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In-plane mechanical behavior of novel auxetic hybrid metamaterials

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ABSTRACT

We present in this paper two novel concepts of hybrid metamaterials that combine a core unit cell of re-entrant or cross-chiral shape and lateral missing ribs. The first topology is a hybrid between an anti-tetrachiral and a missing rib (cross-chiral) configuration; the second one has a variable cross-chiral layout compared to the classical missing rib square structure. Their in-plane mechanical properties have been investigated from a parametric point of view using finite element (FE) simulations. The two classes of metamaterials have been benchmarked to obtain optimized designs and specific effective properties. Nonlinear simulations and experimental tests of the new re-entrant missing rib metamaterials featuring optimized geometry parameters have been performed to understand the behavior of these architectures under large deformations.

Keywords: re-entrant; cross; missing rib; metamaterial; in-plane mechanical property; deformation mechanism

1. Introduction

Auxetic materials or structures with a negative Poisson's ratio (NPR) possess the counter intuitive property of contracting laterally when uniaxially compressed [1]. NPR materials feature enhanced mechanical like enhanced in-plane indentation resistance, enhanced fracture toughness and transverse shear modulus. NPR solids also possess interesting dynamic properties from enhanced impact energy absorption to strong acoustic/wave signature. Pioneering work on NPR was conducted by Lakes [2], Wojciechowski [3], Evans [4] and Alderson [5] et al; those works have not only inspired scores of researchers to work on auxetics, but they also provided a significant contribution to elevate auxetics as one of the most important classes of mechanical metamaterials [6]. Two-dimensional auxetic lattice metamaterials present in open literature feature architectures like the re-entrant one, the rigid (or semi-rigid) rotational unit layout, structures with perforated/cutting slits, the missing-rib, and the chiral ones [7-21]. Missing rib models based on square grid models have however received little attention so far, as well as hybrid new designs based on the combination of the missing rib model with other architectures. Smith et al. [22] developed novel two-dimensional architectures inspired to the 'missing rib' concept – i.e., broken cell ribs generate auxetic behavior in thermoformed open cell processed negative

Poisson's ratio foams. Gaspar et al. [23] manufactured samples of missing rib models; experimental measures of the Poisson's ratios of these lattices were compared with those from analytical models. Rana et al. [24, 25] have carried out systematic investigations on auxetic structures with missing rib design in braided composites, and noted that in those architectures the auxetic and tensile loading behaviors were more strongly dependent on their structural parameters than the core material properties. Scarpa et al. [26] has investigated the equivalent thermal conductivities of iso-volume periodic honeycomb along the in-plane and out-of-plane principal directions, in which a centre-symmetric 3-connectivity hexagonal layout and a centre-symmetric 4-connectivity cross-chiral configuration were considered. Grima et al. [27] originally reported the 'tetrachiral' system as a 'missing ribs' structure, which can also be regarded as a hierarchical system, to some extent [28]. Then Lim [29] has provided a comprehensive review of 2D auxetic models from the deformation mechanism perspective, and as a result he showed that the tilted 45° and cross-chiral-type missing rib models are analogous to the tetrachiral and anti-tetrachiral ones. This feature suggests that the missing rib models should be able to deform through hinging at the joints, as well as via the rotation of the crossed-ligaments through a rotating center coupled with the bending of the joined ligaments. To assess under which conditions a square grid behaves like an anti-tetrachiral one, Grima et al. [30] have carried out an extensive investigation on the missing rib square grid architecture. Square grids possess a type of deformation primarily dependent on the thickness ratio between different ribs; the overall deformation of these lattices can be represented by the primary bending of the crossed-ligaments, followed by both ligaments and crossed ligaments deforming, and then crossed-ligaments rotating as a rigid unit like anti-tetrachiral configurations. The analysis from Grima and co-workers also showed the ability of the structure to deform predominantly through ligament bending, the latter particular deformation mechanism being responsible for the most negative values of Poisson's ratio in the lattice. Jiang and Li [31] have also developed a new hybrid auxetic mechanical metamaterial in which a re-entrant core cell is present at the center of a missing-rib type chiral configuration. This architecture assembles together the two basic deformation mechanisms related to the re-entrant centre-symmetric lattice and structural chirality. Results from experiments and FE simulations have shown a sequential cell-opening mechanism under large strains, and a broad tunability of the effective stiffness, Poisson's ratio and the cell-opening rate of the new designs via the adjustment of two independent geometric cell parameters. Singamneni et al. [32] have selected a square missing rib structure with fillets of different radii to evaluate stress concentration zones and auxetic responses using both linear and nonlinear FE analyses. Key architectures shown in open literature and related to the evolution of the missing rib models are summarized in Table 1.

This paper presents a novel auxetic metamaterial configuration based on a middle re-entrant core and outer missing ribs, together with a cross missing rib grid with the same dimensions as a benchmark. The design has been inspired by the analogies between deformation mechanisms in anti-tetrachiral, missing rib, and re-entrant models [29, 30] as well as the combination of two basic auxetic open-cell metamaterials, re-entrant angle and chirality together [31]. This particular design is aimed at the development of a new auxetic hybrid metamaterial with broader and tunable ranges of mechanical properties and larger in-plane deformations. A parametric analysis using numerical simulations is also carried out to investigate the importance of the various geometric parameters defining the core unit cell and the surrounding missing ribs on the in-plane mechanical behavior and deformation mechanisms of the new lattices. A comparative study of in-plane mechanical properties between the two classes of mechanical metamaterials are discussed, and a configuration of the new re-entrant missing rib metamaterial leading to optimal structural performances has been identified. We have also considered the nonlinear mechanical behavior of the optimized re-entrant missing rib metamaterial through further analysis of simulation and experimental tests.

2. Methodologies

2.1 Metamaterial design

The designs of the new auxetic re-entrant missing rib representative unit cell (RUC) and corresponding cross missing rib with same dimensions are presented in Fig. 1a and Fig. 1b, respectively. The specific geometry of one quarter of re-entrant missing rib RUC is shown in Fig. 1c, in which three groups of parameters are included. The first group involves the inclined cell wall L , the horizontal cell wall $2H$, the inclined cell angle θ and the cell wall thickness t_1 of re-entrant core unit cell. The second group is described by the middle ribs along x and y directions (b_1 , b_2) with identical wall thickness t_1 that are stretched from the re-entrant core configuration. The lengths of the ribs is expressed as $b_1 = c \times x_A$, $b_2 = c \times y_A$, where c is a constant greater than 1 and x_A, y_A are the coordinates of external edge point A in the re-entrant cell. By using this architecture, the contacts between the re-entrant core and the outer ribs along the x and y directions are automatically eliminated. The third group of parameters is represented by the external ribs with same thickness t_2 along the horizontal and vertical directions b_3 , b_4 which are defined as $b_3 = c \times (b_1 + t_2)$, $b_4 = c \times (b_2 + t_2)$. All these geometric parameters are based on the center line of the structure. The geometry of this new re-entrant missing rib RUC can be further defined by using nondimensional parameters like the cell wall aspect ratio $\alpha = H/L$ and the thickness ratio $\beta = t_1/L$ for the re-entrant core, as well as the thickness ratio of

internal and external ribs $\gamma = t_1/t_2$. The representative geometrical parameters of the two RUCs presented in Fig. 1 are $\alpha = 0.5$, $\beta = 0.1$, $\gamma = 0.5$, $\theta = 20^\circ$, $t_1 = 2$, $c = 1.2$, while the out-plane thickness is $h_z = 4$. To avoid contacts between the re-entrant inclined walls, the geometry relation $H - L \times \sin \theta - t_1/2 \cos \theta > 0$ has to be applied. Similarly, the relation $b_3 > c \times (H + t_1/2 \cos \theta + 0.5t_1 \tan \theta)$, $b_4 > c \times (L \cos \theta + 0.5t_1) + t_2$ needs to hold true to avoid interference caused by the mirroring of one quarter of the RUC.

2.2 Relative density

The relative densities of the two novel missing rib RUCs are expressed as follows:

$$\begin{cases} \rho_r = \frac{\rho_r^*}{\rho_s} = \frac{\beta}{c^2} \times \frac{A\gamma^2 + B\gamma + c\beta \cos \theta}{C\gamma^2 + D\gamma + \beta^2 \cos \theta} \\ \rho_c = \frac{\rho_c^*}{\rho_s} = \frac{\beta}{c^2} \times \frac{A'\gamma^2 + B'\gamma + c\beta(\cos \theta - \sin \theta + 1)}{C'\gamma^2 + D'\gamma + \beta^2(\cos \theta - \sin \theta + 1)} \end{cases} \quad (1)$$

In Eq (1) ρ_r and ρ_c are the relative densities of the re-entrant and cross missing rib RUCs, ρ_r^* and ρ_c^* are the densities of the re-entrant and cross missing rib RUCs, respectively. The density ρ_s is related to the bulk material. The terms $A \sim D$ and $A' \sim D'$ are expressed as:

$$\begin{cases} A = 4(c-1)\cos^2 \theta + 2\cos \theta [2\sin \theta + 2\alpha(1+c) + \beta(c-1) + 4] + \beta[(2c-1)(\sin \theta + 1) - 1] \\ B = 4c^2 \cos^2 \theta + 2\cos \theta (c^2\beta + 2c^2\alpha + 2\beta) + 2c^2\beta(\sin \theta + 1) \\ C = 8c^2\alpha \cos^2 \theta + 4c^2\beta \cos \theta (\sin \theta + 1 + \alpha) + 2c^2\beta^2(\sin \theta + 1) \\ D = 8c\beta \cos^2 \theta + 4c\beta \cos \theta (2\alpha + \beta) + 4c\beta^2(\sin \theta + 1) \end{cases},$$

$$\begin{cases} A' = 2c \cos^2 \theta + \cos \theta [2c(\alpha + \beta + 1 - \sin \theta) - \beta] + [(2c\alpha - \beta)(1 - \sin \theta) + 2c\beta] \\ B' = 2c^2 \cos^2 \theta + 2\cos \theta [-c^2 \sin \theta + (\alpha + \beta + 1)c^2 + \beta] + 2[(c^2\alpha + \beta)(1 - \sin \theta) + c^2\beta] \\ C' = 2c^2(2\alpha + \beta)\cos^2 \theta + c^2 \cos \theta [(2\alpha + \beta)(\beta + 2 + 2\sin \theta) - 8\alpha \sin \theta] + c^2\beta[(2\alpha + \beta)(1 + \sin \theta) - 4\alpha \sin \theta] \\ D' = 4c\beta \cos^2 \theta + 4c\beta \cos \theta (\alpha + \beta + 1 - \sin \theta) + 4c\beta(\alpha + \beta - \alpha \sin \theta) \end{cases}$$

From inspecting Eq (1) one can readily observe that the relative densities of the two RUC ρ_r^* and ρ_c^* are affected by the non-dimensional parameters α , β , γ , as well as by the inclined angle θ and the constant c .

2.3 Linear Finite Element Simulations

Linear elastic FE models have been developed using the Abaqus/CAE 6.13-4 commercial package to simulate the in-plane properties of the new mechanical

metamaterials. The core material properties (Young's modulus and Poisson's ratio) are assumed to be $E_c = 2265\text{MPa}$ and $\nu_c = 0.25$. The models have been developed using C3D8R quadrilateral 8 nodes linear brick with reduced integration; the dimensions of the model are the same as illustrated in Fig.1. Periodic boundary conditions for in-plane tensile loading are applied based on [33, 34] and shown in Fig. 2a. The nodes at the left and bottom edges are constrained from the out-plane rotation and translation normal to the edge direction. Displacements along the x (or y) -direction are applied to the ligament nodes on the right (top) edge, which are also constrained from in-plane rotation. The effective Young's moduli E_i ($i=1,2$) of one RUC is then obtained as:

$$E_i = \frac{F_i L_i}{h_z L_j u_i} \quad (2)$$

Where F_i is the total reaction force along the i direction on the prescribed boundary, L_i and L_j are the lengths of one RUC along the i and j direction ($j=2,1$). The parameter h_z is the thickness along 3 direction, u_i is the applied displacement along the i direction. The effective Poisson's ratio is therefore obtained as:

$$\nu_{ij} = -\frac{u_j L_i}{u_i L_j} \quad (3)$$

Where u_j is the lateral displacement perpendicular to the loading i direction.

For the in-plane shear simulations, biaxial loading is introduced to represent a pure shear deformation field [35]. The corresponding boundary conditions and imposed displacements along both the 1 and 2 directions are applied as presented in Fig. 2b. The effective shear modulus G_{12} , defined as $G_{12} = \tau_{12}/\gamma_{12}$, is determined in the following way:

$$G_{12} = \frac{F_i L_i - F_j L_j}{2h_z (u_i L_j - u_j L_i)} \quad (4)$$

where F_i and F_j are obtained as reaction forces from the finite element analysis

A mesh size of $t_1/4 = 0.5$ was applied to guarantee a good convergence towards the norm of the displacement. The models consisted therefore of 212,103 elements (Fig.2c). According to Grima's missing rib square grid concept [30], the Poisson's ratio and the deformation mechanism are primarily dependent on the thickness ratio of the different ribs. In this work, the effects on the mechanical properties of the RUCs given by both the rib thickness ratio $\gamma = t_1/t_2$ (range of 0 and 2) as well as other core unit parameters have been therefore evaluated. To guarantee the feasibility of the geometries, the inclined angle θ for the core re-entrant unit cell varies with α , in such a way that $\theta \leq 20^\circ$ when $\alpha = 0.5$.

3. Parametric study and discussions

3.1 Relative density

The effects on relative density ρ_c^r of re-entrant missing rib RUC provided by the γ parameter and different constant nondimensional variables are shown in Fig.3. The relative density first decreases with increasing γ and then becomes gradually stable for $\gamma > 1$ when c is constant. Similarly, the increase of the c parameter induces a decrease of the relative density for γ constant (seen in Fig.3a). Fig.3b shows the variation of ρ_c^r versus γ with different α and θ values. The relationship between ρ_c^r and γ are similar to those shown in Fig.3a with fixed α and θ . However, when γ is constant ρ_c^r increases for increasing θ values and decreases for increasing α . Within the interval of parameters adopted in this work, the extreme values of the relative density for the re-entrant missing rib RUC are $\rho_{c\max}^r = 0.284$ (with $\alpha = 1$, $\theta = 60^\circ$) and $\rho_{c\min}^r = 0.149$ (for $\alpha = 2$, $\theta = 0^\circ$). Those values are valid when the other parameters are $\gamma = 0.5$, $\beta = 0.1$ and $c = 1.2$. From inspecting Eq (1), the relative density of the cross missing rib RUC ρ_c^c has the same variations as those of the re-entrant missing rib RUCs. The comparison between the relative densities for two RUCs with extreme α and θ values are shown in Fig.4. The cross missing rib RUC has a smaller relative density and a steeper dependency versus γ compared to the re-entrant RUC case.

3.2 Effect of the c constant

Parameter c is designed to adjust the distance between core unit cell and external rib as well as to change the length of external ribs, which plays an important part on both structural design and mechanical property intuitively. When c is increasing, the effect of core unit cell on structural mechanical property is diminishing, the influence of ligaments is increasing however, making the structure more analogous to anti-tetrachiral honeycomb. Therefore, the effect of the parameter c on the two classes of metamaterials is studied initially considering the first uniaxial loading along the x direction. One can observe from Fig.5 that the normalized stiffness versus γ for the two RUCs first increases, and then decreases when c is kept constant, with the maximum values all obtained for $\gamma = 0.5$. When γ remains constant, the increase of c makes the non-dimensional Young's modulus E_1^*/E_c decrease and accrete the specific stiffness $(E_1^*/E_c)/(\rho/\rho_c)$ for both two RUCs, shown in Fig.5a and Fig.5b. For E_1^*/E_c , there is no discrepancy between re-entrant and cross missing RUCs

(Fig.5a). For $(E_1^*/E_c)/(\rho/\rho_c)$, however, the cross missing RUC has a higher specific stiffness compared to the re-entrant missing RUC; this is attributed to its lower relative density ρ_c^c (Fig.5b). The variations of the effective Poisson's ratio versus the parameter c for the two RUCs are however quite different. When the parameter c increases the ν_{12}^* of the re-entrant missing rib RUC decreases and exhibits a greater auxetic behavior; the Poisson's ratio is also quite sensitivity to the parameter c . The Poisson's ratio ν_{12}^* of the cross missing rib RUC however increases with increasing c values and shows a smaller sensitivity to the latter parameter compared to the previous lattice architecture (Fig.5c). Considering that a larger c value will contribute to enhance the deformation of the ligaments and diminish the effect of the core re-entrant unit cell, a value of $c=1.2$ has then been used in the following studies to better represent the two deformation functions of both the core unit cell and the surrounding ribs.

3.3 The effect of the α and θ parameters

The effects of α and θ on the in-plane engineering constants for two RUCs are presented in Fig.6-Fig.9. When α and θ are kept constant the non-dimensional Young's moduli E_i^*/E_c as well as the specific stiffness $(E_i^*/E_c)/(\rho/\rho_c)$, ($i=1,2$) for the two RUCs are in the two cases first increasing and then decreasing with larger values of the γ parameter. The shear moduli G_{12}^*/E_c and $(G_{12}^*/E_c)/(\rho/\rho_c)$ of two RUCs however exhibit a monotonic rise followed by a gradual increase when γ increases. For a fixed γ , the E_1^*/E_c and $(E_1^*/E_c)/(\rho/\rho_c)$ values of two RUCs increase with increasing α and θ ; the maximum values of the uniaxial moduli are reached for $\alpha=2$ and $\theta=60^\circ$ (Fig.6). On the contrary, for two RUCs the variations of E_2^*/E_c and G_{12}^*/E_c with α and θ , their corresponding specific stiffnesses are opposite to the uniaxial modulus along the 1-direction; the maximum normalized modulus along the 2-direction and the shear one are in that case obtained for $\alpha=0.5$, $\theta=0^\circ$ (Fig.7 and Fig.8).

The effects of α and θ on the Poisson's ratio of two RUCs are shown in Fig. 9. From Fig.9a, when $\alpha=0.5$ the Poisson's ratio ν_{12}^* of the re-entrant missing rib RUC firstly increases with γ and θ ($\theta>0^\circ$) and then declines with different slopes. When $\theta=0^\circ$ ν_{12}^* however increases continuously and tends to be flatline; when $\alpha>0.5$, the ν_{12}^* of the re-entrant missing rib RUC declines with increasing γ , α and θ values. Similarly, the Poisson's ratio ν_{12}^* of the cross missing rib RUC exhibits a monotonic decline with increments of α and θ for a constant γ value.

When γ however increases, one can observe an up-and-down trend for $\alpha < 2$, $\theta < 60^\circ$, and a down-up-down tendency when $\alpha = 2$, $\theta \geq 0^\circ$, as well as for when $\alpha < 2$, $\theta = 60^\circ$ (Fig.9b). The resulting max values of ν_{12}^* for two RUCs are both obtained for $\alpha = 2$, $\theta = 60^\circ$. The variation of ν_{21}^* for re-entrant missing rib RUC is illustrated in Fig.9c. When $\alpha = 0.5$, $\theta > 0^\circ$, the Poisson's ratio decreases with increasing γ values; the variation of ν_{21}^* with θ is opposite to the one versus γ , and the gradient of the curve for $\alpha = 0.5$, $\theta = 0^\circ$ is larger than the one related to the other cases. When $\alpha > 0.5$, $\theta > 0^\circ$ the Poisson's ratio ν_{21}^* increases and then gets close to a flatline for larger γ α and θ values; the variations of $\alpha > 0.5$, $\theta = 0^\circ$ with γ display however a trend described by an increase first, and then a decrease. The effective Poisson's ratio ν_{21}^* of the cross missing rib RUC exhibits an up-down variation with increasing γ values, and the slope of the curve becomes larger with smaller α values when α and θ ($\theta < 60^\circ$) are constant. Additionally, the Poisson's ratio of architectures with $\alpha > 0.5$, $\theta = 60^\circ$ tend to be constant for $\gamma > 0.5$; for a fixed γ , the values of ν_{21}^* for the cross missing rib RUCs rise with increasing α and θ (Fig.9d). Finally, the maximum of the Poisson's ratio ν_{21}^* for the two classes of RUCs is reached when $\alpha = 0.5$, $\theta = 0^\circ$.

3.4 Comparison between the two metamaterials

A direct comparison between the optimal mechanical properties of the two RUCs architectures versus the parameters α and θ are shown in Fig.10 and Table 2. Fig.10a indicates that E_1^*/E_c and $(E_1^*/E_c)/(\rho/\rho_c)$ of the cross missing rib RUC is 12% and 50% larger than the ones of the re-entrant missing rib RUC, when $\alpha = 2$, $\theta = 60^\circ$ and $\gamma = 0.5$. The re-entrant missing rib RUC however exhibits an excellent mechanical behavior along the 2-direction and the in-plane shear; in those cases E_2^*/E_c , $(E_2^*/E_c)/(\rho/\rho_c)$ and ν_{21}^* are 24%, 9% and 18% higher than those of the cross missing rib RUC. Furthermore, the normalized shear moduli G_{12}^*/E_c and $(G_{12}^*/E_c)/(\rho/\rho_c)$ of the re-entrant missing rib RUCs are almost 5 times higher than those of the cross missing rib RUC for $\alpha = 0.5$, $\theta = 0^\circ$ and $\gamma = 0.5$ (see Fig.10b-Fig.10d). Additionally, the re-entrant missing rib RUC also presents a greater auxetic performance of ν_{12}^* for $\gamma \geq 0.5$ (Fig.10d).

3.5 The effect of β

The increase of the cell wall aspect ratio provides a way to tailor the in-plane

tensile modulus [36]. The influence of the parameter $\beta = t_1/L$ on two RUCs has been therefore evaluated for optimal configurations characterized by $\alpha = 0.5$, $\theta = 0^\circ$, $\gamma = 0.5$, $t_1 = 2$, $c = 1.2$ (Fig.11). In general, the increase of β leads to higher values of the elastic non-dimensional moduli, the specific stiffness and the effective Poisson's ratio when other parameters remain constant; the resulting mechanical metamaterial is stiffer with a less pronounced auxetic behavior along the 2-direction. Fig.11a and Fig.11b show that the gradient of the normalized modulus E_2^*/E_c , the specific stiffness $(E_2^*/E_c)/(\rho/\rho_c)$ and the Poisson's ratio ν_{21}^* for two RUCs are essentially consistent, while the gradients related to G_{12}^*/E_c and $(G_{12}^*/E_c)/(\rho/\rho_c)$ for the re-entrant missing rib RUC are different from those pertaining the cross missing rib RUC shown in Fig. 11c; this indicates that the shape of core unit cell in the RUCs plays a significant part in the shear resistance.

4. Nonlinear analysis of FEM and experiments

Re-entrant missing rib RUCs with linear elastic in-plane optimized mechanical properties have been used to build models simulating nonlinear loading. Here, RUCs with $\alpha = 0.5$, $\theta = 0^\circ$, $\beta = 0.1$, $\gamma = 0.5$, $t_1 = 1$, $c = 1.2$ have been used to evaluate the nonlinear deformation along the 2-direction.

4.1 Experimental material and mechanical tests

Thermoplastic polyurethane elastomer (RS PRO TPU 98A) has been used as the solid material phase. Five dog-bone specimens have been fabricated by 3D printer (Witbox2), seen in Fig. 12a. The elastic mechanical properties of the TPU material have been determined under uniaxial tension loading using Shimadzu test machine (10KN load cell, 50mm/min) following the ASTM D638-14 standard. A video extensometer (Imetrum video gauge system) was used to track the strains along the loading direction and the transverse direction. The TPU material showed a Young's modulus $E = 90\text{MPa}$ and a Poisson's ratio $\nu = 0.38$ with 0.2% offset. A typical stress-strain curve under uniaxial tension test is shown in Fig. 12b. To verify the numerical approaches, structure samples consisting of 2×2 optimized RUCs were printed with dimensions of $160.16\text{mm} \times 79.12\text{mm} \times 5\text{mm}$. The in-plane uniaxial tension tests of the structure samples were performed by Shimadzu test machine with a capacity of 100N load cell and a constant displacement rate of 60 mm/min. A HD video camera was located to record the deformation process of structure.

4.2 Finite element analysis

FEA model of structure was developed using the same dimensions of the experimental sample. A 4-node bilinear reduced integration plane stress element

CPS4R has been applied with the accuracy verified by a mesh refinement study. The bottom edge of the structural model was fixed and a prescribed displacement $U_2 = 40$ was set on the top edge without in-plane rotation constraint. Self-contact between the inclined cell walls and the external ribs were defined, as well as the tangential behavior for the frictionless and the normal behavior of the hard contact, although attention here is focused on the mechanical behaviors of the cells before contact. Fig. 13 shows a good consistency between experimental and FEA deformation of the structure with 2×2 cells under different tensile strains. We have also investigated the stress-strain relationships between the experimental sample and FE models with both 2×2 cells and 4×4 cells (Fig.14). The experimental effective stress of the 2×2 cells structure is slightly larger than the FEM one for $\varepsilon < 25\%$ (Fig. 14a); this can be attributed to the experimental test errors as well as the samples being manufactured using FDM techniques with layerwise deposition and an additional degree of internal porosity that do not meet the assumption of a homogeneous and isotropic honeycomb materials [37]. The FEM stress-strain curve shows a softening response under tension, which is similar to centre-symmetric hexagonal auxetic configurations and opposite to the densification effect observed in hexagonal honeycombs under tensile loading [38]. Considering the influence of the boundaries on the mechanical properties of the structure, configurations of both 2×2 cells and 4×4 cells have been simulated (Fig.14b) and demonstrated a good agreement within large deformations; it is apparent that the nonlinear behavior was mainly caused by the structural configuration rather than the boundary effect. To study the strength of these auxetic metamaterials, a 4×4 cells configuration under large deformations ($\varepsilon = 35\%$) has been also numerically simulated taking the center RUC as a reference (Fig. 15a). Von Mises stresses related to one quarter of the centre RUC have been taken into account because of symmetry (Fig. 15b). It can be noticed that the four joints between the cross ligaments and the middle re-entrant cell are the components mainly bearing the strength of the configuration, which are likely to trigger the onset of failure. Additionally, the horizontal and vertical walls of the optimized re-entrant cell all exhibit bending deformations rather than deforming into the traditional hexagonal shape; this would maintain the auxetic behavior under large strains.

5. Conclusions

This work presents novel classes of mechanical metamaterials based on auxetic re-entrant and missing rib structures. A cross missing rib metamaterial with same dimensions is developed to benchmark the in-plane mechanical properties of the two new metamaterials. Parametric analysis of the in-plane mechanical properties of the two metamaterials are performed and compared by FE method first. The non-dimensional cell wall aspect ratio α and the inclined angle θ of the re-entrant

core unit cell, as well as the thickness ratio of internal and external ribs γ are the most important parameters that govern the in-plane mechanical behaviors and contribute to the two deformation mechanisms (bending in the re-entrant scale and chirality) synchronously. From FE simulations we have found that the re-entrant missing rib metamaterial with parameters $\alpha=2$, $\theta=60^\circ$, $\gamma=0.5$ possess the best tension along the 1-direction, with little difference compared to the behavior of the cross missing rib RUC with the same size. The re-entrant missing rib metamaterial with $\alpha=0.5$, $\theta=0^\circ$, $\gamma=0.5$ exhibits the optimal mechanical behavior in tension along the 2-direction, as well as in-plane shear; that configuration possesses a superior mechanical behavior to those of cross missing rib metamaterials with same dimensions, especially in terms of shear resistance. Finally, the nonlinear behaviors of configurations with both 2×2 and 4×4 optimized RUCs along the 2-direction have been also simulated, with the 2×2 configuration being benchmarked by uniaxial tensile tests. The results from the mechanical experiments and the systematic FE simulations show that the in-plane effective stiffness and Poisson's ratio of the hybrid new design can be widely enhanced and tuned by tailoring the independent geometric parameters under a large interval of strains. The new design provides potential for applications in biomechanics, electronics and aerospace, as the thermo-mechanical performance of these mechanical metamaterials can be arbitrarily manipulated by introducing – for example – bi-material structured ligaments to design advanced structures with zero thermal expansion for enduring critical environments with sharp changes of temperature.

CRedit authorship contribution statement

Wenjiao Zhang: Conceptualization, Methodology, Software, Writing - review & editing. **Fabrizio Scarpa:** Supervision, Writing - review & editing. **Rujie Sun:** Validation. **Shuyuan Zhao:** Methodology, Funding acquisition. **Jinwu Wang:** Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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FIGURES

Fig. 1. Design of the two novel metamaterials: (a) re-entrant missing rib RUC, (b) cross missing rib RUC, (c) one quarter of the re-entrant missing rib RUC with $\alpha = H/L = 0.5$, $\beta = t_1/L = 0.1$, $\gamma = t_1/t_2 = 0.5$, $\theta = 20^\circ$, $t_1 = 2$ and $c = 1.2$.

Fig. 2. Finite element models with BCs related to the re-entrant missing rib RUC: (a) boundary conditions under tensile loading, (b) boundary conditions for in-plane shear, (c) one quarter of meshing RUC.

Fig. 3. Variation of relative density for re-entrant missing rib RUC vs γ with: (a) different c , (b) different α and θ , when $\beta = 0.1$, $c = 1.2$.

Fig. 4. Comparison between ρ_c^r and ρ_c^c with variable γ under extremum values of α and θ , when $\beta = 0.1$, $c = 1.2$.

Fig. 5. Effect of the c parameter on the mechanical property along x direction for re-entrant and cross missing rib RUCs, when $\alpha = 0.5$, $\beta = 0.1$, $\theta = 20^\circ$, $t_1 = 2$: (a) E_1^*/E_c , (b) $(E_1^*/E_c)/(\rho/\rho_c)$, (c) v_{12}^* .

Fig. 6. Effect of α and θ on the mechanical properties along the x direction for the two RUCs when $\beta = 0.1$, $t_1 = 2$, $c = 1.2$: (a-b) E_1^*/E_c , $(E_1^*/E_c)/(\rho/\rho_c)$ of re-entrant missing rib RUC, (c-d) E_1^*/E_c , $(E_1^*/E_c)/(\rho/\rho_c)$ of cross missing rib RUC.

cross missing rib RUC.

Fig. 7. Effect of α and θ on the mechanical properties along the y direction for two RUCs, when $\beta = 0.1$, $t_1 = 2$, $c = 1.2$: (a-b) E_2^*/E_c , $(E_2^*/E_c)/(\rho/\rho_c)$ of re-entrant missing rib RUC, (c-d) E_2^*/E_c , $(E_2^*/E_c)/(\rho/\rho_c)$ of cross missing rib RUC.

Fig. 8. Effects of α and θ on the shear properties of the two RUCs when $\beta = 0.1$, $t_1 = 2$, $c = 1.2$: (a-b) G_{12}^*/E_c ,

$(G_{12}^*/E_c)/(\rho/\rho_c)$ of re-entrant missing rib RUC, (c-d) G_{12}^*/E_c , $(G_{12}^*/E_c)/(\rho/\rho_c)$ of cross missing rib RUC.

Fig. 9. Effect of α and θ on the Poisson's ratio of two RUCs, when $\beta = 0.1$, $t_1 = 2$, $c = 1.2$: (a) ν_{12}^* of re-entrant missing rib RUC, (b) ν_{12}^* of cross missing rib RUC, (c) ν_{21}^* of re-entrant missing rib RUC, (d) ν_{21}^* of cross missing rib RUC.

Fig. 10. Comparison of the in-plane mechanical behavior between the re-entrant and the cross missing rib RUCs with specific α and θ values when $\beta=0.1, t_1=2, c=1.2$: (a) E_1^*/E_c and $(E_1^*/E_c)/(\rho/\rho_c)$, (b) E_2^*/E_c and $(E_2^*/E_c)/(\rho/\rho_c)$, (c) G_{12}^*/E_c and $(G_{12}^*/E_c)/(\rho/\rho_c)$, (d) ν_{12}^* and ν_{21}^* .

Fig. 11. Effect of β on the mechanical properties along the 2 direction for two RUCs, when $\alpha=0.5, \theta=0^\circ, \gamma=0.5, t_1=2, c=1.2$: (a) E_2^*/E_c and $(E_2^*/E_c)/(\rho/\rho_c)$, (b) ν_{21}^* , (c) G_{12}^*/E_c and $(G_{12}^*/E_c)/(\rho/\rho_c)$

Fig. 12. Material property of thermoplastic polyurethane elastomer (TPU): (a) five dog-bone specimens, (b) stress-strain curve of uniaxial tension test.

Fig. 13. Verification of numerical nonlinear deformation utilizing experimental test for the optimized re-entrant missing rib structure at different levels of tensile strain: (a) $\varepsilon = 10\%$, (b) $\varepsilon = 20\%$, (c) $\varepsilon = 30\%$.

(a)

(b)

Fig. 14. Stress-strain curves of different models under uniaxial tension: (a) experimental test and FE simulation for structure with 2×2 cells, (b) FE simulation for structures with 2×2 cells and 4×4 cells.

Fig. 15. FE simulation of structure with 4×4 cells under $\varepsilon = 35\%$: (a) displacement field along y direction for the center RUC, (b) mises stress field for the center RUC

TABLES

Table 1. Key missing rib metamaterials architectures

Smith ^[22]	Gaspar ^[23]	Rana ^[24,25]	Scarpa ^[26]	Lim ^[29]	Grima ^[30]	Li ^[31]	Singamneni ^[32]

Table 2 Comparison between the optimized re-entrant and cross missing rib RUCs configurations when $\beta = 0.1$, $\gamma = 0.5$, $t_1 = 2$ and $c = 1.2$

	$\alpha = 2, \theta = 60^\circ$			$\alpha = 0.5, \theta = 0^\circ$				
	E_1^*/E_c	$(E_1^*/E_c)/(\rho/\rho_c)$	ν_{12}^*	E_2^*/E_c	$(E_2^*/E_c)/(\rho/\rho_c)$	ν_{21}^*	G_{12}^*/E_c	$(G_{12}^*/E_c)/(\rho/\rho_c)$
re-entrant	4.15×10^{-3}	1.76×10^{-2}	-9.17	5.05×10^{-3}	2.01×10^{-2}	-2.83	1.24×10^{-2}	4.912×10^{-2}
cross	4.65×10^{-3}	2.65×10^{-2}	-8.53	3.85×10^{-3}	1.82×10^{-2}	-2.33	2.48×10^{-3}	1.17×10^{-2}
%	12%↓	50%↓	7%↑	24%↑	9 %↑	18%↑	80%↑	76%↑