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Drag Reduction through Shape Optimisation for Satellites in Very Low Earth Orbit

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Abstract

Operating satellites at altitudes in Very Low Earth Orbit (VLEO) has many advantages. However, due to the higher atmospheric density of this region, satellites encounter significantly higher atmospheric drag. Depending on the mission, this may require a propulsive system to maintain the orbit which costs both fuel mass and volume. It is therefore desirable to reduce the drag in order to either reduce these costs or to extend the operational life. In this paper a series of viable aeroshell profiles are identified for satellites operating in VLEO using a Radial Basis Function-based surrogate model with data generated using both Panel Methods and Discrete Simulation Monte Carlo simulations. It was demonstrated that a maximum drag reduction of between 21% and 35% was achievable for the profiles when optimizing a bi-conic profile for minimum drag based on Discrete Simulation Monte Carlo simulations with an energy accommodation coefficient of 0.95. Accounting for the loss of internal volume and assuming the reduction in fuel mass results in an equally proportioned reduction in fuel system volume it was observed that only a 13% to 27% reduction was achieved.

Keywords: VLEO, Shape Optimisation, Surrogate Modelling, DSMC

2010 MSC: 00-01, 99-00

1. Introduction

There is increasing interest in improving the capabilities and reducing the cost of Earth observation platforms by operating in orbits with much lower altitudes in a region commonly referred to as Very Low Earth Orbit (VLEO)[1]. VLEO provides a number of advantages over higher altitude orbits, such as reduced payload size and mass[1, 2] as well as reduction in the payload power requirements, and improvements in the data rate [3, 4]. It therefore provides the opportunity to reduce the overall cost of a platform for a given mission as compared to similar platforms at more common higher altitudes.

There are many different definitions for the upper extent of VLEO with common definitions including 300km[5] and 450km[6]. The defining characteristic of VLEO is the increased effects of the Earth's atmosphere on the satellite design. One of the key challenges is combating the higher levels of atmospheric drag that are experienced which would otherwise limit the operational life of satellites[1, 7, 8].

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In some case a shorter operational life can be desirable, such as the US Keyhole and USSR Zenit reconnaissance satellites during the cold war. In other cases a longer operational life maybe necessary. To achieve this could require regular orbit-raising manoeuvres or to adopt a low thrust drag compensation scheme in order to maintain altitude. An example of such a satellite is ESA’s Gravity Field and Steady-State Ocean Circulation Explorer (GOCE), a 1077kg Earth observation satellite launched in 2009 to examine the Earth’s gravitational field [9]. Operating at an altitude of 260 km, it employed an ion propulsion system to provide the necessary thrust to compensate for the drag it experienced. Under this regime it was able to maintain orbit for 55 months using 40kg Xenon fuel, before deorbiting within 2 weeks of fuel depletion. JAXA have also sent a satellite to this region of orbit called the Super Low Altitude Test Satellite (SLATS) with the aim to studying the effects of atomic oxygen in VLEO. Launched in late 2017, this 400kg satellite descended to its research orbit between 180-250km where it employed a hybrid chemical and electrical propulsion system to remain there for 90 days[10, 11].

Due to their high specific impulse (I_{sp}), electrical propulsion systems are the ideal choice for supporting long duration drag compensation. However, they still require fuel which becomes the limiting factor on the operational life of the platform[12] as was the case of with both GOCE and SLATS. The fuel required is proportional to the thrust and by extension the drag that the satellite experiences [2], therefore it is desirable to minimise the drag as much as practicable in order to reduce the fuel mass fraction or to extend the life of the platform. Since the drag a satellite experiences is linked to its geometry[13], it is necessary to identify configurations that may prove beneficial within the rarefied gas of the thermosphere of VLEO. Reducing the drag is important for extending the operational life, but this does not guarantee the usability of the geometry selected. It is therefore also important to consider other factors such as the internal volume of the bodies.

The atmospheric density in VLEO, while sufficient to deorbit a satellite within days, is also sufficiently rarefied that the gas flow the satellite experiences can be described as a molecular flow (Knudsen number >10). As summarised by Vallado & Finkleman [8] there are a number of methods available for calculating the aerodynamic forces for this regime. Analytical methods such as those presented by Sentman [14] and Fujita[10] can provide estimates of the aerodynamic forces a body might experience within a rarefied flow. However, while suitable to simple convex geometries they have limited value with more complex shapes, especially where secondary particle-surface interactions may occur[8]. Additionally, the lower the altitude at which the analysis is performed the less accurate analytical methods become as non-linear aspects of the flow begin to dominate[15]. Alternatively, particle simulators such as the Direct simulation Monte Carlo (DSMC) methods pioneered by Bird [16, 17] can provide a more accurate assessment of the aerodynamic forces, as they endeavour to replicate the physics behind molecular flow, however, this comes at the cost of longer simulation time. Examples of the DSMC codes available for this application included ‘SPARTA’ (Stochastic PARallel Rarefied-gas Time-accurate Analyzer)[18, 19], the ‘dsmcFoam’ part of the openFOAM CFD toolkit[20], and DS2V and DS3V1, the original DSMC codes developed by Bird [16, 17]. For the wider work being performed, the ability to capture the non-linear aspects of the flow around complex bodies was highly desirable. As such developing a framework in which

DSMC simulations could be used to assess the viability of satellite geometries was necessary.

Performing the simulations using DSMC methods can be time intensive and computationally expensive. This would normally make it prohibitive to explore an entire design space with sufficient fidelity. As outlined by Forrester et al.[21], this problem can be effectively managed by employing a surrogate model framework. This reduces the number of simulations required through careful selection of the sample points and effective interpolation of the data. Much of the current development in surrogate based modelling builds on the work of Sacks et al. [22] and there have been a number of advances in the field as reported in the review papers Queipo et al. [23] and Simpson et al.[24].

The work presented here forms part of a larger body of work assessing the effectiveness of altering the geometry of the satellite in order to reduce its drag in an effort to either prolong its operational life or increase its payload capacity. The novelty of the work lies in identifying viable aeroshell profiles for satellites operating in VLEO using a Radial Basis Function-based surrogate model with data generated using DSMC. To test and verify the methodology, the results were compared with a similar model created using panel methods. The set-up for both methods are described in Section 4 and will be used to generate samples to train a set of surrogate models as described in Section 5. The aeroshell profiles and their associate design space are described in Sections 3. The optimisation strategy will be described in Section 7 with the resulting optimised profiles presented and explored in section 8.

2. Workflow

The work presented in this paper focuses on how the geometry of the satellite can be altered to minimise the aerodynamic drag it experiences in VLEO, thereby saving fuel mass or extending the system's operational life while not limiting the internal volume. An overview of how this was achieved is provided in Figure 1. As can be seen on the right of Figure 1, to explore how the geometry affected the drag it was first necessary to identify what profiles are to be used in the analysis. This required performing a brief review of typical satellite morphologies, including dimensions, of systems that have previously been launched into Low Earth Orbit, thereby informing the choice of dimensions for the design space.

A key characteristic of VLEO is the higher atmospheric density experienced by the satellites. It was therefore necessary to identify and employ a method for determining the properties of the atmosphere in this region. These properties informed the choice of the fluid simulation technique used, as well as defining the fluid properties for the simulations themselves. To facilitate the analysis, the satellite was assumed to be in a circular orbit of 200km, placing it roughly in the centre of the VLEO region.

Typically aerodynamic simulations are computationally heavy, so to reduce the number of simulations required a surrogate model was generated. This surrogate model was based on a Radial Basis Function (RBF) formulation with an adaptive sampling techniques to populate the design space. Each sample site represented a unique satellite geometry within the design space and its associated atmospheric forces.

Figure 1: Overview of workflow

Figure 2: Geometry of simulated body, showing key areas of variation

The objective was to identify viable aeroshell profiles for satellites operating in VLEO, in this instance by minimising the drag while maximising the volume. Using this as the premise for a cost function, the surrogate model was interrogated, yielding the desired profiles.

3. Satellite Geometry

When designing a satellite, the aerodynamic effectiveness of its geometry is not usually considered since the atmospheric density at the more common higher LEO orbits is low enough that it can be effectively ignored during the early design. The structural/geometric design of the satellite is primarily driven by the considerations of launch and the various constraints of its subsystems. At the time of writing, GOCE and SLATS are the only none-military platforms that have sustained circular orbits below 300km in altitude and were designed with some consideration for the effect of the geometry on the forces they would experience as a result of the higher atmospheric density.

A common satellite layout is that of a central body with solar panels mounted on either side. The central body is often cuboid in nature, but other body types including hexagonal and cylindrical are also used. Typically the solar panels have very large surface areas that may change angle in order to maximise power collection which means their contribution to the drag varies over the course of an orbit. Alternatively the solar panels may be permanently aligned on the body, thus minimising their frontal area to the flow, however, this requires a much larger area to achieve the same amount of power as sun tracking arrays. The power requirements are predominately driven by the payload and, if a drag compensation scheme is adopted, the choice of propulsion system, both of which have not been specified here. Simulating the arrays is thus beyond the scope of this current project but is a potential further application of the methodology. Therefore for this work, the central body will be treated in isolation with the focus on improving its contribution to the overall aerodynamic drag of the satellite.

The initial central body will be assumed to be a cuboid with smooth surfaces and thus lacking any extrusion, recesses or holes. The surface properties used in the simulations will be discussed more fully in section 4.3. Given these assumptions, the analysis was simplified to a 2D axisymmetric body which reduced computation time per sample. Since the smaller the frontal area, the lower the atmospheric drag [13], the bodies will be longer than they are high, with their long axis parallel to the velocity vector.

The geometric features to be examined were conic sections on the front (positive velocity direction) and rear (negative velocity direction) of the satellite, referred to here as the ‘nose’ and ‘tail’ respectively. Collectively this described a diamond-wedge body with variable nose and tail radius as shown in Figure 2. The nose profile had two parameters that controlled its shape: nose length (L_N) and nose radius (R_N). Similarly the tail profile was described by the two parameters: tail length (L_T) and tail radius (R_T).

A brief study was performed to determine typical dimensions for the central body of small spacecraft. Previous work [3, 4, 7] has shown that satellites in the Microsatellite category (10-100kg) may be commercially feasible in VLEO so will be the focus of this line of work. This is significantly smaller than either GOCE or SLATS, which were small and minisatellites respectively, but presents a good starting point for the analysis. Data from 179 satellites was compiled using sources from the original manufacturer and supplemented with data from Encyclopaedia Astronautica[25] and the Union of Concerned Scientists[26]. The results of this study are summarised in Table 1 for nanosatellites, microsatellites and minisatellites.

Table 1: Typical dimensions for each satellite category used in the simulations

Category	Mass Range [kg]	Body Length Range* [m]	Body Diameter Range* [m]
Nanosatellite	1 - 10	0.1 - 0.5	0.1 - 0.3
Microsatellite	10 - 100	0.2 - 1.5	0.2 - 0.75
Minisatellite	100 - 500	0.5 - 5	0.5 - 2.5

* Where possible satellite dimensions were sourced from the original manufacturer and supplemented with data from Encyclopaedia Astronautica[25] and the Union of Concerned Scientists[26].

As the primary geometric features that are being explored are the conic nose and tail, it is useful to constrain the design space. In this case the bodies have been constrained to discrete aspect ratios with exterior dimensions as follows: $0.25m \times 0.5m$, $0.25m \times 1m$, $0.25m \times 1.5m$, $0.5m \times 1m$, and $0.5m \times 1.5m$. This provided a range of aspect ratios at both ends of the microsatellite scale, such that the capabilities of these aerodynamic features can be examined. From a design perspective these constraints could be consider as a result of internal and external geometric constraints placed on the design: internally this could stem from fixed system dimensions such as a minimum diameter for the payload, while externally constraints maybe set by the packing requirements within the fairing. Constraining the design space in this way has the additional advantage of reducing the number of parameters and by extension the number of samples (and simulations) that need to be generated for the surrogate model. This allows faster generation of the surrogate model for both the panel methods and DSMC, which enables testing

and refinement of the method.

4. Rarefied Gas Modelling

4.1. Atmosphere

VLEO sits within the lower portion of the Earth’s thermosphere. As shown in Figure 3, this region has higher air density than higher orbits. This has the effect of limiting the life of satellites in VLEO to the order of weeks or days without active drag compensation. There are multiple models available to approximate the fluid properties of this region, namely the density, temperature and chemical composition. For the work performed here the atmospheric model NRLMSISE-00 was used follows the guideline recommended by the European Space Agency Standard ECSS-E-ST-10-04C[27]. This is an empirical model of the Earth’s atmosphere produced by the US National Research Laboratory and extends from the sea-level to the upper limit of the thermosphere at 1000km above local sea level.

The NRLMSISE-00 model has a number of inputs including the time, location (altitude, latitude and longitude) as well as recent solar activity and terrestrial magnetosphere strength. The number of inputs reflects the significant variability of the atmosphere and represents a level of complexity that is not needed for this current study. To simplify, a global average of the atmospheric properties was taken at the desired altitude of 200km during high solar activity on the 1st March 2000. This date was chosen to be close to the solar maximum of solar cycle 23, a recent active solar cycle, and thus provided a worse case scenario for the conditions at the desired altitude. The gas species included in the model were: Oxygen (O₂), Nitrogen (N₂), Atomic Oxygen (O), Atomic Nitrogen (N), Argon (N), Helium (He) and Hydrogen (H). A summary of the derived properties used are presented in Table 2.

Table 2: Properties of the atmosphere at 200km on the 1st March 2000 at 00:00:00 UTC as given by the spherically weighted mean of NRLMSISE-00 model[28] using sampling points at 1° intervals in Latitude and Longitude

Mission Profile			
Altitude		200 km	
Date		1 st March 2000	
Time		00:00:00 UTC	
Bulk Fluid			
Number Density		$1.06 \times 10^{16} \text{ m}^{-3}$	
Temperature		1063 K	
Composition			
Atomic Oxygen (<i>O</i>)		56.195	%
Nitrogen (<i>N</i> ²)		39.893	%
Oxygen (<i>O</i> ²)		2.972	%
Helium (<i>He</i>)		0.585	%
Atomic Nitrogen (<i>N</i>)		0.278	%
Argon (<i>Ar</i>)		0.064	%
Hydrogen (<i>H</i>)		0.013	%

Figure 3: Variation of atmospheric density with altitude, showing the range of densities during the last solar max (March 2000) and solar min (January 2008) according to the atmospheric model NRLMSISE-00

4.2. Methods for Determining drag in VLEO

A way of characterising the low density flows in VLEO is to use the non-dimensional Knudsen number (K_n), given by Equation 1, which describes the ratio between the molecular mean free path λ , and the size of the object l . The mean free path describes the average distance that particles travel between collisions and under the conditions described in table 2, its value is 139m. This was calculated using the mean free path equations for gas mixtures using the Variable Hard Shell radii model provided in “The DSMC Method” by Bird[17]. This is larger than the objects to be simulated (0.5m and 1.5m) and yields a Knudsen number between 278 and 92 respectively for these two sizes.

$$K_n = \frac{\lambda}{l} \quad (1)$$

Fluids which have a high Knudsen number ($K_n \gg 10$) are described as a ‘free molecular flows’. Continuous flow methods such as Navier-Stokes are not valid for simulating this type of fluid. There are a number of alternative methods available to model free-molecular flow, such as panel methods and DSMC

170 techniques. Panel methods, such as those developed by Sentman [14], use probability distributions to model the internal particle velocities. These methods tend to be fast, with simulation speed scaling with the number of faces on the mesh. However, they do not account for particle reflections or particle-particle interactions in the bulk fluid, so their accuracy in some circumstances is uncertain. The alternative is to simulate the particles directly (or a statistically significant group of particles), as is the case with DSMC. 175 Since this method captures more of the particle interactions, it has improved accuracy over analytical models. However, the simulation time scales with the number of particles, and thus the density of the simulated fluid. This is fine for regions high in VLEO, but becomes more of a concern lower in the atmosphere.

The geometries for the satellite body described in Section 3 are 2D convex shells and so do not experience 180 secondary particle impacts from prior particle-surface interactions. This would usually suggest that panel methods could be sufficient for determining the aerodynamic forces on the body. However, it is intended to scale the surrogate model methodology for more complex geometries, such as concave hulls and full three dimensional bodies, where non-linear interactions such as secondary reflections may have a greater impact on the resulting aerodynamic forces[15].

185 There are also local effects to consider, such as the accumulation of particle around the front of the satellite caused by the slower velocity of particles remitted from the surface following a diffuse interaction. These particles linger around the front of the satellite creating a region of higher density which raises the probability of particle-particle collisions taking places. This was observed during initial simulations of blunt test bodies using the DSMC methods under the atmospheric conditions in Table 2 where the local 190 Knudsen Number dropped from 139 in the far field to as low as 2 in the region immediately ahead of the test body. As the Knudsen Number is close to unity the flow in the region ahead and around the satellite could be considered to be transitional rather than fully free-molecular[29].

where the mean free path dropped as low as 2 metres immediately ahead of the test body (compared to 139 metres for the conditions in Table 2). This is of a similar order of magnitude to the body tested so 195 the flow in the region ahead and around the satellite could be considered to be transitional rather than fully free-molecular. [29]. It was also observed that this minimum was depended on the geometry with the lowest observed mean free path rising to about 4.5 metres for the sharpest profiles. Therefore, given the variability and the higher frequency of particle-particle interactions immediately ahead and around the satellite, particle accumulation may influence the drag calculations.

200 It is therefore useful to test the surrogate model methodology and perform the analysis of the satellites geometry using data generated by both panel and DSMC methods. Data from the DSMC simulations will provide a platform on which to build future work as the geometries become more complex. Meanwhile data from the panel method simulations will serve to provide a point of comparison to validate the results from the DSMC and to identify any non-linearities between the two methods for the conditions in Table 2.

205 The DSMC tool used for the studies in this paper was SPARTA, developed by Sandia National Laboratories[19]. It is an open source tool which offers both two-dimensional and three-dimensional capability and has been

tested on other spacecraft [30, 31] The panel method used in this work was based on method developed by Sentman [14]. Both the DSMC and panel method simulations return the forces acting on the body, however the drag will be sampled as the drag coefficient C_D . This is defined by equation 2 where F_D is the drag force, ρ is the atmospheric density, v is the orbital velocity and A_{ref} is the reference area. Typically the reference area is taken to be the frontal area of the satellite, however, since the simulations were performed in two dimensions, the reference area was taken to be the height of the satellite body.

$$C_D = \frac{2F_D}{\rho v^2 A_{ref}} \quad (2)$$

4.3. Surface Properties

An important aspect of modelling rarefied gases is the interaction between the particle and the surfaces of the test body. Both the Sentman Panel Methods and the DSMC tool SPARTA use a diffuse model to replicate the particle-surface interactions[14, 18] within a rarefied gas. In this model, these interactions are governed by the surface temperature and the energy accommodation coefficient [16]. The coefficient a describes how much energy the incoming particle loses to the surface and is defined as

$$a = \frac{E_i - E_r}{E_i - E_w} \quad (3)$$

where E_i is the kinetic energy of the incident molecule, E_r is the kinetic energy of the reemitted molecule, and E_w is the kinetic energy the re-emitted molecule would have if it were reemitted from the surface at the surface's temperature. When a equals 1, complete thermal accommodation has occurred, i.e. the particle has been reemitted at the same temperature as the impacted surface and therefore has lost some energy to the surface.

At present, there is some uncertainty around the energy accommodation coefficient of materials in VLEO. Empirical findings from satellites in orbit have shown that the energy accommodation coefficients ranges in value from 0.86 to 1, with limited dependences on the underlying material[13, 32]. This contrast however with early ground testing, which has shown some dependence on the material choice though often these are not comparable conditions to those seen in orbit. While it is likely that many materials will still exhibit mostly diffuse or quasi specular behaviour, this is not known with any great certainty. Recent work by Roberts et al. [6, 33], seeks to improve this understanding by examining the gas-surface interactions between the material and the atmosphere under the conditions observed in orbit. As of writing, the Rarefied Orbital Aerodynamics Research facility (ROAR) is still being commissioned but once operational will be capable of simulating the free molecular flow and levels of atomic oxygen flux observed in VLEO.

For the work presented here an energy accommodation coefficient of 0.95 was selected for both methods, representing a mostly diffuse environment with some specular reflections occurring. This also allows the results to be compared with previous work by the authors. Future work could explore if there is any

dependence between the selected profile and the energy accommodation coefficient. The surface temperature of the satellites is taken to be $300K$, which is broadly within the operational range of most terrestrial satellites. These values were used to define the properties of the surface for both the Panel Methods and the DSMC simulations.

5. Surrogate Model Generation and Validation

5.1. Overview

There are three aspects to a successful surrogate model: the interpolation method, the sampling strategy, and the data source. Perhaps the two most common forms of interpolation method are Radial Basis Function (RBF) [34] and Kriging [35]. Both methods share similar model formulations, with key differences arising from the different branches of mathematics used for their derivation. Simpson et al. [36] demonstrated that RBF and Kriging performed well over a range of sampling strategies and sample size when compared to other interpolation methods such as response surface models. Simpson also noted that while larger sample size generally improved the accuracy of the model, above a certain number of samples, the improvement was limited.

RBF is best described as the weighted sum of simple functions that approximate the function of the design space. The weightings are then calculated based on a set of training data, with the output of each function varying with the distance from the training point. This is functionally similar to a single level neural net. Examples of commonly used basis functions include linear, Gaussian or multi-quadratic, though recent work demonstrated the use of Wendland's functions [37] as effective basis functions.

5.2. Interpolation

The interpolation approach adopted for this body of work was RBF [34]. The following section presents the general formulation of the RBF model. The code was implemented by the authors, using the theory presented by Wendland [37], Fasshauer [38] and Mackman [39].

If $y(\mathbf{x})$ is the original function to be approximated, let y_i be the result of the i^{th} sample point $\mathbf{x}_i$, where $\mathbf{x}_i$ is a vector of the n dimensional design space such that $\mathbf{x}_i = \{x_{i,1}, x_{i,2}, \dots, x_{i,n}\}$. If the training data is taken to be a set of N samples, such that $\mathbf{X} = \mathbf{x}_1, \mathbf{x}_1, \dots, \mathbf{x}_N$, with corresponding results $\mathbf{Y} = y_1, y_1, \dots, y_N$, evaluating the RBF interpolation for the sample set $[\mathbf{X}, \mathbf{Y}]$ at an arbitrary point $\mathbf{x}_*$ takes the form

$$\hat{y}(\mathbf{x}_*) = \sum_{i=1}^N \beta_i \phi(\|\mathbf{x}_* - \mathbf{x}_i\|) + p(\mathbf{x}_*) \quad (4)$$

where $\beta_i, i = \{1, \dots, N\}$ are the model coefficients for the basis function ϕ , and $\|\cdot\|$ is used to denote the euclidean norm. $p(\mathbf{x})$ is an optional polynomial with M components with the general form

$$p(\mathbf{x}_*) = \sum_{k=1}^M \gamma_k f_k(\mathbf{x}_*) \quad (5)$$

270 where $f_k(\mathbf{x})$ are the monomial components, and γ_k are the polynomial coefficients.

This provides N basis coefficient (β) and M polynomial coefficients (γ) that must be solved for using the training data $[\mathbf{X}, \mathbf{Y}]$. This can be achieved by requiring the exact recovery of the data at each sample point such that $\hat{y}(\mathbf{x}_i) = y(\mathbf{x}_i)$ for all data points $\mathbf{x}_i$ in $\mathbf{X}$. An additional constraint, given by equation 6, ensures that the polynomial is orthogonal to the basis functions.

$$\sum_{i=1}^N \beta_i f_k(\mathbf{x}_i) = 0, \quad k = 1, \dots, M \quad (6)$$

275 The problem can then be written in matrix form as follows

$$\begin{bmatrix} \mathbf{Y} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R} & \mathbf{F} \\ \mathbf{F}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} \quad (7)$$

where

$$\mathbf{R} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \dots & \phi_{1,N} \\ \phi_{2,1} & \phi_{2,2} & \dots & \phi_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{N,1} & \phi_{N,2} & \dots & \phi_{N,N} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} f_{1,1} & \dots & f_{1,M} \\ f_{2,1} & \dots & f_{1,M} \\ \vdots & \ddots & \vdots \\ f_{N,1} & \dots & f_{N,M} \end{bmatrix} \quad (8)$$

$\mathbf{R}$ contains all pairwise combinations for the basis function such that $\phi_{i,j} = \phi(\|\mathbf{x}_i - \mathbf{x}_j\|)$ where $\mathbf{x}_i$ and $\mathbf{x}_j$ are both points in the training data $\mathbf{X}$. $\mathbf{F}$ contains the monomial results for each sample point $\mathbf{x}_i$ such that $f_{i,k} = f_k(\mathbf{x}_i)$. β and γ are the column vectors of coefficient for the basis function and polynomial respectively.

Obtaining the solution to this equation can cause issues with conditioning, as a result of the large zero block, so the problem is manipulated to give

$$\gamma = (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{R}^{-1} \mathbf{Y} \quad (9)$$

$$\beta = \mathbf{R}^{-1}(\mathbf{y} - \mathbf{F}\gamma) = (\mathbf{R}^{-1} - \mathbf{R}^{-1} \mathbf{F} (\mathbf{F}^T \mathbf{R}^{-1} \mathbf{F})^{-1} \mathbf{F}^T \mathbf{R}^{-1}) \mathbf{Y} \quad (10)$$

Thus, for an arbitrary point $\mathbf{x}_*$, the predicted result $\hat{y}$ can be determined as follows

$$\hat{y}(\mathbf{x}_*) = \mathbf{r}(\mathbf{x}_*)^T \beta + \mathbf{f}(\mathbf{x}_*)^T \gamma \quad (11)$$

285 where $\mathbf{r}$ and $\mathbf{f}$ are vectors of the basis function and monomial terms for the point $\mathbf{x}_*$, respectively, given by

$$\mathbf{r} = \begin{bmatrix} \phi_{*,1} \\ \phi_{*,2} \\ \vdots \\ \phi_{*,N} \end{bmatrix}, \mathbf{f} = \begin{bmatrix} f_{*,1} \\ f_{*,2} \\ \vdots \\ f_{*,N} \end{bmatrix} \quad (12)$$

where $\phi_{*,i} = \phi(\|\mathbf{x}_* - \mathbf{x}_i\|)$ and $f_{*,k} = f_k(\mathbf{x}_*)$.

5.2.1. Choice of basis function ϕ

The choice of basis function (ϕ) was critical to the effectiveness of this method. Many examples of potential basis functions exist, including linear and polynomial, as well as more complex functions such as Gaussian and multi-quadratic. The latter have additional degrees of freedom that allow for more precise fine tuning of the function.

A further example of commonly used basis functions are the Wendland's functions [37]. For a stated number of continuous derivatives C^{2k} in n dimensions, the Wendland's functions are compact functions of minimal degree which decay to zero at a given distance from the centre. This distance is known as the support radius R_s , and allows the model to be tuned to achieve better accuracy. For this body of work Wendland's C^2 functions were used, as they provide a compromise between matrix conditioning and modelling behaviour [40]. Equation 13 gives the general form of Wendland's C^2 for n spatial dimensions, where $l = \lfloor n/2 \rfloor + 2$. The scaled distance from the sample point r is given by the Euclidean norm between the evaluation point $\mathbf{x}_*$ and the control point $\mathbf{x}_i$, and scaled by the support radius R_s such that at the support radius the value of $\phi_{n,1} = 0$, as given by equation 14.

$$\phi_{n,1} = \begin{cases} (1-r)^{l+1} [(l+1)r+1], & 0 \leq r \leq 1 \\ 0, & r > 1 \end{cases} \quad (13)$$

$$r = \frac{\|\mathbf{x}_* - \mathbf{x}_i\|}{R_s} \quad (14)$$

5.3. Sampling

Once the design space has been established, the other key aspect of the surrogate model generation was the selection of the sample locations within the domain. The most notable methods include Latin Hypercube Sampling and orthogonal array as described by Simpson et al.[36]. These methods are single stage whereby all sample points are selected before the simulations are performed. When correctly implemented, these methods can provide consistent coverage of the whole domain, though may require a relatively large number of samples to achieve a low overall error.

The alternative is to implement a multi-stage adaptive sampling process. The goal of these adaptive sampling methods is to achieve a better distribution of sampling points to more accurately and efficiently capture the shape of the design space. These processes begin by taking an initial sample of the domain using one of the methods above and collecting the data for these sample points, then new sample points are generated using some criteria based on the previously generated data [22]. This process is repeated until the maximum allocation of sample points is reached. It was suggested by Forrester et. al. [41] to allocate one-third of the total sampling points, or as close as the method employed would allow, to the

320 initial sample.

The adaptive strategy employed for this line of work used a multi-criteria method based on the work of Mackman et. al.[39]. An initial sample of the domain was achieved using an Optimised Latin Hypercube sampling. New sample points were then added to the model based on a criterion C , given by

$$C(\mathbf{x}_*) = (|\nabla^2 \hat{y}(\mathbf{x}_*)| + \epsilon) (1 - h(\mathbf{x}_*))^2 \quad (15)$$

325 where $\nabla^2 \hat{y}$ is the Laplacian of the surrogate model $\hat{y}$, ϵ is an offset parameter to ensure a non-zero value when $|\nabla^2 \hat{y}(\mathbf{x}_*)| = 0$, and $h(\mathbf{x})$ is the separation function defined such that $(1 - h)$ grows with increased distance from the data sites. Large values of C indicate potential new sampling sites. The criterion is the product of the Laplacian and the separation function and thus gives a balance between adding points in locations of high detail but also in unsampled regions of the domain, thus ensuring a degree of space-filling to minimise uncertainty in less detailed regions.

330 The Laplacian of the interpolation model is defined using Equation 16. Applying this to equation 11, the derivative of the model for a given parameter x_i can be evaluated using equation 17. For a given data set, the coefficients β and γ remain constant, thus evaluations of the laplacian of the data is no more expensive than evaluations of the model itself.

$$\nabla^2 \hat{y} = \sum_{j=1}^n \frac{\partial^2 \hat{y}}{\partial x_j^2} \quad (16)$$

335

$$\frac{\partial^2 \hat{y}}{\partial x_j^2} = \frac{\partial^2 \mathbf{r}^T}{\partial x_j^2} \beta + \frac{\partial^2 \mathbf{f}^T}{\partial x_j^2} \gamma \quad (17)$$

The separation function h can be constructed as a RBF surrogate model allowing a similar framework to be employed to that of the primary surrogate model. In this instance, the data sites are given a value of 1 with the separation function decaying with increased distance from these points. Wendland's C^2 (equation 13) was chosen for the radial function based on Mackman et. al.[39] who suggested that a smooth function such as the Wendland's function would provide better sampling as they would blend better with the Laplacian term. Given that Wendland's C^2 decays to zero at the support radius, the separation function h lies in the range $0 \leq h \leq 1$ everywhere in the domain, with a value of one indicating a point coincident with a sample location and a value of zero indicating a region far from all sample points.

345 As with the primary surrogate model, it was necessary to set an appropriate support radius for the separation function, though in this case the h should decay to a minimum (ideally zero) at a point that is furthest from all points on the domain. This was achieved by setting the support radius proportional to the fill distance. The fill distance was the largest nearest 'neighbour distance' of the current set of samples, which was given by

$$d = \max \{i = [1, 2, \dots, N] | \min \|\mathbf{x} - \mathbf{x}_i\|\} \quad (18)$$

and the support radius R , used in the separation function interpolation was then a scalar constant k times the fill distance d

$$R = kd \quad (19)$$

Each time a new sample point was added, the support radius of the separation function was recalculated.

5.4. Testing and validation

At each stage of development, the surrogate model generation was tested and validated. This was firstly achieved by performing the standard validation case as used in Mackman et al.[40], however these are not presented here. Further testing was achieved using the analytical equations developed by Sentman[14] to describe the aerodynamic forces acting on objects moving through a rarefied gas. These were ideal for testing the generator as they provided quick results thus allowing more specific errors and inaccuracies to be isolated and eliminated before being applied to a DSMC model.

To validate a surrogate model it is often necessary to have two data sets, one from which the model is generated and separate unique data set used to evaluate the model. For analytical models, such as Sentman model, this was easily achieved and took the form of a coarse grid of sample points. However, if the means to acquire the sample point has a large computational load, such as in the case of DSMC, it is inefficient to generate additional sample points that would not be used in the model itself. The Leave one out Cross-Point Validation (LOOCPV) [42], addresses this by allowing the data set to act as both the training and validation data set. Under this scheme a number of Evaluation points were selected at random from the data set, excluding those that fell on the sample space boundary. Each evaluation point is excluded from the training data in turn and the surrogate model was regenerated. The surrogate model was then interrogated at the location of the evaluation point and the result compared to its known value. The Root Mean Square Error (RMSE) was then calculated for the set of evaluation points.

The LOOCPV method provided an efficient way to validate the surrogate model and could also be implemented during the sampling process to track the progression of the error in the model as more samples were added. Figure 4 shows the RMSE progression for the data set used in this work, described in section 6, as determined by the LOOCPV.

6. The Surrogate Model

A surrogate model was generated for the design space outlined in section 3 using the method outlined in section 5 for each body. The design space consisted of the 4 variables representing the nose length, nose radius, tail length and tail radius. The outputs of the surrogate model are the coefficient of drag C_D and internal volume of the geometry represented by the evaluation point. The coefficient of drag for each sample point was acquired using the DSMC software SPARTA as outlined in section 4.

For the training data set used in this work, it was estimated that around 2000 samples would be required, based on observations from previously generated surrogate models while validating the process.

As outlined in section 5.3, this gave an initial sample size of 667 generated using the optimised Latin Hypercube scheme. Further samples were added using the adaptive criteria outlined in section 5.3. While the maximum number of samples was set as 2000, this condition was relaxed to achieve a lower RMSE and data collection was terminated when the system determined the RMSE could not be improved upon.

Slices through the design space are presented in Figures 5 and 6. Figure 5 shows how the coefficient of drag and internal volume varies for nose length and radius while the tail is held in a blunt configuration (tail length and radius both equal to 0). Similarly figure 6 shows these variations for tail length and radius while the nose is held in a blunt configuration (nose length and radius both equal to 0).

The progression of the RMSE of the coefficient of drag as more samples are added is shown in Figure 4. This figure also shows the progression of the 95% confidence bracket as well as the maximum error observed. As seen in both Figure 4 and Table 3, for the $0.5m \times 1m$ profile the final RMSE is 2.88×10^{-3} , with a 95% confidence bracket of 8.63×10^{-3} . Thus, for a 95% confidence interval, this implies the model coefficient of drag is accurate to within ± 0.00863 of the DSMC simulations. In some circumstances greater accuracy can be achieved by increasing the number of sampling points. However, this presents the risk of over-fitting the problem, which may ultimately increase the error margin. In the case of this data set, the generator terminated sampling as it determined no further improvement in the accuracy was possible.

Table 3: properties of the $0.5m \times 1m$ data set

Initial number Sample Points	667
Number of Sample Points	2635
Number of Evaluation Points	500
Final RMSE	2.88×10^{-3}
95% Confidence Bracket	8.63×10^{-3}

7. Optimisation Strategy

7.1. The Cost Function

As outlined at the beginning of the paper there are two key objectives here: to minimise the drag on the satellite body, and to maximise the internal volume of the satellite. The cost function for this is given by

$$f_c = \alpha f_{C_D}^A(G) - (1 - \alpha) f_V(G) \quad (20)$$

The objective function for the atmospheric drag $f_{C_D}^A(G)$ will take the satellite geometry and return the coefficient of drag (C_D) for the orbital environment by interrogating the surrogate model. The objective function for the internal volume $f_V(G)$ takes the satellite geometry and returns the volume. As the simulations were performed in 2D, the area of the geometry was used to represent the volume. As the outputs of these functions are of a similar orders of magnitude it was not deemed necessary to scale them in this case. The parameter α , was a relative weighting of the two objectives. When $\alpha = 0$, the optimizer optimized for minimum drag, while if $\alpha = 1$, the optimizer optimized for maximum internal volume. The

Figure 4: Error history of the surrogate model data set used in this work

selection of an appropriate value for alpha is covered in the discussion.

The optimizer used in this work is global optimisation strategy, particle swarm. DSMC is a stochastic method, which means there will be a random error component to the data used to train the surrogate model. While efforts were made to minimise this error, a risk of local minima forming remains, especially if two sample points are in close proximity. Therefore, to avoid getting stuck, a global optimization strategy was needed. Particle swarm was chosen as it can traverse large multi-variable problems quickly.

Before optimising the whole geometry, it is useful to assess individual elements of the geometry to ascertain their impact on the drag. The first section of the results and discussion focus on the elements that describe the nose profile, namely the nose length and radius. The second section focuses on the elements that describe the tail profile, the tail length and radius. In the third and final section, the geometry and its elements were assessed together for all the sizes simulated. A trade-off between minimising the drag and maximising the internal volume is then presented.

8. Results and Discussion

8.1. Nose Profile

This section will focus on the impact the nose profile will have on the atmospheric drag on the body. The nose profile is described by the nose radius and the nose length, as described in section 3. Figures 8 and 7 show the minimum drag achievable when the nose radius and length are fixed respectively for both panel methods (Left) and DSMC (right). In both figures, the other element was allowed to vary to achieve the lowest drag, while the rear of the body was held in a blunt configuration. As can be seen from the two figures, 8 and 7, while panel methods and DSMC predict similar trends there are notable differences. For instance, panel methods predict a far more linear response to changing the nose

Figure 5: A slice through the surrogate model hyperspace, showing variation of C_D with nose geometry with fixed tail geometry

length constraint as compared to the DSMC method. This is likely as a result of particle pileup ahead of the satellite which is more completely captured in the DSMC method but not normally considered in panel methods. This piling up results in an increase in localised density around the front of the satellite and thus a reduction in the mean free path of the particles. This would result in a greater number of particle-particle interactions taking place in the region ahead of the satellite and thus effect the particle-surface interactions in this region. As can be seen by Table 4, panel methods also predicts higher drag in the fully nose pointed configuration and slight lower drag in the cuboid configuration. In the case of of the fully pointed configuration the inbound particles must traverse more of the high density region before interacting with the surface greatly increasing the opportunity to interact with other slower moving particle. The difference in values between the two methods are relatively small, between 1-3% in the cuboid configuration and 5-6% in the fully pointed configuration. However, since these errors compound over the operational life of the satellite, this can significantly effect the predicted end date of the mission. Additionally, as will be seen later in the paper this impacts the relative the effectiveness of

the nose profile.

In the case of the fixed nose radius, for both methods, the lowest drag was achieved at the maximum nose length, in this case the full length of the body. Similarly, in the case of the fixed nose length, the minimum drag was generally achieved when the nose radius was minimised. The exception to this was
450 when the nose radius was large or the nose length was small respectively, in other words, as the profile approached a simple cuboid. This was a limit of the model since, as is seen in figure 5, the lower right boundaries of the graph defined the simple cuboid body, and as the profile approached this boundary there was much less variation in the coefficient of drag and thus the optimizer was more susceptible to local minima. Critically this was observed for both panel methods and DSMC which implies it is a
455 limitation of the surrogate model with this design space.

The results from figures 8 and 7 imply that in the absence of constraints on the nose geometry, for a blunt tail configuration the optimum shape should be the sharpest nose profile possible. This is what was

Figure 6: A slice through the surrogate model hyperspace, showing variation of C_D with tail geometry with fixed nose geometry

Table 4: Unconstrained optimised bodies for minimum drag for varying nose geometry and fixed (blunt) tail geometry

Profile Size (height $\times$ length)	Sentman				DSMC			
	Cuboid C_D	Opti. Profile	Opti. C_D	% Red. in Drag	Cuboid C_D	Opti. Profile	Opti. C_D	% Red. in Drag
$0.25m \times 0.5m$	2.383		2.043	14.3	2.428		1.932	20.4
$0.25m \times 1.0m$	2.647		2.070	21.8	2.679		1.953	27.1
$0.25m \times 1.5m$	2.905		2.186	24.8	2.930		2.066	29.5
$0.50m \times 1.0m$	2.378		2.043	14.1	2.433		1.933	20.5
$0.50m \times 1.5m$	2.517		2.044	18.8	2.563		1.931	24.6

* direction of flight left-to-right with particle flow from the right

observed in result in Table 4 which shows the profile and coefficient of drag achieved when optimizing for minimum coefficient of drag for the different bodies while holding the tail in a blunt configuration.

Results are provided for both panel methods and DSMC. The table also provides the coefficient of drag of the unoptimised cuboid, i.e. the untapered body, for comparison, along with the relative reduction in drag coefficient.

As was discussed earlier and as can be seen from table 4, the results from the panel methods predicts a higher coefficient of drag in the fully pointed nose configuration as compared to the results collected using DSMC. As a result a reduction in the drag of about 14-25% was achieved when compared to the original cuboid bodies, as compared to the DSMC results which achieved a 20-30% reductions. This change in the effectiveness of the reduction in drag will impact the results of the multi-objective optimisation, the results for which will be discussed in section 8.3. These results are valid assuming an energy accommodation coefficient of 0.95.

The results presented so far call into question the usefulness of including a nose radius in the profile definition. The nose radius acted to blunt the profile, increasing the frontal area that was fully reflecting the flow back on itself. However, it also had the effect of decreasing the internal angle of the nose for a given nose length. As was seen in Figure 8, for the lower aspect ratio profiles, there was a modest increase in the drag below 50% nose radius. For the 1:2 profiles this increase in drag is about 2.8% and 7.3% at 50% nose radius compared to 16.3% and 25.9% at 100% nose radius for panel methods and DSMC respectively. There appears to be less of an advantage for bodies with large aspect ratios as the internal angle was already quite small when the nose is as large as it can be and thus has limited variation over the range of nose radii. However, it was observed that if the nose length was constrained in an effort to increase the internal volume, the drag curve became similar to that of the lower aspect ratio bodies (though these values were higher due to the increased length of the profile). So, while it was not ideal for reducing the drag, it was good for maximising volume which will be useful in section 8.3.

(a) Panel Method

(b) DSMC

Figure 7: Optimum Coefficient of Drag for varying Nose Length with fixed (blunt) Tail Geometry

(a) Panel Method

(b) DSMC

Figure 8: Optimum Coefficient of Drag for varying Nose Radius with fixed (blunt) Tail Geometry

8.2. Tail Profile

The drag the body experiences is predominantly determined by the body's frontal projected area. However, due to the random motion of the particles in the gas, the incoming flow is not perfectly parallel,

Table 5: Unconstrained optimised bodies for minimum drag when varying the tail

Profile Size (height $\times$ length)	Sentman				DSMC			
	Cuboid C_D	Opti. Profile	Opti. C_D	% Red. in Drag	Cuboid C_D	Opti. Profile	Opti. C_D	% Red. in Drag
$0.25m \times 0.5m$	2.388		2.130	10.8	2.435		2.194	9.9
$0.25m \times 1.0m$	2.649		2.170	18.1	2.689		2.237	16.8
$0.25m \times 1.5m$	2.906		2.291	21.2	2.933		2.358	19.6
$0.50m \times 1.0m$	2.383		2.130	10.6	2.437		2.192	10.0
$0.50m \times 1.5m$	2.518		2.139	15.0	2.567		2.209	14.0

* direction of flight left-to-right with particle flow from the right

and so there are particle-surface interactions taking place along the side of the body as well. In some cases this could contribute up to 10% of the atmospheric drag that the body experienced [8] and as can be seen by Table 4, the longer the body the higher the drag coefficient. While this was not necessarily a large contribution, it was still worth asking whether the rear profile could be altered in any way to reduce its contribution to the atmospheric drag. This section will therefore focus on the impact the tail profile had on the atmospheric drag the satellite body would experience. In this instance the tail profile was defined by the tail radius and the tail length (as outlined in section 3).

The graphs in figures 9 and 10 show the minimum drag achievable when the tail radius and length are fixed respectively for both panel methods (Left) and DSMC (right). In both cases, the other element was allowed to vary to achieve the lowest drag, while the front of the body was held in a blunt configuration. Unlike the nose profile, there was less differentiation between panel methods and DSMC as a result of the tail geometry. In the fully tail pointed configuration the difference between the two methods was about 3%. This difference is likely as a result of particle accumulation on the forward facing surface rather than any non-linearities taking place on the rear surfaces. In contrast to the front of the satellite and as a direct result of being in a non-molecular flow the satellite leave a fluid void directly behind it. This has a much lower density and as a result a higher mean free path and Knudsen number than the surrounding fluid. This means the chances of particle-particle interactions taking places is much lower and thus has less of an impact on the predicted coefficient of drag. This would explain why the resulting curves in figures 9 & 10 are broadly parallel if slightly offset.

From figures 9 and 10, it was clear that altering the tail of the satellite would indeed reduce the drag that the body experienced. For instance when the length of the tail was fixed, as was seen in figure 10, the coefficient of drag varied proportionally to the length of the Tail, decreasing as the length increased. On the other hand, as seen in figure 9, the coefficient of drag appears to have an exponential relationship with the tail radius when it is fixed. In fact, for profiles with an aspect ratio of 1:2, there is no significant increase in coefficient of drag below 50% Tail radius. Some of the other profiles exhibit similarly plateauing at lower tail radii though this becomes less pronounced as the aspect ratio increases.

Figure 9: Optimum Coefficient of Drag for varying Tail Radius with fixed (blunt) Nose Geometry

Optimizing the whole tail profile (radius and length) for minimum coefficient of drag while holding the nose in a blunt configuration and with an energy accommodation coefficient of 0.95 gave a reduction of between 10-21% depending on the aspect ratio. Table 5 shows the result of this optimisation for the 5 bodies as well as the profiles for both panel methods and DSMC. The table also provides the coefficient of drag of the unoptimised cuboid, i.e. the untapered body, for comparison along with the reduction in drag coefficient. Comparing the results to those in Table 4, it can be seen that while a reduction in drag was achieved it was about half the reduction achieved by the nose profile alone. This being the case, if a simple conic section were required, a tapered nose would be favourable over a tapered tail as it provides a better reduction in drag for the same volume. Similarly, this would suggest that in a straight optimisation for minimum coefficient of drag, the optimizer would likely favour a sharper nose profile over a tail profile of any kind. This will be explored further in section 8.3.

From Table 5 it can be seen that in the case of the fixed nose configuration, the optimizer favoured a tail with a sharp profile, as was seen in the case of the nose profiles. However, as discussed above and as was seen in Figure 9, at low tail radii there is no significant change in the coefficient of drag which could imply the existence of a line of local minima. While not shown here and in contrast with the profiles in Table 5, it was also observed that the tail radius would bounce up and down as the fixed tail length was varied. This appeared to be the case for all the aspect ratios tested which further supports the existence of a line of local minima.

The implication of this was that below a certain tail radii there is no further improvement in drag performance, a feature that is exemplified by the profiles with a 1:2 aspect ratio as seen in figure 9. Above 50%, the relationship appeared to be broadly exponential, which meant that for these bodies the majority of the drag reduction could be achieved with a relatively small reduction in the tail radius. For

Figure 10: Optimum Coefficient of Drag for varying Tail Length with fixed (blunt) Nose Geometry

the other aspect ratios in Figure 9, the radius at which the plateauing occurred reduced as the aspect ratio of the body increased. This was possibly because these larger aspect ratios passed through a smaller range of tail angles assuming the tail length was at its maximum. In cases where the length of the tail was limited the flat region was much broader. This is a useful result since it implies that significant reductions in coefficient of drag can be achieved with limited loss of internal volume. Being able to use larger tail radii has additional advantages, such as providing space for important equipment such as a propulsion system and a launch adaptor, which would typically be mounted to the rear surface of a satellite.

This raises the further question of how the reduction in the drag was achieved and why the coefficient appeared to plateau at lower tail radii. As discussed above, the particle velocities were not parallel and within the moving frame of the fluid have a random motions as described by the Maxwell-Boltzmann distribution [16]. These random motions had a component perpendicular to the flow which brought some of the particles into contact with the surface, causing an interaction and the transfer of energy. This contributed towards the drag and results in the longer the bodies experiencing high drag [8]. Another property of this gas was its high mean free path (as described in section 4.1), which meant there is limited interaction between particles at this altitude and thus any object passing through the fluid at speed will leave a void behind it. This void will slowly refill, with the refill rate governed by ratio between the bodies velocity and the perpendicular component of the random particle motions. The angle at which this occurs is the molecular Mach angle and can be calculated using the mean internal velocities from the Maxwell-Boltzmann distribution[43]. This is given by equation 22, where the most probable thermal speed v' is given by equation 21 and where v_{inf} is the free stream speed, k_B is the Boltzmann constant, T is the temperature of the gas and m is the particle mass of the gas species.[43, 16]

$$v' = \sqrt{\frac{2k_B T}{m}} \quad (21)$$

Table 6: Approximate refill angles for gas species during the atmospheric condition described in table 2

Gas Species	Number Density [1/m]	Refill Angle [°]
Atomic Oxygen	6.08×10^{15}	7.7
Nitrogen	4.31×10^{15}	5.8
Oxygen	3.21×10^{14}	5.5
Helium	6.33×10^{13}	15.1
Atomic Nitrogen	3.01×10^{13}	8.2
Argon	6.97×10^{12}	4.9
Hydrogen	1.43×10^{12}	28.3

$$\theta_{refill} = \arctan\left(\frac{c'}{v_{inf}}\right) \quad (22)$$

By assuming these internal velocities act perpendicular to the flow, a very rough estimate of the refill angles for each particle species was derived, as presented in Table 6. Physically, these refill angles represent the angle an average particle would take relative to the velocity vector of the spacecraft. Atomic Oxygen and Nitrogen comprise the highest percentage of particles in the fluid and thus contribute the most to the forces on the satellite. As can be seen from the table, they had a refill rate of between $6^\circ - 8^\circ$. It would therefore be expected that if the tail angle was larger than $6^\circ - 8^\circ$, there should only be a limited improvement in the drag. Indeed in figure 11, it can be seen that in general there was no further improvement in the coefficient of drag above about 7° of internal tail angle. In the figure, the data from figure 9 was replotted against the internal tail angle in figure 11. There were a few anomalies near the right of the graph, which resulted from the shape tending towards a cuboid at high tail radii and thus being more susceptible to local minima. This trend was more obvious in bodies with lower aspect ratios, though high aspect ratio bodies appeared to follow the same trend. It should be noted that the plateau angles presented here were unique to the particular atmospheric conditions simulated. As described by the Maxwell-Boltzmann distribution, the random internal velocity of the particles is dependent on the temperature of the gas.

The rate of refill also has implication for other aspects of the satellite, for instance the ingress of Atomic Oxygen to the interior of the satellite especially around sensitive equipment such as payloads. It was therefore of interest to properly describe the refill rate and explore its impact on other aspects of the satellite design. Further research is required to determine whether a reliable relationship can be determined between the atmospheric conditions and the refill angle.

8.3. An Optimal Profile?

In the previous sections, the effects of individual elements of the shape of the satellite on the drag were explored. It was identified that while some features did not improve the drag, they did relieve pressure on the internal volume. In this section, a multi-objective optimisation was performed to minimise drag,

Figure 11: Optimum Coefficient of Drag for varying tail angle with fixed (blunt) Nose Geometry

while maximising the internal volume of the space using the cost function in equation 20 for the whole central body. Figure 12 shows the Pareto-optimal fronts for the five bodies. This is, the line of optimal solutions whose objective functions cannot be improved upon without detriment to at least one of the other objectives. These were obtained by varying the relative weight of the two objectives using the α variable in equation 20. Table 7 provides examples of the shapes achieved across the fronts for both 1:2 (left) and 1:6 (right) bodies with surrogate models generated from panel methods and DSMC. The top profile in each table represents bodies optimised purely for maximum volume ($\alpha = 0$), while the lowest bodies were optimised purely for minimising drag ($\alpha = 1$).

Figure 12 show that the resultant curve was broadly exponential, which meant that the further the body is reduced in volume, the less the achievable improvement in the drag of the body. For instance, as seen in Table 7, a 30% reduction in the 1:2 body's volume would achieve a 13.8% and 18.6% reduction in the drag based on the results from panel methods and DSMC respectively. In both cases, this is nearly 90% of the total achievable reduction in the drag for that aspect ratio. A further 10% reduction in volume only offers an additional 1-2% reduction in the drag.

The exact point at which the further loss of volume is deemed unacceptable will largely be dependent on the payload and subsystem configuration. However, assuming the reduction in fuel mass results in an equally proportioned reduction in fuel system volume, then it is desirable for this to be equal to or less than the loss of internal volume due to the aeroshell. This boundary is represented by the diagonal dashed line in Figure 12. Those points below and to the right of the line receive a net benefit from the optimized profile. Those above and to the left receive a net penalty, with fuel volume potentially using up space for the payload and other subsystems. Using this assumption, an optimised aeroshell provides more benefit for longer bodies, i.e. those with a higher aspect ratio, as they can achieve a much greater

(a) Panel Method

(b) DSMC

Figure 12: Graph showing the Pareto-optimal fronts for the bodies simulated when optimising for minimum drag and maximum Volume

reduction in drag before making a net loss as a result of lost volume. This is seen in the data from both panel methods and DSMC, as can be seen in Figure 12. However, the methods differ in the exact point at which this occurs. For instance, the 1:6 profile could achieve as much as a 27% reduction in drag before there is a potential net loss according to the DSMC results while panel methods predicts a lower

605

21%. For comparison, the shorter 1:2 profiles could only achieve a 10% and 13% reduction before it makes a net loss for panel methods and DSMC respectively.

It is worth highlighting which features were selected in the case of the optimal profiles in table 7. For instance, it was shown that the nose profile had a greater influence on reducing drag than the tail profile. This was certainly seen in the 1:2 body, where the nose profile dominated the overall geometry of the body, however, with the longer 1:6 aspect ratio body, the nose and tail accounted for nearly equal proportions of the overall geometry. In previous sections, the benefits of including features that provide no immediate benefit to the drag within a molecular flow was discussed. Notably in Section 8.2 it was demonstrated that on its own the maximum the tail profile could improve the drag by was less than half the maximum improvement achieved by the nose profile. However, as can be seen by Table 7 and discussed above, most of the profiles feature some degree of tapering of the tail section. For the 1:6 aspect ratio profile this does appear to allow it to achieve a much lower drag coefficient overall (2.05 and 1.93) than either the nose (2.19 and 2.07) or the tail geometries (2.29 and 2.36) in isolation (for panel methods and DSMC respectively, see tables 4 and 5 for details).

Herein lies the value of including the tapered tail section, it effectively reduced the length of the body presented to the flow while maintaining a greater volume. Since it accounts for the rear half of the body the drag experienced is equivalent to a 1:3 profile with a fully tapered nose. Similarly as the volume begins to dominate the optimisation, the nose radius becomes more prominent. The larger the nose radius, the smaller the achievable nose angle, thus mitigating some of the performance lost due to the blunter nose. This has a number of advantages, since not only does it provide more absolute internal volume overall, it also provides more usable volume, particularly in the nose of the satellite.

9. Future Work

In order to carry out the work for this paper a number of assumptions were made about the design space and in particular the geometries of the satellite. First and foremost, the profiles examined here were all two dimensions. As a first step it would be useful to extend the analysis to three dimensional geometries to see if the relationships identified in this paper hold. This would also allow the examination of the choice of cross-section and its impact on the nose and tail profiles.

Another key assumption in this work is that the external dimensions are held constant, thus allowing an analysis of the aerodynamics of a bi-conic wedge. This also affords the opportunity to as well to test and validate a surrogate model with data from DSMC simulation as compared to data collected from panel methods. An important next step would be to relax this constraint to see if the relationships identified in this paper hold when the external dimensions are allowed to vary.

In this paper, the refill rates of the gas species behind a ‘blockage’ was briefly explored. These refill rates have other implications on the design, for instance the ingress of Atomic Oxygen to the interior of the

Table 7: Example optimal profiles at varying points along the Pareto-optimal fronts for 1:2 profiles (left) and 1:6 profiles (right)

	1:2 Shape	Volume		C_D		1:6 Shape	Volume		C_D	
		$[m^3]$	$[\%]$	$[-]$	$[\%]$		$[m^3]$	$[\%]$	$[-]$	$[\%]$
Panel Method		0.50	0	2.394	0.0		0.38	0	2.918	0
	0.45	10	2.147	10.3		0.34	10	2.491	14.7
	0.40	20	2.086	12.9		0.30	20	2.312	20.8
	0.35	30	2.063	13.8		0.26	30	2.176	25.4
	0.30	40	2.043	14.7		0.23	40	2.089	28.4
	0.25	50	2.028	15.3		0.19	50	2.048	29.8
DSMC		0.50	0	2.446	0		0.38	0	2.954	0
	0.45	10	2.168	11.4		0.34	10	2.480	16.0
	0.40	20	2.060	15.8		0.30	20	2.269	23.2
	0.35	30	1.992	18.6		0.26	30	2.114	28.4
	0.30	40	1.945	20.5		0.23	40	1.985	32.8
	0.25	50	1.926	21.2		0.19	50	1.933	34.6

satellite. It is therefore of interest to properly describe the refill rate and explore their impact on other aspects of the design.

Finally, the work presented in this paper focused exclusively on the drag of the central body. Other subsystem such as the solar panels have not been considered as part of the study. Given their large surface area, solar panels represent a significant contribution to the drag of the satellite and represent a vital link in the drag-power optimisation loop. Therefore, leading from the desire to extend the analysis to three dimensional geometries, further work assessing the interaction between the central body and the solar panels is warranted. This could then lead to a broader systems analysis with the satellites geometry as part of the loop.

10. Conclusion

The work presented here forms part of a larger body of work assessing the effectiveness of altering the geometry of the satellite in order to reduce its drag in an effort to either prolong its operational life or increase its payload capacity. In this paper a series of satellite aeroshell profiles were presented that will reduce the drag experienced in VLEO while ensuring the usability of the geometries. This was achieved by first creating a RBF-based surrogate model which could be interrogated by an optimizer to identify viable aeroshell profiles. To help test and validate the methodology, results from both panel methods and DSMC simulations were presented throughout the paper. It was observed that there was broad agreement between the surrogate models generated by the two methods, though the panel methods often predicted much higher coefficients of drag. This was most notably observed while analysing the nose profile where there was a difference of 6% between the results.

In data obtained using the DSMC surrogate model, it was demonstrated that a maximum drag reduction of 21% to 35% was achievable for the 1:2 and 1:6 profiles respectively (15% and 30% from panel methods), when optimizing a bi-conic profile for minimum drag with an energy accommodation coefficient of 0.95. However, this often came with a loss of internal volume due to the constraints on the geometry, namely a fixed height and length. Assuming the reduction in fuel mass results in an equally proportioned reduction in fuel system volume, then the longer the profile the greater the achievable reduction in coefficient of drag. For the bi-conic profiles this was 13% and 27% for the 1:2 and 1:6 aspect ratio profiles respectively based on DSMC data. Using data obtained using panel methods the reductions reduced to 10% and 21% respectively. These values are highly dependent on the satellite configuration, as there will be other constraints on the design which might drive the selection, however they provides a useful consideration for future VLEO design.

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References

- [1] J. Virgili Llop, P. C. Roberts, Z. Hao, L. Ramio Tomas, V. Beauplet, Very Low Earth Orbit mission concepts for Earth Observation: Benefits and challenges, in: 12th Reinventing Space Conference, London, 2014.

- 685 [2] D. G. Fearn, Economical remote sensing from a low altitude with continuous drag compensation, *Acta Astronautica* 56 (5) (2005) 555–572. doi:10.1016/j.actaastro.2004.09.052.
- [3] A. Green, An Ultra Low Altitude Synthetic Aperture Radar Micro-Satellite, Tech. rep., University of Bristol (2011).
- [4] A. Grasso, Feasibility Study of an Ultra-Low Altitude Hyperspectral Micro-Satellite, Tech. rep.,
690 University of Bristol (2011).
- [5] ESA, ARTES Advanced Technology Statement of Work Very low latency satellite communication system 3A.115 (2020).
- [6] P. C. Roberts, N. H. Crisp, F. Romano, G. H. Herdrich, V. T. Oiko, S. Edmondson, S. J. Haigh, C. Huyton, S. Livadiotti, R. E. Lyons, K. L. Smith, L. A. Sinpetru, A. Straker, S. D. Worrall, J. Becedas, R. M. Domínguez, D. González, V. Cañas, V. Hanessian, A. Mølgaard, J. Nielsen, M. Bisgaard, A. Boxberger, Y.-A. Chan, S. Fasoulas, C. Traub, D. Garcia-Almiñana, S. Rodriguez-Donaire, M. Sureda, D. Kataria, R. Outlaw, B. Belkouchi, A. Conte, J. S. Perez, R. Villain, B. Heißen, A. Schwalber, DISCOVERER – Making Commercial Satellite Operations in Very Low Earth Orbit a Reality, in: 70th International Astronautical Congress, no. October, Washington, 2019.
- 700 [7] A. Bacon, B. Olivier, Skimsats: bringing down the cost of Earth Observation, in: S. Hatton (Ed.), *Proceedings of the 12th Reinventing Space Conference*, Springer International Publishing, Cham, 2017, pp. 1–7.
- [8] D. A. Vallado, D. Finkleman, A critical assessment of satellite drag and atmospheric density modeling, *Acta Astronautica* 95 (1) (2014) 141–165. doi:10.1016/j.actaastro.2013.10.005.
- 705 [9] GOCE flight control team (HSO-OEG), GO-RP-ESC-FS-6268: GOCE End-of-mission operations report, Tech. rep., ESA (2014).
- [10] K. Fujita, A. Noda, T. Abe, Aerodynamics of Satellites on a Super Low Earth Orbit, in: *Rarefied Gas Dynamics*, 26th International Symposium, no. 1, Kyoto, 2009, pp. 772–777. doi:10.1063/1.3076580.
- 710 [11] ESA, SLATS (Super Low Altitude Test Satellite) (2017).
URL <https://directory.eoportal.org/web/eoportal/satellite-missions/s/slats>
- [12] J. A. Walsh, L. Berthoud, Is it possible to integrate Electric Propulsion thrusters on Very-Low Earth Orbit Microsatellites?, in: *Space Propulsion 2016*, Rome, 2016.
- [13] K. Moe, M. M. Moe, Gas-surface interactions and satellite drag coefficients, *Planetary and Space Science* 53 (8) (2005) 793–801. doi:10.1016/j.pss.2005.03.005.
- 715 [14] L. H. Sentman, Free Molecule Flow Theory and its Application to the Determination of Aerodynamic Forces, Tech. rep., Lockheed Missile and Space Co., Sunnyvale, CA (1961).

- [15] V. K. Dogra, J. N. Moss, Hypersonic rarefied flow about plates at incidence, *AIAA Journal* 29 (8) (1991) 1250–1258. doi:10.2514/3.10729.
- 720 [16] G. A. Bird, *Molecular gas dynamics and the direct simulation of gas flows*, 2nd Edition, Sydney, 1994.
- [17] G. A. Bird, *The DSMC Method*, 1st Edition, Sydney, 2013.
- [18] Sandia National Laboratories, SPARTA Direct Simulation Monte Carlo Simulator.
URL <https://sparta.sandia.gov/>
- 725 [19] M. A. Gallis, J. R. Torczynski, S. J. Plimpton, D. J. Rader, T. Koehler, Direct simulation Monte Carlo: The quest for speed, in: *AIP Conference Proceedings*, Vol. 1628, 2014, pp. 27–36. doi:10.1063/1.4902571.
- [20] T. J. Scanlon, E. Roohi, C. White, M. Darbandi, J. M. Reese, An open source, parallel DSMC code for rarefied gas flows in arbitrary geometries, *Computers and Fluids* 39 (10) (2010) 2078–2089.
730 doi:10.1016/j.compfluid.2010.07.014.
- [21] A. I. J. Forrester, A. J. Keane, Recent advances in surrogate-based optimization, *Progress in Aerospace Sciences* 45 (1-3) (2009) 50–79.
- [22] J. Sacks, W. J. Welch, T. J. Mitchell, H. P. Wynn, Design and Analysis of Computer Experiments, *Statistical Science* 4 (4) (1989) 409–423. doi:10.1214/ss/1177012413.
- 735 [23] N. V. Queipo, R. T. Haftka, W. Shyy, T. Goel, R. Vaidyanathan, P. Kevin Tucker, Surrogate-based analysis and optimization, *Progress in Aerospace Sciences* 41 (1) (2005) 1–28. doi:10.1016/j.paerosci.2005.02.001.
- [24] T. Simpson, V. Toropov, V. Balabanov, F. Viana, Design and Analysis of Computer Experiments in Multidisciplinary Design Optimization: A Review of How Far We Have Come - Or Not, 12th
740 AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference (September) (2008) 1–22.
doi:10.2514/6.2008-5802.
URL <http://arc.aiaa.org/doi/10.2514/6.2008-5802>
- [25] M. Wade, *Encyclopedia Astronautica*.
URL <http://www.astronautix.com/>
- 745 [26] Union of Concerned Scientists, UCS Satellite Database.
URL https://www.ucsusa.org/nuclear-weapons/space-weapons/satellite-database?{_}ga=2.108595713.2003365126.1536848252-1039379185.1536848252{#}.W5pzac5KiUk
- [27] European Cooperation for Space Standardization, ECSS-E-ST-10-04C Space environment, Tech. Rep. November (2008).

- [28] J. M. Picone, A. E. Hedin, D. P. Drob, A. C. Aikin, NRLMSISE-00 empirical model of the atmosphere: Statistical comparisons and scientific issues, *Journal of Geophysical Research: Space Physics* 107 (A12) (2002) 1–16. doi:10.1029/2002JA009430.
- [29] T. I. Gombosi, *Gaskinetic Theory*, Cambridge University Press, 1994. doi:10.1017/CB09780511524943.
 URL <https://www.cambridge.org/core/product/identifier/9780511524943/type/book>
- [30] A. Marín C., I. B. Sebastião, S. Tamrazian, D. Spencer, A. Alexeenko, DSMC-SPARTA aerodynamic characterization of a deorbiting CubeSat, *31st International Symposium on Rarefied Gas Dynamics* 2132 (August) (2019) 070024. doi:10.1063/1.5119578.
- [31] G. March, E. Doornbos, P. Visser, High-fidelity geometry models for improving the consistency of CHAMP, GRACE, GOCE and Swarm thermospheric density data sets, *Advances in Space Research* 63 (1) (2019) 213–238. doi:10.1016/j.asr.2018.07.009.
- [32] K. Moe, M. M. Moe, Gas-Surface Interactions in Low-Earth Orbit, in: *AIP Conference Proceedings*, no. 1333, Pacific Grove, 2011, pp. 1313–1318. doi:10.1063/1.3562825.
- [33] V. T. A. Oiko, Peter C. E. Roberts, S. D. Worrall, S. Edmondson, S. J. Haigh, N. H. Crisp, S. Livadiotti, C. Huyton, R. E. Lyons, K. L. Smith, L. A. Sinpetru, B. E. Holmes, A. Straker, J. Becedas, R. M. Domínguez, D. González, V. Cañas, V. Hanessian, A. Mølgaard, J. Nielsen, M. Bisgaard, A. Boxberger, Y.-A. Chan, G. H. Herdrich, F. Romano, S. Fasoulas, C. Traub, D. Garcia-Almiñana, S. Rodriguez-Donaire, M. Sureda, D. Kataria, R. Outlaw, B. Belkouchi, A. Conte, J. S. Perez, R. Villain, B. Heißerer, A. Schwalber, ROAR – A Ground-Based Experimental Facility for Orbital Aerodynamics Research Vitor, in: *70th International Astronautical Congress*, no. October, Washington, 2019.
- [34] D. S. Broomhead, D. Lowe, Multivariable Functional Interpolation and Adaptive Networks, *Complex Systems* 2 (3) (1988) 321–355.
- [35] D. G. Krige, A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand, *Journal of the Chemical, Metallurgical and Mining Society of South Africa* 52 (9) (1952) 201–215. doi:10.2307/3006914.
- [36] T. W. Simpson, D. K. J. Lin, W. Chen, Sampling Strategies for Computer Experiments: Design and Analysis, *International Journal of Reliability and Applications* 2 (3) (2001) 209–240. doi:citeulike-article-id:1454478.
- [37] H. Wendland, *Scattered Data Approximation*, Cambridge University Press, 2005.
- [38] G. E. Fasshauer, *Meshfree Approximation Methods with Matlab*, Vol. 6 of *Interdisciplinary Mathematical Sciences*, World Scientific, 2007. doi:10.1142/6437.

- [39] T. J. Mackman, C. B. Allen, Investigation of an adaptive sampling method for data interpolation using radial basis functions, *International Journal for Numerical Methods in Engineering* (March) (2010). doi:10.1002/nme.2885.
- [40] T. J. Mackman, C. B. Allen, M. Ghoreyshi, K. J. Badcock, Comparison of Adaptive Sampling Methods for Generation of Surrogate Aerodynamic Models, *AIAA Journal* 51 (4) (2013) 797–808. doi:10.2514/1.J051607.
- [41] A. I. J. Forrester, A. Sóbester, A. J. Keane, *Engineering Design via Surrogate Modelling*, Wiley, 2008. doi:10.1002/9780470770801.
- [42] M. Stone, Cross-Validatory Choice and Assessment of Statistical Predictions, *Royal Statistical Society* 36 (2) (1974) 111–147.
- [43] G. Koppenwallner, W. Wuest, Concave corner flow problem, *Acta Astronautica* 6 (9) (1979) 1227–1234. doi:10.1016/0094-5765(79)90067-5.