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Truck Suspension Incorporating Inerters to Minimise Road Damage

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Abstract

Road damage caused by heavy vehicles is a serious problem experienced worldwide. This paper investigates the potential for reduction in road damage by incorporating the inerter element into truck suspension systems. Initially, quarter-car, pitch-plane and roll-plane models with two low-complexity inerter-based linear suspension layouts are investigated in the frequency domain. Reductions of the J_{95} road damage index for each model are identified against conventional parallel spring-damper truck suspension layouts. It is also shown that proposed suspensions are capable to enhance the roll stability while keeping the road damage at a given level. Subsequently, the nonlinear relationship between force and displacement as manifested by leaf springs is incorporated into the pitch-plane and roll-plane time-domain models. These confirm the potential advantage of inerter-based suspension layouts for road damage reduction.

Keywords

passive suspension, suspension dynamics, road damage, heavy vehicle, inerter

1 Introduction

Maintenance of the UK road network during 2017/2018 cost approximately £1.2 billion.¹ Whilst this is partially attributable to ungovernable factors such as bad weather, the vertical tyre force from heavy vehicles has been identified as a significant source of road damage.² The static tyre force is determined by the vehicles weight, but the dynamic tyre force depends *inter alia* on the suspension system and is one of the most critical causes of road damage.³

The ‘dynamic load coefficient’⁴ is known as the first criterion used to measure dynamic tyre force. However, the effect of correlation between axles (whereby rear wheels encounter the same road profiles as the front wheels after a speed-dependent delay) on heavy vehicles has not been taken into account by this measure. To achieve a more realistic

evaluation, the ‘road stress factor’⁵ is defined, based on the fourth power of the dynamic tyre forces in frequency domain. Subsequently, the effect of ‘spatial repeatability’⁶ has been identified to describe the phenomenon of aggregate peak dynamic tyre forces on certain points along the road surface. The 95th percentile aggregate fourth power force (J_{95}) was proposed to be the corresponding road damage index and has been adopted in many relevant works.^{8,9}

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The possibility of reducing road damage by adjusting the spring stiffness and damping rate in conventional passive truck suspensions has been investigated by Cole and Cebon,⁶ as well as Yang et al.¹⁰ More significant benefits obtained from active and semi-active suspensions have also been identified in a series of works.^{11–13} For more commonly used suspension performance criteria, such as ride comfort and handling, many studies have been carried out on both passive suspensions^{14–20} and (semi-)active suspensions.^{21–31} A good summary can be found in the literature.³² In general, the (semi-)active approach can provide more significant improvements. However, passive suspensions are still attractive in being less expensive and more reliable. Stimulated by the analogy between electrical and mechanical systems, the inerter, a new passive mechanical element, has been introduced by Smith in 2002.³³

The inerter is a two-terminal device, with the property that the applied force is proportional to the relative acceleration between the terminals.^{33,34} Inerters have been used to enhance passive suspension systems within a wide range of mechanical structures, such as motorcycles,³⁵ railway vehicles,^{36–40} tall buildings,^{41–43} landing gear systems⁴⁴ and suspension bridges.^{45,46} Whilst inerters have been shown to be beneficial for the ride comfort and handling of road vehicles,^{47,48} as well as the dynamic tyre load of trucks,^{49,50} the investigation of road damage J_{95} reduction was only studied preliminarily in a conference paper.⁵¹ The current paper aims to illustrate systematically the potential for road damage reduction offered by inerter-based passive suspensions. Three simplified truck models are investigated: quarter-car, pitch-plane and roll-plane models. While the correlation between road damage and vehicle speed are considered for both half-vehicle models, the trade-off between road damage and roll stability is also analysed. Besides, the typical nonlinearities due to leaf springs are further been incorporated. The design of suspension systems for road vehicles (especially for trucks) is often subject to

constraints such as space, weight and cost. Hence in the present paper, studies are focused on two low-complexity inerter-based suspension layouts, which will be compared with conventional parallel spring-damper layout. It also worth to mention that an adaptive inerter concept has recently been introduced by Li et al.⁵² In this work, the performance advantages on ride quality, tyre deflection and suspension deflection have been identified by using the force-tracking control strategy and adjustable inertance, compared with the passive inerter-based suspension. While the layouts proposed in the present work are pure passive with constant element values, the adaptive inerter concept can be directly applied, where more significant road damage reductions are anticipated.

This paper is organised as follows. Following the introduction, the road damage index is reviewed. The candidate low-complexity linear suspension layouts and the nonlinear characteristics of leaf springs are then introduced. In the next section, the potential for road damage reduction using inerter-based suspensions is investigated using three simplified models. The improvements that can be delivered by low-complexity inerter based layouts are computed with corresponding optimal parameter values identified. In addition, the influence of nonlinear leaf springs is investigated. Conclusions are drawn in the final section.

2 Performance criterion and candidate suspension layouts

In this section, the J_{95} road damage criterion⁶ will be recalled using a quarter-car model as an example. The derivation of J_{95} for the pitch-plane and roll-plane models will be discussed in the next section after each model being explained. This performance index is adopted by the present study. Candidate suspension layouts and the optimisation approach used are then presented.

2.1 Road damage index calculation

Consider a typical quarter-car model with conventional parallel spring-damper suspension layout shown in Figure 1, where z_r , z_u and z_s represent vertical displacements of road surface, axle (unsprung mass m_u), and vehicle body (sprung mass m_s) respectively. The suspension strut consists of a spring with stiffness k_s and a damper with damping c_s . The tyre is assumed to be a linear spring with stiffness k_t , while the damping effect of the tyre is ignored in this paper due to the research focus on suspension.

Based on Newton's second law and taking Laplace transforms, the equations of motion for the sprung and unsprung masses are (s denotes complex number frequency, and $\hat{\cdot}$ denotes Laplace transform):

$$m_u \hat{z}_u s^2 = k_t(\hat{z}_r - \hat{z}_u) - c_s(\hat{z}_u - \hat{z}_s)s - k_s(\hat{z}_u - \hat{z}_s) \quad (1)$$

$$m_s \hat{z}_s s^2 = c_s(\hat{z}_u - \hat{z}_s)s + k_s(\hat{z}_u - \hat{z}_s) \quad (2)$$

The first step in derivation of the road damage index (J_{95}) is the calculation of dynamic tyre force based on specified road inputs. Equation 3 shows a single-sided spectrum (S_{Z_r}) introduced by Kamash and Robson⁵³ to describe the road surface input (z_r):

$$S_{Z_r}(\omega) = \frac{G_0 \left| \frac{\omega}{2\pi V} \right|^{-2w}}{2\pi V} \quad (3)$$

where ω is the angular frequency, V is the vehicle speed, w is the spectrum slope, and G_0 is the road roughness. The value of G_0 is typically between 0.03×10^{-6} to 0.5×10^{-6} for motorways and 0.5×10^{-6} to 30×10^{-6} for minor roads.⁵³ In this paper, G_0 is chosen to be 0.5×10^{-6} , while w is 1.25. The corresponding units of G_0 are $\text{m}^{0.5}\text{s}^{-2}$.

Based on the assumption of linear tyre stiffness, the dynamic tyre force is proportional to the tyre deflection,

which is calculated by subtracting road displacement from axle displacement. To obtain axle displacement, Newland⁵⁴ derived the direct spectral density of axle displacement (S_{Z_u}) by applying the formula between the input and output spectral density for a linear system:

$$S_{Z_u}(\omega) = H_{Z_r Z_u}(\omega)^* S_{Z_r}(\omega) H_{Z_r Z_u}(\omega)^T \quad (4)$$

where $*$ denotes complex conjugate, T denotes matrix non-conjugate transpose, and $H_{Z_r Z_u}(\omega)$ is the transfer function between road input (z_r) and axle output (z_u), which can be derived by combining Equation 1 and 2:

$$\begin{aligned} H_{Z_r Z_u} &= \frac{\hat{z}_u}{\hat{z}_r} \\ &= \frac{k_t(k_s + c_s s + m_s s^2)}{m_u m_s s^4 + (m_u + m_s)c_s s^3 \\ &\quad + (k_t m_s + (m_u + m_s)k_s)s^2 + c_s k_t s + k_s k_t} \end{aligned} \quad (5)$$

The cross-spectral densities between road input and axle output are defined as:

$$S_{Z_r Z_u}(\omega) = S_{Z_r}(\omega) H_{Z_r Z_u}(\omega)^T \quad (6)$$

$$S_{Z_u Z_r}(\omega) = S_{Z_r Z_u}(\omega)^* \quad (7)$$

Then the spectral density of the dynamic tyre force can be derived as:

$$S_F(\omega) = H_{Z_r F, Z_u F}(\omega)^* S_{Z_r, Z_u}(\omega) H_{Z_r F, Z_u F}(\omega)^T \quad (8)$$

where $S_{Z_r, Z_u}(\omega)$ is a matrix containing the direct and cross-spectral densities of road input and axle output:

$$S_{Z_r, Z_u}(\omega) = \begin{bmatrix} S_{Z_r}(\omega) & S_{Z_r Z_u}(\omega) \\ S_{Z_u Z_r}(\omega) & S_{Z_u}(\omega) \end{bmatrix} \quad (9)$$

and $H_{Z_r F, Z_u F}(\omega)$ is the frequency response transfer function between tyre deflection and dynamic tyre force, which can be expressed as:

$$H_{Z_r F, Z_u F}(\omega) = \begin{bmatrix} k_t & -k_t \end{bmatrix} \quad (10)$$

The aggregate fourth power tyre force S_{A^4} is defined as:⁵⁴

$$S_{A^4}(\omega)|_{\omega>0} = 16F_s S_F(\omega) \quad (11)$$

$$S_{A^4}(\omega)|_{\omega=0} = F_s^8 \delta(\omega) \quad (12)$$

where F_s is the static tyre force, and $\delta(\omega)$ is Dirac delta function.

Subsequently, the standard deviation of aggregate fourth power tyre force σ_{A^4} is derived to be equal to the square root of the area under the spectral density over the positive frequency range:

$$\sigma_{A^4} = \sqrt{\int_{\omega=0}^{\infty} S_{A^4}(\omega)|_{\omega>0} d\omega} \quad (13)$$

and the static aggregate fourth power tyre force m_{A^4} is calculated as:

$$m_{A^4} = \sqrt{\int_{\omega=0^-}^{0^+} S_{A^4}(\omega)|_{\omega=0} d\omega} = F_s^4 \quad (14)$$

Following the results by Cole and Cebon,⁶ the road damage index is defined as the 95th percentile aggregate fourth-power tyre force normalised by the static component, which can be expressed as:

$$J_{95} = \frac{1.65\sigma_{A^4}}{m_{A^4}} + 1 \quad (15)$$

In this paper, the adopted road damage index J_{95} is calculated for a given speed V , road surface profile (G_0, w), and vehicle model (mass, suspension parameter values).

2.2 Candidate suspension layouts

Three low-complexity linear passive suspension layouts S1, S2 and S3 are shown in Figure 2(a), (b) and (c), respectively representing a conventional parallel spring(k_s)-damper(c_s) layout, a parallel inerter(b)-damper(c_s)-spring(k_s) layout and a series inerter(b)-damper(c_s) in parallel with a spring(k_s) layout.

The admittance $Y(s)$ of each suspension layout is defined as the strut force (F) divided by the strut relative velocity (v) in the Laplace domain, given by:

- S1: $Y_1(s) = \frac{F}{v} = \frac{k_s(\hat{z}_u - \hat{z}_s) + c_s s(\hat{z}_u - \hat{z}_s)}{s(\hat{z}_u - \hat{z}_s)} = \frac{k_s}{s} + c_s$
- S2: $Y_2(s) = \frac{F}{v} = \frac{k_s(\hat{z}_u - \hat{z}_s) + c_s s(\hat{z}_u - \hat{z}_s) + bs^2(\hat{z}_u - \hat{z}_s)}{s(\hat{z}_u - \hat{z}_s)} = \frac{k_s}{s} + c_s + bs$
- S3: $Y_3(s) = \frac{F}{v} = \frac{k_s(\hat{z}_u - \hat{z}_s) + \frac{c_s s \times bs^2}{c_s s + bs^2}(\hat{z}_u - \hat{z}_s)}{s(\hat{z}_u - \hat{z}_s)} = \frac{k_s}{s} + \frac{c_s bs}{bs + c_s}$

As a common nonlinear component in heavy vehicles, it is important to test the influence of leaf springs on road damage performance. One of the most frequently used models of leaf spring was introduced in the book,⁵⁵ in which an empirical equation was used to characterise the force-deflection relationship:

$$F_i = F_{ENV_i} + (F_{i-1} - F_{ENV_i})e^{-\frac{|\delta_i - \delta_{i-1}|}{\beta}} \quad (16)$$

In Equation 16, for $\delta_i > \delta_{i-1}$,

$$F_{ENV} = k_u \delta + F_u$$

and for $\delta_i < \delta_{i-1}$,

$$F_{ENV} = k_l \delta + F_l$$

where F_i , F_{i-1} and δ_i , δ_{i-1} are the spring force and displacement during the current and previous time steps, F_{ENV_i} is the upper and lower boundary for the envelope of the measured force-deflection curves, β is a spring parameter describing the rate of hysteresis as the force approaches the boundary of the envelope and differs for ascending and descending deflections, k_u is upper envelope stiffness, k_l is lower envelope stiffness, F_u is upper envelope friction, and F_l is lower envelope friction.

Figure 3 illustrates the force-deflection characteristics of a typical leaf spring.⁵⁵ It is clear that the spring rate changes continuously during compression and extension.

2.3 Optimisation approach

For each vehicle model, across a wide range of static stiffness values, Matlab was used to calculate the standard deviation of aggregate fourth power tyre force. For linear models this was carried out in the frequency domain as per Equation 13, across a frequency range from 0.2 Hz to 30 Hz with step of 0.2 Hz and assuming a vehicle speed of 22 m/s (80 km/h), similar to the approach used by Cole and Cebon.⁶ Non-linear leaf spring models were necessarily studied in the time domain, for which a time interval of 20 seconds with sample time 0.01 seconds was used. In both domains, Matlab's *fminsearch* algorithm was used to calculate the damper (c_s) and inerter (b) values which minimised the road damage value J_{95} . It is also noted that the stiffness of a monoleaf steel spring is typically 2 MN/m to support the self-weight

of the truck within a reasonable suspension deflection, while the stiffness of a typical air spring is around 0.4 MN/m and can be adjustable.

3 Optimisation results for linear suspension layouts

Previous studies on the influence of suspensions to road damage performance are typically using three types of models, quarter-car, roll-plan, and pitch-plan.^{6,7} In order to form a fair comparison, the potential performance improvement of inerter-based suspensions will be demonstrated with these three models in this section.

3.1 Quarter-car model analysis

A quarter-car model with 2 degrees-of-freedom (DoF) is shown in Figure 4, which is used to describe the bounce motion of a heavy truck.⁶ The same quarter-car model is used in this work, with sprung mass m_s of 4500 kg, unsprung mass m_u of 500 kg, and tyre stiffness k_t of 2 MN/m. Its suspension with admittance $Y(s)$ can be selected from the three candidate layouts introduced in the previous section.

Based on Newton's second law and taking Laplace transforms, the equations of motion for the sprung and unsprung masses are:

$$m_u \hat{z}_u s^2 = k_t (\hat{z}_r - \hat{z}_u) - Y(s) (\hat{z}_u - \hat{z}_s) s \quad (17)$$

$$m_s \hat{z}_s s^2 = Y(s) (\hat{z}_u - \hat{z}_s) s \quad (18)$$

The transfer function $H_{Z_r Z_u}$ from $\hat{z}_r$ to $\hat{z}_u$ can be derived by combining Equations 17 and 18:

$$\begin{aligned} H_{Z_r Z_u} &= \frac{\hat{z}_u}{\hat{z}_r} \\ &= \frac{k_t (Y(s)s + m_s s^2)}{m_u m_s s^4 + (m_u + m_s) Y(s) s^3 + k_t m_s s^2 + Y(s) k_t s} \end{aligned} \quad (19)$$

The corresponding road damage is calculated based on Equation 15. The optimisation results for this quarter-car model are plotted in Figure 5 across a wide range of static stiffness for the three candidate suspension layouts. Figure 5(a) shows the optimised J_{95} values, while Figure 5(b) displays the percentage improvements of S2 and S3 compared with S1, respectively. The improvement of J_{95} increases with static stiffness for both S2 and S3. In the latter case the road damage is reduced by about 12% for a static stiffness of 2 MN/m, which represents the stiffness of typical leaf springs for a heavy vehicle. Figures 5(c) and 5(d) present the corresponding optimum damper and inerter values respectively. At very low stiffness values, inerters are found to be incapable of reducing road damage, hence optimal inerter values are then infinity for S3 and zero for S2, yielding similar road damage to the S1 case. It is also notable that the parallel S2 layout requires less damping and lower inertance than the series S3 layout, which is to be expected because the displacements (and hence velocities and accelerations) across the terminals of the damper and inerter will generally be greater in the parallel case. Figures 5(a) and 5(b) show that S3 has better performance than S2 over the whole range of static stiffness. But for a given loading or stiffness value, S3 needs much higher inertance, which may have practical or financial implications. It is worth to pointed out that in the work by Smith and Wang,⁴⁷ the r.m.s dynamic tyre load, which represents the tyre grip performance, was studied for S1-S3. Due to the close link between these two performance indices, similar trends of the optimisation results for both performance indices are expected.

3.2 Pitch-plane model analysis

A pitch-plane model with 4 DoF is shown in Figure 6 to describe both the pitch and vertical motions of a heavy truck. The parameters of this model are specified based on the same heavy truck in the previous subsection, including half-truck

sprung mass $m_s = 9000$ kg, front/rear unsprung masses $m_{u1} = m_{u2} = 500$ kg, and front/rear tyre stiffness $k_{t1} = k_{t2} = 2$ MN/m (as in the work⁶). The distances between the centre of gravity of the sprung mass to both suspensions are defined to be identical: $a = 4$ m, and the sprung mass is assumed to be uniformly distributed over its length, with ratio $r = 0.58$, such that its moment of inertia $I_s = m_s a b r^2$. The admittances of the front and rear suspensions are assumed to be the same: $Y_1(s) = Y_2(s)$, consequently the front and rear static tyre forces F_{s1} and F_{s2} are also equal.

Four equations of motion can be constructed as below, corresponding to pitch and vertical motions of the masses:

$$m_{u1} \hat{z}_{u1} s^2 = k_{t1} (\hat{z}_{r1} - \hat{z}_{u1}) - Y_1(s) (\hat{z}_{u1} - \hat{z}_s + a \hat{\theta}_s) s \quad (20)$$

$$m_{u2} \hat{z}_{u2} s^2 = k_{t2} (\hat{z}_{r2} - \hat{z}_{u2}) - Y_2(s) (\hat{z}_{u2} - \hat{z}_s - a \hat{\theta}_s) s \quad (21)$$

$$m_s \hat{z}_s s^2 = Y_1(s) (\hat{z}_{u1} - \hat{z}_s + a \hat{\theta}_s) s + Y_2(s) (\hat{z}_{u2} - \hat{z}_s - a \hat{\theta}_s) s \quad (22)$$

$$I_s \hat{\theta}_s s^2 = -a Y_1(s) (\hat{z}_{u1} - \hat{z}_s + a \hat{\theta}_s) s + a Y_2(s) (\hat{z}_{u2} - \hat{z}_s - a \hat{\theta}_s) s \quad (23)$$

In this pitch-plane model, the road inputs z_{r1} and z_{r2} represent the vertical road disturbances of the front wheel and the rear wheel, respectively. The axle outputs z_{u1} and z_{u2} denote the vertical displacements of the unsprung mass at the front and rear axles respectively. The vehicle body outputs θ_s and z_s represent the rotational and vertical displacements of the sprung mass.

To derive the value of road damage index J_{95} for this pitch-plane model, the direct spectral densities of axle displacements (S_{Z_u}) needs to be extended as there are now two road inputs and two axle outputs:⁵⁴

$$S_{Z_u}(\omega) = H_{Z_r Z_u}(\omega)^* (S_{Z_r}(\omega) P(\omega)) H_{Z_r Z_u}(\omega)^T \quad (24)$$

where $H_{Z_r, Z_u}(\omega)$ here is a matrix of frequency response functions, including four transfer functions between road inputs (z_{r1} and z_{r2}) and axle outputs (z_{u1} and z_{u2}) based on Equations 20 to 23. Matrix $P(\omega)$ represents the relations between each of the road displacement inputs, which are identical, but with a time delay between them. The matrix $P(\omega)$ is specified based on this time delay:

$$P(\omega) \doteq P_{pitch}(\omega) = \begin{bmatrix} 1 & e^{-i\omega T_{12}} \\ e^{i\omega T_{12}} & 1 \end{bmatrix} \quad (25)$$

where T_{12} represents the time delay between road inputs to the front and the rear axle that can be derived from the vehicle speed and the wheelbase ($2a$) of the vehicle:

$$T_{12} = \frac{2a}{V} \quad (26)$$

Then the matrix $S_{Z_r, Z_u}(\omega)$ in Equation 8 needs to be updated to include all the terms for direct and cross-spectral densities of two road inputs and two axle outputs:

$$S_{Z_r, Z_u}(\omega) = \begin{bmatrix} S_{Z_{r1}Z_{r1}}(\omega) & S_{Z_{r1}Z_{u1}}(\omega) & S_{Z_{r1}Z_{r2}}(\omega) & S_{Z_{r1}Z_{u2}}(\omega) \\ S_{Z_{u1}Z_{r1}}(\omega) & S_{Z_{u1}Z_{u1}}(\omega) & S_{Z_{u1}Z_{r2}}(\omega) & S_{Z_{u1}Z_{u2}}(\omega) \\ S_{Z_{r2}Z_{r1}}(\omega) & S_{Z_{r2}Z_{u1}}(\omega) & S_{Z_{r2}Z_{r2}}(\omega) & S_{Z_{r2}Z_{u2}}(\omega) \\ S_{Z_{u2}Z_{r1}}(\omega) & S_{Z_{u2}Z_{u1}}(\omega) & S_{Z_{u2}Z_{r2}}(\omega) & S_{Z_{u2}Z_{u2}}(\omega) \end{bmatrix} \quad (27)$$

and the frequency response transfer functions ($H_{Z_r, F, Z_u, F}(\omega)$ in Equation 8) between tyre deflections and dynamic tyre forces are extended to:

$$H_{Z_r, F, Z_u, F}(\omega) = \begin{bmatrix} k_{t1} & -k_{t1} & 0 & 0 \\ 0 & 0 & k_{t2} & -k_{t2} \end{bmatrix} \quad (28)$$

The aggregate fourth power tyre force S_{A^4} is defined for two correlated tyre forces:⁵⁴

$$S_{A^4}(\omega)|_{\omega>0} = \sum 16F_{s12} \otimes P_{pitch}(\omega)^* S_F(\omega) \quad (29)$$

$$S_{A^4}(\omega)|_{\omega=0} = \left(\sum_{j=1}^2 F_{sj}^4 \right)^2 \delta(\omega) \quad (30)$$

where Σ indicates the sum of all the elements of the matrix, $\otimes$ indicates element by element multiplication, F_{sj} is static tyre force on the j th tyre, and $\mathbf{F}_{s12}$ is a matrix that includes all the static tyre forces (F_{s1} and F_{s2}).

$$\mathbf{F}_{s12} = \begin{bmatrix} F_{s1}^3 & F_{s2}^3 \end{bmatrix} \quad (31)$$

Having modified the above expressions, the road damage index J_{95} is again calculated through Equation 15 for the pitch-plane model. According to Equations 25 and 26, there will be a correlation between vehicle speed and road damage. With fixed suspension stiffness $k_{s1} = k_{s2} = 0.4$ MN/m and fixed suspension damping $c_{s1} = c_{s2} = 10$ kNs/m, the minimum road damage can be achieved by only optimising inertance b for suspension layouts S2 and S3, are plotted in Figure 7 versus vehicle speed, compared with the road damage caused by conventional layout S1. The plots of road damage with correlated inputs and outputs display peaks at some speeds across the ascending curve of road damage, while the value for the case without correlation (when $P(\omega)$ is an identity matrix) grows monotonically with speed. Such differences demonstrate the effect of the interaction between the vehicles front and rear axles on road damage, which can be explained by a wheelbase filtering effect related with the model's natural frequencies. For

example, there is a pitch mode at 2.35 Hz, which equates to 18.8 m/s for a wheelbase of 8.0 m, at which speed the ratios of correlated to uncorrelated road damage are approximately local maxima for each of S1-S3. Hence, as a simple design approach for heavy vehicle manufacturers, placing the front and rear wheels in different wheel paths may effectively reduce the road damage. Meanwhile, the optimum road damage for suspension layout S3 illustrates that the achievable reduction of road damage by inerter-based suspensions varies with speed.

For brevity, the optimisation results for the pitch-plane model will be discussed together with roll-plane model in the next subsection for all three candidate suspension layouts.

3.3 Roll-plane model analysis

For the roll-plane case, due to the more serious road damage caused by rigid axle structures,⁶ it is more persuasive to choose this axle type for investigation. In order to calculate the dynamic tyre force for road damage assessment, a 4 DoF roll-plane model of a heavy truck is depicted in Figure 8. The parameters of the model are selected based on a single axle of a typical heavy truck, therefore the model is symmetrical and all the parameters are identical for both sides. Similar assumptions are adopted to those in the study by Cole and Cebon:⁶ pitch-plane correlation between axles and load transfer within the axles are neglected during the analysis for simplicity. The parameters of this roll-plane model are specified from the same heavy truck in the previous subsections with a typical rigid axle suspension, including half-truck sprung mass $m_s = 9000$ kg, single axle unsprung masses $m_u = 1000$ kg, distances between tyres and unsprung mass roll centre $l_t = 0.925$ m, distances between suspensions and sprung mass roll centre $l_s = 0.5l_t = 0.4625$ m, height of sprung mass roll centre $h_s = 2$ m, height of sprung mass centre of gravity $h_{sr} = 0.6$ m, and tyre stiffness $k_t = 2$ MN/m. In addition, we assume that

the unsprung mass roll centre is at ground level, hence the height of unsprung mass centre of gravity $h_u = 0$ m.

The vertical and angular equations of motion for both the sprung and unsprung masses are shown below:

$$m_s \ddot{z}_s = Y(s)(\ddot{z}_u - \ddot{z}_s - l_s \ddot{\theta}_s - l_s \ddot{\theta}_u)s \quad (32)$$

$$+ Y(s)(\ddot{z}_u - \ddot{z}_s + l_s \ddot{\theta}_s + l_s \ddot{\theta}_u)s$$

$$m_u \ddot{z}_u = -Y(s)(\ddot{z}_u - \ddot{z}_s - l_s \ddot{\theta}_s - l_s \ddot{\theta}_u)s \quad (33)$$

$$- Y(s)(\ddot{z}_u - \ddot{z}_s + l_s \ddot{\theta}_s + l_s \ddot{\theta}_u)s$$

$$+ k_t(\ddot{z}_{r1} - \ddot{z}_u + l_t \ddot{\theta}_u)$$

$$+ k_t(\ddot{z}_{r2} - \ddot{z}_u - l_t \ddot{\theta}_u)$$

$$I_s \ddot{\theta}_s = -l_s Y(s)(\ddot{z}_u - \ddot{z}_s - l_s \ddot{\theta}_s - l_s \ddot{\theta}_u)s \quad (34)$$

$$+ l_s Y(s)(\ddot{z}_u - \ddot{z}_s + l_s \ddot{\theta}_s + l_s \ddot{\theta}_u)s$$

$$I_u \ddot{\theta}_u = l_s Y(s)(\ddot{z}_u - \ddot{z}_s - l_s \ddot{\theta}_s - l_s \ddot{\theta}_u)s \quad (35)$$

$$- l_s Y(s)(\ddot{z}_u - \ddot{z}_s + l_s \ddot{\theta}_s + l_s \ddot{\theta}_u)s$$

$$- l_t k_t(\ddot{z}_{r1} - \ddot{z}_u + l_t \ddot{\theta}_u)$$

$$+ l_t k_t(\ddot{z}_{r2} - \ddot{z}_u - l_t \ddot{\theta}_u)$$

where θ_s is sprung mass roll angle, z_s is sprung mass vertical displacement, θ_u is unsprung mass roll angle and z_u is unsprung mass vertical displacement. For consistency with the variables specified in the derivation of road damage index J_{95} , the corresponding z_{u1} and z_{u2} are expressed by combining l_t with θ_u and z_u (assuming $\sin(\theta_u) \approx \theta_u$ when θ_u is small):

$$z_{u1} = z_u - \theta_u l_t \quad (36)$$

$$z_{u2} = z_u + \theta_u l_t \quad (37)$$

Also the moments of inertia (I_s and I_u) for the sprung and unsprung masses are:

$$I_s = m_s l_s^2 + m_s (h_s - h_{sr})^2 \quad (38)$$

$$I_u = m_s l_t^2 + m_s h_u^2 \quad (39)$$

3.3.1 Road damage analysis The vehicle's left and right wheels are subject to identical road excitations on a homogeneous and isotropic random surface. Therefore, the matrix $P(\omega)$ in Equation 24 representing the relationship between each of the road displacement inputs is:

$$P(\omega) \doteq P_{roll}(\omega) = \begin{bmatrix} 1 & g_n \\ g_n & 1 \end{bmatrix} \quad (40)$$

where the coherence function g_n between each height of the parallel profiles can be represented as:⁵⁶

$$g_n = \left\{ \frac{2|\pi l_t n|^w}{\Gamma(w)} \right\} K_w(2\pi l_t n) \quad (41)$$

where n is wave number, $\Gamma(\cdot)$ denotes the gamma function, and $K_w(\cdot)$ denotes the modified Bessel function of order w . Additionally, due to the lack of correlation between the two road inputs, the aggregate fourth power force S_{A^4} reduces to:

$$S_{A^4}(\omega)|_{\omega>0} = \sum 16F_s \otimes S_F(\omega) \quad (42)$$

Optimisations were carried out to achieve minimum road damage following the procedure in the section of 'Optimisation approach'. For the quarter-car, pitch-plane and roll-plane models, the optimum values of road damage index J_{95} for layout S1 are demonstrated in Figure 9(a). The general trends of the results for both pitch-plane and roll-plane models are similar to those for the quarter-car model discussed before. However, for the pitch-plane model the road damage is reduced, which must be attributable to the effect of correlation between front and rear axles. The percentile improvements of J_{95} for layouts S2 and S3 in comparison with S1 for each vehicle model are plotted in Figure 9(b). It is clear that the improvements are further enhanced in half-vehicle models, except for soft suspensions

with stiffness approximately less than 0.7 MN/s for the S3 layout in the pitch-plane model. For roll-plane model, both S2 and S3 layouts can reduce road damage across a wider range of static stiffness than for pitch-plane and quarter-car models. Meanwhile, greater road damage reduction is achieved by the S3 layout across the majority of the static stiffness range for all three models. The corresponding values of damping and inertance for optimum J_{95} are shown in Figure 9(c) and 9(d), respectively. Figure 9(d) shows that generally lower inertance values are optimal for pitch-plane than for quarter-car models. The nearly infinite value of inertance for low stiffness values reveals approximately a rigid rod. Therefore S3 works similarly to S1 below stiffness of 0.5 MN/s for the pitch-plane model. In addition, in contrast to the pitch-plane model with S2 and S3 layouts, the required damping and inertance for roll-plane models are larger compared to S1 layout. The reason could be the superimposed dynamic tyre force from the interactive road inputs across the rigid axle.

3.3.2 Rollover threshold analysis Vehicle rollover during cornering is one of the most serious safety concerns for heavy trucks and hence merits investigation. Rotations of the sprung and unsprung masses about the roll axis are the dominant causes of rollover. According to the method presented in the work by Chondros et al.,⁵⁷ the quasi-static roll process of the vehicle is governed by Equations 43 and 44, which are derived from the roll-plane model in Figure 8:

$$m_s q_s h_s + m_u q_s h_u + m_s g(h_s - h_{sr})\theta_s \quad (43)$$

$$= (R_u - m_s g h_{sr} - m_u g h_u)\theta_u$$

$$m_s(h_s - h_{sr})q_s + (m_s g(h_s - h_{sr})\theta_s + R_s)\theta_s = -R_s\theta_u \quad (44)$$

where q_s denotes vehicle lateral acceleration, R_u is roll stiffness about the unsprung mass roll centre resulting from the vertical stiffness of the tyres, and R_s is roll stiffness

about the sprung mass roll centre resulting from the vertical stiffness of the suspension. They are given by:

$$R_u = 2k_t l_t^2 \quad (45)$$

$$R_s = 2k_s l_s^2 \quad (46)$$

Similar to the assumptions made in the study by Chondros et al.,⁵⁷ ignoring other dynamic effects, the static rollover threshold is defined as the lateral acceleration (q_s) required to reduce the vertical tyre force on one of the tyres to zero, in other words, the maximum lateral acceleration before rollover. Taking the contact point between the other tyre and the road as the pivot, the moment about it arises from gravity, so the maximum unsprung mass roll angle is (assuming the centre of total mass is at the centre of track width):

$$\theta_u = \frac{(m_s + m_u)g}{k_t l_t} \quad (47)$$

Equations 43 and 44 can be used to derive the lateral acceleration q_s :

$$q_s = \frac{m_s g(h_s - h_{sr})(R_u - m_s g h_{sr} - m_u g h_u) + R_s(m_s g h_s + m_u g h_u - R_u)}{m_s g(h_s - h_{sr})(m_s g h_{sr} + m_u g h_u) - R_s(m_s g h_s + m_u g h_u)} \theta_u \quad (48)$$

Substituting Equations 45, 46, and 47 into Equation 48, the static rollover thresholds (q_s) can be calculated. Therefore, the optimised road damage and static rollover threshold in response of a wide range of stiffness can be derived for a given speed of 22 m/s. Accordingly, three curves are plotted in Figure 10, to represent the relationship between J_{95} and q_s for suspension layouts S1, S2 and S3, respectively. The curves show the boundaries of the

achievable performance. The top left area indicates the achievable performance and the inerter-based suspension layouts (S2 and S3) extend the area towards the bottom right, which is more desirable for both the road damage improvement and the rollover threshold. For all three layouts, the maximum lateral accelerations before rollover tend to constant values as the road damage increases. For any given rollover threshold, S3 effectively reduces the road damage compared to S1, and S2 is also beneficial for static rollover thresholds above 3 m/s². It is also notable that a trade-off must be made between lower road damage and higher static rollover threshold.

4 Optimisation results of models with leaf spring nonlinearities

The previous investigations were carried out in the frequency domain. In order to assess the effect of nonlinear leaf springs, time domain Simulink models for the pitch-plane and the roll-plane were developed.

In the time domain, the road profile needs to be time dependent. Based on Equation 3, a filtered white noise profile was introduced to simulate the roads vertical input z_0 .⁵⁸

$$\dot{z}_0(t) = -2\pi f_0 z_0(t) + 2\pi \sqrt{G_0 V} w(t) \quad (49)$$

where f_0 denotes lower boundary of frequency ($f_0 = 0.2$ Hz is chosen in this paper), V is the vehicle speed, and $w(t)$ represents a random time-dependent white noise.

Using Equation 16, a Simulink model was built with the deflection of the suspension as its input and the force produced by a leaf spring as its output. The random road inputs give rise to variable amplitudes of spring deflection for each cycle, but all the parameters in Equation 16 are sensitive to the amplitude of deflection and the preload. The road damage was optimised for three different sets of leaf spring parameters (shown in Table 1) corresponding to

soft, medium and relatively stiff leaf springs respectively, based on the parameters in the relevant literatures.^{55,59,60} The optimised results for S1, S2 and S3 for each of these three leaf springs are listed in Table 2. It is also worth mentioning that the fixed time step is used when run the simulation due to the values of β are chosen as constant.

Table 1. Three sets of parameter values for typical soft, medium and stiff leaf springs.

Parameters	Leaf spring 1 (soft)	Leaf spring 2 (medium)	Leaf spring 3 (stiff)
k_u (MN/m)	0.58	1.35	2.06
k_l (MN/m)	0.47	1.14	1.77
F_u (kN)	15.8	36.5	55.4
F_l (kN)	11.5	29.0	46.0
β (mm)	2.03	1.03	0.1

The general trends of road damage are similar to those from the frequency-domain analysis for pitch-plane and roll-plane models with linear springs: at low values of stiffness (leaf spring 1), layouts S2 and S3 do not provide any benefit over S1, but at intermediate and high stiffness (leaf springs 2 and 3), layout S3 in particular reduces road damage by up to 19.3% for the pitch-plane model and 12.2% for the roll-plane model. It is also worth mentioning that smaller damping is required for a given reduction in road damage since the damping effect of leaf springs contributes to the total damping force. There are some other uncertainties that may affect the results, such as the amplitude sensitivity of the leaf spring parameters, which means the current optimised suspension parameters will vary when the amplitude of road input changes.

In terms of the investigation of the leaf spring in both pitch-plane and roll-plane models, the road damage improvements are not strongly affected by this nonlinear factor, especially the benefit obtained for the pitch-plane model, which is even bigger than the linear model for very stiff suspension.

5 Conclusions

This paper demonstrates that road damage can be significantly reduced by incorporating inerter into the suspension systems of heavy trucks. Matlabs *fminsearch* algorithm was used to determine the values of suspension elements (i.e. spring, damper and inerter), which minimise the J_{95} road damage index. Based on the results obtained, the following design guidelines for inerter-based suspensions for heavy vehicles have been concluded: the inerter-integrated suspension can provide more significant road damage reduction when the static stiffness are higher, and in general, the series inerter-damper layout (S3) can provide more road damage reduction than the parallel inerter-damper layout (S2). Therefore, it has been shown that there are promising benefits for reduction in road damage by incorporating the inerter element into truck suspension systems. The present work has also demonstrated that a step change in the trade off between roll stability and road damage can be achieved with inter-based suspensions for heavy vehicles. Furthermore, similar levels of improvement have been shown to arise when the leaf-spring nonlinearities are incorporated into inerter-based suspensions. It is also anticipated that the road damage reduction can be further enlarged with more complicated inerter-based layout, the approach described in this work can be directly applied to identify their exact parameter values. Building on this work, performance evaluation with realistic full truck models, e.g. an articulated truck, and physical realisations of identified inerter-based suspension configurations, are both essential next steps for industrial applications.

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Table 2. Optimum road damage, together with corresponding damper and inerter values for the pitch-plane and the roll-plane models based on three sets of leaf spring parameters (% improvement are given in bracket compared with S1).

Leaf spring	Layout	Pitch-plane Model			Roll-plane Model		
		Road Damage J_{95}	Damping (kNs/m)	Inertance(kg)	Road Damage J_{95}	Damping (kNs/m)	Inertance (kg)
1	S1	1.66	28.5	N/A	1.71	41.4	N/A
	S2	1.66 (0%)	28.5	0	1.71 (0%)	41.4	0
	S3	1.66 (0%)	27.7	16622	1.71 (0%)	40.7	52000
2	S1	1.93	41.2	N/A	1.83	44.2	N/A
	S2	1.93 (0%)	41.2	0	1.83 (0%)	44.1	0
	S3	1.78 (7.7%)	45.8	3888	1.80 (2%)	51.1	10705
3	S1	2.54	79.3	N/A	2.30	80.8	N/A
	S2	2.10 (17.3%)	46.7	1303	2.08 (9.6%)	41.0	3437
	S3	2.05 (19.3%)	111.3	5354	2.02 (12.2%)	101.1	8248

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List of notations

J_{95}	The 95th percentile aggregate fourth power force
z_r	Vertical displacements of road surface
z_u	Vertical displacements of axle
z_s	Vertical displacements of vehicle body
m_u	Unsprung mass
m_s	Sprung mass
k_s	Static stiffness
c_s	Damper damping
k_t	Tyre stiffness
V	Vehicle speed
ω	Angular frequency
w	Spectrum slope
G_0	Road roughness
S_{A^4}	Aggregate fourth power tyre force
F_s	Static tyre force
F_{ENV}	The upper and lower boundary for the envelope
β	The spring parameter for the rate of hysteresis
k_u	Upper envelope stiffness
k_l	Lower envelope stiffness
F_u	Upper envelope friction
F_l	Lower envelope friction
a	Half wheelbase
r	Sprung mass distributed ratio
I_s	Sprung mass moment of inertia
T_{12}	Time delay
b	Inertance
l_t	Distances between tyres and unsprung mass roll centre
l_s	Distances between suspensions and sprung mass roll centre
h_s	Height of sprung mass roll centre
h_{sr}	Height of sprung mass centre of gravity
h_u	Height of unsprung mass centre of gravity
θ_s	Sprung mass roll angle
θ_u	Unsprung mass roll angle
I_u	Unsprung mass moment of inertia
n	Wave number
q_s	Vehicle lateral acceleration
R_u	Roll stiffness about the unsprung mass roll centre
R_s	Roll stiffness about the sprung mass roll centre
f_0	Lower boundary of frequency