Round-1 DM under limited monitoring

Here we consider the case where the first DM round is monitored, in the baseline model where trades are determined by TIOLI offers. In this case, it is straightforward to adapt Lemma 3 and obtain that, if $r \leq \hat{r}_{cm}$, the optimal allocation without intervention is given by $(y_1^{cm}, y_2^{cm}) = (y_1^*, y_2^m)$, where

$$\hat{r}_{cm} = \frac{u_1(y_1^*) - \rho_1 y_1^*}{\rho_1 y_1^*} \quad (68)$$

becomes the new threshold. If, instead, $r > \hat{r}_{cm}$, the optimal allocation is given by $y_2^{cm} = y_2^m$, and y_1^{cm} is the unique positive solution to

$$-r\rho_1 y_1 + u_1(y_1) - \rho_1 y_1 = 0.$$

Conditional on this adaptation, Theorem 1 holds without any change.

In turn, Theorem 2 still holds but it requires a genericity condition:

$$\frac{u'_2(y_2^{cm}) - \rho_2}{u''_2(y_2^{cm})y_2^{cm}} + \frac{u'_1(y_1^{cm}) - \rho_1}{(1+r)\rho_1 - u'_1(y_1^{cm})} \frac{1}{1+r} \neq 0. \quad (69)$$

This condition is generic in the following sense. Suppose that (69) is violated for some u_2 . Perturb u_2 by

$$v_2(y_2; \epsilon) = u_2(y_2) + \epsilon \frac{y_2^{1-\theta}}{1-\theta},$$

where $\theta \in (0, 1)$. Then, except for possibly one knife-edge θ , for all $\theta \in (0, 1)$, (69) holds under $v_2(\cdot; \epsilon)$ for an open interval of ϵ around zero.

B2. Perfect Enforcement

Here we consider our environment but assume that credit trades face no limited commitment friction. More precisely, we shall assume that buyers always pay back their debt made in monitored meetings in the next CM. We maintain the assumption that buyers cannot issue private debt in non-monitored meetings and hence money is still needed to conduct trades in those meetings. For simplicity, we assume that debt is only feasible in the second DM, and money is required to finance consumption in the first DM. The trading mechanism is very simple: buyers always make take-it-or-leave-it offers.

No intervention

As a benchmark, let us first assume a constant money supply. The problem of the buyer in the non-monitored meeting is exactly as in the no monitoring economy in the main text. Due to perfect enforcement, in the monitored round, a buyer can use both money and debt. However, since there is no enforcement issue w.r.t. debt, the buyer can issue as much of debt as he wishes, and hence it is never optimal to use money in that round. Moreover, this implies that the trade in the second DM is given by

$$\max_{y_2 \geq 0} u(y_2) - D \text{ s.t. } D \geq \rho_2 y_2^*,$$

with the solution

$$u'(y_2) = \rho_2, \quad D = \rho_2 y_2. \quad (70)$$

The choice of real balances by the buyer in the CM is then determined by

$$\max_{z_1 \geq 0} -r z_1 + u_1\left(\frac{z_1}{\rho_1}\right) - z_1,$$

where TIOLI offers imply that $\frac{z_1}{\rho_1}$ is the consumption of the buyer if he brings z_1 real balances to use in the first DM round. The following FOC then determines the buyer's real balances:

$$\frac{u'_1(y_1) - \rho_1}{\rho_1} = r. \quad (71)$$

Thus, the trades are then determined by (70) and (71). Note that (70) implies that $y_2 = y_2^*$ and hence round-2 DM credit trade achieves the first-best outcome; this is a straightforward consequence of the fact that there is no enforcement issue in the credit trade.

Optimal intervention

In the main text we considered two classes of interventions: either the monetary authority uses inflation tax to forgive private debt or uses credit tax to increase the real rate of return on money. Since $y_2 = y_2^*$, the inflationary policy that forgives debt cannot be optimal. Hence, we only consider credit taxes. Let χ denote the credit tax (in terms of the CM good).

As in the main text, when solving his problem, each buyer takes as given the nominal interest rate i and the credit tax χ . The solution for choice of real balances in the CM is then determined by the following FOC:

$$\frac{u'_1(y_1) - \rho_1}{\rho_1} = i, \quad (72)$$

the solution of which is denoted $y_1(i)$. Different from the main text, however, there is no enforcement issue for the credit taxes here. Nevertheless, buyers may simply skip the credit trade and do not pay taxes. This gives rise to the following participation constraint:

$$\chi \leq u(y_2) - \rho_2 y_2. \quad (73)$$

The right-side of (73) is the gain from trade for the buyer in the round-2 DM, and (73) simply says that this gain has to be greater than the tax paid. Since the tax does not depend on the size of the trade,¹⁹ it follows that in equilibrium round-2 DM trade is still determined by (70) and hence $y_2 = y_2^*$.

Finally, as in the main text, for any targeted nominal interest rate i , feasibility of the policy still requires

$$\chi(i) = \left(\frac{r-i}{1+r} \right) \rho_1 y_1(i), \quad (74)$$

which is strictly decreasing in i for $i \geq 0$. As a result, we may simply denote a policy by i , and the policy i is implementable if and only if

$$\chi(i) \leq u_2(y_2^*) - \rho_2 y_2^*. \quad (75)$$

We have the following result:

Theorem 7 *The optimal policy is deflationary with $i < r$.*

(a) *Suppose that*

$$\frac{r}{1+r} \rho_1 y_1^* \leq u_2(y_2^*) - \rho_2 y_2^*. \quad (76)$$

Then, the optimal policy is $i = 0$ and the first-best is implementable.

(b) *Suppose that (76) does not hold. Then, the optimal i is given by*

$$\chi(i) = u_2(y_2^*) - \rho_2 y_2^*.$$

Proof. (a) It is straightforward to verify that if (76) holds, then (75) holds for $i = 0$, in which case $y_1(i) = y_1^*$.

(b) Otherwise, we choose the lowest i subject to (75). This gives the result. ■

¹⁹Alternatively, one can consider a proportional tax to credit trades. It can be shown that such tax scheme will lead to lower welfare, but it will not change the qualitative results here.