Dynamic analysis of coupling misalignment and unbalance coupled faults

Haifei Wang¹,² and Junjie Gong¹

Abstract
Misalignment is a common fault occurring in the rotor system. However, the response characteristics have not been understood comprehensively, especially the relation between forces or torques and displacements, accelerations, or moments. First, misalignment modeling is investigated in this paper. Two coupled rotor system is modeled by six degrees of freedom. Misalignment effects are considered at coupling location using nodal force vectors and moment vectors. Second, Newmark-β method is used to solve the nonlinear equations. Acceleration, displacement, and force or moment response characteristics are discussed. Some results are obtained as follows: (1) $\frac{2}{C^2}$ will appear in the parallel misalignment forces spectrum, and $0\times$ will appear in the vertical force spectrum; $2\times$, $4\times$, $6\times$ will appear in the angular misalignment moment spectrum. (2) In parallel misalignment simulation, it is found that multifrequency components are more obvious, static components are showed in vertical forces and displacements, $1\times$ is dominated and $2\times$ is weak in the displacement spectrum, and $2\times$ is obvious in the force spectrum; acceleration is periodic impulse signal and $1\times$ and $2\times$ are dominated in its spectrum; vertical displacement is truncated and its values are positive, the orbit looks like an inverted triangle. (3) In angular misalignment simulation, it is found that multifrequency components of response are more obvious, $2\times$ is obvious in the vertical displacement spectrum, and $2\times$ is dominated in the moment spectrum; acceleration is periodic impulse signal, horizontal and vertical displacements are periodic, the orbit looks like a moon or an eight shape, and $2\times$ is obvious in the moment spectrum.

Keywords
Dynamic model, parallel misalignment, angular misalignment, numerical method, nonlinear response

Introduction
Misalignment is a common fault occurring in rotary machine, which is caused by unequal foundation movement or uneven thermal heating in rotor system. Misalignment can be divided into two types, one is the angular misalignment and the other one is the parallel misalignment. Misalignment cannot be reduced wholly. However, the relation between response and forces or moments has not been known comprehensively. Although many researchers acknowledge the adverse effect of misalignment on the dynamic performance of the rotor system, relative fewer studies have been conducted in this regard.

Experimental and theoretical works have been conducted by many researchers. Tejas et al.¹ used the experimental approach for the first time for the determination of magnitude and harmonic nature of the misalignment excitation. The force vector is used for the misalignment coupling stiffness matrix, derived from the experimental data and applied the misalignment coupling stiffness rotor. Alok et al.² used stator current signature to diagnose misalignment.

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It is found that misalignment is responsible for the cause of instability. Hariharan et al.\textsuperscript{3} used experiments and ANSYS analysis to study the shaft misalignment. It is found that the misalignment effect can be amplified when the $2\times$ running speed is close to the system natural frequency. Amit et al.\textsuperscript{4} used Lagrangian approach to derive the misalignment equation. Newton–Raphson method is obtained from the dimensionless form. Discrete wavelet transform is used to analyze the signal. Wang et al.\textsuperscript{5} studied the dual-rotor system with unbalance-misalignment coupling faults from simulation and experiments. Prabhakar et al.\textsuperscript{6} analyzed the transient response of a misalignment rotor–coupling–bearing system passing through the critical speed. The continuous wavelet transform has been used to extract the silent features. Saavedra and Ramirez\textsuperscript{7} deduced a new coupling finite element stiffness matrix and considered the frequency response functions from theory and experiments. Janusz\textsuperscript{8} analyzed the character of the rotor’s longitudinal vibration with large misalignment. Guan et al.\textsuperscript{9} proposed two kinds of dynamic models of shaft misalignment. Simulation and experiments have been done. Michael et al.\textsuperscript{10} compared the spectral analysis, orbitals, and full spectrum. It is found that full spectra can distinguish imbalance from misalignment by looking at forward and reversed phenomena. Sekhar et al.\textsuperscript{11} used higher order finite elements and developed the reaction forces and moments due to flexible coupling misalignment. Wan et al.\textsuperscript{12} used a numerical integration method to analyze the nonlinear dynamic behaviors and stability of a multidisk rotor supported by oil-lubricated journal bearing with coupling misalignment. Li et al.\textsuperscript{13} used simulations and experiments to research the dynamic behaviors and vibration mechanisms of multirotor bearing system with parallel and angular misalignments. Feng et al.\textsuperscript{14} studied the bearing misalignment of an inner-and-outer dual-rotor system of an aero-engine with numerical calculation. Li et al.\textsuperscript{15} established a dynamic model of the misalignment fault of a twin spool rotor with inter-shaft bearing. The vibration features of high- and low-pressure rotors were revealed by numerical analysis and experiment.

In previous research, many researchers have conducted some simulation and experiments on angular and parallel misalignments. However, the relation between forces or torques and response has not been investigated comprehensively. In order to better understand the misalignment faults, a rotor-coupling coupled model is established by six degrees of freedom, and angular and parallel misalignment model is used to simulate fault characteristics. The response characteristics are analyzed to reveal the mechanism of the effect of misalignments with respect to forces (torques) and response.

Misalignment modeling

A rotor model

Figure 1 shows the sketch of two rotors with three disks. Each rotor is supported by two bearings and two rotors are connected by coupling. Figure 2 is the finite element model (FEM) of the two rotor systems where the 6th and 12th nodes of rotor 1 is fixed on the ground, the 3rd and 7th nodes of rotor 2 is fixed on the ground, the first disk is at the 3rd node of rotor 1, the second disk is at the 15th node of rotor 1, and the third disk is at the 6th node of rotor 2. The bearing is simplified as string element. Without misalignments, the two rotor systems are linear systems; otherwise, the whole system with misalignment fault is a nonlinear system.
The main parameters of rotor 1 are as follows: the diameter of the shaft is 30 mm; the length of the shaft is 880 mm, the mass of the two disks is \(m_{p1} = 10\) kg and \(m_{p2} = 10\) kg; the polar moment of inertia of the two disks is \(J_{dp1} = 0.05\) kg/m\(^2\) and \(J_{dp2} = 0.05\) kg/m\(^2\); and the equatorial moment of inertia of the two disks is \(J_{dd1} = 0.025\) kg/m\(^2\) and \(J_{dd2} = 0.025\) kg/m\(^2\), for \(E = 2.1 \times 10^{11}\) Pa, \(\mu = 0.3\), \(\rho = 7800\) kg/m\(^3\).

The main parameters of rotor 2 are as follows: the diameter of the shaft is 40 mm; the length of the shaft is 40 mm, the mass of the disk is \(m_{p3} = 2\) kg; the polar moment of inertia of the disk is \(J_{dp3} = 0.05\) kg/m\(^2\); and the equatorial moment of inertia of the disk is \(J_{dd3} = 0.025\) kg/m\(^2\), for \(E = 2.1 \times 10^{11}\) Pa, \(\mu = 0.3\), \(\rho = 7800\) kg/m\(^3\).

The bearing line stiffness in \(x\) and \(y\) directions are \(7.5 \times 10^6\) N/m and \(5 \times 10^7\) N/m. The line damping is \(2000\) N\cdot m/s. The bearing angular stiffness in \(x\) and \(y\) directions are \(1 \times 10^7\) N/m and \(1 \times 10^8\) N/m. The angular damping is \(100\) N\cdot m/s.

**Misalignment modeling**

**Parallel misalignment.** Figure 3 shows the coordinate system for parallel misalignment of two rotors where two rotors are forced by \([F_{x1} F_{y1} F_{z1} M_{x1} M_{y1} M_{z1}]^T\), \([F_{x2} F_{y2} F_{z2} M_{x2} M_{y2} M_{z2}]^T\); Rotor 1 is driven by motor. There is parallel misalignment \(\Delta x_1, \Delta y_1, \Delta x_2, \Delta y_2\) in \(x\) direction and \(\Delta y_1, \Delta y_2\) in \(y\) direction between two rotors.

Figures 4 and 5 show the mechanical and movement diagram of coupling misalignment fault. In Figure 4, \(O_1\) and \(O_2\) are the rotating center of left and right shaft, \(F_R\) and \(F_L\) are the forces of the left and right parts acting on the bolt, \(P\) is the bolt connector on the coupling, and \(\theta\) is the angle of \(F_R\) and \(y\)-axis. In Figure 5, \(\Delta E\) is the amount of misalignment, the rotating speed of the bolt in misalignment direction is \(\Omega t\), \(R\) and \(r\) are the distance of left and right parts and bolt center, \(F_x\) and \(F_y\) are forces of the left and right parts acting on the bolt in \(x\)- and \(y\)-axes.

In Figure 5, when the coupling is rotating, the rotating radius \(R > r\), so there is a tendency to pull two shafts together and two half couplings are deformed compressively. In Figure 5, \(O_2B\) is perpendicular to \(O_1P\),

![Figure 3. Coordinate system for parallel misalignment of two rotors.](image)

![Figure 4. Mechanical diagram of parallel misalignment fault.](image)
as \( r \gg \Delta E \), \( PB = r \) and \( O_1B \) can be expressed as follows

\[
O_1B = R - r = \Delta E \cos(\Omega t)
\]  

(1)

If the size and material are the same for the two half couplings, their deformations are the same, which can be expressed as follows

\[
d = O_1B / 2 = \Delta E \cos(\Omega t) / 2
\]  

(2)

Assuming that the stiffness of coupling bolt in \( O_1P \) direction is \( k_b \), the force acting on two half couplings can be expressed as follows

\[
F = k_b d = k_b \Delta E \cos(\Omega t) / 2
\]  

(3)

The force is decomposed in \( x \) and \( y \) directions, which can be expressed as follows

\[
F_{x1} = F \sin(\Omega t) = k_b \Delta E \cos(\Omega t) \sin(\Omega t) / 2 = k_b \Delta E \sin(2\Omega t) / 4
\]  

(4)

\[
F_{y1} = F \cos(\Omega t) = k_b \Delta E \cos(\Omega t) \cos(\Omega t) / 2 = k_b \Delta E / 4 + k_b \Delta E \cos(2\Omega t) / 4
\]  

(5)

These two forces acting on the left half coupling will act on the right half coupling, which can be expressed as follows

\[
F_{x2} = -F_{x1}, \quad F_{y2} = -F_{y1}
\]  

(6)

In equations (4) and (5), the forces are varying with \( 2\pi \), i.e. the radius forces will change two times rotating speeds when the coupling rotates for a revolution.

Angular misalignment. Figure 6 shows the sketch of forces and moments for angular misalignment. There is a slight amount of angle misalignment \( \alpha \) between two coupling shafts. It is assumed that the coordinate system \( X_1Y_1Z_1 \) is established in rotor 1, and coordinate system \( X_2Y_2Z_2 \) is established in rotor 2. The centerline of rotor 1 is axis \( Z_1 \) and the centerline of rotor 2 is axis \( Z_2 \) where two rotors are forced by \([F_{x1} \ F_{y1} \ M_{x1} \ M_{y1} \ M_{z1}]^T \) and \([F_{x2} \ F_{y2} \ M_{x2} \ M_{y2} \ M_{z2}]^T \). Rotor 1 is driven by the motor.

Figure 7 shows torque decomposition of angular misalignment in three directions. Torque \( T \) from rotor 1 is transmitted to rotor 2, which can be decomposed into two parts, and can be expressed as follows

\[
\begin{align*}
T_x &= T \cos \alpha \\
T_y &= T \sin \alpha
\end{align*}
\]  

(7)
Ts can be decomposed into two components in x and y directions, which can be expressed as follows

\[
\begin{align*}
T_x &= T \sin \alpha \cos \beta \\
T_y &= T \sin \alpha \sin \beta
\end{align*}
\]  

(8)

As rotor 2 rotates around axis Z, \( T \cos \alpha = J_R \dot{\omega} \) where \( J_R \) is the polar moment of inertia and \( \dot{\omega} \) is the angle acceleration of rotor.

As for rotors with angle misalignment \( \alpha \), the relation between two angle velocities is expressed as follows

\[
\frac{\omega_{R2}}{\omega_{R1}} = \frac{C}{1 + D \cos(2\Omega t)}
\]  

(9)

where \( \omega_{R1} \) and \( \omega_{R2} \) are the two rotor’s angle velocities and \( C \) and \( D \) can be expressed as follows

\[
\begin{align*}
C &= \frac{4 \cos \alpha}{3 + \cos 2\alpha} \\
D &= \frac{1 - \cos \alpha}{3 + \cos 2\alpha}
\end{align*}
\]  

(10)

Equation (9) can be expanded as follows

\[
\frac{\omega_{R2}}{\omega_{R1}} = A_0 - A_2 \cos 2\Omega t + A_4 \cos 4\Omega t - \cdots + (-1)^n A_{2n} \cos 2n\Omega t + \cdots
\]  

(11)

\[
\begin{align*}
A_0 &= C \left( 1 + \frac{D^2}{2} + \frac{3D^4}{8} + \frac{5D^6}{16} + \frac{35D^8}{128} + \cdots \right) \\
A_2 &= C \left( D + \frac{3D^3}{4} + \frac{5D^5}{8} + \frac{35D^7}{64} + \cdots \right) \\
A_4 &= C \left( \frac{D^2}{2} + \frac{D^4}{2} + \frac{15D^6}{32} + \frac{7D^8}{16} + \cdots \right)
\end{align*}
\]  

(12)
where $n = 1, 2, \ldots$.

Equation (11) can be differentiated as follows

$$
\frac{\partial^2 \phi_{R2}}{\partial \phi_{R1}} = B_2 \sin 2\Omega t - B_4 \sin 4\Omega t + \cdots + (-1)^{n+1} B_{2n} \sin 2n\Omega t + \cdots
$$

(13)

where $B_2 = 2A_2\Omega$, $B_4 = 4A_4\Omega$, $B_6 = 6A_6\Omega$.

Equation (13) is substituted in equation (8), and input torque can be expressed as follows

$$
T = \frac{J_R \Omega^2}{\cos \alpha} \sum_{n=1}^{\infty} (-1)^{n+1} B_{2n} \sin 2n\Omega t
$$

(14)

Equation (14) is substituted in equation (7) and torques in $x$ and $y$ directions can be expressed as follows

$$
\begin{cases}
T_x = \sum_{n=1}^{\infty} E_{2n} \sin 2n\Omega t \\
T_y = \sum_{n=1}^{\infty} G_{2n} \sin 2n\Omega t
\end{cases}
$$

(15)

where $E_{2n} = (-1)^{n+1} \frac{J_R \Omega^2}{\cos \alpha} B_{2n} \tan \alpha \cos \beta$

$G_{2n} = (-1)^{n+1} \frac{J_R \Omega^2}{\cos \alpha} B_{2n} \tan \alpha \sin \beta$

In equation (15), the misalignment forces can be expressed as follows

$$
F = \sum_{n=1}^{\infty} F_{2n} \sin 2n\Omega t
$$

(16)

where $F_{2n} = [0, \cdots, E_{2n}, G_{2n}]^T$.

**Solving method**

Newmark-$\beta$ method is used to solve the differential equation of two rotor systems, and the movement equation can be illustrated as follows

$$
M_s \ddot{q} + (C_s - \omega G_s) \dot{q} + K_s q = Q_s
$$

(17)

where $Q_s$ is the force vector, $M_s$ is the mass matrix, $G_s$ is the gyroscopic torque matrix, $K_s$ is the stiffness matrix, and $C_s$ is the damper matrix.

In this paper, $C_s$ is assumed to be proportional damping, which can be expressed as

$$
C_s = z_0 M_s + z_1 K_s
$$

(18)

where $z_0, z_1$ are constants, which can be obtained from modal experiments. In this paper, $z_0, z_1$ are set as 5 and $1.35 \times 10^{-5}$, respectively.

**Results and discussion**

**Characteristics analysis of parallel misalignment**

In the simulation of parallel misalignment, misalignment amount $\Delta E$ is $1 \times 10^{-3}$ m and stiffness $k_k$ is $1 \times 10^5$ N/m. The acceleration, displacements, and forces of coupling of rotor 1 (node 1) are obtained.
Figure 8(a) to (e) shows the waterfall of acceleration, displacement, and forces. In Figure 8(a), \( \frac{1}{C^2} \), \( \frac{2}{C^2} \), and \( \frac{4}{C^2} \) will appear in the acceleration waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min; \( \frac{6}{C^2} \) and \( \frac{8}{C^2} \) will appear when the rotating speeds are between 5000 r/min and 6000 r/min in the acceleration waterfall due to the resonance of rotor system.

In Figure 8(b) and (c), \( \frac{1}{C^2} \) and \( \frac{2}{C^2} \) will appear in the displacement waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min. Meanwhile, \( \frac{0}{C^2} \) will appear in the vertical displacement waterfall. Also, when the rotating speed is at the first-order resonance speed 2400 r/min, \( \frac{0}{C^2} \) is dominant.

In Figure 8(d) and (e), \( \frac{2}{C^2} \) is dominant in the force waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min. \( \frac{4}{C^2} \) and \( \frac{6}{C^2} \) will appear in the force waterfall when the rotating speeds are around the second-order resonance speed 6000 r/min. \( \frac{0}{C^2} \) will appear in the vertical force waterfall.

Figure 8(a) to (e) shows the waterfall of acceleration, displacement, and forces. In Figure 8(a), \( 1 \times, 2 \times \) will appear in the acceleration waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min; \( 4 \times \) will appear in the acceleration waterfall when the rotating speeds are between 5000 r/min and 10,000 r/min; \( 6 \times \) and \( 8 \times \) will appear when the rotating speeds are between 5000 r/min and 6000 r/min in the acceleration waterfall due to the resonance of rotor system.

In Figure 8(b) and (c), \( 1 \times, 2 \times \) will appear in the displacement waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min. Meanwhile, \( 0 \times \) will appear in the vertical displacement waterfall. Also, when the rotating speed is at the first-order resonance speed 2400 r/min, \( 0 \times \) is dominant.

In Figure 8(d) and (e), \( 2 \times \) is dominant in the force waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min. \( 4 \times \) and \( 6 \times \) will appear in the force waterfall when the rotating speeds are around the second-order resonance speed 6000 r/min. \( 0 \times \) will appear in the vertical force waterfall.

Figure 9(a) to (k) shows the acceleration, displacements, orbits, and force and response spectrums when the rotating speed is 3800 r/min. In Figure 9(a) and (b), acceleration is periodic impulse signal and \( 1 \times \) and \( 2 \times \) are dominated in its spectrum. In Figure 9(c) to (f), horizontal and vertical displacements are periodic signal. However, vertical displacement is truncated and its values are positive, which means its \( 0 \times \) is obvious in its spectrum. The orbit looks like an inverted triangle in Figure 9(g). In Figure 9(h) to (k), the horizontal and vertical force waves are periodic signals and \( 2 \times \) is obvious. However, the vertical force is positive and its \( 0 \times \) is obvious in its spectrum.

Figure 10(a) to (k) shows the acceleration, displacements, orbits, and force and response spectrums when the rotating speed is 5200 r/min. In Figure 10(a) and (b), acceleration is the periodic impulse signal and \( 1 \times, 2 \times 4 \times, \ldots \),
Figure 9. Acceleration, displacements, orbits, and forces and response spectrum when the rotating speed is 3800 r/min:
(a) acceleration wave; (b) acceleration spectrum; (c) horizontal displacement wave; (d) horizontal displacement spectrum;
(e) vertical displacement wave; (f) vertical displacement spectrum; (g) orbit; (h) horizontal force wave; (i) horizontal force spectrum;
(j) vertical force wave; and (k) vertical force spectrum.
Figure 10. Acceleration, displacements, orbits, and force and response spectrum when the rotating speed is 5200 r/min:
(a) acceleration wave; (b) acceleration spectrum; (c) horizontal displacement wave; (d) horizontal displacement spectrum;
(e) vertical displacement wave; (f) vertical displacement spectrum; (g) orbit; (h) horizontal force wave; (i) horizontal force spectrum;
(j) vertical force wave; and (k) vertical force spectrum.
6\times, and 8\times are dominated in its spectrum. In Figure 10(c) to (f), horizontal and vertical displacements are periodic signals. The orbit looks like an inverted triangle in Figure 10(g). In Figure 10(h) to (k), the horizontal and vertical force waves are periodic signals and 2\times is obvious. However, the vertical force is positive and its 0\times is obvious in its spectrum.

Characteristics analysis of angular misalignment

In the simulation of angular misalignment, the angular misalignment amount is 5°, the polar initial moment \( J_R \) is \( 5 \times 10^{-3} \) kg/m², and angle \( \beta \) is 30°. The acceleration, displacements, and moments of coupling of rotor 1 (node 1) are obtained.

Figure 11(a) to (e) shows waterfall of acceleration, displacement, and moments. In Figure 11(a), 1\times, 2\times will appear in the acceleration waterfall when the rotating speeds are between 5000 r/min and 10,000 r/min. In Figure 11(b) and (c), 1\times, 2\times will appear in the displacement waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min. In Figure 11(d) and (e), 2\times is dominant in the moment waterfall when the rotating speeds are between 1000 r/min and 10,000 r/min.

Figure 12(a) to (k) shows the acceleration, displacements, orbits, and moments and response spectrums when the rotating speed is 6400 r/min. In Figure 12(a) and (b), acceleration is periodic impulse signal and 1\times, 2\times are dominated in its spectrum. In Figure 12(c) to (f), horizontal and vertical displacements are periodic signals and 1\times, 2\times are dominated in their spectrums. The orbit looks like a moon in Figure 12(g). In Figure 12(h) to (k), the horizontal and vertical moment waves are periodic signals and 2\times is obvious in their spectrums.
Figure 12. Acceleration, displacements, orbits, and force and response spectrum when the rotating speed is 6400 r/min:
(a) acceleration wave; (b) acceleration spectrum; (c) horizontal displacement wave; (d) horizontal displacement spectrum; (e) vertical displacement wave; (f) vertical displacement spectrum; (g) orbit; (h) horizontal moment wave; (i) horizontal moment spectrum; (j) vertical moment wave; and (k) vertical moment spectrum.
Figure 13. Acceleration, displacements, orbits, and force and response spectrum when the rotating speed is 7600 r/min:
(a) acceleration wave; (b) acceleration spectrum; (c) horizontal displacement wave; (d) horizontal displacement spectrum; (e) vertical displacement wave; (f) vertical displacement spectrum; (g) orbit; (h) horizontal moment wave; (i) horizontal moment spectrum; (j) vertical moment wave; and (k) vertical moment spectrum.
Figure 13(a) to (k) shows the acceleration, displacements, orbits, and moments and response spectrum when the rotating speed is 7600 r/min. In Figure 13(a) and (b), acceleration is the periodic impulse signal and $1 \times, 2 \times$ are dominated in its spectrum. In Figure 13(c) to (f), horizontal and vertical displacements are periodic signals and $2 \times$ is dominated in their spectrums. The orbit looks like an eight shape in Figure 13(g). In Figure 13(h) to (k), the horizontal and vertical moment waves are periodic signals and $2 \times$ is obvious in their spectrums.

**Conclusion**

In this paper, parallel and angular misalignment models are established. A rotor system is established by FEM considering coupling misalignments. Some results are obtained as follows:

1. In parallel misalignment simulation, $2 \times$ will appear in the horizontal and vertical misalignment forces, and there is $0 \times$ in the vertical force. In angular misalignment simulation, $2 \times, 4 \times, 6 \times$ will appear in the horizontal and vertical misalignment moments.

2. In parallel misalignment simulation, it is found that multifrequency components of response are more obvious, static components ($0 \times$ in the frequency domain) will appear in the vertical force and displacement spectrums, $1 \times$ is dominated and $2 \times$ is weak in the displacement spectrum, and $2 \times$ is obvious in the force spectrum. Acceleration is periodic impulse signal and $1 \times$ and $2 \times$ are dominated in its spectrum. Vertical displacement is truncated and its values are positive. Its orbit looks like an inverted triangle. The vertical force is positive and $0 \times$ is obvious in its spectrum.

3. In angular misalignment simulation, it is found that multifrequency components of response are more obvious, $2 \times$ is obvious in the vertical displacement spectrum, and $2 \times$ is dominated in the moment spectrum. Acceleration is periodic impulse signal, horizontal and vertical displacements are periodic signals, the orbit looks like a moon or an eight shape, and $2 \times$ is obvious in the moment spectrum.

**Declaration of conflicting interests**

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