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Stiffness of artificially cemented sands: insight on characterisation through empirical power relationships

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ABSTRACT

Empirical power relationships linking the initial small strain shear stiffness of artificially cemented soils under unconfined loading conditions to a porosity over cement content ratio factor are convenient relations for the design and control quality of such reinforced soils. This paper will present a theoretical justification for the existence of such relationships and will demonstrate that they can be obtained through manipulation and simplification of well-established Hardin’s type formulas for cemented soils. In the process, the meaning and significance of the terms composing the empirical power relationships will be discussed. The proposed theoretical developments are validated against published data of the initial shear stiffness of two different artificially cemented soils under unconfined loading conditions.
1 INTRODUCTION

The reinforcement technique based on the mixing of soils with small amounts of cement material is an effective means of improving stiffness characteristics, enabling the reuse of locally available soils in many engineering projects. For example, highly compacted mixtures of soil/aggregate, cement and water are widely used as a low-cost pavement base for roads, residential roads, parking areas, airports and storage zones, among others (e.g. Hein et al. 2016). Other applications include reinforced excavations, soil-cement columns, jet-grouting, slope protection for embankments and dams, soil-stabilisation below superficial foundations (e.g. Fan et al. 2018, Sariosseiri and Muhunthan 2009). The soil material used in soil-cement mixtures can be any combination of clay, silt, sand, gravel or crushed stone. However, higher mechanical performances are expected for non-fine soils, i.e. sands and coarser materials.

The testing of cemented soils under unconfined loading condition provides design data for low stress level practical applications of cemented soils or, alternatively, for material quality control assessment (e.g. Saxena et al. 1988; Porbaha et al. 1998; Gallagher and Mitchell 2002; Thomé et al. 2005; Mitrani and Madabhushi 2010; Gomez and Anderson 2012). This particular test has also been frequently used in many experimental programmes reported in the literature (e.g. Consoli et al. 2012; Kaniraj and Havanagi, 1999; Ayeldeen et al. 2016; Yi et al. 2014) in order to verify the effectiveness of the soil stabilisation with cement, or to explore the relative importance of the factors controlling the stiffness of the cemented soils.

Hardin’s type formulas (Hardin, 1978) are well-established within the soil mechanics research and practice community, and describe the small strain shear stiffness of geomaterials. These relations are generally given in the following format:

$$ G_0 = \frac{S f(e) R^k}{p_r} \left( \frac{p'}{p_r} \right)^n $$

(1)

where $S$, $n$ and $k$ are dimensionless material parameters, $f(e)$ is a decreasing function of the void ratio, $R$ is a measure of the overconsolidation ratio, $p'$ is the mean effective stress and $p_r$ is a reference pressure which makes the expression independent of the choice of units. However, some experimental and theoretical studies (e.g. Weiler 1998, Houlsby and Wroth 1991) have contended that the use of three variables, $e$, $R$ and $p'$, is redundant. Following suggestions by Viggiani (1992) and Viggiani and Atkinson (1995), similarly to the relationship given by Cafaro and Cotecchia (2001), Trhlíková et al. (2012) have proposed and validated the following equation for cemented/structured soils:

$$ G_0 = A \left( \frac{p'}{p_r} \right)^n \left( \frac{p_e'}{p_r} \right)^m \left( \frac{s}{s_f} \right)^l $$

(2)

where $A$, $n$, $m$ and $l$ are model constants, $p_e'$ is the Horslev equivalent pressure (thus the ratio $(p_e' / p')^m$ represents to some extent the $f(e)$ or overconsolidation function), $s$ and $s_f$ are measures of structure for the cemented and uncemented soil, respectively. The effect of the progressive cementation breakage is accounted for by the ratio $s / s_f \geq 1$, being equal to 1 when the soil is uncemented or the cementation is fully broken. The relation (2) can predict the small strain stiffness over a comprehensive range of pressures and has been validated against both structured clays and cemented sand, with satisfactory results (Trhlíková et al. 2012).
This paper demonstrates that relationships of type (2) can also be applied for the prediction of small strain stiffness of cemented soils under unconfined conditions. However, this relationship does not provide an explicit account for the soil density and cement content variables, which are key ingredients in the design and control of artificially cemented soil mixtures. The application of such a relationship also requires the knowledge of the soil’s normal compression line (NCL), in both its uncedmented and cemented state, which demands careful isotropic compression testing.

Recently, it has been demonstrated that the initial small strain stiffness of cemented soils under unconfined loading can solely be described by the soil density and cement content variables through the factor \( \eta /C_{iv}^b \), where \( \eta \) is the porosity of the material, \( C_{iv} \) is the volumetric cement content (defined as the volume of cement divided by the initial total sample volume), and \( b \) is an empirical exponent (e.g. Consoli et al. 2010, Consoli et al. 2012):

\[
G_0 \rho_r = C \left( \frac{\eta}{C_{iv}} \right)^d
\]  

where \( \rho_r \) is a reference pressure, and \( C \) and \( d \) are additional material parameters. Note that the relationship between \( C_{iv} \) and the cement content by weight of sand \( CC \), widely used in practice is given by:

\[
CC = \frac{1}{C_{iv}\gamma_{sc}}
\]  

where \( \gamma_d \) is the dry density of the mixture and \( \gamma_{sc} \) represents the specific gravity of cement. The empirical relationship (3) has been deduced and validated for a wide range of data considering different cement and soil matrix types and curing conditions, such as temperature and time (e.g. Consoli et al. 2012, Consoli et al. 2016). A similar relationship function of the soil porosity and volumetric cement content has also been derived for the description of the unconfined compressive(203,543),(989,907) and tensile strengths of various cemented soils (Consoli et al. 2007, Consoli et al. 2017, Festugato et al. 2018) and recent theoretical derivations by Diambra et al. (2017) and Diambra et al. (2018) have clearly provided physical insight of the material coefficients.

Relationships as given by Eq. (3) have the advantage of providing a good estimation of the initial small strain stiffness of the cemented soil using a limited dataset of measurements. In fact, the parameters \( C \), \( b \) and \( d \) in Eq. (3) can be determined by fitting data provided by just a few tests - for example, nine small strain stiffness measurements on unconfined samples with different \( \eta /C_{iv} \) could be sufficient (e.g. Consoli et al., 2010, Consoli et al., 2016). Once the relationship is defined, the values of the variables \( \eta \) and \( C_{iv} \) can then be conveniently selected to satisfy the stiffness design requirements of pavements, superficial soil layer or material quality control (e.g. AASHTO, 2011, NCHRP, 2004).

Although small strain stiffness relationships such as Eq. (3) are very convenient in practice because of their simplicity, their derivation was solely based on empirical approaches through experimental data fitting. This paper seeks to provide a theoretical justification for such type of relationship and, in the process, demonstrates: (i) the existence of a direct link between the terms and coefficients of the relations type (2) and type (3); and (ii) that the empirically derived Eq. (3) is a mathematical simplification of Eq. (2). Published data for initial shear stiffness of two different artificially cemented soils under unconfined loading conditions are used for the validation of the proposed theoretical developments.
2 MATERIALS AND (NCL) PROPERTIES

This study looks at data for initial small strain stiffness of artificially cemented Osorio sand and Porto silty sand under unconfined condition provided by Consoli et al. (2012). Osorio sand is a non-plastic uniform fine sand from the region of Osorio, near Porto Alegre in southern Brazil. The sand particles are predominantly quartz. The Porto silty sand originates from weathered granite in the region of Porto, in northern Portugal. According to ASTM D 1497-93 (ASTM, 1993), it is a very well-graded silty sand, made up predominantly of kaolinite for the soil fraction smaller than 2μm and quartz for the larger grains. Previous studies show that among other factors, the effect of cementation on soil behaviour depends on the type of cementing agent (Haeri et al, 2006). In this study, Portland cement of high initial strength (Type III, ASTM C 150-09; ASTM, 2009) was used as a cementation agent. Cemented samples were cured for 7 days before initial shear stiffness measurements were carried out through shear wave propagation measurements based on bender element testing on unconfined cemented soil samples.

The theoretical developments in the following parts of this paper require the knowledge of the Normal compression line (NCL) for uncemented and cemented states of both sand materials. Isotropic compression loading up to high pressures of both uncemented and cemented materials investigated in this study are shown in Figure 1, following the data published by Dos Santos et al. (2010) for Osorio Sand and Rios et al. (2012) for Porto silty sand. For the cemented Osorio sand, the experimental data of the (NCL) is available only for one cement content. For Porto silty sand more data corresponding to several cement contents are available.

It is assumed that the (NCL) for the uncemented soil is linear in the $v$-$\ln p'$ and can be described by the following relationships:

$$v = T - \lambda \ln p'$$  \hspace{1cm} (5)

with the values of $T$ and $\lambda$ for Osorio sand equal to 3.06 and 0.156 respectively, and 2.44 and 0.12 for the Porto silty sand respectively. The presence of the cementation is usually described by an upward shift of the (NCL) of the uncemented soils (e.g. Rotta et al. 2003, Rios et al. 2012). Consoli and Foppa (2014) demonstrated that the shift of the (NCL) line is solely dependent on the cement content. The material conditions and the values of $T$ and $\lambda$ for the two soil types are summarised in Table 1.

![Figure 1](image-url)

Table 1. NCLs for uncemented and cemented Osorio sand and Porto silty sand.

<table>
<thead>
<tr>
<th>$p'$ (kPa)</th>
<th>NCL (Civ=0%)</th>
<th>NCL (Civ=1.21%)</th>
<th>NCL (Civ=2.21%)</th>
<th>NCL (Civ=3.07%)</th>
<th>NCL (Civ=4.41%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>Exp data</td>
<td>Exp data</td>
<td>Exp data</td>
<td>Exp data</td>
<td>Exp data</td>
</tr>
<tr>
<td>1000</td>
<td></td>
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<tr>
<td>10000</td>
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<tr>
<td>100000</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

(a) Exp Data (Civ=0%)
(b) Exp data (Civ=1.21%)
(c) Exp data (Civ=2.21%)
(d) Exp data (Civ=3.07%)
(e) Exp data (Civ=4.41%)

Figure 1 Isotropic compression data and NCLs for uncemented and cemented (a) Osorio sand (after Dos Santos et al. 2010) and (b) Porto silty sand (after Rios et al. 2012)
<table>
<thead>
<tr>
<th>Cemented Osorio Sand</th>
<th>Cemented Porto Silty Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{iv}$</td>
<td>$T$</td>
</tr>
<tr>
<td>0%</td>
<td>3.06</td>
</tr>
<tr>
<td>$\approx 1.4%$</td>
<td>3.16</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
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</tr>
</tbody>
</table>

3 APPLICATION OF TYPE (2) RELATIONSHIPS

This section assesses whether the established relationships for small strain stiffness of geomaterials, such as those in the form of Eq. (2), can describe the experimental data under unconfined conditions for the two selected materials: Osorio sand and Porto silty-sand. Following the developments proposed by Trhlíková et al. (2012), the ratio $s/s_f$ in Eq. (2), defining the degree of structure/cementation of the material, can be replaced by the $\frac{p_e^*}{p_e'}$ ratio, with $p_e^*$ being the Horslèv equivalent pressure for an un cemented soil. Schematic representations of $p_e^*$ and $p_e'$ mean pressures for a given density and stress level are shown in Figure 2.

![Schematic representation of $p_e^*$ and $p_e'$ mean pressures in specific volume, $v$, - mean effective pressure, $p'$, plane for a given specific volume.](image)

As such, it is possible to rewrite Eq. (2) in the following form:

$$\frac{G_0}{p_r} = A \left( \frac{p_e^*}{p_r} \right)^m \left( \frac{p_e'}{p_r} \right)^q$$

with the new constant $q=m+l$, and $p_r=1 \text{kPa}$ as a reference pressure.

At low stress levels or unconfined stress conditions, the initial stiffness of cemented soils is independent of the confining pressure (e.g. Fernandez and Santamarina, 2001; Rinaldi and Santamarina 2008; Trhlíková et al., 2012). Therefore, a non-null value of $p' = p_r$ can be conveniently imposed to simulate the unconfined stress conditions leading to:

$$\frac{G_0}{p_r} = A \left( \frac{p_e^*}{p_r} \right)^m \left( \frac{p_e'}{p_r} \right)^q$$

Using the expression (5) of the (NCL) with the parameters provided in Table 1 and interpolating the values of $T$ for different cement contents other than those tabulated (for Osorio sand, it had to be simplistically assumed to be a linear variation of $T$ with $C_{iv}$), a good fit of the experimental data can be obtained using the values of the parameters $A, m$ and $q$ given in Table 2. The qualitative agreement...
between experimental and simulations of the stiffness data for Osorio sand and Porto silty sand is shown in Figure 3.

Table 2. Values of model parameters used in Eq. (7) for both the investigated cemented soils.

<table>
<thead>
<tr>
<th></th>
<th>Cemented Osorio sand</th>
<th>Cemented Porto silty sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>5.2</td>
<td>11</td>
</tr>
<tr>
<td>$m$</td>
<td>0.5</td>
<td>0.47</td>
</tr>
<tr>
<td>$q$</td>
<td>0.8</td>
<td>0.63</td>
</tr>
</tbody>
</table>

$\frac{G_0}{p_r} = 5.2 \left( \frac{p_e^*}{p_r} \right)^{0.5} \left( \frac{p_e^*}{p_e^\prime} \right)^{0.8}$

$\frac{G_0}{p_r} = 11 \left( \frac{p_e^*}{p_r} \right)^{0.47} \left( \frac{p_e^*}{p_e^\prime} \right)^{0.63}$

Figure 3 Predicted (Eq. 7) versus experimental $G_0$ values for (a) cemented Osorio sand and (b) Porto silty sand.

4 LINK BETWEEN THE RELATIONSHIPS TYPE (2) AND TYPE (3)

The small strain stiffness relationship provided by Eq. (7) depends on two pressure dependent terms:

- one related to the equivalent pressure for the uncemented soil ($p_e^*/p_r$ with $p_r = 1\, kPa$);
- one related to the ratio between the equivalent pressures of cemented and uncemented soil ($p_e^*/p_e^\prime$).

For unconfined loading conditions, the $p_e^*$ value is solely dependent on the sample porosity, $\eta$ (which is directly related to the specific volume, $\nu$, through $\eta=100(\nu-1)/\nu$). Thus, the (NCL) for the uncemented soil can be plotted in the $\eta - p_e^*$ plane. In such representation, the (NCL) compression line can be well approximated by a relation of the following form:

$p_e^* = D\eta^{-f}$

(8)

where $D$ and $f$ are fitting parameters. The Figures 4a and 4b show the fit of the function (8) with the experimentally obtained (NCL)s for Osorio sand and Porto silty sand respectively, leading to the values of the parameters $D$ and $f$ as given in Table 3.
Figure 4 Approximation of NCL using Eq. (8) for (a) Osorio sand and (b) Porto silty sand.

Table 3. Formulas used to link empirical and theoretical relationships.

<table>
<thead>
<tr>
<th>NCL for uncemented soil Eq. (8)</th>
<th>Cemented Osorio sand</th>
<th>Cemented Porto silty sand</th>
</tr>
</thead>
</table>
|                                  | \( p_e^* = \frac{7E15}{\eta^{7.5}} \)  
D = 7E15; \( f = 7.5 \)  |
| Approximated ratio \( \frac{p_e'}{p_e^*} \) Eq. (13) | \( p_e' \approx 1.1C_{iv} \)  
\( L = 1.1 \)  |
| Stiffness relationship Eq. (14) | \( \frac{G_0}{p_r} = 6.06E7 \left( \frac{\eta}{C_{iv}} \right)^{-3.29} \)  |
|                                  | \( \frac{G_0}{p_r} = 4.95E8 \left( \frac{\eta}{C_{iv}} \right)^{-3.75} \)  |

For a fixed cement content, the assumptions of a linear (NCL) in the \( v \cdot \ln p' \) plane and parallelism between (NCL)s for uncemented and cemented soils lead to the following expression for the \( p_e'/p_e^* \) ratio of the second pressure dependent term of Eq. (7):

\[
\frac{p_e'}{p_e^*} = e^{\left( \frac{\Delta T}{\lambda} \right)} \tag{9}
\]

which is independent of the actual sample porosity. In relation (9), \( \Delta T \) represents the vertical shift between the (NCL) of a given cemented soil with respect to the (NCL) of the uncemented soil. Following Consoli and Foppa (2014), the (NCL) shift, \( \Delta T \), can be expressed as a function of the cement content only. Therefore, based on the available experimental data of both Osorio and Porto silty sands (Figure 5), a relationship between \( p_e'/p_e^* \) ratio and \( C_{iv} \) of the following form can be defined:

\[
\frac{p_e'}{p_e^*} = 1 + HC_{iv} \tag{10}
\]

where \( H \) is a fitting parameter and takes the values given in Figure 5.
The Eqs. (8) and (10) can now be introduced in Eq. (7), leading to the following relationship between $G_s$ and the parameters $\eta$ and $C_{iv}$:

$$\frac{G_s}{\rho_r} = A \left(\frac{p_e}{p'_e}\right)^m \left(\frac{p'_e}{p_c}\right)^q = A \left(\frac{D\eta^{-f}}{\eta^{\frac{1}{m}}+1}\right)^m \left(1 + H C_{iv}\right)^q = AD^m \left(\frac{1+HC_{iv}}{\eta^{\frac{1}{m}}+1}\right)^q$$

(11)

which can be rewritten in the following form to mirror the form of the empirical Eq. (3):

$$\frac{G_s}{\rho_r} = AD^m \left(\frac{\eta}{(HC_{iv}+1)\eta^{\frac{1}{m}}+1}\right)^{-f/m}$$

(12)

The main difference between Eq. (3) and Eq. (12) is the term ‘+1’ between brackets in the denominator of the relationship (12). For $C_{iv}$ values between 1 and 4, which are usually employed in artificially cemented soils, it appears that the following approximation could be further applied:

$$\frac{p'_e}{p_c} = HC_{iv} + 1 \approx LC_{iv}$$

(13)

where $L$ is a fitting parameter and its value is also reported in Table 3 for both sands. Therefore, Eq. (12) can then be rewritten to fully recover the form of Eq. (3):

$$\frac{G_s}{\rho_r} = AD^m L^q \left(\frac{\eta}{C_{iv}\eta^{\frac{1}{m}}+1}\right)^{-f/m}$$

(14)

From direct comparison between Eq. (14) and Eq. (3), we can now deduce $C$, $d$, and $b$ of Eq. (3) to be $C=AD^m L^q$, $d=\frac{1}{m}$ and $b=q(f/m)$ based on the constants inferred throughout the previous developments (see Tables 2 and 3). These values are also provided on the bottom line of Table 3.

The comparison of the experimental data with Eq. (14) for both cemented materials is shown in Figure 6a (Osorio sand) and Figure 6b (Porto silty sand), where the $G_s$ values are plotted against the adjusted ratio $\eta/C_{iv}^b$. The relationship (14) provides good predictions for both sets of experimental data. This work has also demonstrated that expressing the $G_s$ as function of the $\eta/C_{iv}^b$ ratio appears to be a sensible choice from a theoretical point of view. The exponential term depending on the $\eta$ may be seen as an approximation of the current Horslev pressure for uncemented soils and it is a term which accounts for the overconsolidation state or $f(e)$ function of the material. The exponential term linked to the $C_{iv}$ may represent the shift of the NCL for cemented soil and is a measure of the current state of cementation. For the Porto silty sand, the exponent $b$ for the adjusted ratio $\eta/C_{iv}^b$ assumes a value of 0.19 which is rather similar to the value of 0.21 previously found by Consoli et al. (2012). For the
cemented Osorio sand, the exponent $b$ is equal to 0.24, which is rather different from the value of 1.0 previously found by Consoli et al. (2012).

![Figure 6 Comparison between experimental data and prediction with Eq. (14) for (a) Osorio sand and (b) Porto silty sand.](image)

5 CONCLUSION

The initial small strain stiffness of cemented soil under unconfined conditions is a useful design data for field applications involving low stress levels as well as the material’s quality control assessment. The evaluation of the small strain stiffness through empirical power relationships governed by a porosity over cement content factor, $\eta/C_{iv}^b$, is very convenient because of the explicit account for the two key ingredients controlling the design of artificially cemented soil mixtures: soil density and cement content. This paper has provided for the first time a theoretical justification for the existence of such a type of empirical relationship, which was originally derived through experimental data fitting only. This paper has demonstrated that such a power relationship is a convenient simplification of well-established Hardin’s type relationship for the initial small strain shear stiffness of cemented soils under unconfined loading conditions. The power of the porosity $\eta$ term in the empirical relationship accounts for the shape of the (NCL) of the uncemented soil in the $\eta$-$p$ plane, while the cement content $C_{iv}$ term accounts for the shift of the (NCL) of the cemented soil with respect to its uncemented (NCL) state. Validation of the proposed theoretical derivation has been demonstrated against experimental data for two sets of cemented soil type.

6 REFERENCES


