Zhang, B., Kawashita, L. F., & Hallett, S. R. (2020). Composites fatigue delamination prediction using double load envelopes and twin cohesive models. *Composites Part A: Applied Science and Manufacturing*, 129, [105711].
<https://doi.org/10.1016/j.compositesa.2019.105711>

Peer reviewed version

License (if available):
CC BY-NC-ND

Link to published version (if available):
[10.1016/j.compositesa.2019.105711](https://doi.org/10.1016/j.compositesa.2019.105711)

[Link to publication record in Explore Bristol Research](#)
PDF-document

This is the author accepted manuscript (AAM). The final published version (version of record) is available online via Elsevier at <https://www.sciencedirect.com/science/article/pii/S1359835X19304609#!>. Please refer to any applicable terms of use of the publisher.

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

Composites fatigue delamination prediction using double load envelopes and twin cohesive models

Bing Zhang*, Luiz F. Kawashita, Stephen R. Hallett

Bristol Composites Institute (ACCIS),

University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, UK

Abstract

This paper presents an explicit finite element methodology for predicting fatigue delamination in composite laminates using twin cohesive models and a combined static & fatigue cohesive formulation; one model is loaded under the peak-load envelope, whilst the other model is loaded under the trough-load envelope. The twin models contain pairs of twin cohesive interface elements that predict delamination growth by exchanging data at every time increment. The cohesive formulation evaluates fracture mechanics parameters, e.g. the local minimum to maximum fracture energy ratio via local information associated with the twin cohesive elements, without the need to know the global loading information, e.g. the global R ratio. The method allows predicting the mechanical condition of a laminate at both the peak and trough loads. This method is validated by multiple test cases with varying mode mixities and R ratios, showing a high computation efficiency.

Keywords: B. Delamination; B. Fatigue; C. Cohesive interface modelling; C. Finite element analysis (FEA)

*Corresponding author: b.zhang@bristol.ac.uk (Bing Zhang); +44 (0) 117 33 15311

DECLARATIONS OF INTEREST: NONE

1. Introduction

Fibre-reinforced plastic composites demonstrate excellent in-plane specific stiffness and strength by laminating plies in multiple orientations. However, their relatively lower resistance against interlaminar disbond (delamination) is a major concern for structural engineers. Mechanical fatigue is a typical cause of delamination. Experimental characterisation of a composite structure is always very expensive, which is particularly true for fatigue cases since an extremely large number of cycles may be needed. It is thus particularly important to establish effective numerical models to evaluate the mechanical performance of composite laminates subjected to cyclic loading [1]. Most of the numerical models for fatigue delamination predictions are based on fracture mechanics or damage mechanics. One critical point of the fracture mechanics modelling methods presented in the literature is to determine the energy release rate (G) components, then a modified form of Paris law and G - N curves achieved by experimental measurements are employed for evaluating fatigue crack propagation and initiation, respectively. Two typical approaches to compute the G components are the virtual crack closure technique (VCCT) [2–4] and the level set method [5–7].

Cohesive-zone models (CZMs), built based on damage mechanics, have been widely used to predict delamination. The past two decades have also seen the extension of CZMs into fatigue delamination prediction [1]. The existing fatigue CZMs in the literature are implemented in two approaches: the loading-unloading hysteresis (LUH) method and the single load envelope (SLE) method [8]. In the LUH method, the laminate model is globally loaded in the same manner as in actual tests whilst locally the loading and unloading processes of cohesive elements are fully described, which allows knowing the mechanical condition of the laminate throughout a whole cycle (see Fig. 1a). A cycle jump strategy is usually needed in a LUH model to reduce the computational cost [9,10], as shown in Fig. 1a. Large kinetic energies may be generated if the LUH method is implemented by an explicit FE scheme, thus the LUH method is more preferred in an implicit FE algorithm. The SLE method is a more efficient modelling approach for an explicit FE scheme, as it globally loads a model in a static manner by following the maximum peak envelope of actual cycles (see Fig. 1b). Locally, cohesive elements follow a fatigue constitutive law that is usually an extension of a bilinear static cohesive law [11–18], taking into account an assumed number of fatigue cycles in a given time increment. The number of cycles elapsed in a fatigue model over one second is here defined as ‘numerical fatigue frequency’, which is usually much larger than the fatigue frequency of actual tests. The SLE approach allows the application of a high numerical fatigue frequency without invoking non-trivial kinetic energies, thus reducing the computational cost.

The combined static/fatigue cohesive law used in a SLE model is schematically illustrated in Fig. 2a. Under static loading (see Fig. 2a), damage initiation is determined by the mode I and II strength pair. Crack propagation is controlled by a static damage variable D_s , which linearly degrades the stiffness pair, as shown in Fig. 2b. Under fatigue loading, damage initiation is usually predicted by a failure criterion devised based on experimental S-N curves [16,19,20]; when the initiation criterion is satisfied, the mode I/II strength pair is scaled or reduced to zero [16,20]. Good agreement has been achieved in some cases, however, building up a thoroughly validated fatigue damage initiation approach is still a challenge, which becomes even more difficult due to the generally large scatter in experimental results [21,22]. Compared to static damage propagation, fatigue damage propagation features more rapid degradation of the mode I and II stiffness pair in the softening region of the cohesive law (red lines in Fig. 2a). A fatigue damage variable D_f is usually employed to describe the fatigue stiffness degradation, thus as shown in Fig. 2b the overall stiffness degradation is controlled by the total damage variable D_t , which equals the sum of D_f and D_s :

$$D_t = D_s + D_f \quad (1)$$

The fatigue damage variable D_f can be derived as the numerical integral of the fatigue damage rate dD_f/dN over the number of cycles (N). In the SLE models proposed in Refs. [11,23], dD_f/dN was devised as a function of the interface separation and the total damage variable; satisfactory predictions were demonstrated for mode I, mode II and mixed-mode I/II cases. Most of the SLE cohesive models evaluate dD_f/dN based on a modified Paris law [12–16,24,25]. Turon *et al.* [12] derived a dD_f/dN formula by assuming that the increase in the damaged area along a crack front equals the sum of the damaged areas of all the cohesive-zone elements. The same assumption was also employed by Pironi and Moroni [24], and Harper and Hallett [14]. Bak *et al.* [15] derived a dD_f/dN formula via establishing a link between the local damage evolution and the global crack propagation; the formula has more recently been applied to three-dimensional cases by Carreras *et al.* in Ref. [17], in combination with their efficient approach to evaluate the crack propagation direction as presented in Ref. [26]. Kawashita and Hallett proposed a crack-tip tracking fatigue algorithm that assumes that only the crack-tip element experiences fatigue degradation [13]; the dD_f/dN of a crack-tip element was evaluated by the element length measured along the crack propagation direction and a modified Paris law, without the need to know the total cohesive zone length. More recently, the crack-tip tracking algorithm has been applied to multi-interface cases [16,27].

The SLE methods allow predictions of crack propagation at the maximum load peaks, however, in some circumstances a knowledge of the cohesive model at the minimum (trough) load would also be required. All the

SLE methods require to use the global R ratio to estimate the local strain energy release rate (SERR) ratio, i.e. the ratio (R_G) of the minimum SERR, G_{min} , to the maximum SERR, G_{max} , within one cycle, using Eq. 2 [12–16,24,25,27]. This assumption is derived from linear elastic fracture mechanics (LEFM).

$$R_G = \frac{G_{min}}{G_{max}} = R^2 = \left(\frac{P_{min}}{P_{max}}\right)^2 \quad (2)$$

This paper proposes a double load envelope (DLE) method based on twin cohesive models, which allows evaluating the local SERR ratio (R_G) directly, from connected elements within each model, without the need to know the global R ratio. It also allows predicting the mechanical condition of a laminate at the points of maximum and minimum loads. The main objectives of this study cover: 1) giving very detailed description of the twin-cohesive-model based DLE modelling strategy and 2) applying it to fatigue cases with varying mode mixities and R ratios. Section 2 gives the detailed description of the DLE method for its static and fatigue parts. Section 3 verifies the modelling strategy through mode I Double Cantilever Beam (DCB), mode II four-point End Notched Flexure (4ENF) and Mixed-Mode Bending (MMB) tests. The numerical method is then employed to predict the fatigue performance of a central cut-ply specimen in Section 4, in comparison with experiments.

2. Double load envelope method

2.1 Overall framework

Fig. 3 illustrates the overall framework of the DLE method, which is implemented here by the explicit finite element algorithm. It describes a cyclically loaded composite laminate using twin cohesive models. Firstly, the twin models are respectively loaded to the maximum and minimum load peaks of the first fatigue cycle in a static step. The static step can follow a linear load curve, or a smooth polynomial load curve to reduce possible kinetic effects [13,14]. The two models may need to be held at the load peaks for some time prior to fatigue loading to reduce possible dynamic effects. In the following fatigue step, the twin models are loaded by separate load envelopes, which comprise the maximum and minimum peaks of actual cycles, respectively. Since the twin cohesive models jointly describe the same composite laminate, these two load envelopes remain synchronised throughout the whole loading history. Note that the load R ratio (i.e. the ratio between the minimum and maximum load peaks) is allowed to change in this modelling strategy, since as shown in the next section, the local damage information is purely predicted by the local information associated with twin elements.

Locally, each segment of an interlaminar interface is described by the combination of twin cohesive elements, which follow the mixed mode static and fatigue cohesive law as introduced in Sections 2.2 and 2.3. The less deformed element represents the mechanical condition of the interface segment at the trough of a cycle, while the more deformed element represents the mechanical condition of the interface segment at the peak of a

cycle. Since these two twin cohesive elements jointly describe the same segment of an interlaminar interface, they should share the same static damage variable D_s and fatigue damage variable D_f . For the same reason, they should simultaneously enter the stiffness degradation region of the cohesive law due to either static damage initiation or fatigue damage initiation. The fatigue initiation algorithms are not addressed in the current study, since all cases presented initiate in the static loading phase. The constitutive synchronisations between the twin cohesive elements will be seen in the cohesive law, which is introduced in Sections 2.2 and 2.3, based on single integration-point (IP) elements, and then extended to four-IP elements in Section 2.4.

2.2. Static cohesive formulation

The static and fatigue cohesive law used for the DLE method is also an extension to a bilinear cohesive law, as illustrated in Fig. 2a. Deformation of a cohesive element is described by three orthogonal displacements (δ_{33} , δ_{13} and δ_{23}) between the top and bottom surfaces of the element in a local coordinate system, whose 3-axis follows the thickness direction of the element; the 1-2 plane is orthogonal to the 3-axis, whilst the specific directions of 1-axis and 2-axis are not critical as the resultant shear displacement is used for mode II displacement δ_{II} :

$$\delta_{II} = \sqrt{\delta_{13}^2 + \delta_{23}^2} \quad (3)$$

Mode I (opening) displacement δ_I and mixed-mode displacement δ_m are defined by:

$$\delta_I = \max(0, \delta_{33}), \quad \delta_m = \sqrt{\max(0, \delta_I)^2 + \delta_{II}^2} \quad (4)$$

Prior to damage initiation, the twin elements deform elastically (recall Fig. 2a) by following Eq. 5; σ and K indicate interface traction and elastic stiffness, respectively.

$$\sigma_I = K_I \delta_I, \quad \sigma_{II} = K_{II} \delta_{II} \quad (5)$$

Damage initiation is determined by a quadratic stress criterion [28]:

$$\left(\frac{\max(0, \sigma_I)}{\sigma_I^{\max}} \right)^2 + \left(\frac{\sigma_{II}}{\sigma_{II,E}^{\max}} \right)^2 = 1 \quad (6)$$

where $\sigma_I^{\max}$ is mode I strength; $\sigma_{II,E}^{\max}$ is enhanced mode II strength due to through-thickness compression (TTC), as defined by:

$$\sigma_{II,E}^{\max} = \sigma_{II}^{\max} - \eta \cdot \min(0, \sigma_I) \quad (7)$$

where $\sigma_{II}^{\max}$ is mode II strength without TTC enhancement; η is the enhancement factor, which can be determined by experimental measurements [29].

A static cohesive zone usually covers more than one element to give satisfactory prediction [30,31]. For any two twin cohesive elements in the cohesive zone, once one of the two twin elements satisfies the initiation criterion defined by Eq. 6, it means that damage is initiated in the interface described, thus both elements start entering the stiffness degradation stage (softening region of the cohesive law). This guarantees the synchronisation of the twin elements regarding damage initiation. In the stiffness degradation stage, the stiffness pairs of both elements are controlled by a common damage variable D_s , which increases from 0 at damage initiation to 1 at full failure. The static damage variable is the maximum of the individual damage variables that are obtained by treating the twin elements separately:

$$D_s = \max(^1D_s, ^2D_s) \quad (8)$$

where 1D_s and 2D_s are two transitional damage variables that are separately calculated for two twin elements by Eq. 9. The damage variable definition in Eq. 9 corresponds to the ratio of the damaged area to the total area of a cohesive element [12,28].

$$^iD_s = \frac{\delta_m - \delta_m^0}{\delta_m^f - \delta_m^0}, \quad i = 1, 2 \quad (9)$$

In Eq. 9, δ_m^0 and δ_m^f are the mixed-mode displacements at damage initiation and full failure, respectively. A power law criterion is used here for full failure:

$$\left(\frac{G_I}{G_{IC}}\right)^\alpha + \left(\frac{G_{II}}{G_{IIC,E}}\right)^\alpha = 1 \quad (10)$$

where the power $\alpha \in [1,2]$ is an empirical factor derived from mixed-mode tests for interlaminar fracture [28]. G_{IC} is mode I critical SERR. $G_{IIC,E}$ is the enhanced mode II critical SERR due to TTC, as defined by [32]:

$$G_{IIC,E} = \left(\frac{\sigma_{II,E}^{\max}}{\sigma_{II}^{\max}}\right)^2 G_{IIC} \quad (11)$$

where G_{IIC} is the un-enhanced mode II critical SERR. Mode mixity $\emptyset$ is here defined as the ratio of mode II SERR, G_{II} , to the sum of mode I SERR (G_I) and G_{II} . Mode I and II tractions in the stiffness degradation region are determined for both elements by:

$$\sigma_I = K_I^D \delta_I, \quad \sigma_{II} = K_{II}^D \delta_{II} \quad (12)$$

where the degraded stiffnesses K_I^D and K_{II}^D are determined by the static damage variable D_s for the more deformed one of the twin cohesive elements [12,28]:

$$K_I^D = (1 - D_s) \frac{\delta_m^0}{\delta_m} K_I, \quad K_{II}^D = (1 - D_s) \frac{\delta_m^0}{\delta_m} K_{II} \quad (13)$$

To derive the degraded stiffnesses for the less deformed twin element, it is required to use the damage variable (determined by the more deformed element in Eq. 9) in combination with the deformation condition of the element. One could imagine that the less damaged element is always accompanied by a reference mixed-mode bilinear traction-separation curve. This is illustrated in Fig. 4a. The profile of the reference curve can be determined by the static cohesive formulation as described above, in combination of the cohesive properties (K_I , K_{II} , G_{IC} and G_{IIC}) and the mode mixity of the less deformed element. In detail, the mixed-mode displacements for damage initiation and full failure on the reference curve, i.e. $\delta_m^{0,Ref}$ and $\delta_m^{f,Ref}$ in Fig. 4a are respectively calculated by Eqs. 6 and 10. The traction peak shown on the reference curve, i.e. $\sigma_m^{max,Ref}$ is determined by first decomposing $\delta_m^{0,Ref}$ into individual modes, then using Eq. 5 to get individual mode traction peaks (i.e. $\sigma_I^{max,Ref}$ and $\sigma_{II}^{max,Ref}$), and finally combining these two components by $\sigma_m^{max,Ref} = ((\sigma_I^{max,Ref})^2 + (\sigma_{II}^{max,Ref})^2)^{1/2}$. These two displacements ($\delta_m^{0,Ref}$ and $\delta_m^{f,Ref}$) are substituted into Eq. 9 to get the reference displacement δ_m^{Ref} corresponding to the current damage variable. This is mathematically expressed by:

$$\delta_m^{Ref} = \delta_m^{0,Ref} + (\delta_m^{f,Ref} - \delta_m^{0,Ref})D_s \quad (14)$$

then δ_m^{Ref} and $\delta_m^{0,Ref}$ are substituted into Eq. 13 by replacing δ_m and δ_m^0 to get the degraded mode I and II stiffnesses of the less deformed element, which is illustrated by $K^D(D_s)$ in Fig. 4a. Finally, Eq. 12 is used to calculate the traction components for the less deformed twin element. Fig. 4a illustrates a typical individual mode (I or II) component of the traction versus separation response associated with the less deformed twin element for both static and fatigue parts. The fatigue part of the response will be discussed in the next section. Note that if there is no difference between the twin cohesive elements regarding mode mixity and through-thickness enhancements, the reference bilinear curve will be the same as the mixed-mode traction versus separation response of the more damaged element.

2.3. Fatigue cohesive formulation

The crack-tip tracking algorithm as developed in [13] is used, which means that the crack-tip cohesive elements in cohesive zones follow both the fatigue crack propagation law explained here and the static cohesive law as presented in Section 2.2, whilst the remaining cohesive elements in cohesive zones follow only the static cohesive law. Fatigue delamination propagation is described by a variant of the Paris law:

$$\frac{da}{dN} = C \left(\frac{G_{max}}{G_C} \right)^m \quad (15)$$

where a is the delamination length; G_C is the critical SERR under a mode mixity, assessed by Eqs. 6 and 10. In detail, the mode I and mode II yield stresses at failure initiation determined by Eq. 6 are respectively indicated by

σ_I^Y and σ_{II}^Y in Fig. 2a, while the mode I and mode II displacements at full failure determined by Eq. 10 are respectively indicated by δ_I^f and δ_{II}^f in Fig. 2a. The critical SERR is the sum of the areas associated with the two triangles covered under blue lines in the mode I and mode II subplots in Fig. 2a; this can be mathematically expressed as $G_C = \frac{1}{2}\sigma_I^Y\delta_I^f + \frac{1}{2}\sigma_{II}^Y\delta_{II}^f$. C and m in Eq. 15 are respectively the Paris law constant and exponent. These two parameters, determining the fatigue crack propagation rate (da/dN), are sensitive to both the mode mixity and SERR ratio [8]. Allegri *et al.* [33] proposed to consider the effects of both mode mixity and SERR ratio by formulating the Paris law exponent (m) as:

$$m = \frac{b}{(1-\sqrt{R_G})^{1+\alpha}} e^{-h\phi} \quad (16)$$

where the parameter b controls the slope of the da/dN versus $G_{\min}/G_{\max}$ curve in pure mode I, while the parameter h reflects the effect of SERR ratio on the fatigue crack propagation rate; These three material dependent parameters (b, h, C) need to be calibrated through curve fitting experimental data, which is normally achieved by DCB, ENF and MMB benchmark tests at different R ratios and mode mixities. The parameter α is employed to describe the interaction between SERR ratio and mode mixity:

$$\alpha = \frac{G_C - G_{IC}}{G_{IIc,E} - G_{IC}} \quad (17)$$

In the DLE method, the $G_{\max}$ used in Eq. 15 is the maximum SERR of the twin elements, as mathematically expressed by:

$$G_{\max} = \max(^1G, ^2G) \quad (18)$$

where the SERR (G) is defined as the sum of mode I SERR and mode II SERR, which are separately assessed as the integral of traction times separation on individual mode traction vs. response curves:

$$G_I = \int \sigma_I d\delta_I; \quad G_{II} = \int \sigma_{II} d\delta_{II} \quad (19)$$

$G_{\min}$ indicates the minimum SERR value of the twin elements. Thus, the SERR ratio (used in Eq. 16) can be directly given by the twin elements, *i.e.*

$$R_G = \frac{G_{\min}}{G_{\max}} = \frac{\min(^1G, ^2G)}{\max(^1G, ^2G)} \quad (20)$$

instead of using the global R ratio (Eq. 2). This is a significant asset of the DLE method. It should be highlighted that the fatigue cohesive formulation used in the DLE modelling strategy here is fully independent on the global loading information, including the R ratio.

Note that in this study the real-time SERRs of the twin cohesive elements are used to evaluate both $G_{\max}$ and $G_{\min}$, as illustrated in Fig. 4b by pure mode traction versus separation response of two twin cohesive elements.

This is based on these considerations given here. It can be seen from Fig. 4b that the real-time SERRs increase from the end of static stage to the end of fatigue stage. It has been shown in the literature that a crack-tip cohesive element would give an underestimate SERR for a pure static loading case, unless an extremely fine mesh was used but this is normally unacceptable considering computational cost [31]. It means that the fatigue crack propagation rate will be underestimated if the SERR extracted at the end of static stage was used to represent $G_{\max}$ in Eq. 15 during the fatigue degradation stage [13,16]. On the other hand, the fatigue crack propagation rate may be overestimated if the SERR extracted at the end of fatigue loading is used to represent $G_{\max}$ for the fatigue degradation stage; it may also require to use a more costly nonlocal approach considering neighbouring elements to estimate $G_{\max}$ [13]. Considering the possible disadvantages caused due to these two extreme cases, a balanced approach between these two cases is used. That is, the real-time SERR extracted from the more deformed one of the twin crack-tip elements is used to estimate $G_{\max}$; this approach was also used for the SLE modelling method proposed in Ref. [16]. Since the real-time SERR of the more deformed one of the twin crack-tip cohesive elements is employed to estimate $G_{\max}$, it is straightforward to also using the real-time SERR of the less deformed twin crack-tip element to represent $G_{\min}$. Thus, it is also acceptable to use real-time $G_{\min}$ and $G_{\max}$, to compute local SERR ratio by Eq. 20.

The combination of Eq. 10 with Eqs. 16 to 20 shows that by treating twin elements separately, the DLE method also accounts for mode mixity variations which may occur within one load cycle (*i.e.* due to crack closure effects) by determining Paris law constant and exponent (C , m) for each twin element independently. The contributions from peak and trough states to the global crack growth rate are considered by averaging the Paris law constants and exponents, and critical SERRs in Eq. 21. The simple averaging approach is employed here as it gives satisfactory results for these models presented in Sections 3 and 4.

$$C = (^1C + ^2C)/2, \quad m = (^1m + ^2m)/2, \quad G_c = (^1G_c + ^2G_c)/2; \quad (21)$$

Knowing the fatigue crack propagation rate, the number of cycles that a crack-tip element can sustain before full failure, ΔN_e , can be evaluated by:

$$\Delta N_e = \frac{dN}{da} l_f \quad (22)$$

where l_f is the fatigue characteristic length, representing the length of the crack-tip element in the crack growth direction. For a single-IP crack-tip element, the nonlocal algorithm, as developed in [13] based on comparison between element neighbours, is used here to estimate the characteristic length. Interested readers are referred to [13] for more details. To account for the possible variation of crack propagation direction within one cycle, the averaged l_f value of two twin elements is used in Eq. 22, as expressed by:

$$l_f = ({}^1l_f + {}^2l_f)/2 \quad (23)$$

where 1l_f and 2l_f are the characteristic lengths of the twin elements that are separately estimated using the nonlocal algorithm in [13]. According to Eqs. 15, 18, 21, 22 and 23, the twin elements have the same fatigue life (ΔN_e); this physically make sense since they are used to describe the same segment of a composites interface.

Now it is required to derive the fatigue damage rate (dD_f/dN). At a point when the crack-tip twin elements have the static damage variable D_s , the elements need the additional fatigue damage equivalent to $(1 - D_s)$ for full failure. The number of cycles that the elements can survive at this point is ΔN_e as derived by Eq. 22. Thus, the fatigue damage propagation per cycle, i.e. dD_f/dN can be defined by Eq. 24 [13]. These twin elements share the same fatigue damage rate since they have the same ΔN_e and D_s .

$$\frac{dD_f}{dN} = \frac{1-D_s}{\Delta N_e} \quad (24)$$

In an explicit FE solution, the fatigue damage variable is updated by [13]:

$${}^{t+\Delta t}D_f = {}^tD_f + \frac{dD_f}{dN} f_N \Delta t \quad (25)$$

where f_N is the numerical fatigue frequency; Δt is the time increment. According to Eq. 25, to assure that the twin elements have the same fatigue damage variable, it requires that the twin cohesive models have the same numerical fatigue frequency and the same time increment. This can be easily addressed in a FE programme. The total damage variable D_t for the twin crack-tip elements can thus be obtained by Eqs. 1, 8 and 25. The ‘reference mixed-mode bilinear curve’ idea as explained previously is also employed to evaluate tractions in fatigue step for both the less deformed and more deformed elements. The only change that needs to be made for the fatigue traction evaluation is replacing D_s with D_t in Eq. 14 to get the reference displacement for the fatigue case, which is indicated by $\delta_{m,Fat}^{Ref}$ in Fig. 4a. Eq. 13 is used to get the degraded mode I and II stiffnesses due to fatigue, which correspond to $K^D(D_t)$ in Figs. 4a. The traction components for both twin elements are then computed by Eq. 12. Fig. 4b presents the typical pure mode fatigue response of two twin cohesive elements.

2.4. Extension to 4-IP cohesive elements

4-IP elements are found to be more robust and give higher accuracy than single-IP elements in many cases. It is thus necessary to extend the DLE method to 4-IP cohesive elements. Two twin 4-IP cohesive elements have four pairs of twin IPs. The static formulation as introduced in Section 2.2 can be directly applied to twin 4-IP elements, except that the synchronisation between the static damage variables as achieved by Eq. 8 is realised at the IP level, instead of the element level. It means that each pair of twin IPs are treated independently from another pair by the static law as introduced in Section 2.2.

The implementation of the fatigue law by twin 4-IP elements is based on the averaging strategy presented in [16]. This was proposed due to the observation that the deformation associated with one IP of a 4-IP element non-physically influences another IP, as a result of the element geometry. First, the Paris law constant and exponent (C, m), the critical SERR and the maximum SERR are calculated for each pair of twin IPs by combining Eqs. 16-20. Then, the averaged values of these parameters associated with four IP pairs are applied to the Eq. 15 to achieve a common fatigue crack propagation rate for all four IP pairs. The averaging strategy can be written as:

$$C = \frac{\sum_{i=1}^2 \sum_{j=1}^4 i^j C}{8}, \quad m = \frac{\sum_{i=1}^2 \sum_{j=1}^4 i^j m}{8}; \quad (26)$$

$$G_c = \frac{\sum_{i=1}^2 \sum_{j=1}^4 i^j G_c}{8}, \quad G_{\max} = \frac{\sum_{j=1}^4 \max(1^j G_{\max}, 2^j G_{\max})}{4} \quad (27)$$

where i indicates the element number and j indicates the IP number.

The common characteristic length of two twin 1-IP elements is also the averaged value of the twin elements (Eq. 23). But for a 4-IP element the characteristic length is here calculated by the length of a line that cuts the element mid-plane in the direction of the G/G_c gradient at the element centroid. This is similar to criteria 1 and 2 of the approach proposed in Ref. [26]. In this study, the G/G_c value at a given location on the 4-IP mid-plane is interpolated by the combination of linear shape functions and the G/G_c values associated with four IPs of a 4-IP cohesive element. Thus the G/G_c gradient at the element centroid can be derived with the aid of the Jacobian matrix and the partial derivatives of the shape functions relative to natural coordinates. Then, the averaged characteristic length of the twin 4-IP elements is applied to Eq. 22 to evaluate the fatigue life for all eight integration points. Eqs. 24 and 25 are employed to evaluate the fatigue damage variable for each IP of the twin elements. Thus, each IP of two twin 4-IP elements have the same fatigue damage variable. Finally, the stiffness pair and tractions can be calculated for each IP using the ‘reference mixed-mode bilinear curve’ idea as explained previously.

2.5. Logic flowchart of the static/fatigue cohesive law.

The entire DLE modelling framework is here implemented in the commercial FE software package Abaqus/Explicit 6.14, where the static and fatigue cohesive law were compiled in a user-defined material subroutine (VUMAT). The overall logic of the cohesive law as implemented in the subroutine by one core is outlined in Fig. 5. When the DLE cohesive law was implemented by one core, two twin elements were calculated one after the other, which holds true for both single-IP and 4-IP elements. However, it should be noted that when multiple cores were employed, two twin cohesive elements may be calculated in parallel if they

were not allocated into the same domain (core). As seen in Fig. 5, the interactions between twin IPs/Elements to accomplish the aforementioned averaging strategies are implemented after all the IPs have been called from the subroutine, thus there exists a delay of one time increment (Δt) between the calculations of damage variables and tractions. However, the delay effect is negligible if the time increment is very small, which is normally true in an explicit FE analysis.

3. Model verification

The DLE fatigue prediction strategy is first verified through mode I DCB, mode II 4ENF and 50% mode-mixity MMB fatigue tests of a unidirectional IM7/8552 laminate under 0.1 R ratio. These benchmark tests were used to verify whether the predicted fatigue crack propagation rate could reproduce analytical inputs, prior to applying the DLE method to a more general case. Fig. 6a gives the geometry and dimensions of the IM7/8552 laminate. Recalling the overall framework of the DLE method (Section 2.1), the laminate was described by two twin FE models, which were loaded following the double load envelopes illustrated in Fig. 3b. Each arm was described by regular 1-IP solid elements (C3D8R in Abaqus). There are respectively one element and three elements across the width and thickness of each arm. One layer of 0.01 mm thick cohesive elements was inserted between the top and bottom arms to predict delamination. Two cases, with the two types of cohesive elements, *i.e.* 1-IP (C3D8R) element and 4-IP (COH3D8) element in Abaqus used to describe the interface, were run. These cohesive elements have a size of 0.0625 mm in the laminate length (the delamination propagation direction); such a fine mesh was used to ensure that a sufficient amount (>3) of cohesive elements exist in a cohesive zone for all the DCB, 4ENF and MMB models considered here [30,31,34,35]. Table 1 lists the material properties used for all the models here [16,27,33].

In the mode I case (Fig. 6b), each twin model was loaded at the pre-cracked end by a pair of equal and opposite moments. According to LEFM, this assures a constant mode I crack-tip SERR during crack propagation [12,14], which is given by:

$$G_I = \frac{M^2}{BEI} \quad (28)$$

where M is the applied moment; B is the beam width; E is the longitudinal flexural modulus of the laminate; I represents the second moment of area of a DCB arm. In the mode II case (Fig. 6c), each twin model was loaded by a rigid bar. The left bottom edge of each twin 4ENF model was constrained in the beam length and thickness directions, while its right bottom edge was constrained only in the beam thickness direction. This approach is consistent with the set-up used in Refs. [11,12,14]. Analytical solution of the mode II SERR is given by:

$$G_{II} = \frac{3P^2 a^2}{16BEI} \quad (29)$$

where d indicates the distance from the left edge of the laminate to the left end of the loading bar prior to deformation (Fig. 6c). For the mixed-mode case (Fig. 6d), in each twin model one arm was loaded by a moment at the pre-crack end of the laminate, whilst the other arm was loaded under a lower moment. To achieve a 50% mode mixity, the ratio between the two moments is given by Eq. 30, and the SERRs for mode I and mode II are given by Eq. 31 [36].

$$\rho = \frac{1-\sqrt{3}/2}{1+\sqrt{3}/2} \quad (30)$$

$$G_I = G_{II} = \frac{3}{4(1+\sqrt{3}/2)^2} \frac{M^2}{BEI} \quad (31)$$

The analytically input $G_{\max}/G_C$ ratios were thus derived by Eqs. 28-31 for each case, and the analytically input da/dN values were calculated by Eq. 15. The FE models were run by using both 1-IP and 4-IP cohesive elements with the analytically input $G_{\max}/G_C$ ratios at 0.5, 0.7 and 0.8 for the DCB case, 0.25, 0.35 and 0.5 for the 4ENF case, and 0.25, 0.35 and 0.55 for the MMB case, respectively. The FE crack propagation rate was calculated by counting the number of cycles elapsed over a crack length. The DCB and MMB models give a crack propagation rate independent of crack length. In the 4ENF models, the crack length used to compute the crack propagation rate was 1 mm, to avoid the minor mode-mixity variation due to the slight sliding of loading bar. Fig. 7 compares the FE predictions obtained by both 1-IP and 4-IP cohesive elements with analytical inputs regarding the crack propagation rate. Considering that the SERR of a crack-tip cohesive element varies in the stiffness degradation region of the fatigue cohesive law (as illustrated in Fig. 4b), the averaged value of the SERRs at the points of starting fatigue degradation and full failure was used to indicate $G_{\max}$ in the FE plots (an averaged value was used for 4-IP elements). This leads to slight mismatch between the analytically input and the FE predicted $G_{\max}/G_C$ ratios. The second reason that leads to the difference is that the non-perfect correlation between the SERR of the crack-tip element extracted at the end of the static step and the analytical SERR given by Eqs. 28-31. This point was demonstrated by re-running the DCB model under pure static loading. Fig. 8 compares the analytical SERR and that given by the DCB model using 1-IP and 4-IP cohesive elements. This SERR mismatch during static loading can be resolved by using a very fine mesh [25], but it will significantly increase the computation cost. However, it can be seen in Fig. 8 that the averaged SERR when using the 4-IP cohesive element and the SERR given by the 1-IP cohesive element largely give satisfactory correlation with the analytical SERR, which also supports the averaging strategy used in the Eq. 27 for $G_{\max}$.

The numerical models were run with a sensitivity study regarding the numerical fatigue frequency, which was incrementally reduced until the predicted crack propagation rate converged. Effect of numerical fatigue

frequency will be explained in the next section. Overall, excellent agreement is present between FE predictions and analytical inputs in all the cases, which is true for both 1-IP and 4-IP cohesive models, as shown in Fig. 7. The 1-IP models and the 4-IP models give very close predictions in all the DCB and 4ENF cases. The minor difference in the prediction of these two element types in the MMB case is because that the averaged G_C value of a 4-IP element pair given by Eq. 27 is slightly larger than a 1-IP element pair for a given mixed-mode fatigue severity.

As mentioned above, one significant asset of the DLE method proposed here is to allow predicting the mechanical condition of a laminate at both the minimum and maximum loads of fatigue cycles. Fig. 9 plots the traction versus separation curves of the twin cohesive elements located at 2 mm away from the initial crack tip in the 0.5 $G_{\max}/G_C$ ratio DCB case. The twin elements keep stiffness synchronised throughout the whole loading history, which means that the traction envelope of the less deformed element follows the unloading trace of the more deformed element, as shown by a dashed black line in Fig. 9. Fig. 9 also plots the SERR ratio predicted by the twin elements versus the separation of the more deformed one of the initial crack-tip twin elements. The FE predicted SERR ratio correlates well with the analytical solution given by Eq. 2, and the predicted SERR ratio shows no apparent variation during the fatigue degradation stage; these two points hold for all the models presented in this section. In combination of Eq. 19, Fig. 9 confirms that the SERR shows an increasing trend in the stiffness degradation region due to fatigue.

For further demonstration of the benefit of using double load envelopes and twin cohesive models, Fig. 10a presents the deformation status (thickness-direction displacement) of the 0.55 $G_{\max}/G_C$ ratio MMB laminate at the minimum and maximum loads of the 1000th fatigue cycle, while Fig. 10b gives the failure status of the twin 4-IP interface elements used to predict delamination in the laminate. The slight mode ratio (ϕ) variation of the crack tip during the cycle was also captured by the twin cohesive elements; specifically, at the maximum load, the mode ratio was 0.348 for the two forward integration points of the crack-tip 4-IP cohesive element in the crack propagation direction, while it was 0.640 for the other two integration points (at the crack tip); at the minimum load, the mode ratio was 0.353 for the two forward integration points of the crack-tip 4-IP cohesive element, while it was 0.645 for the two crack-tip integration points.

4. Application to a cut-ply case

The DLE method is further employed to predict the tensile fatigue performance of a unidirectional laminate, which comprises eight continuous plies and two central cut-ply. The central cut-ply (CCP) laminate is described by two twin FE models (Fig. 11), which were loaded following the double load envelope as illustrated

in Fig. 3b. Due to symmetries only one quarter of the CCP laminate needs to be considered in the twin models. By assuming the crack front is uniform across the laminate width, a 0.125 mm wide ‘slice’ mesh is employed to describe the laminate in combination with a generalised plane strain boundary condition applied to both side surfaces of the slice model [13]. The nodes corresponding to the cut are free of any constraint. A layer of 0.01 mm thick 4-IP (COH3D8) cohesive elements was inserted between the continuous plies and the central cut ply to predict delamination. The material properties listed in Table 1 were also used for the CCP modelling [16,27,33].

The CCP laminate was experimentally and numerically characterised under two R ratios (0.3 and 0.5). Experimental characterisation of the CCP laminate has been detailed in [37] with 3 repeats for each R ratio. The effect of fatigue delamination on the mechanical performance of the CCP laminate is evaluated by the effective stiffness E_{eff} as defined by:

$$E_{eff} = \frac{L_0(P_{max} - P_{min})}{A(\Delta L_{max} - \Delta L_{min})} \quad (32)$$

where L_0 and A are the original length and cross-section area of the laminate, respectively. ΔL_{max} and ΔL_{min} are the tensile elongations corresponding to the maximum and minimum loads. As another benefit of using the DLE method, these two elongation parameters can be directly evaluated by the twin models, while in experiments a clip-on extensometer was employed to record these two parameters.

The CCP numerical models were run with a sensitivity study regarding the numerical fatigue frequency, which was incrementally reduced until the predicted results converged. As an example, Fig. 12 presents the global stiffness and the local response of the twin integration points at the initial crack tip, which were predicted for the 0.3 R ratio and 60% severity case by using different numerical fatigue frequencies. The time increment used in all the CCP models was 2.735×10^{-6} s. Both global and local results start to converge from 1×10^6 Hz. Numerical oscillation was present when using a higher than 1×10^7 Hz frequency (see Fig. 12a); this is because that fatigue damage was accumulated faster over one time increment with increasing the numerical fatigue frequency (see Eq. 25). The rapid local degradation in the 1×10^8 Hz case led to a conservative prediction of the global stiffness knock-down versus cycles number. The numerical fatigue frequencies at which the CCP models showed good convergence without non-trivial numerical oscillation were respectively 1×10^6 Hz, 1×10^6 Hz, 1×10^5 Hz, 1×10^4 Hz for the 50%, 60%, 70% and 80% severities in the 0.3 R ratio case. The convergence frequencies for the 70%, 80% and 90% severities in the 0.5 R ratio case were respectively 1×10^6 Hz, 1×10^5 Hz and 1×10^4 Hz.

Fig. 13 compares the normalised stiffness versus cycle number curves that are given by the CCP models

and experimental measurements. The FE predicted E-N curves show excellent agreement to experiment measurements. Both show that at a lower load severity (50% for 0.3 R ratio and 70% for 0.5 R ratio), the CCP laminate shows minimal delamination at runout (10^6 cycles). As expected, the CCP laminate shows earlier and faster delamination propagation from the cut tip with increasing the load severity for a given R ratio. For the same load severity, the fatigue deamination propagation rate decreases with the R ratio. The twin cohesive models allow predicting the mechanical condition of the CCP laminate at both the maximum and minimum loads. Fig. 14a presents the fibre-direction normal stress fields of the laminate at the maximum and minimum loads of the 8000th cycle of 0.5 R ratio and 80% severity fatigue loading. The stress fields at the peak and trough loads show quite comparable contours, however, the stress value at a given location of the laminate is altered by a factor approximately equal to the R ratio for the trough load versus the peak load. As it was observed in experiments, fibre failure was not expected to happen in the models since the maximum fibre-direction stress is much lower than the fibre strength [38]. Fig. 14b shows the through-thickness displacement (δ_t) fields of the interlaminar interface at the peak and trough loads. For both peak and trough loads, there exists through-thickness compression in the cohesive zone, while through-thickness tension prevails in the interface behind the cohesive zone. The through-thickness displacement for a given location of the interface at the peak load is around two times as large as that given at the trough load. In combination with Eqs. 7 and 11, it can be summarised that the mode II strength and toughness enhancements due to through-thickness compression vary within one cycle.

Due to difference existing between the peak load and the trough load regarding both the mode II strength and toughness enhancements, the local SERR ratio given by twin cohesive elements is different from the analytical solution given by Eq. 2. This has been first demonstrated in Fig. 12b by plotting the local SERR ratio given by twin cohesive elements at the initial crack tip in the 0.3 R ratio and 60% severity case. The SERR ratio starts with a value very close to the analytical solution (0.09 for this case), then shows a linear decreasing trend relative to the separation of the more deformed twin cohesive element when it deforms elastically. At the static degradation stage of the cohesive law, the decreasing trend of the SERR ratio becomes nonlinear and slower, while there is no apparent variation during the fatigue degradation stage. In addition, there is change in the local SERR ratio with crack extension. This is illustrated in Fig. 15 by plotting the SERR ratios associated with the first six discretised interface segments from the initial crack tip for the 0.3 R ratio and 60% severity case. There is negligible difference in the SERR ratio among the 4 IP pairs belonging to one pair of the twin cohesive elements in the cut-ply modelling cases. It can be seen from Fig. 15a that the SERR ratio shows variation

throughout the whole global static loading stage, and there is apparent SERR difference between these interface segments. The dashed vertical lines in Fig. 15a indicate when the corresponding cohesive elements start to be statically degraded (i.e. $D_s > 0$). It is interesting to see there is always a sudden change in the SERR ratio of one interface segment when its preceding segment has entered static degradation region. There is also a sudden change in the SERR ratio when the global fatigue loading starts to be applied (i.e. from 2s for this case). The first 5 interface segments had entered static degradation region, while the sixth interface segment was still in the elastic deformation stage at the start of global fatigue loading stage. The SERR ratio of an interface segment shows negligible change during the fatigue degradation stage of the cohesive law, as shown in Fig. 15b. These findings from Fig. 15 regarding the difference between the local SERR ratio given by the twin cohesive elements and the analytical value given by Eq. 2 further explain the necessity of building the twin cohesive models, respectively at the peak and trough loads.

Mesh sensitivity study was carried out for the central cut-ply models. Apart from the baseline mesh size (0.125 mm), all the CCP models were re-run by using the other two mesh sizes (0.25 mm and 0.5 mm). A very fine mesh can capture the slightly local opening at the cut tip [32], however, the global prediction regarding the stiffness knock-down versus cycle number shows relatively less sensitivity to the mesh size. As shown in Fig. 16, all the CCP models started to converge from 0.25 mm. It can also be observed from Fig. 16 that a higher severity model was less sensitive to mesh size than a lower severity model for a given R ratio, e.g. the 0.3 R ratio & 80% severity case and the 0.5 R ratio & 90% severity case.

5. Conclusions

This paper has presented a novel double-load-envelope numerical method to predict fatigue delamination in composite laminates. A fatigue laminate is described by twin models; one is loaded under the peak-load envelope, whilst the other one is loaded under the trough-load envelope. Each material interface is described by twin cohesive elements that exchange data at every time increment of an explicit FE solution and represent the evolution of both peak- and trough-load states as fatigue damage is accumulated according to a modified Paris law. The cohesive law used in the DLE method allows the direct evaluation of strain energy release rates (G_I and G_{II}) and mode mixities [$G_{II}/(G_I+G_{II})$] at peak- and trough-load states, as well as SERR ratios [$G_{\min}/G_{\max}$] independently for crack fronts, without the need to know global loading information, e.g. the load R ratio. The numerical technique features the high computational efficiency of a single load envelope method, since a higher numerical fatigue frequency than the fatigue frequency of actual tests can be used. It also allows predicting the mechanical condition of a laminate at both the peak and trough loads of fatigue cycles.

Implementations of the static/fatigue law by 1-IP elements and 4-IP elements have been explained with the aid of a logic flowchart. As the first demonstration, the fatigue delamination prediction method has been applied to mode I DCB, mode II 4ENF and mixed-mode MMB cases. The predicted da/dN results can accurately replicate the analytical input. The modelling strategy has also been applied to a central cut-ply specimen, which was tested under two different R ratios; good agreement has been achieved between experimentally measured and FE predicted stiffness degradation. The locally determined SERR ratio was used in all these benchmark models without the need to input the global R ratio. This opens the door to being able to model more complicated loading cases, such as fully reversed loading ($R = -1$) or when there is asymmetry in the positive and negative parts of a reversed cycle, such as was tested in Ref. [39]. One thing that needs to be noted is that a modified Paris based law, instead of the one in Eq. 15, may need to be embedded into the twin-cohesive models based DLE modelling strategy for fatigue delamination prediction in a load reversal case. This will be addressed in future work, when more comprehensive experimental results are available.

Acknowledgements

The authors would like to acknowledge Rolls-Royce plc for giving permission to use the cut-ply experimental data from [37], which was measured by the Rolls-Royce Composites University Technology Centre (UTC) at the University of Bristol, UK. The authors also wish to thank Dr Supratik Mukhopadhyay at the University of Bristol for helpful discussion regarding existing fatigue modelling methods.

References

- [1] Pascoe JA, Alderliesten RC, Benedictus R. Methods for the prediction of fatigue delamination growth in composites and adhesive bonds - A critical review. *Eng Fract Mech* 2013;112–113:72–96.
- [2] Krueger R. Virtual crack closure technique: History, approach, and applications. *Appl Mech Rev* 2004;57:109–43.
- [3] Deobald LR, Mabson GE, Engelstad S, Prabhakar Rao M, Gurvich MR, Seneviratne W, et al. Guidelines for VCCT-based interlaminar fatigue and progressive failure finite element analysis. *Tech Rep NASA-TM-2017-219663* 2017.
- [4] De Carvalho NV, Mabson GE, Krueger R, Deobald LR. A new approach to model delamination growth in fatigue using the Virtual Crack Closure Technique without re-meshing. *Eng Fract Mech* 2019;222:106614.
- [5] Latifi M, van der Meer FP, Sluys LJ. A level set model for simulating fatigue-driven delamination in composites. *Int J Fatigue* 2015;80:434–42.
- [6] Latifi M, van der Meer FP, Sluys LJ. Fatigue modeling in composites with the thick level set interface method. *Compos Part A Appl Sci Manuf* 2017;101:72–80.
- [7] Amiri-Rad A, Mashayekhi M, van der Meer FP. Cohesive zone and level set method for simulation of high cycle fatigue delamination in composite materials. *Compos Struct* 2017;160:61–9.
- [8] Bak BL V., Sarrado C, Turon A, Costa J. Delamination under fatigue loads in composite laminates: a review on the observed phenomenology and computational methods. *Appl Mech Rev* 2014;66.
- [9] Paepegem W Van, Degrieck J. Fatigue degradation modelling of plain woven glass/epoxy composites. *Compos - Part A Appl Sci Manuf* 2001;32:1433–41.
- [10] Nojavan S, Schesser D, Yang QD. An in situ fatigue-CZM for unified crack initiation and propagation in composites under cyclic loading. *Compos Struct* 2016;146:34–49.
- [11] Robinson P, Galvanetto U, Tumino D, Bellucci G, Violeau D. Numerical simulation of fatigue-driven delamination using interface elements. *Int J Numer Methods Eng* 2005;63:1824–48.
- [12] Turon A, Costa J, Camanho PP, Dávila CG. Simulation of delamination in composites under high-cycle fatigue. *Compos Part A Appl Sci Manuf* 2007;38:2270–82.

- [13] Kawashita LF, Hallett SR. A crack tip tracking algorithm for cohesive interface element analysis of fatigue delamination propagation in composite materials. *Int J Solids Struct* 2012;49:2898–913.
- [14] Harper PW, Hallett SR. A fatigue degradation law for cohesive interface elements - Development and application to composite materials. *Int J Fatigue* 2010;32:1774–87.
- [15] Bak BLV, Turon A, Lindgaard E, Lund E. A simulation method for high-cycle fatigue-driven delamination using a cohesive zone model. *Int J Numer Methods Eng* 2016;106:163–91.
- [16] Tao C, Mukhopadhyay S, Zhang B, Kawashita LF, Qiu J, Hallett SR. An improved delamination fatigue cohesive interface model for complex three-dimensional multi-interface cases. *Compos Part A Appl Sci Manuf* 2018;107:633–46.
- [17] Carreras L, Turon A, Bak BLV, Lindgaard E, Renart J, Martin de la Escalera F, et al. A simulation method for fatigue-driven delamination in layered structures involving non-negligible fracture process zones and arbitrarily shaped crack fronts. *Compos Part A Appl Sci Manuf* 2019;122:107–19.
- [18] De Moura MFSF, Gonçalves JPM. Cohesive zone model for high-cycle fatigue of adhesively bonded joints under mode I loading. *Int J Solids Struct* 2014;51:1123–31.
- [19] May M, Hallett SR. A combined model for initiation and propagation of damage under fatigue loading for cohesive interface elements. *Compos Part A Appl Sci Manuf* 2010;41:1787–96.
- [20] larve E V, Hoos K, Braginsky M, Zhou E, Mollenhauer DH. Progressive failure simulation in laminated composites under fatigue loading by using discrete damage modeling. *J Compos Mater* 2016;51:2143–61.
- [21] O'Brien TK, Chawan AD, Krueger R, Paris IL. Transverse tension fatigue life characterization through flexure testing of composite materials. *Int J Fatigue* 2002;24:127–45.
- [22] May M, Hallett SR. An assessment of through-thickness shear tests for initiation of fatigue failure. *Compos Part A Appl Sci Manuf* 2010;41:1570–8.
- [23] Tumino D, Cappello F. Simulation of fatigue delamination growth in composites with different mode mixtures. *J Compos Mater* 2007;41:2415–41.
- [24] Pirondi A, Moroni F. A progressive damage model for the prediction of fatigue crack growth in bonded joints. *J Adhes* 2010;86:501–21.
- [25] Tao C, Qiu J, Yao W, Ji H. A novel method for fatigue delamination simulation in composite laminates. *Compos Sci Technol* 2016;128:104–15.
- [26] Carreras L, Bak BLV, Turon A, Renart J, Lindgaard E. Point-wise evaluation of the growth driving direction for arbitrarily shaped delamination fronts using cohesive elements. *Eur J Mech / A Solids* 2018;72:464–82.
- [27] Mukhopadhyay S, Nixon-Pearson OJ, Hallett SR. An experimental and numerical study on fatigue damage development in laminates containing embedded wrinkle defects. *Int J Fatigue* 2018;107:1–12.
- [28] Jiang W-G, Hallett SR, Green BG, Wisnom MR. A concise interface constitutive law for analysis of delamination and splitting in composite materials and its application to scaled notched tensile specimens. *Int J Numer Methods Eng* 2007;69:1982–95.
- [29] Gan KW, Hallett SR, Wisnom MR. Measurement and modelling of interlaminar shear strength enhancement under moderate through-thickness compression. *Compos Part A Appl Sci Manuf* 2013;49:18–25.
- [30] Turon A, Dávila CG, Camanho PP, Costa J. An engineering solution for mesh size effects in the simulation of delamination using cohesive zone models. *Eng Fract Mech* 2007;74:1665–82.
- [31] Harper PW, Sun L, Hallett SR. A study on the influence of cohesive zone interface element strength parameters on mixed mode behaviour. *Compos Part A Appl Sci Manuf* 2012;43:722–34.
- [32] Li X, Hallett SR, Wisnom MR. Predicting the effect of through-thickness compressive stress on delamination using interface elements. *Compos Part A Appl Sci Manuf* 2008;39:218–30.
- [33] Allegri G, Wisnom MR, Hallett SR. A new semi-empirical law for variable stress-ratio and mixed-mode fatigue delamination growth. *Compos Part A Appl Sci Manuf* 2013;48:192–200.
- [34] Turon A, Camanho PP, Costa J, Renart J. Accurate simulation of delamination growth under mixed-mode loading using cohesive elements: Definition of interlaminar strengths and elastic stiffness. *Compos Struct* 2010;92:1857–64.
- [35] Harper PW, Hallett SR. Cohesive zone length in numerical simulations of composite delamination. *Eng Fract Mech* 2008;75:4774–92.
- [36] D6671/D6671M - 06. Standard test method for mixed mode I-mode II interlaminar fracture toughness of unidirectional fiber reinforced polymer matrix composites. *ASTM Stand* 2006.
- [37] Allegri G, Jones MI, Wisnom MR, Hallett SR. A new semi-empirical model for stress ratio effect on mode II fatigue delamination growth. *Compos Part A Appl Sci Manuf* 2011;42:733–40.
- [38] HEXCEL. HexTow® IM7 carbon fibre product data sheet. 2014.
- [39] Kuehlewind R, Kawashita LF, Hallett SR. Fully reversed mode-II fatigue testing. 17th Eur. Conf. Compos. Mater., Munich: 2016.

Fig. 1. Schematic illustration of (a) the loading-unloading hysteresis (LUH) method and (b) the single load envelope (SLE) method used in the literature for fatigue delamination prediction, as summarised in Ref. [8].

Fig. 2. Illustrations of (a) the mixed mode bilinear static and fatigue law, and (b) the effects of static and fatigue damage variables on the interface individual mode stiffnesses.

Fig. 3. Overall framework of the double load envelope (DLE) method, including (a) two twin cohesive models and (b) two synchronised load envelopes.

Fig. 4. (a) Determining the individual mode response of the less deformed twin element by a reference mixed-mode bilinear curve, and (b) pure mode static and fatigue traction vs. separation response of two twin cohesive elements.

Fig. 5. Flowchart of the DLE static and fatigue cohesive law when implemented by an Abaqus/Explicit VUMAT subroutine on one core; N_e indicates the number of cohesive elements in each twin model; N_{IP} indicates the number of IPs for each cohesive element.

Fig. 6. (a) Geometry of the laminate used for (b) mode I DCB, (c) mode II 4ENF and (d) 50% mode mixity MMB virtual tests (unit: mm).

Fig. 7. Comparison between FE predicted and analytically input fatigue crack propagation rates in DCB, 4ENF and MMB cases. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 8. FE predicted versus analytically input energy release rates in the static DCB case; IP 1 and IP 2 indicate the two integration points of the 4-IP cohesive element at the initial crack tip, while IP 3 and IP 4 indicate other two integration points of the 4-IP cohesive element. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 9. Traction vs. separation curves of two DCB twin elements; the dashed black line indicates the stiffness synchronisation between the twin elements. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(a)

(b)

Fig. 10. (a) Deformation of the $0.55 G_{\max}/G_c$ ratio MMB laminate and (b) the corresponding failure status of its interlaminar interface, at the (top) maximum and (bottom) minimum loads of the 1000th fatigue cycle.

Fig. 11. The twin fatigue models used to describe the central cut-ply specimen.

(a)

(b)

Fig. 12. Effects of numerical fatigue frequency on (a) the global stiffness and (b) the local traction vs. separation curves of the initial crack-tip twin integration points in the 0.3 R ratio and 60% severity central cut-ply model. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(a)

(b)

Fig. 13. Comparison between experimentally measured and FE predicted stiffness degradation curves at (a) 0.3 and (b) 0.5 R ratios. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(a)

(b)

Fig. 14. (a) Fibre-direction normal stress fields of the 0.5 R ratio and 80% severity loaded cut-ply laminate and (b) the through-thickness displacement fields of the interlaminar interface at the (top) maximum and (bottom) minimum loads of the 8000th cycle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(a)

(b)

Fig. 15. SERR ratios (R_G) given by twin cohesive elements for the first six discretised interface segments from the initial crack tip in the 0.3 R ratio and 60% severity case: (a) overall plots for both static and fatigue stages and (b) localised plots for the fatigue stage; ‘IS’ is the abbreviation for ‘interface segment’, which is described by two twin cohesive elements in this study. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 16. Mesh sensitivity study on the cut-ply models; 0.125 mm curves in green; 0.25 mm curves in blue and 0.5 mm curves in red. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 1. Material properties for IM7/8552 [16,27,33].

Ply properties		Interface properties	
E_{11} (MPa)	161,000	G_{IC} (N/mm)	0.2
$E_{22} = E_{33}$ (MPa)	11,380	G_{IIC} (N/mm)	1.0
$G_{12} = G_{13}$ (MPa)	5170	$\sigma_I^{\max}$ (MPa)	60
G_{23} (MPa)	3980	$\sigma_{II}^{\max}$ (MPa)	90
$\nu_{12} = \nu_{13}$	0.32	K_I (N/mm ³)	4.67×10^5
ν_{23}	0.436	K_{II} (N/mm ³)	1.75×10^5
		η	0.3
Paris law parameters for Eqs. 16 and 17 [33]			
C (mm/cycle)	3.51×10^{-2}	b	14.05
h	1.457		