A Cautionary Tale of Topography and Tilt from Kilauea Caldera

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Abstract We conduct finite element analysis to investigate the effect of sharp topography on surface ground deformation caused by pressure changes in a magma reservoir. Tilt data express the horizontal gradient of vertical displacement and therefore can emphasize small variations in deformation that go unnoticed using other methods. We find that the vertical displacement profile at a surface with a cliff can be thought of as the superposition of the deformation from shallow and deeper sources. This combination can create a small peak in vertical displacement that acts as a pseudo-source, creating a reversal of the deformation gradient and therefore anomalous tilt magnitude and a rotation of up to 180°. We apply these models to Kilauea Caldera and find that surface geometry creates a tilt rotation of ∼10°, partially explaining anomalous tilt that has been observed. Our analysis highlights the importance of considering topography when assessing tilt measurements at active volcanoes.

1. Introduction

Surface deformation is often observed in connection with volcanic unrest. Common methods of monitoring volcano deformation include Global Navigation Satellite System (GNSS), synthetic aperture radar interferometry (InSAR) and tilometers (e.g., Dzurisin, 2006). Tiltmeters measure horizontal gradients (derivatives) of vertical displacements. As such, tilt can emphasize small variations in deformation that might go unnoticed in GNSS or InSAR data.

Observed volcano ground deformation has been attributed to a variety of mechanisms including magma intrusion (Dzurisin, 2003). Analytical solutions, such as the commonly used Mogi (1958) point-source model, can be used to predict deformation patterns. However, these analytical models assume a deep source in a homogeneous, elastic half-space—assumptions which are often violated in the real world (e.g., Cayol & Cornet, 1998). This analytical estimation breaks down under several conditions, including when the reservoir is shallow or the topography is steep. Models have been developed to overcome some of these assumptions, such as nonspherical source geometries (e.g., Yang et al., 1988), subsurface heterogeneity (e.g., Masterlark, 2007), viscoelasticity (e.g., Del Negro et al., 2009), and topographic corrections (Williams & Wadge, 1998, 2000). Williams and Wadge (1998) introduced a simple method of adjusting the elevation of the reference surface using analytical equations based on McTigue (1987) to account for topography. They later introduced a second method that can be used to account for topography by calculating higher-order corrections to approximate the slope (Williams & Wadge, 2000). However, the latter method is only effective when slopes are small.

The surface expressions of many volcanoes feature steep walls or cliffs. These cliffs are often part of calderas or caldera complexes but can also be caused by other processes such as rifting or sector collapse. Many calderas have steep bounding walls hundreds of meters high that plausibly could affect tilt measurements. Rhyolitic calderas can have >1 km of subsidence of the caldera floor (Cole et al., 2005). The effect of sharp variations in topography such as cliffs has not been previously considered in studies of surface deformation in volcanic regions, even though they are a common feature. In addition, monitoring equipment is commonly placed on caldera rims as these locations are often more accessible (especially if the caldera is lake-filled) and have relatively less risk than more proximal locations, such as the caldera floor. Tilt measurements have played a significant role in the understanding of volcanic processes on at least 40 volcanoes worldwide (Gambino & Cammarata, 2017). Many volcanoes with tilt networks have steep topography.
Cayol and Cornet (1998) constructed axis-symmetric models using finite element analysis (FEA) to investigate the effect of slopes up to 30° on tilt and found that in some cases, tilt at the summit of a volcano can be reversed relative to what would be expected with no topography. Neuberg et al. (2018) demonstrated that shear stress from magma ascending in a conduit can affect tilt measurements on the sloping (<30°) flank of the cone at Tungurahua Volcano in Ecuador. Tilt records at several caldera volcanoes are difficult to explain with simple analytical models, including Campi Flegrei (Orsi et al., 1999), Miyakejima (Yamamoto et al., 2001), and Rabaul (McKee et al., 1984). Here, we examine the effect of a step in topography on ground tilt caused by a simple inflation source and apply the model to Kilauea Volcano in Hawai‘i.

2. Models

To assess the control of sharp topography (i.e., a cliff) on surface deformation due to a pressurizing magma reservoir, we constructed a 3-D finite element model using COMSOL Multiphysics. We used three dimensions to allow an azimuthal variation in tilt, which cannot be accounted for using axis-symmetric models (e.g., Cayol & Cornet, 1998; Hickey & Gottsmann, 2014). A simple model was constructed using the methods described in the supporting information (S1) and parameters listed in Table S9, which were chosen to represent Kilauea but are typical of basaltic shield volcanoes. Here we normalized distances by the depth of the pressure source (zsphere) for ease of application to other systems. We used a fixed sphere radius (rsphere) of 0.025 times zsphere, and the height of the cliff (C) and the lateral distance of the cliff from the pressure source (D) were allowed to vary between 0.025–2 times zsphere and 0–10 times zsphere, respectively. We also normalized tilt and vertical displacement in our results because the outcomes scale with the ratio of the pressure of the source (ΔP) to the shear modulus (μ) (McTigue, 1987), and therefore are independent of the magnitude of deformation.

Vertical displacement and tilt vectors resulting from our finite element models can be seen in Figure 1. We observed that tilt, when measured just above the cliff, was different to that with no topography for all cliff geometries. This anomalous tilt is due to a small secondary peak in vertical displacement. This secondary peak is two orders of magnitude smaller than the peak deformation (Figure 1) and so is unlikely to be noticed in GNSS or InSAR measurements, but is visible in tilt measurements as tilt measures the gradient of displacement, rather than absolute ground displacement.

We suggest that the profile of vertical displacement in the presence of a cliff can be thought of as a combination of the displacement profiles from a deep source (depth z_1 = zsphere) and a shallower source (depth z_2 = zsphere - C, where C is the height of the cliff) with no topography. This is because a shallow Mogi-type inflation source in a homogeneous elastic half-space with no surface topography creates a profile of vertical deformation that has a relatively large maximum and a relatively narrow peak (e.g., Figure 2, top, red). In contrast, when a source is deeper, the maximum vertical deformation is smaller and the curve is broader (e.g., Figure 2, top, blue). When the two vertical deformation profiles are plotted together, they will cross at a distance r_c where:

\[ r_c^2 = (1 - C)^{2/3} + (1 - C)^{2/3}. \]  

In this equation, r_c and C are both normalized by zsphere. Figure 2 (top) displays the vertical displacement profiles for inflation sources at z_1 = 0.95 (red) and z_2 = zsphere = 1 (blue) depth. The profile of vertical displacement in the presence of a cliff can be thought of as a superposition of both of these profiles (Figure 2, top green, orange, and magenta). In this case z_1 is the depth of the source beneath the caldera floor, and z_2 is the depth of the source beneath the top of the cliff, which is equal to z_1 + C, where C = 0.05 is the height of the cliff. Figure 2 (middle) displays the radial tilt profiles for the same sources. Most of the tilt is positive because the ground is tilting away from the center of inflation.

If the cliff is approximately the same distance away as the crossover of the profiles (D ∼ r_c), there will not be a sharp change in deformation gradient but rather a smooth transition from one profile to the next (Figure 2 orange at D = 1.38). This can be seen in the transition from one tilt profile to the next without a significant change in magnitude. If the cliff is closer to the source than the crossover of the profiles (D < r_c), there will be a sudden decrease in uplift with distance, seen in the tilt as a sharp peak, but the gradient will not change sign and so the tilt will stay positive (Figure 2 magenta). However, this necessarily means that if the cliff is farther away from the source than the crossover of the profiles (D > r_c), there will be a local secondary
Figure 1. Deformation using parameter values from Table S9. Black arrows show tilt vectors, with red arrow indicating 0.1 μrad. Colors show normalized vertical displacement with cold colors showing full deformation field and bright colored contours showing a narrow range around the lobe displacement. (a) Deformation above a linear cliff with normalized height of 0.25, at a normalized distance of 2.5 horizontally away from the source. (b) Deformation above a linear cliff with normalized height of 0.25, at a normalized distance of 1.25 horizontally away from the source. (c) Deformation above a circular (caldera-like) cliff with normalized height of 0.25, at a normalized distance of 2.5 horizontally away from the source.
Figure 2. Deformation profiles for spherical inflation sources using parameter values from Table S9. Left panels (a and c) show results from finite element analysis. Right show results from an analytical model similar to Williams and Wadge (1998). Top panels (a and b) show the vertical displacement profiles at the surface, normalized by the sphere depth. Middle panels (c and d) show the normalized radial tilt profile. Bottom panels (e and f) show schematics of the models with gray circles representing the inflating pressure source. (e) and (f) are the same but are plotted with different topographies illustrated. In all plots red shows the profiles with the source $0.95 \times z_{\text{sphere}}$ below a flat surface, blue shows the profiles with the source $1 \times z_{\text{sphere}}$ below a flat surface. Green, orange, and magenta show the profiles for a source depth of $1 \times z_{\text{sphere}}$ with a cliff located at 0.5, 1.38 and $2 \times z_{\text{sphere}}$ away, respectively.

In our example, when the cliff is 2 times $z_{\text{sphere}}$ away from the center (green), the inversion of the deformation gradient can be seen where the tilt becomes negative, which means that the ground is tilting toward the inflation source.

The effect of the cliff in the tilt can be seen in both the FEA (Figure 2, left) and the analytical solutions (Figure 2, right) using a method similar to Williams and Wadge (1998). However, using FEA, each element communicates with its neighbors, resulting in the reversal of tilt being smoothed across a wider distance compared to the analytical models, where each point is calculated individually.
Equation (1) can be used to predict the existence of the secondary lobe, but does not contain information about the magnitude of the lobe, nor where the peak is relative to the cliff. The magnitude of the secondary maximum or lobe is dependent on the difference between the cliff-free deformation using source depths of $z_1$ and $z_2$. This is a function of the difference in depths ($z_2 - z_1 = C$, height of the cliff) and the horizontal distance of the cliff from the source ($D$). Figure 3 (blue) shows the maximum vertical displacement in the secondary lobe for $C = 0.025$, $0.5$, and $1.5$ times $z_{sphere}$, for $D = 0 - 5$ times $z_{sphere}$. This secondary lobe of deformation will have a maximum magnitude if the cliff is located where the difference between the profiles is the greatest while $D > r_c$.

The distance of the lobe from the cliff is also dependent on $C$ and $D$. Figure 3 (red) shows the distance of the lobe from the cliff for $C = 0.025$, $0.5$, and $1.5$ times $z_{sphere}$, and $D = 0 - 5$ times $z_{sphere}$. As $D$ increases, the distance of the lobe from the cliff increases.

Figure 1 displays oblique views of the 3-D FEA models for a 1 MPa inflation source with a normalized cliff height of 0.25. Colored contours show the small interval of vertical displacement in which the secondary lobes are visible, and black arrows show tilt azimuth and magnitude. When a linear cliff is used, the secondary lobe creates an elongated virtual deformation source on the cliff (Figures 1a and 1b). This virtual source causes tilt vectors to be rotated from their expected azimuth. When $D > r_c$ (Figure 1a), the secondary lobe of deformation causes nearby tilt vectors to rotate away from it. When $D < r_c$ (Figure 1b), the gradient of deformation is not reversed but there is a steepening in an elongated area. This causes the tilt vectors to have a greater magnitude further away from the cliff and to rotate toward the expected deformation pattern from the primary source.

An axisymmetric model allows the effect of the secondary lobe on a circular caldera to be viewed (Figure 1c). In this case the lobe is circular and therefore does not affect the azimuth of the tilt vectors except for the space between the cliff edge and the lobe maximum. In this region, the tilt vector will be rotated by 180°. Further away from the cliff edge, vector azimuths can be well approximated by a half-space analytical solution. The tilt magnitude is also affected by the presence of the secondary lobe, with it tending to zero close to the peak of the lobe. Above the cliff, tilt magnitude will be slightly larger than expected but the two profiles become more similar as they tend toward zero deformation.
Figure 4. Map of Kīlauea Caldera prior to May 2018, showing tiltmeters (black circles) and hypothesized Halema’uma’u reservoir as red circle. Black vectors indicate averaged tilt data for DID-type events with 95% error ellipses. Red vectors indicate modeled tilt from best fitting Mogi-type inflation source (red circle) from Anderson et al. (2015). Blue vector indicates modeled tilt vectors with simple cliff topography. Inset shows study area location on the Island of Hawai‘i.

3. Case Study: Kīlauea Volcano

Kīlauea Volcano is a basaltic shield volcano on the Island of Hawai‘i (Figure 4). Between 1983 and 2018, eruptive activity was fairly stable with occasional minor shifts (Orr et al., 2015). During that time period there were two primary eruptive centers: along the East Rift Zone centered on the vicinity of the Pu‘u ‘Ō‘ō cone starting in 1983 and at the summit from a lava lake contained within a vent along the southeast side of Halema‘uma‘u crater starting in 2008. Both of these vents ceased activity in mid-2018 due to a major Lower East Rift Zone lava effusion and summit collapse.

Evidence for the geometry of the magmatic plumbing system at Kīlauea comes largely from deformation data (Poland et al., 2014). The so-called Halema‘uma‘u (HMM) deformation source is the shallowest magma reservoir at approximately 1 km below the surface, centered just to the east of Halema‘uma‘u crater (Figure 4). Large-scale deformation during eruption and intrusion events has been attributed to this hypothesized reservoir (e.g., Lundgren et al., 2013). Several authors have estimated the depth of the HMM reservoir using geodetic, seismic, and petrological evidence, and depths range from 0.2 to 5 km below the surface, with the majority of estimates around 1 km below the floor of Kīlauea Caldera (e.g., Almendros et al., 2002; Battaglia et al., 2003; Cervelli & Miklius, 2003; Chouet et al., 2010; Dawson et al., 1999; Dzurisin et al., 1980; Johnson et al., 2010; Ohminato et al., 1998; Poland et al., 2014; Ryan, 1988; Thornber et al., 2015).

Transient deformation events with shorter durations and smaller magnitudes have also been observed as originating from the HMM source. These so-called deflation-inflation (DI) events have been detected with GNSS and InSAR, but have been particularly well recorded by the network of borehole electronic tiltmeters since 1999. The deformation source appears constant over time (Anderson et al., 2015). These repeating events have the benefit that the data can be stacked to increase the signal-to-noise ratio and can be used to accurately locate the HMM source. Anderson et al. (2015) used a Bayesian inverse formulation with a Markov Chain Monte Carlo algorithm to locate the source of DI events to within 600 m horizontally. Several
factors including the geometry of the tilt network, however, prevented the accurate estimation of the depth of this source. Anderson et al. (2015) also noted that, although inversions using most of the summit tiltmeters yielded low errors, one tiltmeter (SMC, Figure 4) consistently degraded the result of the inversions. This was because the vectors from the DI events were consistently rotated by about 25° counterclockwise from that predicted by analytical models (Figure 4). Anderson et al. (2015) proposed several possible reasons for the consistent misfit of SMC, including the effect of local topography.

Tiltmeter SMC is located near a section of the caldera rim that is more linear than other parts of the caldera (Figure 4). The difference of the azimuth and magnitude of tilt at tiltmeter SMC relative to that predicted from a simple analytical model (Figure 4) has been calculated as 28 ± 2° and 20 ± 5%, respectively (Anderson et al., 2015). Therefore, to model the effect of the linear portion of the caldera rim and investigate whether the difference in tilt data can be explained by topography, the top boundary was approximated with a single linear vertical cliff. We conducted a grid search over the depth of the pressure source to find a model that best fits the data. For these models, we no longer normalize the distances.

If we assume that the horizontal location of the HMM source is well constrained (Anderson et al., 2015), we can rotate the reference frame so that the center of the deformation source is at $x = 0, y = 0$, the top of the cliff is at $z = 0$, and the cliff in the vicinity of SMC runs parallel to the $y$ axis. The cliff is known to be 80 m high ($C = 80\text{ m}$) and tiltmeter SMC is approximately 200 m from the cliff edge ($y_{\text{tilt}} = 200\text{ m}$). The distance between the HMM reservoir and the cliff ($D$) is approximately 1,000 m and tiltmeter SMC is approximately 1,500 m along the cliff ($y_{\text{tilt}} = 1,500\text{ m}$).

Using these values, the depth at which the crossover distance ($r_c$) equals $D$ is $z_2 = 748\text{ m}$ from the caldera floor, using equation (1). Therefore, the maximum $z_1$ is 750 m. We assign the minimum $z_2$ as 500 m (the a priori limit set by previous observations, see Anderson et al., 2015, for details). Using $D, C, x_{\text{tilt}}$, and $y_{\text{tilt}}$ defined above and $z_1 = 500\text{ m}$, a secondary lobe is created with its peak only 40.9 m away from the cliff edge. The tiltmeter is far enough away from the secondary lobe that the effect of the secondary lobe is much less than if the tiltmeter were closer, with a tilt rotation of only 10° and a change in tilt magnitude of only 20%. With $z_2 = 750\text{ m}$, the deformation of the lobe is not greater than the deformation at the cliff edge, and the tilt at SMC is affected even less than with a shallower source. We found that changing the radius of the source did not significantly affect these results (see supporting information S6 for more information).

The same analysis for tiltmeter UWE, using $D = 1,500\text{ m}, C = 85\text{ m}, x_{\text{tilt}} = 660\text{ m},$ and $y_{\text{tilt}} = 0$, does not rotate the tilt vector as the cliff is perpendicular to the source-tiltmeter line. However, the magnitude of the tilt at UWE is 6% larger, indicating that inversions for the pressure source using this tiltmeter could also be influenced by the topography. There is not a significant cliff between SDH and the source (17 m), so this analysis would not elucidate any discrepancies in the data from SDH. Tiltmeter IKI is about 500 m away from the edge of Kilauea caldera and so the topography of that caldera does not have a significant effect on the tilt here when the deformation is caused by the HMM source. However, IKI is also near to the edge of Kilauea Iki crater, which is over 100 m deep in places. If there was a deformation source related to Kilauea Iki crater, then it is likely that the more complex topography around IKI would influence the tilt there. Models with realistic topography (supporting information S7) also indicate that the rotation at these other tiltmeters is negligible.

Despite the models of idealized topography displaying tilt rotations of up to 180°, the geometry at Kilauea only allows a maximum rotation of tilt at SMC of 10°. Models using the same source geometry with realistic topography (supporting information S9) agree with this rotation. Therefore, the anomalous tilt data at Kilauea cannot be completely explained by the presence of a topographic step, although we have shown that it exerts significant influence. The simple model with homogeneous physical properties and a spherical pressure source is not adequate to fully explain the anomalous tilt data. We suggest that a more complex source geometry, as suggested by the shape of the partial caldera collapse during the 2018 Lower East Rift Zone eruption, and material heterogeneity, are likely to contribute to the rotation of the tilt data.

The 2018 collapse at Kilauea summit has reshaped the cliffs around the caldera (Wasser & Benitez, 2018). The new geometry has near-vertical cliffs of up to 500 m and terrace-like steps of 50–150 m. These new structures may have an impact on tilt measured at the existing network of tiltmeters and have implications for any new monitoring equipment that is installed.
4. Conclusions

We have conducted FEA of deformation due to a shallow pressure source to characterize the effect of sharp changes in topography. Our results show that steps in topography such as caldera rims can create a secondary lobe of deformation, which can affect tilt data. We have devised a simple relationship between geometry elements (the depth of the pressure source, the height of the cliff, and the distance of the cliff from the pressure source) that allows us to predict the existence of the secondary lobe. Where a secondary lobe is created, its size is as much as two orders of magnitude smaller than the main deformation and so is unlikely to be noticed in GNSS or InSAR measurements, but will be visible in tilt measurements, as tilt measures the gradient of displacement, rather than absolute ground displacement. Our models show that when a cliff runs perpendicular to a line between the source and a tiltmeter then only the tilt magnitude is affected. However, if the cliff is oblique then the tilt azimuth can be rotated by up to 180°, which may introduce errors in data inversion.

During 1999–2018, borehole tilt data at Kilauea Volcano were often characterized by small deformation events that were highly repeatable. These repeating deformation events allow the magma reservoir, in which the pressure transients were occurring, to be well characterized except for the depth of the reservoir. One tiltmeter, located near a linear section of the caldera rim, persistently displayed deformation that does not fit with other data. Our finite element models were applied to a simplified Kilauea summit caldera to investigate whether the anomalous data from this tiltmeter could be due to topography. We found that the geometry of Kilauea Caldera up to early 2018 meant that the maximum tilt rotation from topographic effects was 10°, compared to an observed discrepancy of about 25° between the anomalous tiltmeter data and analytical models that best fit data from other tiltmeters. Therefore, the anomalous tilt data at Kilauea cannot be completely explained by topography, although that may exert some influence. Nevertheless, our analysis does point to the importance of considering topography when assessing tilt measurements at active volcanoes.

These findings have implications for network design and show that sharp topography can have dramatic effects on tilt data. This also implies that other tiltmeters around Kilauea and other volcanoes globally could be affected by caldera rims and other sharp topography, and so tilt magnitude and azimuth should be treated with caution.

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References


