Meibodi, A. A., & Alexander, N. A. (2020). Exploring a generalized nonlinear multi-span bridge system subject to multi-support excitation using a Bouc-Wen hysteretic model. *Soil Dynamics and Earthquake Engineering*, 135, [106160].
<https://doi.org/10.1016/j.soildyn.2020.106160>

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[10.1016/j.soildyn.2020.106160](https://doi.org/10.1016/j.soildyn.2020.106160)

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Exploring a generalized nonlinear multi-span bridge system subject to multi-support excitation using a Bouc-Wen hysteretic model

A.A. Meibodi¹, N.A. Alexander²

Abstract

This paper presents a generalized reduced-order nonlinear model subjected to multi-support seismic excitation. The hysteretic, nonlinear, relationship of piers is phenomenologically captured by a calibrated Bouc-Wen model. This generalized reduced-order model is benchmarked against legacy physical experimental tests performed at the University of Bristol. A deterministic approach using real spatiotemporal ground motions recorded at the SMART-1 array, Taiwan, is employed as an alternative to a stochastic methodology used in current provision codes. This is so that the influence of nonlinearity and ground motion aleatory and epistemic effects are fully captured. Incremental Dynamic Analysis (IDA) is then performed to identify the performance levels at which this system transitions from elastic to inelastic behaviour. A parametric study is then performed to explore the effect of the spatial variability of the ground motion while bridge alignment, valley profile and ground motion intensity are modified. Results indicate that bridges over shallow valleys with a central rise are prone to significant analysis errors if multi-support excitation is not employed.

Keywords: Multiple excitations; Nonlinear Time-history analysis; Bridge dynamics; Hysteretic MDOF structure; Bouc-Wen model; Bridge Assessment

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Nomenclature

A, β, γ	Basic hysteresis shape control []
$\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3$	State-space system matrices []
$\mathbf{C}$	Non-dimensional damping matrices []
$\mathbf{C}_{11}, \mathbf{C}_{12}, \mathbf{C}_{22}$	Non-dimensional damping block matrices []
d_c	Depth of the pier's sections [L]
$\mathbf{M}$	Non-dimensional mass matrix []
$\bar{\mathbf{M}}$	Generalized mass matrix []
$\mathbf{M}_{11}, \mathbf{M}_{12}, \mathbf{M}_{22}$	Non-dimensional block mass matrices []
EI_d	Deck flexural rigidity (about z-axis) [F][L] ²
I_d	The moment of inertia of deck [L] ⁴
I_{ci}	The moment of inertia of i^{th} pier [L] ⁴
$f_l(x, t)$	inertia force [F]
F_i	hysteretic spring force at i^{th} pier [F]
F_{yi}	Yield force of i-th pier [F]
$\mathbf{g}$	Ground DOFs vector []
$\mathbf{g}_E$	Ground DOFs vector at exterior/end supports []
$\mathbf{g}_I$	Ground DOFs vector at interior pier supports []
h_{ci}	Height of i^{th} pier [L]
k_i	Pier lateral flexural stiffness [F][L] ⁻¹
$\mathbf{K}_d$	Non-dimensional deck stiffness matrix []
$\bar{\mathbf{K}}_d$	Generalized deck stiffness matrix [F][L] ⁻¹
$\mathbf{k}_{d,11}, \mathbf{k}_{d,12}, \mathbf{k}_{d,22}$	Non-dimensional block deck stiffness matrices []
$\mathbf{K}_g$	Non-dimensional ground stiffness matrix []
$\bar{\mathbf{K}}_g$	Generalized ground stiffness matrix [F][L] ⁻¹

$\mathbf{k}_{g,11}, \mathbf{k}_{g,12}, \mathbf{k}_{g,22}$	Non-dimensional block ground stiffness matrices []
$\mathbf{K}_u$	Non-dimensional pier stiffness matrix []
$\bar{\mathbf{K}}_u$	Generalized pier stiffness matrix [F][L] ⁻¹
$\mathbf{k}_{u,11}, \mathbf{k}_{u,12}, \mathbf{k}_{u,22}$	Non-dimensional block Pier stiffness matrices []
$\mathbf{K}_z$	Non-dimensional hysteretic stiffness matrix [L]
$\mathbf{k}_{z,11}, \mathbf{k}_{z,12}, \mathbf{k}_{z,22}$	Non-dimensional block hysteretic stiffness matrices [L]
$\bar{\mathbf{K}}_z$	Generalized hysteretic stiffness matrix [F]
L	Span length [L]
$\bar{\mathbf{L}}$	Generalized excitation factor []
$\mathbf{L}$	Non-dimensional excitation factor []
$\mathbf{L}_{11}, \mathbf{L}_{12}, \mathbf{L}_{22}$	Non-dimensional block excitation factor []
m_d	Deck mass per unit length [F][T] ² [L] ⁻²
m	Number of deck DOFs []
n	Sharpness of yield []
PGA	Peak ground acceleration [L] [T] ⁻²
s	Number of Piers []
t	Time [T]
$\mathbf{u}$	Bridge deck DOFs vector (including free and constrained DOFs) []
u_{yi}	yield displacement of i-th pier [L]
$\mathbf{X}(t)$	the state vector of deck DOFs []
$y^t(x, t)$	lateral total displacement at time t [L]
$y(x, t)$	lateral displacement relative to the ground at time t [L]
$y_g(x, t)$	The lateral deformation of the ground at time t [L]
$\mathbf{w}$	Deck displacement DOFs (including only free DOFs) []
$z_i(t)$	dimensionless auxiliary variable []
$\mathbf{z}(t)$	Hysteretic vector of Deck DOFs []

$\mathbf{z}_E$	Hysteretic DOFs vector at exterior/end supports []
$\mathbf{z}_I$	Hysteretic DOFs vector at interior pier supports []
α	Pier to deck stiffness ratio []
β_m, β_k	Rayleigh damping coefficients []
χ	Percentage error in pier deformation when using ISE [%]
γ_i	i^{th} modal damping ratio []
δW_I	Internal virtual work [F][L]
δW_E	External virtual work [F][L]
$\delta y(x)$	Virtual displacement [L]
δ_v	Strength degradation []
δ_n	Stiffness degradation []
ϵ_n	Normalized dissipated energy []
ϵ_y	Nominal yield strain []
Φ_Y	idealized curvature at the yield point [L] ⁻¹
η_i	i^{th} pier to deck stiffness ratio []
ρ	Pier to pier stiffness ratio []
ξ	Non-dimensional x coordinate along the length of the bridge []
Π	Lagrangian [F][L]
Ψ	Bridge deck Ritz vector []
Ψ_g	Ground DOFs Ritz vector []
ω	System frequency parameter [Rad][T] ⁻¹

1. Introduction

The influence of the spatial variation of seismic ground motion timeseries on the dynamic response of generic life-line structures, such as bridges has been studied extensively for several decades [1-9]. A large number of studies assume that the structural model behaves linearly to make use of the principle of superposition [10-12]. However, for a design level seismic event,

it is often the case that the design philosophy will make use of the energy dissipation of ductile piers [13-18]. Hence, a system-level reduced-order model needs to accommodate the nonlinear inelastic behavior of the piers. It must provide the necessary hysteretic mechanism to dissipate the considerable amounts of earthquake energy under an extreme seismic event [19]. This is why seismic code provisions favor the utilization of nonlinear methodologies for bridge assessment with the aim of identifying the configuration of plastic hinges in piers, estimate post-yield deformation capacity of ductile members and determine the required strength for the avoidance of failure in the demand-protected elements [13, 20, 21].

Amongst all parameters in the modeling of a dynamic system, ground motion input exhibits the highest level of uncertainty. A variety of methodologies and analytical tools are currently available to address these uncertainties. The spatial incoherence of the ground motion are modelled by empirical method (calibrated through regression analyses) [22-26], semi-empirical approach [8, 27, 28] and fully theoretical models [29, 30]. However, these parametric approaches cannot capture the full complexity real ground motion observations, see [31]. Therefore, the authors suggest that real spatiotemporal ground motions, from multi-station arrays records, are employed as an alternative to parametric coherency model.

The proposed methods for analysis can be deterministic or statistical [32]. In practical design, a probabilistic approach developed by structural engineers generally involves defining both ground excitation and seismic response through stochastic (frequency domain) procedures such as power spectra, transfer function estimates and coherency [33-38]. The proposed analysis schemes are based on statistical linearization in order to decompose an inelastic multi-degree-of-freedom system (MDOF) into a limited set of equivalent linear single-degree-of-freedom system (SDOF) with effective linear characteristics [39-43]. However, there are two major limitations associated with linearization techniques. First, the uncertain spatiotemporal nature of real seismic ground motions may not be fully captured through an approximate stochastic

empirical idealisation. Second, the nonlinear coupling effects of nonlinear structural components cannot be accurately captured by pseudo-nonlinear (linearized) systems.

Various analytical procedures ranging from elastic (equivalent static or dynamic) to nonelastic (pushover or dynamic time-history) analysis have been proposed to estimate the post-elastic response of a structure. Among all proposed methods, nonlinear dynamic (time-history) analysis has been recognized as the most rigorous method, although considerable time and effort is required to apply. The leading code provision such as Eurocode 8 [13] and CALTRANS [20] and the analytical tools developed by different researchers [5, 44] aim to employ idealized bilinear models for the stress-strain relationship of ductile members. The main drawback of the above methods is to not capture as well the hysteretic, stiffness and strength degrading behavior of nonlinear piers.

1.1 Aims of paper

Conventional nonlinear finite element analyses uses tens of thousands of degrees of freedom for large bridge in 3D and must have a prescribed explicit geometry. Thus, any parametric exploration using large numbers of ground motions places a huge computational burden on any analyst. This makes such systematic and extensive parametric explorations of geometric effects, such as valley profile, etc. very difficult to undertake in practice. As an alternative, we seek to develop a reduced-order nonlinear model that enables parametric explorations to be undertaken in a more timely fashion, and thus allows far more parametric cases to be considered.

Hence, the main aim of this paper is to first develop a novel reduced order model that is capable of incorporating hysteretic and inelastic behavior of bridge piers. This reduced order model should contain the smallest reasonable set of degrees of freedom and system parameters. [45] developed and benchmarked a linear generalized bridge model and this is extended to the case

of bridge with ductile piers. All analyses are conducted in the time domain using actual seismic data recorded at the SMART-1 array, Taiwan [46]. We make use of standard correction procedures, amplitude scaling and spatial interpolation. Next, generalized reduce order model is exemplified a prototype four-span bridge structure [47]. Finally, an extensive parametric analysis scheme is conducted in order to explore the parametric behavior of this nonlinear bridge system. The purpose of this paper is to quantify the response differences between multi-support excitation (MSE) and identical support excitation (ISE) analyses. With the specific objectives:

- (i) What is the effect of valley profile on the accuracy of ISE analyses? For this objective, we parametrically vary pier-to-deck stiffness ratio and inter-pier stiffness ratio parameters.
- (ii) What is the effect of bridge alignment on the accuracy of ISE analyses? For this objective, we explore 12 different horizontal bridge alignments.
- (iii) Does ground motion amplitude affect the accuracy of ISE analyses? For this objective, we explore low (elastic response), medium (a moderate ductile response) and high (a large ductile response) cases.

2. A minimal generalised reduced bridge model

The proposed model concerns the seismic analysis of ordinary standard highway bridges. In the area with medium to high seismicity, it is generally advisable and cost-effective that a bridge is designed for ductile behavior [48]. Under the no-collapse requirement, flexural hinges are permitted to form in the piers to provide energy dissipation and to limit the overall design seismic action. The superstructure (a capacity-protected member) should be designed to remain essentially elastic in order to retain its functionality during/after seismic action. Therefore, in this context, nonlinear inelastic piers and a linear elastic deck shall be assumed in our proposed

model as shown in Figure 1. This figure is a 2D plan view which shows the total deformation $y_t(x,t)$ of the bridge and piers in the horizontal x - y plane when subjected to ground deformations $y_g(x,t)$.

Figure 1; the x - y plan view of bridge layout including lateral deformation of pier, ground and deck

The generalized bridge system consists of m degree of freedom resting on s support ($m > s$) nonlinear springs. F_i denotes hysteretic spring force at i^{th} pier located at x_i coordinate. $m_d(x)$ is mass per unit length and $EI_z(x)$ is the deck lateral flexural rigidity. It is assumed that the bridge deck is continuous longitudinally (with respect to lateral flexure) and ductile members are assumed as Cantilever Column with Fixed Base therefore, there is shear but no significant moment transfer from deck to piers.

The bridge has $s-1$ equal span with the length of length L and the overall bridge length is $(s-1)L$. The total bridge deck displacements can be re-expressed in terms of a moving coordinate frame as follows

$$y_t = y + y_g \quad (1)$$

where $y(x,t)$ is the deck's lateral displacements relative to the moving ground. Using

D'Alembert's principle [49] , the seismic acceleration induced inertial force per unit length on the deck is given by:

$$f_I = -m_d \ddot{y}_t \quad (2)$$

where, $m_d(x)$ is deck mass per unit length. The piers masses can be assumed lumped decked and ground levels and hence although superficially neglected they are present parametrically through $m_d(x)$. Note the Newtonian prime notation is used for derivatives of a spatial variable whereas the dot notation is use for derivatives of a temporal variable. External virtual work done by inertia force f_I acting through the virtual displacement $\Delta y(x)$:

$$\Delta W_E = \int_0^{(s-1)L} f_I \Delta y \, dx \quad (3)$$

Similarly, the internal virtual work due virtual displacement $\Delta y(x,t)$ is given as:

$$\Delta W_I = \int_0^{(s-1)L} EI_z y_t'' \Delta y'' \, dx + \sum_{i=1}^s \{F_i \Delta y(x_i)\} \quad (4)$$

The first term in above expression is due to deck flexure, where EI_z is the lateral flexural rigidity of the deck, y_t'' is the deck curvature. The second term in the above equation is due to the work done by the nonlinear stiffness forces F_i in the i th pier in moving through a virtual displacement (deformation) $\Delta y(x_i)$. We introduce a spatiotemporal orthogonal basis (Rayleigh-Ritz) form for the ground and relative displacement respectively, as follows:

$$y(x, t) = \boldsymbol{\psi}(x)^T \mathbf{u}(t), \quad y_g(x, t) = \boldsymbol{\psi}_g(x)^T \mathbf{g}(t) \quad (5)$$

147

148 Where column vector $\mathbf{u} \in \mathbb{R}^{m \times 1}$ denotes the degrees of freedom (dofs) on superstructure (deck)
 149 and vector $\mathbf{g} \in \mathbb{R}^{s \times 1}$ denotes restrained dofs of ground. The orthogonal shape function vector for
 150 deck and ground is defined by $\boldsymbol{\psi}(x) \in \mathbb{R}^{m \times 1}, \boldsymbol{\psi}_g(x) \in \mathbb{R}^{s \times 1}$ respectively. Now if a Lagrangian
 151 interpolation scheme is used for ψ as follows:

$$\boldsymbol{\psi}^T = [\psi_1 \quad \psi_2 \quad \cdots \quad \psi_m], \quad \psi_i(x) = \frac{\ell_i(x)}{\ell_i(x_i)}, \quad \ell_i(x) = \frac{\prod_{p=1}^m (x - x_p)}{(x - x_i)} \quad (6)$$

153

154 These Lagrangian interpolants have the added property that $\psi_i(x_i) = 1$ and $\psi_i(x_j) = 0$ ($\forall j \neq i$)
 155). Similarly, the known ground DOFs which include the coordinate of piers and abutments at
 156 ground level are defined by the similar orthogonal shape function vector as:

$$\boldsymbol{\psi}_g^T = [\psi_1 \quad \psi_2 \quad \cdots \quad \psi_s], \quad \psi_i(x) = \frac{\ell_i(x)}{\ell_i(x_i)}, \quad \ell_i(x) = \frac{\prod_{q=1}^s (x - x_p)}{(x - x_i)} \quad (7)$$

158

159 Now let the virtual displacements be defined as follows

160

$$\Delta y = \boldsymbol{\psi}^T \Delta \mathbf{u} \quad (8)$$

162

163 Substituting Eq (5) and Eq (8) in Eq (3) and Eq (4) is given respectively:

164

$$\Delta W_E = - \int_0^{(s-1)L} m_d \left(\boldsymbol{\psi}^T \ddot{\mathbf{u}} + \boldsymbol{\psi}_g^T \ddot{\mathbf{g}} \right) \left(\boldsymbol{\psi}^T \Delta \mathbf{u} \right) dx \quad (9)$$

$$\Delta W_I = \int_0^{(s-1)L} EI_z (\boldsymbol{\psi}''^T \mathbf{u} + \boldsymbol{\psi}_g''^T \mathbf{g}) (\boldsymbol{\psi}''^T \Delta \mathbf{u}) dx + \sum_{i=1}^s \left\{ F_i (\boldsymbol{\psi}(x_i)^T \Delta \mathbf{u}) \right\} \quad (10)$$

167

168 For a product of a row and column vector $\mathbf{a}^T \mathbf{b}$ is a scalar hence, $\mathbf{a}^T \mathbf{b} = \mathbf{b}^T \mathbf{a}$. Therefore, the
 169 above the above work Eq (9) and (10) are re-expressed as follows

170

$$\Delta W_E = -\Delta \mathbf{u}^T \left[\left(\int_0^{(s-1)L} m_d \boldsymbol{\psi} \boldsymbol{\psi}^T dx \right) \ddot{\mathbf{u}} + \left(\int_0^{(s-1)L} m_d \boldsymbol{\psi} \boldsymbol{\psi}_g^T dx \right) \ddot{\mathbf{g}} \right] \quad (11)$$

172

$$\Delta W_I = \Delta \mathbf{u}^T \left[\left(\int_0^{(s-1)L} EI_z \boldsymbol{\psi}'' \boldsymbol{\psi}''^T dx \right) \mathbf{u} + \left(\int_0^{(s-1)L} EI_z \boldsymbol{\psi}'' \boldsymbol{\psi}_g''^T dx \right) \mathbf{u} + \sum_{i=1}^s F_i \boldsymbol{\psi}(x_i) \right] \quad (12)$$

174

175 By employing virtual work principle ($\Delta W_I = \Delta W_E$), the final form of the equation of motion
 176 can be expressed as:

177

$$\overline{\mathbf{M}} \ddot{\mathbf{u}} + \overline{\mathbf{K}}_d \mathbf{u} + \sum_{i=1}^s F_i \boldsymbol{\psi}(x_i) = -\overline{\mathbf{L}} \ddot{\mathbf{g}} - \overline{\mathbf{K}}_g \mathbf{g} \quad (13)$$

179

180 where the dimensional system matrices are defined as follows,

$$\begin{aligned} \overline{\mathbf{M}} &= \int_0^{(s-1)L} m_d \boldsymbol{\psi} \boldsymbol{\psi}^T dx, \quad \overline{\mathbf{K}}_d = \int_0^{(s-1)L} EI_z \boldsymbol{\psi}'' \boldsymbol{\psi}''^T dx \\ \overline{\mathbf{L}} &= \int_0^{(s-1)L} m_d \boldsymbol{\psi} \boldsymbol{\psi}_g^T dx, \quad \overline{\mathbf{K}}_g = \int_0^{(s-1)L} EI_z \boldsymbol{\psi}'' \boldsymbol{\psi}_g''^T dx, \end{aligned} \quad (14)$$

182

183 **2.1 Incorporation of piers hysteretic energy to the** 184 **system by Bouc-Wen model**

185 The inelastic behavior of a dynamic system is associated with a hysteretic correlation between

the resisting pier lateral force and deck-ground displacement [50]. Under assumption of Bouc–
Wen class models, the resisting force of i^{th} pier is derived by a summation of a hysteretic and
an elastic component as [51]:

$$F_i = k_i \{ \alpha u_i + (1 - \alpha) u_{yi} z_i \} \quad (15)$$

$$F_{yi} = k_i u_{yi} \quad (16)$$

Where u_{yi} and k_i denote the pseudo-yield displacement and linear elastic stiffness of the i^{th}
pier. Hence, F_{yi} denotes the pseudo-yield force for the i^{th} pier. α is the ratio of post- to pre-
yield (elastic) stiffness and $z_i(t)$ is dimensionless auxiliary variable that describes hysteretic
behavior. The following auxiliary ordinary differential equation is employed to determine the
hysteretic variable.

$$\dot{z}_i = \frac{1}{1 + \delta_n \epsilon_n} \left\{ A - (1 + \delta_v \epsilon_n) (\gamma \operatorname{sgn}(\dot{u}_i z_i) + \beta) |z_i|^n \right\} \frac{\dot{u}_i}{u_{yi}} \quad (17)$$

where A, β, γ, n are dimensionless parameters controlling the shape of hysteretic loops
(δ_v, δ_n) are dimensionless parameters to incorporate strength deterioration and stiffness
degradation respectively. These values are identical for all piers as they are only dependent on
the physical characteristic of a column. The parameter ϵ_n is dissipated energy normalized by

$F_{yi} u_{yi}$ given as:

$$\epsilon_n(t) = (1 - \alpha) \int_0^t \frac{\dot{u}_i}{u_{yi}} z_i \, dt \quad (18)$$

208

209 With suitably chosen values of parameters γ and β , the model can generate a hysteretic loop
 210 corresponding to the physical properties of the system [52]. It is convenient that conditions
 211 $A = \beta + \gamma = 1$ simplify the representation of the Bouc-Wen model for engineering purposes
 212 provided compatibility with smooth softening hysteretic characteristic of structure [51, 53].
 213 The suggested range of degradation parameters are $0 \leq \delta_n \leq 0.3$ and $0 \leq \delta_v \leq 0.05$ [54, 55]. The
 214 Bouc-Wen parameters $A = 1, \beta = \gamma = 0.5, \alpha = 0.15$ are adopted to exhibit smooth softening-
 215 strain model [53], as well as the degradation parameters, δ_n, δ_v is taken equal to 0.3 and 0.05
 216 respectively throughout analyses in this study. Figure 2 shows the schematic of the hysteretic
 217 system as well as hysteretic loops of different restoring force peak normalized to the yielding
 218 force where the bridge piers are subjected to synchronous harmonic excitation.

219

220

221 Figure 2 ; (a) the idealized hysteretic system with a post-yielding and a hysteretic spring;
 222 (b) force-displacement cycle represented by the Bouc-Wen model with strength
 223 deterioration and stiffness degradation under synchronous excitation.

223

224 Vectors $\psi(x_i) \in \mathbb{R}^{m \times 1}$ and $\psi_g(x_i) \in \mathbb{R}^{s \times 1}$ are single-entry vectors containing a single unity at the

dof locations corresponding to the i^{th} pier. All other vector elements are zero which is a natural consequence of Eq (6). So, we can introduce the following relationships,

$$u_i = \boldsymbol{\psi}(x_i)^T \mathbf{u}, \quad z_i = \boldsymbol{\psi}_g(x_i)^T \mathbf{z} \quad (19)$$

Where the vector containing all the auxiliary variables is defined as follows

$$\mathbf{z} = [z_1(t), z_2(t), \dots, z_s(t)]^T \in \mathbb{R}^{s \times 1} \quad (20)$$

Substituting Eq (15) and Eq (19) into the hysteretic term in the equation (13) we obtain

$$\sum_{i=1}^s \boldsymbol{\psi}(x_i) F_i = \bar{\mathbf{K}}_u \mathbf{u} + \bar{\mathbf{K}}_z \mathbf{z} \quad (21)$$

Where matrices are defined as follows

$$\bar{\mathbf{K}}_u = \alpha \sum_{i=1}^s k_i \boldsymbol{\psi}(x_i) \boldsymbol{\psi}(x_i)^T \in \mathbb{R}^{m \times m}, \quad \bar{\mathbf{K}}_z = (1 - \alpha) \sum_{i=1}^s \left(k_i u_{yi} \boldsymbol{\psi}(x_i) \boldsymbol{\psi}_g(x_i)^T \right) \in \mathbb{R}^{m \times s} \quad (22)$$

By adding this hysteretic term, Eq (21) to the dynamical system, the final form would be as:

$$\bar{\mathbf{M}} \ddot{\mathbf{u}} + \bar{\mathbf{K}}_d \mathbf{u} + \bar{\mathbf{K}}_s \mathbf{u} + \bar{\mathbf{K}}_z \mathbf{z} = -\bar{\mathbf{L}} \ddot{\mathbf{g}} - \bar{\mathbf{K}}_g \mathbf{g} \quad (23)$$

2.2 Defining system parameters of the nonlinear system

We introduce a dimensionless coordinate $x = \xi L$ and assume that the mass per unit length m_d and flexural rigidity EI_z remain constant. By application of change of variable rule from Differential calculus [56], The dynamical equation (23) is re-cast as

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{K}_d + \mathbf{K}_u)\mathbf{u} + \mathbf{K}_z\mathbf{z} = -\mathbf{L}\ddot{\mathbf{g}} - \mathbf{K}_g\mathbf{g} \quad (24)$$

Where matrices are defined as follows

$$\mathbf{M} = \int_0^{(s-1)} \boldsymbol{\psi}\boldsymbol{\psi}^T d\xi \in \mathbb{R}^{m \times m}, \quad \mathbf{K}_d = \omega^2 \int_0^{(s-1)} \boldsymbol{\psi}''\boldsymbol{\psi}''^T d\xi \in \mathbb{R}^{m \times m} \quad (25)$$

$$\mathbf{K}_u = \alpha\omega^2 \sum_{i=1}^s \eta_i \boldsymbol{\psi}\boldsymbol{\psi}^T \in \mathbb{R}^{m \times m}, \quad \mathbf{K}_z = (1-\alpha)\omega^2 \sum_{i=1}^s \eta_i u_{yi} \boldsymbol{\psi}(x_i) \boldsymbol{\psi}_g(x_i)^T \in \mathbb{R}^{m \times s} \quad (26)$$

$$\mathbf{L} = \int_0^{(s-1)} \boldsymbol{\psi}\boldsymbol{\psi}_g^T d\xi \in \mathbb{R}^{m \times s}, \quad \mathbf{K}_g = \omega^2 \int_0^{(s-1)} \boldsymbol{\psi}''\boldsymbol{\psi}_g''^T d\xi \in \mathbb{R}^{m \times s} \quad (27)$$

Where, frequency system parameters ω and dimensionless system parameter η_i which denotes the i^{th} pier to deck stiffness ratio, are defined as follows,

$$\omega^2 = \frac{EI_z}{m_d L^4}, \quad \eta_i = \frac{k_i L^3}{EI_z} \quad (28)$$

2.3 Rayleigh damping model

In this study, it is assumed that elasto-mechanical system follows the classical, proportional damping model [57]. The basic assumption is that a classical damping matrix can be diagonalized throughout a dynamic matrix of the undamped system. For the sake of simplicity, it is considered that hysteretic force would not contribute to the damping mechanism and

stiffness of completely linear-system are considered to calculate proportional damping matrix.

Hence:

$$\mathbf{C} = \beta_m \mathbf{M} + \beta_k (\mathbf{K}_d + \mathbf{K}_u) \quad (29)$$

where β_m and β_k are mass-proportional and stiffness-proportional damping coefficient respectively.

It is rational that the units of system equation are partially or fully removed by nondimensionalization process. This technique provides a simplification to present the problem throughout system parameters, where the intrinsic properties of the system would be recovered.

2.4 Adding abutment constraints and the final form of the equation of motion

The two DOFs at ends of deck (the abutments) have the same displacement as the ground. Therefore, we should either augment equations of motions (24) with Lagrange multiplier and constraint equations or partition and condense equations of motion (24); the latter is chosen as it reduces the sizes of all matrices and aid computational efficiency.

After applying boundary conditions to the dynamic system, the final form of the equation of motion would take form as:

$$\mathbf{M}_{22} \ddot{\mathbf{w}} + \mathbf{C}_{22} \dot{\mathbf{w}} + (\mathbf{K}_{d22} + \mathbf{K}_{u22}) \mathbf{w} + \mathbf{K}_{z22} \mathbf{z}_I = -\mathbf{L}_{12}^T \ddot{\mathbf{g}}_E - \mathbf{L}_{22} \ddot{\mathbf{g}}_I - \mathbf{K}_{g12} \mathbf{g}_E - \mathbf{K}_{g22} \mathbf{g}_I \quad (30)$$

Where $\mathbf{w} \in \mathbb{R}^{(m-2) \times 1}$ denotes the relative displacement at free (unconstraint) DOFs. The details and the derivation of dynamic blocks are described in Appendix A. Hence equations of motion are defined by (17) and (30).

The key feature of the proposed dynamic system is to represent the inelastic structural equation of motion throughout a few numbers of system parameters. These parameters consist of two parts: (i) Linear system parameters including frequency system parameters ω and pier to deck stiffness ratio η_i Which express linear properties of the bridge and (ii) Hysteretic parameters which quantify deformation ductility capacity in vertical component (Piers). Note that all Bouc-Wen model parameters $A, \beta, \gamma, \alpha, \delta_n, \delta_v$ are dependent on the physical characteristic of the concrete structure and are generically considered constant for all piers of a bridge. However, yield displacement u_{yi} (of the i^{th} pier) depends on design philosophy/geometry. This is discussed further in the next section.

2.5 Estimation of yielding displacements from bridge design philosophy/geometry

In order to estimate of local ductility capacity of piers, the yielding displacement should be calculated. It is assumed that no bearing and isolation are incorporated into the bridge structure. Hence, all piers are designed for the ductile behavior necessary for a dissipating mechanism. The yield displacement of the piers at point of the intended plastic hinge is estimated in our study as [20] :

$$u_{yi} = \frac{h_i^2}{3} \phi_{yi} \quad (31)$$

Where h_i is the height of the i^{th} column, ϕ_{yi} denotes idealized curvature at yield point

determined by M- ϕ curve of the cross-section at the formation of plastic hinge. The yield curvature of the rectangular column is estimated by following dimensionless formulation [58]:

$$\phi_{yi} = \frac{2.12 \varepsilon_{yi}}{d_i} \quad (32)$$

Where ε_{yi} denotes the yield strain of the reinforcement bar in the columns and d_i is the depth of the i^{th} pier's sections. Substituting Eq (32) **Error! Reference source not found.** in Eq **Error! Reference source not found.**(31) gives:

$$\frac{u_{yi}}{h_i} = 0.71 \varepsilon_{yi} \frac{h_i}{d_i} \quad (33)$$

where design yield drift ratio (u_{yi}/h_i) is a function of pier span/depth ratio (h_i/d_i) and yield strain ε_{yi} .

3. Real ground excitation from the SMART-1 array

In this paper, the real ground motion obtained from seismic records at SMART-1, Taiwan is used rather than artificial ground motion estimates. It would capture fully spatial variability effects of ground excitation and avoid further discussion on unknown features of seismic data inputs. The SMART-1 Array is one of the largest seismic databases of digital accelerometers specifically arranged to explore the near-field properties of seismic ground excitation [59]. It is situated on the northeast side of Taiwan. Herein, the seismic data from event 43 is used. The details of the correction process are fully explained in [60-62]. In order to generate ground displacement inputs, refined acceleration time-series are numerically integrated twice using a Simpson's 1/3rd rule. Low-cut filtering is applied on acceleration-time histories and the

velocity-time histories to mitigate the effect of spurious growing trends in ground displacement time-series[63, 64]. We make no attempt to recover fling displacements in this paper. Figure 3(a) shows the location of the studied bridge. All twelve orientation is employed in the parametric study. Three time series at stations [I03], [I09] and [C00] are used to generate seismic input data at piers and abutments for studied bridge. The seismic ground motion series at intermediate bridge support is obtained by a biharmonic interpolation [31]. The same procedure, for other bridge alignments with three stations along a circle diameter. The ground motion time history series for selected stations are shown in Appendix C.

Figure 3 (b) depicts the elastic response spectra of acceleration at direction north-south and east-west. These plots reveal the frequency content in which the seismic energy of ground motion series is significant.

Figure 3; (a) example of bridge position in the inner ring of SMART-1 array (b) Elastic response spectra for SMART-1 Array inner ring stations I00 to I12 for event 43

4. Exploration of a heuristic bridge reference model

4.1 Reference prototype bridge definition

The reference bridge studied in [47, 65, 66] is selected to exemplify the proposed procedure. The benchmark experimental model (1:50 scale) of this bridge was constructed in the laboratory (University of Bristol). The prototype bridge contains continuous deck integral supported on three cantilever piers with an equal height of 10m. The total length bridge is 200m with four equal spans (Figure 4).

Figure 4; configuration of studied real bridge; (a) bridge arrangement; (b) deck section; (c) pier section.

The uniform concrete deck is pre-stressed box girders and Bridge Piers has a rectangular hollow section. The Nominal yield strain is $\varepsilon_y = 0.0021$ and modulus elasticity of concrete is 34 GPa. The damping ratio of first and second modes are assumed equal to 0.05 in the Rayleigh damping Eq **Error! Reference source not found.**(29). System frequency parameter ω equal

to 5 is adopted [45] which is consistent with the prototype bridge frequency range. The values of lateral stiffness k_i for cantilever piers are taken equal to $3EI_i/h_i^3$. The yielding displacement for all columns is calculated to equal 6cm. The summary of section details and dynamic parameters are given in Table 1.

Parameters	Value	Units
m_b	19.6	[ton/m]
I_z (deck)	156	[m ⁴]
I_i (Pier)	1	[m ⁴]
ω	5	[rad/s]
η_i	2.4	[]

Table 1. the section property and dynamic parameters for bridge prototype

4.2 Benchmarking using scaled experimental reference bridge data

The proposed mathematical model of a nonlinear dynamic system Eqs (17) and (30) will be benchmarked. The displacement time-series data from Experimental bridge tests conducted previously at the University of Bristol [65, 66] has been used to validate the proposed dynamic system. The dynamic response quantities of the bridge experiment are limited to the linear region, therefore the linear version of the proposed analytical model is compared with experimental data. An accurate calibration of a hysteresis loop with the experimental data has been established for a single column in the recent studies [67, 68]. The Bouc-Wen parameters implemented in our reduced order system (Eq(15) to (18)) lie within the range previously reported in these recent studies. The details of the section and dynamic parameters of the experimental bridge can be found in the previous study [45]. The updating of linear dynamic system parameters with respect to the experimental model was performed in the

previous study [45]. This is achieved via inverse system identification that makes use of the Grey-Box Model Estimation. The same values of dynamic coefficients are employed for the nonlinear system. The values for the damping ratios were determined experimentally and are 0.0103 and 0.0202 for first and second mode respectively. From all Bouc-Wen parameters that can be chosen suitably from the recommended range, yielding displacement should be estimated from the pier dimension (see section 2.5). As the experimental piers do not have the real dimension and the experimental bridge was designed to be excited in the elastic region only, therefore the yielding displacement for all three piers is assumed to be 6 mm which enforces the dynamic nonlinear system behaves linearly. The response quantity of interest is obtained by solving the first-order state-space form of the dynamic system. The example of representation of nonlinear differential equations of motion in the state-space form of the 5-DOFs system is shown for prototype bridge in appendix C. The ODE solver (ode45 in Matlab) was employed. The dynamical linear model of the bridge system is obtained from Eq (23) **Error! Reference source not found.** where α is taken equal to 1. The scaled maximum of the PGA of 0.023g is employed at stations. Figure 5 confirms that the experimental results were accurately simulated by the linear space-state form (where $\alpha=1$) of the proposed analytical model. The full nonlinear equations (where $\alpha=0.05$) results in solutions that exhibit a small amount of ductility during the high amplitude part of the earthquake as expected. Thus, we conclude that the equations (17) and (30) are reasonable.

Figure 5; (a) collaboration of dynamic nonlinear and linear system by Benchmark experimental test data; (b) force-deformation curve of proposed Bouc-Wen inelastic model for the linear experimental bridge. PGA=0.023g.

4.3 Nonlinear time history analysis of prototype reference bridge

Figure 6 shows the inelastic responses of the nonlinear time history analysis for the studied bridge when is subjected to real asynchronous excitation. The seismic ground motion data is chosen from the first alignment of the SMART-1 Array in order to represent the spatial variability effect. The maximum of the PGA at stations is equal to 0.23g. The displacement time history for linear-model where α in Eqs. (17) and (30) is set equal to 1 has been also plotted for the sake of comparison.

Figure 6; (a) response of middle piers under asynchronous excitation; (b) normalized displacement-force cycle of middle piers; PGA=0.23g.

As it can be seen, this configuration, the piers exhibits only slightly hysteresis while the peak lateral deformation does not reach the yielding point (typical of RC sections [69]). The results suggest that piers exhibit limited ductile behavior therefore, the unscaled amplitude of the SMART-1 event 43 is not large enough to demonstrate significant energy dissipation. For a

deeper understanding of hysteretic effects, we assumed that the bridge is constructed in medium to high seismicity area with a design PGA equal to 0.69g. By a simple ground motion amplitude scaling an example of this case is shown in Figure 7.

Figure 7; (a) Response of middle piers under asynchronous excitation; (b) normalized displacement-force cycle of middle piers; PGA=0.69g.

We know explore examples of non-uniform valley profiles. In this set of dynamic configurations, the middle pier exhibits larger ductility where the peak response exceeded the yield displacement. Incorporation of hysteresis pier would increase slightly the peak dynamic response in this case. Let consider another case Figure 8 where the middle pier height is reduced to 6m.

Figure 8; bridge configuration with the shorter middle pier.

Figure 9; (a) response of middle piers under asynchronous excitation; (b) normalized displacement-force cycle of middle piers; PGA=0.69g,

The analysis in Figure 9 shows that the peak linear displacements responses are greater than those where nonlinearity is taken to account suggesting that pier hysteresis reduces the displacement responses while Figure 7 suggests the opposite. Thus, suggesting that the so-called “equal displacement rule” is not generally valid. Results presented in this section are particular anecdotal examples; so, we seek to extend the generality of these analyses by the large-scale parametric study.

5. Large-scale parametric study

5.1 Parametrically exploration of valley profile.

In this section, we investigate the effect of spatial variability on the dynamic response when the pier hysteresis is taken to account. In this setting, we explore such effects where the geometrical configuration of the bridge due to the valley profile of ground is altered. This is shown in Figure 10. Herein it is supposed that due to topographical change at supports, the height of piers is altered while the section of the piers remains constant. This assumption is expedient for construction purposes as well as it makes a use simplified form of yield displacement representation of Eq (33). Moreover, it is assumed that bridges are symmetrical, so that deck to pier stiffness ratios for the outer piers is equal and the ratio of stiffness between central and outer pier is defined as:

469

$$470 \quad \rho = \eta_2 / \eta_1, \quad \eta = \eta_1 = \eta_3 \quad (34)$$

471

472 Note that the pier-to-deck stiffness ratio η is inversely proportional to pier height. The
 473 parameter ρ is the ratio (of pier-to deck stiffness ratios) between central and outer piers. So,
 474 when $\rho > 1$ this implied the central pier is shorter than the outer piers and the valley profile is
 475 like that shown in Figure 8. Figure 11 demonstrate the how the parameters η, ρ can define
 476 different configurations, both deep and shallow valleys and ones with and without a central
 477 rise.

478

479 Figure 10; Different bridge configuration (a) a shallow valley with the a high value η and
 480 $\rho < 1$, (b) a shallow valley including a raise in the middle with a high value η and
 481 $\rho > 1$, (c) deep valley with the low value η and $\rho < 1$, (d) deep valley including a
 raise in middle with the low value η and $\rho > 1$.

482 The frequency parameters ω and span length for different arrangement are assumed to remain
 483 constant equal to 5[rad/s] and 50 m respectively.

484

5.2 Defining error in ductility estimates using MSE

and ISE

To compare the dynamic response of the bridge under sets of ground motion (ISE and MSE), we introduce the performance measure as a follow:

$$\chi(\eta, \rho, i, j) = 100 \left\{ \frac{\max_t |u_i(t)|_{MSE} - \max_t |u_i(t)|_{ISE}}{\max_t |u_i(t)|_{ISE}} \right\} \quad (35)$$

where $\chi(\eta, \rho, i, j)$ is the percentage difference (error) in pier ductility, for a bridge with (i) parameters η and ρ and (ii) pier i and bridge orientation j , between a MSE and ISE simulations. The number j denotes the bridge orientation which varies from 1 to 12. SMART-1 array ground motion for event 43 is chosen to generate MSE simulations where abutments and pier supports are exposed to different ground motion time-series. For the ISE case, the ground excitation from center station I00 is employed at supports and abutments. As we are only interested in the range of errors with regard to bridge orientation j and pier i , we consider the following statement:

$$\min_{\forall i, \forall j} \{\chi\} \leq \chi \leq \max_{\forall i, \forall j} \{\chi\} \quad (36)$$

The minimum of this range corresponds to the cases where MSE simulations are non-conservative. The maximum of these ranges corresponds to the cases where MSE is conservative. To evaluate the effect of nonlinearity, we perform Incremental Dynamic Analysis (IDA) to estimate the level of intensities where the bridge excites (i) in an elastic (ii) and a nonelastic range. In the following section, the procedure is explained in full details.

508

509 **5.3 Incremental dynamic analysis (IDA)**

510 IDA is a parametric study of the structural model involving a series of nonlinear analyses
511 performed under a scaled image of ground motion records [70]. The range of scaled intensity
512 levels is wisely selected to cover an entire range of elastic to the plastic region and finally to
513 structural instability (collapse state). With the suitable chosen damage index quantity (i.e.
514 maximum ductility), it is possible to identify the linear and nonlinear region and collapse state
515 for a structural system with mild degradation in IDA curve[54]. The reference bridge explained
516 in section 4.1 is chosen for IDA analysis. The MSE ground motion series used for the studied
517 bridge is selected as unscaled time history. To perform IDA, first, the time series at pier
518 supports and abutments are simultaneously multiplied by a scale factor with the range from
519 0.01 to 5 to obtain a set of ground motion with different intensity level. Then, a series of
520 nonlinear analysis are performed under scaled ground motion records to obtain the maximum
521 ductility of the middle pier from time history response at different Peak ground motion (PGA).
522 IDA curve of the middle pier with Bouc-Wen model of the studied bridge is shown in Figure
523 11.

524

Figure 11; Example of IDA curves for central piers with strength deterioration and stiffness degradation for the prototype bridge with $\eta=2.4$, $\rho=1$ and alignment 3.

As it can be observed from the IDA curve, the pier exhibits almost pure elastic behavior at $\text{PGA}=0.2\text{g}$. From this point onwards, the IDA curve is subject to small fluctuate and curvature as hysteresis initiates in piers. In this study, we use a PGA of 0.2g (low seismicity), 0.69g (medium seismicity) and 1.1g (high seismicity) as qualitative estimates of the different regions of system behavior.

5.4 Effect of different valley profiles, alignment and ground motion intensity.

Figure 12 shows the sample of inelastic time-history analyses for outer pier 3 under MSE and ISE simulation with $\text{PGA}=0.69\text{g}$. Figure 12(a) indicates that maximal lateral ductility for MSE case is 19% greater than one for ISE case. Figure 12(b) shows the difference of -22% between ISE and MSE cases implying that ISE is more conservative for this alignment.

Figure 12; Examples of the outer pier 3 displacement responses for parameters $\eta = 15, \rho = 6, PGA = 0.69g$, (a) alignment 1 with an error in ductility of $\chi = 19\%$, (b) alignment 5 with an error in ductility of $\chi = -22\%$

As discussed in section 5.1 and Figure 10, the dynamic parameters ρ, η can be viewed as a proxy for the valley profile. To explore such effects for a wider range of dynamic parameters ρ, η , let's consider the cases with $PGA=0.2g$ where bridges mostly excite in a linear-elastic region. Figure 13 displays the errors in ductility χ (of any pier and alignment) if MSE is not employed. It required a substantial computational effort of 90000 full nonlinear time-history analyses that make use of 192 different ground motions. Figure 13(a) shows the lower bound of error for ductility and Figure 13(b) displays the upper bound of error ductility. From these analyses, it is clear that for any configuration of the bridge $\{\rho, \eta\}$ ISE could be either conservative or nonconservative. From the Figure 13(b), it is possible to identify the critical regions (the area I with $4 < \rho < 10, 4 < \eta < 30$ and area II with $70 < \eta$) where MSE analyses are necessary.

Figure 13; Error range in pier ductility χ for symmetrical bridges for a low seismicity

(a) Lower bound of errors (b) upper bound of errors; $-47 \leq \chi \leq 63.7$; PGA=0.2g,

Now, consider the analyses for nonlinear cases with the medium and high-intensity level as shown in Figure 14 and Figure 15.

Figure 14; Error range in pier ductility χ for symmetrical bridges for medium seismicity

(a) Lower bound of errors (b) upper bound of errors; $-50.93 \leq \chi \leq 66.08$; PGA=0.69g,

Figure 15; Error range in pier ductility χ for symmetrical bridges for high seismicity

(a) Lower bound of errors (b) upper bound of errors; $-54.5 \leq \chi \leq 65.8$; PGA=1.1g,

Maximal errors in Figures 13 to 15 occur for high $\eta \approx 20$ and $\rho > 1$, which from Figure 10(b) is a shallow valley with a central rise. The response patterns in Figures 14 and 15, for nonelastic cases, are similar to the linear one. However, consider the size of the parametric region that shows an error above 50%, in plots Figure 13-15. As the PGA increase, there is a small increase in the parametric bridge/valley configurations that show more error in ISE analyses. This trend is also seen for lower bound plots where the maximal negative error slightly decreases to -50.9 and -54.5 for PGA=0.69g and PGA=1.1g respectively.

Conclusions

A generalized reduced-order model of a multi-support bridge under spatial variable ground motions is set up to account for the pier hysteresis. The simulation of the simplified bridge physics is not based on either modal projection or static condensation in a classic sense as these

techniques are limited to elastic analysis. We reduce the number of system dofs by using explicit assumptions, namely limiting plasticity to piers via Bouc-Wen element, that provides a very efficient way of reducing system complexity and numerical run-times. Note that in our heuristic case we were able to model this multispan bridge effectively with only 3 dofs rather than thousands dofs in a typical FEA. Thus, the computational saving for extensive parametric studies could be considerable.

The linear form of this analytical model is validated by physical experimental data. Then, the extensive parametric analyses using the proposed nonlinear dynamic model was performed to explore the detrimental effects of asynchronous excitation while the geometrical configuration of the bridge (due to valley profile) and bridge alignment are varied. Within the context of the parametric study presented in this paper, the following conclusion can be drawn:

- The effect of variation of bridge geometry due to the valley profile on a bridge response subjected to spatially variable ground motion input is significant.
- The evidence from this study suggests that the bridge constructed in a shallow valley with central rise is more susceptible to spatial incoherence of ground excitations. In these cases the contribution of alignment to the error in ISE is up to 80% of the maximal error. In these cases the bridge is very sensitive to input ground motion variation caused by alignment.
- The result indicates that for each geometry parameter set, the error engendered in ISE varied substantially when the bridge orientation is altered. The large range between minimum and maximum errors in ISE suggest that bridge alignment is an arguably more important factor than valley profile.
- Pier nonlinearity (with increasing the ground motion intensity) causes a 3% increase in

the unconservative maximal error in ISE which is insignificant. However, it does also produce a -16% change in the conservative minimal error in ISE which is significant. Generally, nonlinearity tends to increase the range between maximal and minimal errors in ISE.

- The maximal error in using ISE rather than MSE analyses was found to be of the order of approximately 65% when employing the ground motions in this paper. Further work is required in order to validate the consistency of the above observations using a larger database of real recorded spatiotemporal ground motion and the wider range of bridge configurations.

Appendix A. Matrix partitioning and condensation to enforce abutment constraints

Herein, it is assumed that the bridge deck lays on the valley at its two ends and therefore the relative lateral displacement at two ends is known $u_1(t) = u_m(t) = 0$. Hence, the DOFs of ground and deck are reordered as:

$$\mathbf{u} = \begin{bmatrix} u_1=0 \\ \vdots \\ u_m=0 \\ \vdots \\ u_{m-1} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{w} \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} \mathbf{0} \\ \mathbf{z}_I \end{bmatrix}, \quad \ddot{\mathbf{g}} = \begin{bmatrix} \ddot{g}_1 \\ \vdots \\ \ddot{g}_m \\ \vdots \\ \ddot{g}_{m-1} \end{bmatrix} = \begin{bmatrix} \ddot{\mathbf{g}}_E \\ \ddot{\mathbf{g}}_I \end{bmatrix} \quad (37)$$

where $\ddot{\mathbf{g}}_E \in \mathbb{R}^{2 \times 1}$ denotes the ground acceleration time-series at exterior supports of the bridge,

$\ddot{\mathbf{g}}_I \in \mathbb{R}^{(s-2) \times 1}$ denotes the ground acceleration time-series at the inner pier columns and

$\mathbf{w} \in \mathbb{R}^{(m-2) \times 1}$ is the vector of relative displacement at free (unconstrained) DOFs of the

superstructure (deck). Similarly, $\mathbf{z}_I$ denote imaginary hysteretic displacements of deck DOFs at interior piers. Now, matrix partitioning is employed to decouple the constrained from unconstrained DOFs as follow:

$$\begin{aligned} & \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{12}^T & \mathbf{M}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \ddot{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{12}^T & \mathbf{C}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \dot{\mathbf{w}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{d11} & \mathbf{K}_{d12} \\ \mathbf{K}_{d12}^T & \mathbf{K}_{d22} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{w} \end{bmatrix} + \dots \\ & \begin{bmatrix} \mathbf{K}_{u11} & \mathbf{K}_{u12} \\ \mathbf{K}_{u12}^T & \mathbf{K}_{u22} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{w} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{z11} & \mathbf{K}_{z12} \\ \mathbf{K}_{z12}^T & \mathbf{K}_{z22} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{z}_I \end{bmatrix} = - \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{12}^T & \mathbf{L}_{22} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{g}}_E \\ \ddot{\mathbf{g}}_I \end{bmatrix} - \begin{bmatrix} \mathbf{K}_{g11} & \mathbf{K}_{g12} \\ \mathbf{K}_{g12}^T & \mathbf{K}_{g22} \end{bmatrix} \begin{bmatrix} \mathbf{g}_E \\ \mathbf{g}_I \end{bmatrix} \end{aligned} \quad (38)$$

and partitioned to achieve the reduced and final form of the equations of motion for this generalized nonlinear bridge system as follow:

$$\mathbf{M}_{22} \ddot{\mathbf{w}} + \mathbf{C}_{22} \dot{\mathbf{w}} + (\mathbf{K}_{d22} + \mathbf{K}_{u22}) \mathbf{w} + \mathbf{K}_{z22} \mathbf{z}_I = -\mathbf{L}_{12}^T \ddot{\mathbf{g}}_E - \mathbf{L}_{22} \ddot{\mathbf{g}}_I - \mathbf{K}_{g12} \mathbf{g}_E - \mathbf{K}_{g22} \mathbf{g}_I \quad (39)$$

Figure 16 represents the Corrected time histories series in bridge tangential direction at center station and selected stations of inner ring (I03, I06, I09 and I12). The ground motion acceleration records are directly used in this study as seismic input for end supports.

Appendix B. Time histories series of seismic input

Figure 16 represents the Corrected time histories series in bridge tangential direction at center station and selected stations of inner ring (I03, I06, I09 and I12). The ground motion acceleration records are directly used in this study as seismic input for end supports.

Figure 16; Corrected aligned time histories recorded at selected stations of the inner ring from SMART-1 array (event 43), (a) ground motion displacement input (b) ground motion velocity input (c) ground motion acceleration input.

Appendix C. Explicit state space form for prototype bridge system

In order to solve the proposed mathematical model, the nonlinear differential equation of motion is represented in the first-order state-space framework. it is necessary to reduce the second-order differential equation to the first order indenting to use a numerical algorithm. To achieve this goal, the state vector is defined as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{w} \\ \dot{\mathbf{w}} \end{bmatrix} \quad (40)$$

Therefore, by substituting of the above state vector in Eq

661 , State-space models for the adopted dynamical system would take form as:

662

663

$$664 \quad \dot{\mathbf{x}} = \mathbf{B}_1 \mathbf{x} + \mathbf{B}_2 \mathbf{z}_l + \mathbf{B}_3 \mathbf{f} \quad (41)$$

665

$$666 \quad \mathbf{B}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}_{22}^{-1}(\mathbf{K}_{d22} + \mathbf{K}_{u22}) & -\mathbf{M}_{22}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} \mathbf{0} \\ -\mathbf{M}_{22}^{-1} \mathbf{K}_{z22} \end{bmatrix} \quad (42)$$

$$\mathbf{B}_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{M}_{22}^{-1} \mathbf{L}_{12}^T & -\mathbf{M}_{22}^{-1} \mathbf{L}_{22} & -\mathbf{M}_{22}^{-1} \mathbf{K}_{g12}^T & -\mathbf{M}_{22}^{-1} \mathbf{K}_{g22} \end{bmatrix}$$

667

668 where the ground motion input vector $\mathbf{f}(t) = [\ddot{\mathbf{g}}_E; \ddot{\mathbf{g}}_I; \mathbf{g}_E; \mathbf{g}_I]$, and $\mathbf{z}_l(t)$ is the state vector

669 representing hysteretic displacement at piers. For the proposed bridge system in this study, 4th

670 order Lagrangian interpolating function is used to define the shape function of both ground and

671 deck DOFs as:

$$672 \quad \boldsymbol{\Psi} = \boldsymbol{\Psi}_g = \begin{bmatrix} \frac{(\xi-1)(\xi-2)(\xi-3)(\xi-4)}{24} \\ \frac{\xi(\xi-1)(\xi-2)(\xi-3)}{24} \\ -\frac{\xi(\xi-2)(\xi-3)(\xi-4)}{6} \\ \frac{\xi(\xi-1)(\xi-3)(\xi-4)}{4} \\ -\frac{\xi(\xi-1)(\xi-2)(\xi-4)}{6} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1=0 \\ u_5=0 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} g_1 \\ g_5 \\ g_2 \\ g_3 \\ g_4 \end{bmatrix} \quad (43)$$

673

674 Hence, the number of deck DOFs (m) is equal to the number of ground DOFs (s). The dynamic

675 matrices of the system are calculated by algebra toolbox in Matlab for prototype bridge as:

676

$$\begin{aligned}
\mathbf{M} &= \begin{bmatrix} 0.206 & 0.209 & -0.123 & 0.0395 & -0.0205 \\ 0.209 & 1.26 & -0.271 & 0.181 & 0.0395 \\ -0.123 & -0.271 & 1.32 & -0.271 & -0.123 \\ 0.0395 & 0.181 & -0.271 & 1.26 & 0.209 \\ -0.0205 & 0.0395 & -0.123 & 0.209 & 0.206 \end{bmatrix}, \quad \mathbf{K}_d = \begin{bmatrix} 103.0 & -287.0 & 301.0 & -153.0 & 36.2 \\ -287.0 & 880.0 & -1066.0 & 613.0 & -153.0 \\ 301.0 & -1066.0 & 1500.0 & -1066.0 & 301.0 \\ -153.0 & 613.0 & -1066.0 & 880.0 & -287.0 \\ 36.2 & -153.0 & 301.0 & -287.0 & 103.0 \end{bmatrix}, \\
\frac{\mathbf{K}_u}{\alpha} = \frac{\mathbf{K}_z}{1-\alpha} = \mathbf{K}_g &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_1 & 0 & 0 \\ 0 & 0 & 0 & \eta_2 & 0 \\ 0 & 0 & 0 & 0 & \eta_3 \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 0.206 & 0.209 & -0.123 & 0.0395 & -0.0205 \\ 0.209 & 1.26 & -0.271 & 0.181 & 0.0395 \\ -0.123 & -0.271 & 1.32 & -0.271 & -0.123 \\ 0.0395 & 0.181 & -0.271 & 1.26 & 0.209 \\ -0.0205 & 0.0395 & -0.123 & 0.209 & 0.206 \end{bmatrix}
\end{aligned} \tag{44}$$

Where $\mathbf{M}$ is the mass matrix and $\mathbf{K}_u, \mathbf{K}_z, \mathbf{K}_g$ denote the pier, the hysteretic and the ground stiffness matrix respectively. $\mathbf{L}$ is denominated in this study as the excitation factor matrix. The imaginary hysteresis displacements in $\mathbf{z}_i(t)$ are determined from following auxiliary ordinary differential equation for the studied bridge.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{1+0.3\epsilon_1} \{1 - (1+0.05\epsilon_1)(0.5\text{sgn}(\dot{u}_1 z_1) + 0.5)|z_1|\} \frac{\dot{u}_1}{u_{y1}} \\ \frac{1}{1+0.3\epsilon_2} \{1 - (1+0.05\epsilon_2)(0.5\text{sgn}(\dot{u}_2 z_2) + 0.5)|z_2|\} \frac{\dot{u}_2}{u_{y2}} \\ \frac{1}{1+0.3\epsilon_3} \{1 - (1+0.05\epsilon_3)(0.5\text{sgn}(\dot{u}_3 z_3) + 0.5)|z_3|\} \frac{\dot{u}_3}{u_{y3}} \end{bmatrix} \tag{45}$$

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