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Imperfection-Insensitive Continuous Tow-Sheared Cylinders

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ABSTRACT

The design of thin-walled cylinders in axial compression is limited by sensitivity to geometric imperfections. This paper focuses on reducing this imperfection sensitivity from a design perspective. By using variable-angle composites, the load paths are tailored to reduce the effective area over which imperfections can initiate buckling. Continuous Tow Shearing (CTS) is one such variable-angle manufacturing technique. It does not cause manufacturing defects associated with Automated Fibre Placement and entails a fibre angle-thickness coupling that results in a local thickness build-up—used as a design feature to embed stiffeners in the cylinder. Nonlinear finite element models with seeded imperfections are used to calculate knockdown factors (KDF). It was found that there is an inverse trend between embedded stiffener frequency and the KDF. The best performing CTS cylinder has a KDF 30% greater and a specific buckling load 4% greater than a QI cylinder, the optimal straight-fibre laminate for axial cylinder buckling. It was also found that, the smaller the effect of geometric imperfections on the pre-buckling strain field, the greater the KDF of the cylinder. This novel finding shows that a computationally inexpensive nonlinear analysis can provide a first-order approximation of the imperfection sensitivity of a cylinder in axial compression.

1. Introduction

The largest percentage (60–70%) of the dry mass of heavy launch vehicles is made up by the fuel and oxidizer tanks [1]. Using composite materials instead of current generation Li-Al fuel tanks may save up to 30% in mass and 25% in recurring manufacturing costs [2]. Recent research programs (Composite Cryotank Technology and Demonstration [3] and Shell Buckling Knockdown Factor [4]) are addressing various issues pertaining to the use of composite materials in these structures. Yet the aforementioned projects all use blade-stiffened shells or foam/honeycomb sandwich structures. Monocoque shells have lagged behind these designs mainly due to the highly sensitive nature of unstiffened shells in compression buckling—the predominant design load case of launch vehicles.

Imperfection sensitivity in shell buckling is well documented. Typical test results show both large discrepancies compared to the classical prediction based on a linear eigenvalue problem, and large scatter among cylinders of various sizes, as shown in Figure 1.

The well-known NASA SP-8007 space vehicle design criteria [6] outline a conservative empirical formula that captures this behaviour in a “lower bound design philosophy”. This conservative guideline was based on tests carried out between the 1930s to 1960s on isotropic cylinders. According to the guideline, the critical buckling stress, σ_{cr} of an isotropic cylinder with Young's Modulus E , thickness t , and radius r , is given by the formula

$$\sigma_{cr} = \frac{E\gamma}{\sqrt{3(1-\nu^2)}} \frac{t}{r}, \quad (1)$$

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Figure 1: Variation in knockdown factor for a range of radius-to-thickness ratios for isotropic cylindrical shells, adapted from [5]. P is the experimental value of critical buckling load and P_{cl} is the classical critical buckling load calculated by linear stability theory.

where γ is the knockdown factor (KDF), a ratio between experimental buckling load and theoretical buckling load, and is given by

$$\gamma = 1 - 0.901(1 - e^{-\phi}), \quad (2)$$

and the exponent factor, ϕ , is

$$\phi = \frac{1}{16} \sqrt{\frac{r}{t}} \quad \text{for} \quad \frac{r}{t} < 1500. \quad (3)$$

Much research into the cause of the discrepancy between theory and experiment has followed, with many avenues pursued: the post-buckling behaviour of a perfect shell [7]; the post-buckling behaviour of an imperfect shell [8]; and the effect of loading and boundary conditions [9]. These studies showed that the buckling of cylinders is sensitive to many of these issues, but struggled to completely explain the large discrepancy between theory and experiments. It was

Koiter that established the relationship between geometrical imperfections and low buckling loads in his PhD thesis [10, 11]. The application of Koiter's geometric imperfection theory accurately explained the discrepancy between theory and experiment. When his thesis was translated from Dutch to English it confirmed the research of Almroth *et al.* [12] where initial imperfections causing a buckling collapse were recorded in slow motion.

Despite the effect of imperfections on cylinders now being well understood, the design of cylindrical shells still hinges largely on the guidelines stipulated by NASA. Due to the limitations of these guidelines (from outdated test data and data based on isotropic cylinders), application to modern composite launch structures has been shown to be too conservative [13]. Much research has gone into developing new design guidelines with many numerical techniques such as the Reduced Stiffness Method [14], Localised Reduced Stiffness Method [15], Single Boundary Perturbation Approach, Single Perturbation Displacement Approach and Single Perturbation Load Approach [16], and Worse Multiple Perturbation Load Approach [17], being developed. These techniques aim to mimic the behaviour of a cylinder with an imperfection—thus determining its sensitivity to imperfections—to derive a revised and less conservative knockdown factor.

Despite the effort to develop new and less conservative knockdown factors, only a limited amount of research has focused on removing the sensitivity to imperfections of monocoque cylinders. Analytical work by Mang *et al.* [18] and Schranz *et al.* [19] has shown that the transition from an imperfection-sensitive structure to an imperfection-insensitive one is possible through the use of a stiffening spring attached to the structure, and has also elucidated that the post-buckling path of such a structure can be influenced by the “load coefficients associated with the secondary (post-buckling) path” [18]. From a practical viewpoint, recent advancements in fibre placement manufacturing techniques offer to eliminate the need to rely heavily on empirical or computational knockdown factor design guidelines by creating a cylinder that is inherently imperfection insensitive. These manufacturing techniques create composite parts that are known as Variable Angle Tow (VAT) composites, which steer fibre-reinforced tapes in curvilinear paths rather than straight trajectories. VAT composites enable tailoring of load paths, which can reduce the likelihood of instabilities, and are well documented to increase the mechanical performance of a composite plate without a weight penalty [20, 21, 22, 23]. VAT composites have also been shown to improve the buckling performance of elliptical cylinders [24]. The post-buckling behaviour of cylindrical panels has also been improved by White and Weaver [25] by tailoring the membrane stress distribution during pre-buckling, which leads to localisations of the buckling modes. These two features result in a stable, plate-like post-buckling response of the tow-steered cylindrical panel. White *et al.* [26] also studied the post-buckling response of a VAT cylinder. One of the VAT cylinders in-

vestigated showed a consistent, small (relative to the circumference of the cylinder) dimple that had a limited impact on the post-buckling regime. The limited influence on the post-buckling regime was from the “direct membrane load path through the structure” [26] when compared to a VAT cylinder that had a relatively large dimple. The imperfection insensitivity of cylinders does not necessarily have to come from tailoring material systems within the cylinder, as Ning and Pellegrino [27, 28] have shown by investigating a wavy cylindrical structure. The authors found that KDFs could reach above 1 and the overall response was comparable to that of a stiffened cylindrical structure. Recent studies by Yadav and Gerasimidis [29, 30] on similarly wavy cylinders under axial compression or bending have established similar conclusions.

The VAT concept has also been investigated in relation to cutouts, often necessary for inspection or safety reasons. Passos *et al.* [31] have determined a 61% increase (when compared to a QI laminate) in critical axial buckling load when optimising a VAT layup for a large oblong cutout within a stiffened composite panel. The influence of the VAT concept is also not limited to static buckling, as Labans and Bisagni [32] have revealed good correlation between experimental and numerical results pertaining to the free-vibration of variable-stiffness cylinders. These recent investigations show that the implementation of VAT composites as solutions to complex structural problems is stimulating promising research.

The most established VAT manufacturing method is currently Automated Fibre Placement (AFP). AFP has been used since its development in the 1980s [33], however, process-induced defects restricted the uptake of this manufacturing method. These process-induced defects are due to the in-plane bending of the tows as the AFP head moves along the curved tow path. The distance of each fibre from the reference path is different and this can cause fibre buckling on the inside of the tow and fibre straightening on the outside of the tow [34]. One downside of this process is that the tessellation of laid tows is not possible, leading to either tow gaps or tow overlaps. Additionally, the in-plane bending of tows inherently limits the steering radius prior to any process-induced defects occurring. Fibre wrinkling, tow gaps and discontinuities are common defects that still challenge the use of curvilinear fibre paths created with AFP. However, the work of Wu *et al.* [35, 36] has shown that using AFP with large steering radii to create VAT composite cylinders reduces the sensitivity to geometric imperfections by a large margin—producing linear bifurcation analysis knockdown factors with an average of 0.99 (comparing linear finite element analysis and experimental results).

To combat the aforementioned process-induced defects associated with AFP, Kim *et al.* [37] developed the Continuous Tow Shearing (CTS) process. A full description of the mechanisms that create the curvilinear paths using this process can be found in References [37] and [38]. A key feature of this manufacturing technique is that the common process-induced defects are eliminated by shearing the tow instead

Figure 2: CTS tow being sheared through a shearing angle θ .

of bending it. A by-product of this action is that there is a thickness build-up perpendicular to the fibre path, as shown in Figure 2.

The thickness build-up is given by

$$t = \frac{t_0}{\cos(\theta)}, \quad (4)$$

where t is the thickness of the lamina once sheared, t_0 is the nominal thickness of the unsheared lamina and θ is the shearing angle. Note that this secant relationship implies a significant thickness build-up of 3x if tows are sheared through 70°. The thickness build-up adds another dimensionality to the design of cylinders, as embedded stiffeners (local areas of increased thickness) can be created by shearing the tows, and opens up the ability to embed hoops, stringers, orthogrids and isogrids within the structure during manufacture. The embedding of stiffeners was previously not possible as AFP does not have the thickness coupling characteristic of CTS and the steering radius limits the ability to have multiple regions with a thickness build-up. One caveat to the thickness build-up is that the shearing angle cannot currently be greater than 70°, and clearly a shearing angle of 90° would create a tow of infinite thickness. Therefore, the design space is limited to a maximum shearing angle of 70°.

It is helpful to briefly describe the nomenclature used within the paper, which is adapted from Gurdal and Olmedo [20]. Their nomenclature was sufficient to describe the curvilinear fibre paths for a flat composite. For CTS cylinders, the coupling between fibre variation and thickness means that an enhanced system for the description of fibre paths is needed. The adaptation defines the number of times a lamina is sheared, n , hereafter called periods and the order of polynomial that defines the fibre trajectory, j . The rest of the nomenclature is consistent with that of Reference [20], so that the fibre angle path is described as

$$\phi \langle T_0 | T_1 \rangle_j^n, \quad (5)$$

where ϕ is the clockwise angle from the global x -axis that defines the shearing direction. The ϕ direction defines the relative location of ‘periods’, shown by the transition between $0 \rightarrow 30 \rightarrow 0$ in Figure 3. The dashed line in Figure 3 defines

Figure 3: Diagram of a ‘rolled out’ cylinder showing how the variation in fibre angle is defined for a single layer of $90\langle 0|30 \rangle_1^2$ with global axis system. Thickness variation is exaggerated for clarity.

the embedded stiffener region. T_0 is the counter-clockwise angle from ϕ representing the starting angle of the variation of fibre path; T_1 is the counter-clockwise angle from T_0 representing the ending angle at the middle of a period; n is the periodicity of the CTS fibre path; and j is the order of polynomial that defines the fibre angle trajectory. The change in fibre angle is graphically shown in Figure 3.

Currently, the CTS manufacturing method cannot shear angled plies. For a starting angle of $T_0 = 30^\circ$, a ‘lead-up’ from 0° to 30° is required. Figure 4 shows this feature graphically. The present paper considers both the current capability of the CTS method (where $T_0 \neq 0^\circ$ results in a thickness build up) and the idealistic manufacturing case (where $T_0 \neq 0^\circ$ is considered an unsheared ply). The idealistic case is used as a challenge to the manufacturing community to showcase the increase in design landscape and superior buckling performance when off-axis plies are not inherently penalised and are used in the CTS manufacturing method. In addition to the design benefits, by shearing off-axis plies the amount of waste material and time to manufacture would decrease.

Inherent in the angle-thickness coupling is the increase in average wall thickness, leading to a mass penalty. For the idealistic manufacturing method, this average thickness increase peaks at 42%, if all plies are sheared through 70° ($|T_1 - T_0| = 70^\circ$). However, for the current manufacturing method, the average increase in thickness can be up to 192%, if all plies are sheared 70° ($T_0 = T_1 = 70^\circ$). A comparison of average thicknesses (calculated from the thickness integral in Equation 6) as the manufacturing method changes is found in Table 1. For the idealistic method, the absolute difference in angle ($|T_0 - T_1|$) is the sheared value and therefore the parameter that contributes to the thickness increase. For the current manufacturing method, it is both shearing angles, T_0 and T_1 , that contribute to the thickness increase. For a $\langle 30|50 \rangle$ ply, current manufacturing constraints cause a 30% increase in average thickness when comparing the same angles across an idealistically manufactured ply. The increase

Figure 4: Comparison between idealistic and current manufacturing methods where: (a) is a $90\langle 0|50 \rangle$ laminate for both methods, (b) is a $90\langle 30|50 \rangle$ laminate for the current manufacturing method and (c) is shearing $90\langle 30|50 \rangle$ for the idealistic manufacturing method. For all lamina sheared, the enclosed area is the final ply size. Note the ‘preshearing’ necessary in Figure 4b to start at 30° where the grey area would be scrap material.

Table 1

Thickness, t_α , and angle, T_α , at point α in Figure 4. $\bar{t}$ is the average thickness as calculated from the thickness integral of Equation 6.

Point	4a		4b		4c	
	t [mm]	T [°]	t [mm]	T [°]	t [mm]	T [°]
1	1.00	0	1.00	0	1.00	30
2	1.56	50	1.15	30	1.06	50
3	1.00	0	1.56	50	1.00	30
4	-	-	1.15	30	-	-
$\bar{t}$ [mm]	1.16	-	1.32	-	1.02	-

in average thickness translates to a direct mass increase of 30%, hindering the mass-specific mechanical performance of the CTS cylinders designed with the current manufacturing method.

In both cases, the tow shearing process creates a non-uniform stiffness profile that results in a non-uniform pre-buckling strain field when the cylinder is under uniform compression. The working hypothesis of this paper is that the non-uniformity of the initial pre-buckling strain field reduces the imperfection sensitivity of the structure. It has been shown that the onset of unstable cylinder buckling due to localisations is governed by “spatial chaos” [39]. That is, the onset of unstable buckling due to localisations is highly dependent on initial conditions (*i.e.* imperfections). If the non-uniform pre-buckling strain field reduces the symmetry of the cylinder and chaos of the post-buckling paths, the imperfection sensitivity of the cylinder is reduced. This hypothesis is also inspired by Johann Arbocz’s PhD thesis [40], where the circumferential strain field of a buckling shell was experimentally measured. Arbocz found that as the cylinder buckled, the strain field associated with a small imperfection initiated local buckling. To this end, this paper also investigates the possibility of analysing pre-buckling strain fields to inform the sensitivity of a cylinder to imperfections.

This paper explores the design space of a CTS cylinder, derived from an interstage structure for the Ariane 6 [41]. Section 2 discusses the numerical model used, the designs

explored and the types of analyses employed. Section 3 collates the results of the three different analyses used, discusses them and aims to extract and understand future work in designing cylinders with the CTS manufacturing technique. Section 4 concludes the paper and the findings of the research.

2. Model

Abaqus 2018 [42] was used to model the CTS cylinders. The material used for the cylinder is general T-800/MTM49-3 prepreg, taken from the data sheet [43] listed in Table 2. The mesh size was determined after a mesh convergence study. The reference surface is the surface defined by the radius of the cylinder. The stacking sequence of each cylinder was directed outwards radially with respect to the reference surface. The laminate offset was defined to create a stacking sequence where the bottom layer was at the radius of the cylinder. Reduced integration four-noded elements (S4R) were used with enhanced hour-glassing control.

The design space of CTS cylinders is large, as ϕ , T_0 , T_1 , n and j can be varied in each lamina, and these lamina combined in multiple ways. Additionally, t is varied as it is a function of T_0 , T_1 and n . The chosen designs fit conventional laminate criteria: balanced and symmetric. Additionally, as the mass of the CTS cylinders is invariant to the number of stringers or hoops created due to the shearing effect, see Appendix A for a proof of this observation, an arbitrary number of stringers and/or hoops are used for any given design. For the present paper, j is kept to be 1 for all laminates, implying a linear fibre angle variation. The designs chosen are listed in Table 3. For all analyses, a custom Python [45] script was written to generate input files that were run in Abaqus. The script created an orphan mesh based on a predefined geometry and element requirements. Distributions were defined for shell thickness, shell orientation and fibre angle. The method was effective for customising and editing the necessary input files. One nuance of designing CTS cylinders in a finite element analysis software is that the model must conserve various symmetries to ensure a symmetric buckling mode. For example, for a circumferential variation in

Table 2

Material and geometry properties used within Abaqus, taken from [43] and [44].

E_1 [GPa]	E_2 [GPa]	G_{12} [GPa]	ν_{12} -	ρ [g/cm ³]	t_{ply} [mm]	t_{lam} [mm]	r [mm]	L [mm]	Ele size [mm]
122	7.32	4.9	0.31	1.54	0.13	1.04	400	1200	10.6

Figure 5: Schematics of boundary conditions and cross-sectional area/mass increase where: **(a)** is the wagon-wheel type boundary conditions and load application, **(b)** is the cross-sectional view of a $90(0|30)^1$ lamina. Clearly the cross-section has increased from a nominal thickness to the sheared thickness, thus causing an increase in mass. Thickness increase is exaggerated for explanation and **(c)** shows the normalised mass increase for a variation in maximum shearing angle. for the idealistic manufacturing case.

Table 3

Designs explored with layup and CTS variation detailed. H = hoop stiffener, St = stringer stiffener.

Design	Layup	Stiffener
1	$[\pm 0(T_0 T_1)_1^8]_{2s}$	H
2	$[\pm 90(T_0 T_1)_1^{12}]_{2s}$	St
3	$[\pm 45, \pm 0(T_0 T_1)_1^8]_s$	H
4	$[\pm 45, \pm 90(T_0 T_1)_1^{12}]_s$	St
5	$[\pm 0(T_0 T_1)_1^8, 0, 90]_s$	H
6	$[\pm 90(T_0 T_1)_1^{12}, 0, 90]_s$	St
7	$[\pm 0(T_0 T_1)_1^8, \pm 90(T_0 T_1)_1^{12}]_s$	H / St
8	$[\pm 90(T_0 T_1)_1^{12}, \pm 0(T_0 T_1)_1^8]_s$	St / H

fibre angle, the model must be symmetric about the global $x - y$ and $x - z$ planes with respect to the number of elements. The enforced symmetry ensures that the circumferential variation is the same for each quadrant, and one angle does not appear more than another. Figure 3 shows this circumferential variation and visualises that if an element with a 0° fibre angle was created at $y' = 0$ and $y' = C$ where C is the circumference, then the cylinder would not be axisymmetric.

To calculate the buckling load of the cylinders, a Linear Buckling Analysis (LBA) is performed. For the LBA, a 1 mm displacement was applied in the positive x -direction, to act as an end-shortening. The reaction force was measured from the top reference node to calculate the critical buckling load. A 'wagon wheel'-type boundary condition was used

for this analysis. A multi-point constraint beam element was connected from the circumferential nodes along the top and bottom of the cylinder to a central reference point on the top and bottom of the cylinder, respectively. The top and bottom reference points were then constrained. The top reference point was fully constrained, whereas the bottom reference point was constrained in 5 degrees of freedom, leaving the axial (x -direction) free.

To calculate the buckling load when a geometric imperfection is present a Nonlinear Buckling Analysis (NBA) was executed. Abaqus Static General with nonlinear geometry was used. The loading and boundary conditions were identical to the LBA. To investigate the imperfection (in)sensitivity, a pseudo-random variation in radial node coordinate was applied to the entire cylinder where the magnitude of the imperfection varied from 0 to the magnitude of the wall-thickness. A random imperfection was generated and used for all cylinders to allow for a fair comparison. The KDF is calculated as the ratio between the critical buckling load from the NBA and LBA.

2.1. Mass of CTS cylinders

The shearing process of CTS causes a thickness build-up that can cause an overall mass increase, due to the lines of constant thickness that cause a cross-sectional area increase, as shown in Figure 5b. The mass, m , of a CTS cylinder with radius r , length L and nominal lamina thickness t with either axial ($\phi = 0^\circ$) or circumferential variation ($\phi = 90^\circ$) is

$$m = 2\pi r L \rho \sum_{k=1}^N \frac{t_0^k}{\sqrt{\theta_{max}^k}} \int_0^{\sqrt{\theta_{max}^k}} \sec(u^j) du, \quad (6)$$

where k is the superscript for each laminae, N is the total number of lamina, ρ is the density of the lamina and θ_{max} is the maximum shearing angle ($|T_1 - T_0|$) of each lamina (a straight-layer would have zero shearing) in radians. The full derivation of the CTS cylinder mass is found in Appendix A. The mass formula in Equation 6 is for the idealistic manufacturing method. It is imperative to note that the periodicity of the layup, n —that is, how many hoops or stringers are present—is not in this equation. The independence of periodicity on mass contrasts with typical use of stiffening elements, where more stiffeners leads to an increase in mass. The relationship between mass and shearing angle defined by Equation 6 is shown graphically in Figure 5c.

3. Results and discussion

3.1. Linear Buckling Analysis

A linear buckling analysis (LBA) was carried out to explore the interaction between shearing angles and buckling performance of CTS cylinders. Two manufacturing possibilities were considered: current (no shearing of off-axis plies) and idealistic (shearing of off-axis plies). These two manufacturing capabilities are compared to investigate the benefits of opening up the design space to include shearing angled plies. For the current manufacturing capability, T_0 was varied from $0-70^\circ$ for each design in increments of 10° , whereas T_1 angles were varied from $0-70^\circ$ in increments of 5° . For the idealistic manufacturing, T_0 was varied from $0-90^\circ$ for each design in increments of 10° , and T_1 angles were varied from $T_0 - 70 - -T_0 + 70^\circ$ in increments of 5° , representing the theoretical maximum shearing angle possible for the CTS process for a given T_0 . The axial stiffness and buckling loads of all eight designs shown in Table 3 were recorded. The axial stiffness was calculated by recording the reaction force, F , of each cylinder under a 1 mm displacement and then computing an equivalent section stiffness $\tilde{E}^t$ using

$$\tilde{E}^t = \frac{FL}{2\pi Rm} \quad (7)$$

where R is the radius of the cylinder and m is the mass of the cylinder. Both the axial stiffness and buckling load of each cylinder are normalised by the mass of the cylinder. The mass-specific buckling load, $\tilde{P}$, is then normalised by the average thickness of the cylinder using

$$\tilde{P}^t = \frac{\tilde{P}}{t^*} \quad (8)$$

where t^* is the thickness integral of Equation 6. The thickness increase is key for the current manufacturing capability as laminates with high degrees of shearing inevitably have higher buckling loads due to the increase in thickness. Note that the buckling load of an isotropic cylinder is proportional to t^2 , and therefore the mass-specific buckling is proportional to t . The mass-specific results were then normalised by a QI cylinder ($[\pm 45, 0, 90]_s$ where 0° is orientated with the global x -axis). A QI cylinder was used as the benchmark as it has been shown to be the best straight-fibre cylinder

[46] in terms of linearised buckling load. The results for the idealistic manufacturing constraints are shown in Figures 6a and 9a–15a. The results for the current manufacturing constraints are shown in Figures 6b and 9b–15b. Each point on a design plot refers to a specific layup with a T_0 angle given in the legend. These lines have been connected linearly. The minimum and maximum T_1 angles are designated on the plots as squares and diamonds, respectively, to allow the reader to know the specific layup by counting forwards or backwards along a given T_0 line.

3.1.1. Design 1

Figures 6a and 6b show the thickness-independent, normalised results for design 1 under idealistic and current manufacturing ability, respectively. Figure 6a shows a clear trend between the buckling load and axial stiffness of the cylinders for straight-fibre angles: as the straight-fibre angle varies from $\pm 0^\circ$ to $\pm 45^\circ$ (right to left), the buckling load peaks at $\pm 20^\circ$, whilst the stiffness decreases consistently. Then, at $\pm 45^\circ$ the buckling load trends are symmetric, with the buckling load peaking at $\pm 70^\circ$ and reaching a minimum at $\pm 90^\circ$. Hirano showed that the buckling load variation of a $[\pm \theta_2]_s$ laminate is symmetric about the $\theta = \pm 45^\circ$ laminate and that an all 0° laminate and an all $\pm 90^\circ$ laminate have the same buckling load [47]. Both these findings are seen in the present study and are shown clearly in Figure 6a.

One interesting feature of Figure 6a are the peaks for the straight-fibre $\pm 20^\circ$ and a $\pm 70^\circ$ laminates. In the study into angle-ply laminates carried out by Hirano [47], the buckling load either peaked at an all 0° and 90° laminate or a $\pm 45^\circ$ laminate (see Figure 7). The difference in peak values resulted from assuming an axisymmetric or axially unsymmetric buckling mode, which cause peaking at $\pm 0^\circ$ and $\pm 90^\circ$ laminates or at a $\pm 45^\circ$ laminate, respectively. Figure 7 shows the variation in buckling load for these two assumptions. It is hypothesised that the buckling of the current cylinder initially (starting from an all $\pm 90^\circ$ laminate) runs on the unsymmetric path, but that at a critical value, of around $\pm 70^\circ$, the overall buckling mode changes to follow the symmetric path. The traversing is followed in reverse, symmetric about $\pm 45^\circ$. Two other reasons for the peaking behaviour were considered: the anisotropy of the laminates and the L/D ratio. The anisotropy of the laminates comes from the D_{16} and D_{26} coupling of the 8 plies. Hirano's study considered an infinite number of plies to remove anisotropy. However, when 48 plies were used in the present FE model (keeping the overall thickness the same) to reduce anisotropy considerably, negligible difference was observed. The L/D ratio, known to influence the buckling of composite cylinders [48], was also studied but the analyses suggest that reducing the L/D from the current 1.5 to 1 insignificantly affects the results. The transition from unsymmetric buckling mode to symmetric buckling mode is related to the lowest-energy option for the cylinder undergoing compression buckling. The FE solver does not impose a symmetry on the buckling mode and therefore favours a buckling mode that requires less energy.

Figure 6: Variation in buckling load and axial stiffness for CTS design 1 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

Figure 7: A schematic of Hirano's results [47], showing the proposed path that the cylinder buckling has traversed along the highlighted red line. Theta is the angle in the cross-ply laminate $[\pm\theta_2]_s$. U and S stand for unsymmetric and symmetric buckling mode, respectively.

Considering the tow-sheared designs in Figure 6a, it is clear that shearing decreases $\tilde{P}^t$ and $\tilde{E}^t$ compared to straight-fibre designs, due to the mass increase associated with shearing the laminate. The tow-sheared laminates that show the largest decrease in $\tilde{P}^t$ and $\tilde{E}^t$ are those that shear the maximum by 70° . These cylinders feature 8 hoops with a hoop thickness of nearly three times that of the nominal thickness. The thickness and respective fibre variation of such a cylinder

is shown in Figure 8.

Comparing the idealistic manufacturing case (Figure 6a) to the current manufacturing case shown in Figure 6b, the initial shearing penalty is clear. There are fewer sheared laminates in Figure 6b. The smaller design landscape is due to the T_0 angle being sheared instead of an off-axis ply. In the idealistic case, a $T_0 = 50^\circ$ is unsheared, and therefore can shear $+70^\circ$ or -70° . However, in the current manufacturing case, a $T_0 = 50^\circ$ ply is already sheared by 50° , and therefore cannot shear beyond 70° . This consideration limits the number of laminates plotted. It also has the knock-on effect of reducing the design space as a whole, limiting the number of viable laminates that could have high buckling loads.

Compared to the QI laminate—the optimal straight-fibre laminate configuration [46]—the CTS laminates rarely approach more than 60% of the QI buckling load on a mass and thickness normalised basis¹. The buckling load alone does not tell the full story however, as the main aim of CTS designs in this study is to increase the imperfection insensitivity of the cylinders. Thus, the knockdown factor (a direct function of the imperfection insensitivity of a design) must also be considered when deeming a design ‘good’ or not. The trade-off can be summarised in the following inequality:

$$\tilde{P}_{CTS}^t \times \text{KDF}_{CTS} > \tilde{P}_{QI}^t \times \text{KDF}_{QI}. \quad (9)$$

Thus, to ascertain whether a CTS design is ‘better’ than a QI design (*i.e.* has a greater load-carrying capacity before buckling), it does not necessarily transpire that the specific buckling load has to be greater than a QI design; rather, the product of the specific buckling load and respective knock-

¹If mass or thickness is not accounted for in the normalisation process, many laminates outperform the QI cylinder.

Figure 8: Thickness and fibre angle variation for design 1, $0\langle 20| - 50 \rangle^8$, where (a) shows the thickness variation of each element. See the legend in the figure for values of thickness in mm. (b) shows the fibre variation that can be compared with the left-hand side figure to show that where the fibres are most sheared corresponds to the thickest regions.

down factor needs to be greater. The inequality is considered in Section 3.2 when selecting designs to take forward to the NBA, with the caveat that the specific stiffness must be greater than 0.8 so as not to compromise the axial rigidity of the cylinder.

3.1.2. Design 2

Figures 9a and 9b show the thickness-independent, normalised results for design 2 under idealistic and current manufacturing capability, respectively. Similar to design 1, the results for the idealistic case in Figure 9a show a peak of buckling load around $\pm 20^\circ$ and $\pm 70^\circ$ for the straight-fibre designs, with the buckling load symmetric about $\pm 45^\circ$. The straight-fibre designs are the ‘reverse’ of that seen in design 1, with $T_0 = T_1 = 90^\circ$ in design 2 being the same as $T_0 = T_1 = 0^\circ$ for design 1.

The y-axis range for the results in Figure 9a is greater than that of Figure 6a. This result can be explained by to the embedded axial stringers in this design, known to increase the buckling load of cylinders by increasing the cross-sectional area aligned with the load. Interestingly, the peak in buckling load is not in the $T_0 = 20^\circ$ laminate, where the most-sheared fibres would be orientated along the cylinder x-axis, but in the $T_0 = 10^\circ$ laminate, where the heavily sheared laminate ($[90\langle 10| - 60 \rangle^{12}]_s$) has the highest $\bar{P}^t$. This indicates that the thickness build up perpendicular to the shearing direction and the fibre-angle orientation are acting in tandem to increase the buckling load. The disadvantage of this

is that the axial stiffness is low as the fibres are orientated circumferentially.

When considering the current manufacturing capability with design 2 in Figure 9b, the spread of results has decreased. Similar to design 1, the number of CTS laminates has decreased. The x-axis range has decreased to not be greater than 30% of a QI cylinder. This is because the majority of laminates have fibre angles orientated circumferentially, and any increase in axial stiffness due to the average thickness of the laminate has been accounted for by normalising $\bar{E}^t$ by thickness.

3.1.3. Design 3

Figures 10a and 10b show the thickness-independent, normalised results for design 3 under idealistic and current manufacturing ability, respectively. This design has a $[\pm 45, 0 \pm \langle T_0|T_1 \rangle]_s$ layout. Figure 10a shows that no tow-sheared designs show improvements over a QI laminate in buckling load. The laminates with the highest buckling load are typically the cylinders with the least shearing. Whilst the embedded hoops resists the bulging mode of buckling, the average thickness and mass increase penalise these cylinders when normalised. It is clear that the addition of a $\pm 45^\circ$ layer has increased the average buckling load and decreased the average axial stiffness. Between the idealistic case and current manufacturing case, the change in x-axis range is minor, reflecting the fact that the orientation of fibres is axial. The overall trend in Figure 10b is that as the shearing an-

Figure 9: Variation in buckling load and axial stiffness for CTS design 2 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

Figure 10: Variation in buckling load and axial stiffness for CTS design 3 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

gle changes from $T_0 = 0^\circ$ (axially orientated), to circumferentially orientated, the buckling load and axial stiffness decrease. The figure also shows that the laminates exhibit a ‘collecting’ behaviour where the increase in average thickness and mass cause many laminates to show similar properties despite having slightly different fibre angle orientations.

A new feature of Figure 10a is that the behaviour of the tow-sheared layups $\langle 0|T_1 \rangle$ and $\langle 90|T_1 \rangle$ are no longer identical. Whereas in Figures 6a and 9a, these lines traversed the

same path (as the ABD matrix for a $[\pm\langle 0|70 \rangle_2]_s$ layup is the same as $[\pm\langle 0|-70 \rangle_2]_s$), design 3 shows that the direction of the T_1 angle changes the buckling load of the laminate. The stiffness of these laminates does not change however, as the ordering of layers in symmetric, balanced laminates does not influence the in-plane behaviour (A-matrix). The difference in buckling load can be attributed to the differing levels of anisotropy in the $[\pm 45, \pm\langle 0|-70 \rangle]_s$ laminate when compared to the $[\pm 45, \pm\langle 0|-70 \rangle]_s$ laminate. Increased lev-

Figure 11: Variation in buckling load and axial stiffness for CTS design 4 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

els of anisotropy are known to decrease buckling loads [49]. The average D_{16} and D_{26} values for a $[\pm 45, \pm \langle 0 | 70 \rangle]_s$ layup are 70% and 58% greater than a $[\pm 45, \pm \langle 0 | -70 \rangle]_s$ layup, respectively.

3.1.4. Design 4

Figures 11a and 11b show the thickness-independent, normalised results for design 4 under idealistic and current manufacturing ability, respectively. Here, tow shearing is aligned with the circumferential direction, creating embedded stringers due to the thickness build-up. Similar to design 3, the $\pm 45^\circ$ plies on the outside and inside of the laminate are clearly hindering the maximum buckling load and have a lower axial stiffness. Design 4 has the highest buckling load observed for the idealistic manufacturing case, with some laminates approach 85% of a QI cylinder buckling load. However, similar to previous circumferentially orientated designs (design 2), the axial stiffness is low.

In Figure 11b, the performance trend of the CTS layups with the current manufacturing method is similar to previous designs—a less populated design space. The axial stiffness range is greater than design 2 for two reasons. Firstly, as half the number of plies are being sheared, the thickness and mass penalty is decreased. Secondly, the $\pm 45^\circ$ plies contribute to slightly increase the axial stiffness of the cylinder.

3.1.5. Design 5

Figures 12a and 12b show the thickness-independent, normalised results for design 5 under idealistic and current manufacturing ability, respectively. In contrast to previous graphs, the QI laminate that is used as a reference can be plotted on the graphs and compared directly with the other idealistic laminates as design 5 has a $[\pm \Phi, 0, 90]_s$ design.

The QI laminate is shown by the red star on the plots and the star falls neatly between the $[\pm 40, 0, 90]_s$ and $[\pm 50, 0, 90]_s$ laminates. Note, the star does not lie directly on the line between these two laminates as the straight-fibre line is extrapolated linearly.

Considering Figure 12a, many of the tow-sheared layups of design 5 have a stiffness and buckling load close to or worse than the straight-fibre layups when the fibres are aligned with the cylinder x -axis. Some layups with a moderate amount of shearing (30 – 50°) have a normalised buckling load greater than the straight-fibre layups and QI reference.

Figure 12a reveals that the results of this design are the most promising so far in terms of buckling. This laminate, $[0 \langle 50 | 55 \rangle^8, 0, 90]_s$, has a buckling load 2% higher than a QI cylinder and 95% of the axial stiffness. This is a novel and fascinating result: CTS cylinders can outperform a QI cylinder in buckling on a mass-specific and thickness-normalised basis. Granted, these layups perform marginally worse with respect to axial stiffness, but if the imperfection sensitivity of these layups is less than that of a QI layup, then the imperfection insensitivity benefit outweighs the marginal stiffness decrease.

The performance of the CTS cylinders with the current manufacturing ability is less promising. Clearly, the $[0, 90]_s$ set of plies are contributing to the overall performance of the cylinder and reducing the effect of the thickness and mass increase. ‘Peaking’ type behaviour is observed in many of the layups shown in Figure 12b. The apex of the peak appears to trend towards laminates where $T_0 = T_1$. This indicates that the straight-fibre designs are favourable in terms of $\tilde{P}^t$. These laminates have a homogeneous prebuckling strain field and do not have the symmetry-breaking effect of

Figure 12: Variation in buckling load and axial stiffness for CTS design 5 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

Figure 13: Variation in buckling load and axial stiffness for CTS design 6 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

the CTS embedded stiffeners, likely increasing imperfection sensitivity compared to a steered design with embedded stiffeners.

3.1.6. Design 6

Figures 13a and 13b show the thickness-independent, normalised results for design 6 under idealistic and current manufacturing ability, respectively. The QI layup that is used to normalise the results is shown as a red star. The

general shape of the idealistic graph is highly similar to that of design 5, with the arch-type behaviour of the straight-fibre designs peaking with the QI laminate. Similar trends to design 5 are shown in the idealistic case where many laminates have similar mechanical behaviour to the QI laminate, where an increase in KDF would compensate for the lower buckling load or axial stiffness. In design 6, the peak value of $\tilde{P}^t$ is again 2% higher than the QI cylinder with 95% of $\tilde{E}^t$. This laminate has the layup $[90\langle 40|25 \rangle^{12}, 0, 90]_s$.

Figure 13b shows that the circumferentially oriented fibres of design 6 are detrimental to the axial stiffness of the cylinder and any benefit in terms of embedded stiffeners is removed by the thickness normalisation. The shearing limit of 70° about $\phi = 90^\circ$ (*i.e.* the circumferential direction) does not allow the fibres to be oriented with the x -axis, reducing the stiffness of the laminate. This is a key example of how the current manufacturing restrictions limit the design space of possible CTS laminates.

3.1.7. Design 7

Figures 14a and 14b show the thickness-independent, normalised results for design 7 under idealistic and current manufacturing ability, respectively. Design 7 is an embedded orthogrid structure that has hoops on the outside of the laminate and stringers on the inside of the laminate. For the current study, T_0 and T_1 are kept constant between the inside and outside plies to maintain the same number of laminates per design as designs 1–6. The straight-fibre designs are orthogonal cross-ply laminates.

Figure 14a shows that no layups in the idealistic manufacturing case exceed the normalised buckling load of a QI cylinder. Many laminates show a similar trend to previous designs, as the shearing increases, the normalised buckling load and normalised stiffness decreases. However, multiple laminates are in the feasible range of 80% a QI cylinder for buckling load and axial stiffness. The number of laminates within this range is greater than design 1 and 2, which also sheared all the lamina. This shows that the embedded orthogrid structure is more effective at increasing the specific buckling load when compared to just embedded stringers or embedded hoops alone. For $T_0 = T_1 = 0^\circ$ or $T_0 = T_1 = 90^\circ$ ($[\pm 0\langle 0|0\rangle, \pm 90\langle 0|0\rangle]_s$ or $[\pm 0\langle 90|90\rangle, \pm 90\langle 90|90\rangle]_s$), the mechanical properties of the laminate are identical due to the stacking sequence (the ABD-matrix for the laminates is the same). Interestingly, $T_0 = T_1 = 10^\circ$ or $T_0 = T_1 = 80^\circ$ ($[\pm 0\langle 10|10\rangle, \pm 90\langle 10|10\rangle]_s$ or $[\pm 0\langle 80|80\rangle, \pm 90\langle 80|80\rangle]_s$) also have similar buckling loads and axial stiffnesses, despite not being aligned with the cylinder axis. The off-axis pairs of laminates diverge in behaviour when $T_0 = T_1 = 20^\circ$, however, with $T_0 = T_1 = 20^\circ$ having a 16% higher buckling load than $T_0 = T_1 = 70^\circ$.

Similar to design 5, the current manufacturing ability laminates shown in Figure 14b represent a smaller design space and fewer laminates are within 80% of the QI laminate.

3.1.8. Design 8

Figures 15a and 15b show the thickness-independent, normalised results for design 8 under idealistic and current manufacturing ability, respectively. Design 8 is an embedded orthogrid structure that has hoops on the outside of the laminate and stringers on the inside of the laminate. A similar trend in Figure 14a to design 7 is observed: laminates do not outperform the QI cylinder but approach 80–90% of its mechanical properties (on a mass and thickness normalised basis). The straight-fibre cross-ply laminates represent an

‘upper-bound’ on the CTS cylinders.

The current manufacturing CTS cylinders shown in Figure 15b indicate that the direction of shearing is key to the performance of CTS cylinders. As the shearing angle increases from T_0 , the increase in buckling load is smaller than design 7. This is due to the smaller contribution to the bending rigidity increase for circumferentially orientated fibres compared to axially orientated fibres.

3.1.9. Overall Remarks

The design space of 8 architectures of cylinder layups has been explored in terms of the buckling load and axial stiffness with reference to a nominal QI layup. A quick summary is helpful before proceeding to investigate the imperfection sensitivity of a few CTS cylinders.

- The idealistic manufacturing method presents a larger design space with more laminate configurations approaching the QI cylinder buckling load.
- The mass increase due to shearing considerably decreases the relative merits of layups from a stiffness and buckling perspective. This is particularly true for design 1–2 and 7–8, where all of the plies are sheared. This trade-off is less dramatic in design 3–6 as only half of the layers in the stacking sequence are sheared.
- A $\pm 45^\circ$ set of plies on the outside and inside of the laminate does increase the buckling load of more tow-sheared cylinders, but the axial stiffness is the direct trade-off.
- Using tow-sheared plies to mimic the buckling resistance of a $\pm 45^\circ$ set of plies on the outside is more promising in terms of behaving similar to a QI design.
- Aligning shearing so that hoops are created results in more tow-sheared layups that have a higher buckling load with lower axial stiffness.
- Orthogrid cylinders have better mass and thickness normalised properties than just hoop- or stringer-stiffened designs with all plies sheared (designs 1 and 2).
- Axially orientated plies on the outside of the laminate generally have higher buckling loads, due to the larger bending rigidity increase compared to circumferentially orientated plies.

It has been shown that by using Continuous Tow Shearing in the design of thin-walled cylinders (for both idealistic and current manufacturing ability), the mechanical properties of a QI cylinder can be matched within 10–20%. The motivation of this work is to use CTS to create an inherently imperfection-insensitive cylinders. To this end, a nonlinear analysis with geometric imperfections must be carried out on the cylinders to investigate the KDF of these cylinders. As the current design space encompasses 2320 laminates, a sifting of laminates is beneficial.

Figure 14: Variation in buckling load and axial stiffness for CTS design 7 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

Figure 15: Variation in buckling load and axial stiffness for CTS design 8 where (a) shows the idealistic manufacturing capability and (b) shows the current manufacturing capability.

For the sifting process, the idealistic manufacturing method is considered alone (Figures 6a and 9a–15a) as there are more laminates with mechanical properties that could satisfy the inequality of Equation 9. Therefore, all laminates discussed herein are assumed to be manufactured with this idealistic process. The mass-specific properties are considered for the idealistic manufacturing method. As the average thickness of a CTS cylinder under the idealistic manufacturing method can only reach 42% higher than nominal thick-

ness (compared to the 192% increase the current manufacturing method could produce), it is considered to be a better metric. Moving forward, designs 1–4 are not considered as many of their normalised properties are not comparable to a QI laminate, such that even with a high KDF the cylinder mechanical performance would not satisfy the inequality in Equation 9. For designs 5–6, two cut-off criteria are used for

Figure 16: KDF for cylinders with $n_{\phi=0} = 8$, $n_{\phi=90} = 12$.

selecting laminates, that is

$$\tilde{P} > 0.9, \quad (10)$$

and

$$\tilde{E} > 0.9, \quad (11)$$

where $\tilde{P}$ is the mass-specific buckling load and $\tilde{E}$ is the mass-specific axial stiffness. Similarly, two criteria are used for selecting laminates from design 7–8,

$$\tilde{P} > 0.8, \quad (12)$$

and

$$\tilde{E} > 0.8. \quad (13)$$

With the above criteria, 90 laminates remain and taken forward to a full nonlinear analysis within Abaqus.

3.2. Nonlinear analyses

The results for the most promising laminates (*i.e.* those that could easily satisfy the inequality of Equation 9) are shown in Figure 16 for KDF and Figure 17 for specific, imperfect buckling load ($\tilde{P}^{\text{imp}}$). As described previously, the imperfect buckling loads are computed by running a nonlinear analysis with pseudo-random geometric imperfections (maximum amplitude of one nominal wall thickness) in Abaqus. The geometric imperfection choice used within a nonlinear buckling analysis has a large influence on the buckling load [50]. Choosing the ‘worst’ imperfection is typically a conservative approach for calculating the KDF, but these imperfections are rarely seen in manufacturing [51]. A more robust method would be to manufacture many CTS cylinders and use the imperfection signatures from these cylinders within a stochastic analysis [52]. However, as data relating to the manufacturing of CTS cylinders is currently not available and geometric imperfections are known to be of a random nature [53], a pseudo-random imperfection field is used. This gives a fair comparison to demonstrate the imperfection (in)sensitivity of CTS cylinders.

Figure 17: $\tilde{P}^{\text{imp}}$ for cylinders with $n_{\phi=0} = 8$, $n_{\phi=90} = 12$.

Table 4

Laminates with highest KDF and $\tilde{P}^{\text{imp}}$.

Design	Layup	KDF	$\tilde{P}^{\text{imp}}$
QI	$[\pm 45, 0, 90]_s$	0.576	20.6
6	$[\pm 90 \langle 20 75 \rangle^{12}, 0, 90]_s$	0.699	22.3
7	$[\pm 0 \langle 50 110 \rangle^8, \pm 90 \langle 50 110 \rangle^{12}]_s$	0.778	19.8

The general trend of Figure 16 is that many of the CTS laminates have a higher KDF when compared to the QI laminate (plotted as a black cross on the left side of the figure). This is promising as many of these laminates have a similar mechanical performance when compared to a QI cylinder, meaning it is easier to satisfy the inequality of Equation 9. The maximum KDF is 0.778 with laminate $[\pm 0 \langle 50 | 110 \rangle^8, \pm 90 \langle 50 | 110 \rangle^{12}]_s$ — an increase of 30% compared to the QI cylinder (0.576).

The other metric used, specific imperfect buckling load, is a combination of specific buckling load and KDF taken from the inequality of Equation 9. As

$$\tilde{P}^{\text{perf}} \times \text{KDF} = \tilde{P}^{\text{perf}} \times \frac{\tilde{P}^{\text{imp}}}{\tilde{P}^{\text{perf}}} = \tilde{P}^{\text{imp}}, \quad (14)$$

where $\tilde{P}^{\text{perf}}$ is the mass-specific perfect buckling load from a linear eigenvalue analysis. The maximum specific imperfect buckling load of the current laminates studied is 22.3 kN/kg, an improvement of 8% compared to the QI cylinder (20.6 kN/kg). This occurs in the laminate $[\pm 90 \langle 20 | 75 \rangle^{12}, 0, 90]_s$. On average, designs 5 and 6 have a higher specific buckling loads. This is due to two factors: the criteria used to pick these laminates ($\tilde{P}$ and $\tilde{E} > 0.9$) and the smaller mass penalty as only half of the lamina are sheared.

The results for the best KDF and $\tilde{P}^{\text{imp}}$ are in Table 4. Whilst the layup that has the highest KDF does have a lower $\tilde{P}^{\text{imp}}$ compared to the QI cylinder, the difference is 4%. A high KDF is desirable as it reduces the uncertainty of the design, as well as reducing weight and cost. As shown in Figure 16, tow shearing leads to laminates that are more imperfection insensitive. One variable not yet quantified is the

Imperfection-Insensitive Cylinders

Figure 18: KDF and $\bar{P}^{\text{imp}}$ across low-, middle- and high-periodicity cylinders.

periodicity of shearing. The influence of the number of embedded hoops and stringers is found by a basic parametric study on the 90 most promising designs. The periodicity is increased to $n_{\phi=0} = 12, n_{\phi=90} = 24$ and also reduced to $n_{\phi=0} = n_{\phi=90} = 2$. These results are collated along with

the previous $n_{\phi=0} = 8, n_{\phi=90} = 12$ for direct comparison in Figure 18.

Figures 18a and 18b show the KDF and $\bar{P}^{\text{imp}}$ for the low periodicity selected laminates, respectively. These laminates have KDFs that are most often higher than a QI lami-

Table 5

Laminates with highest KDF and $\tilde{P}^{imp}$ for low, middle and high levels of periodicity.

Design	Layup	KDF	$\tilde{P}^{imp}$
QI	$[\pm 45, 0, 90]_s$	0.576	20.6
7	$[\pm 0(30 20)^2, \pm 90(30 20)^2]_s$	0.778	21.5
7	$[\pm 0(0 30)^2, \pm 90(0 30)^2]_s$	0.844	18.5
6	$[\pm 90(20 75)^{12}, 0, 90]_s$	0.699	22.3
7	$[\pm 0(50 110)^8, \pm 90(50 110)^{12}]_s$	0.778	19.8
6	$[\pm 90(20 75)^{24}, 0, 90]_s$	0.654	22.3
7	$[\pm 0(50 110)^{12}, \pm 90(50 110)^{24}]_s$	0.766	20.4

nate. However, many laminates have $\tilde{P}^{imp}$ that is lower than a QI cylinder. Thus, the lower number of embedded hoops and stringers act to decrease the critical buckling load, while at the same time improving the KDF. The peak value of KDF is 0.844 in design 7 $[\pm 0(0|30)^2, \pm 90(0|30)^2]_s$, 38% greater than a QI cylinder. The highest $\tilde{P}^{imp}$ is 21.5 kN/kg in design 7 $[\pm 0(30|20)^2, \pm 90(30|20)^2]_s$, 4% greater than a QI cylinder. This latter laminate has a KDF of 0.778, equal to the highest KDF when there were 8 and 12 embedded hoops and stringers, respectively.

With high periodicity, the KDF and $\tilde{P}^{imp}$ have a tighter range across the chosen laminates, as shown in Figures 18e and 18f. Many laminates have a higher KDF than a QI cylinder. High periodicity improves the specific imperfect buckling load of many laminates, with the average laminate having a higher specific imperfect buckling load than low and medium levels of periodicities. The increase in specific imperfect buckling load is an interesting result as the cross-sectional area is a function of the shearing angle only, not the periodicity. This trend shows that whilst the embedded stiffeners are not acting as ‘true’ stringers (breaking the surface into discrete panels), the EI contributions is sufficient to exhibit a stiffening effect. The highest KDF for the high number of embedded hoops and/or stringers is 0.776 in design 7 $[\pm 0(50|110)^{12}, \pm 90(50|110)^{24}]_s$, 35% greater than a QI cylinder. The largest $\tilde{P}^{imp}$ found is 22.3 kN/kg, 8% greater than a QI cylinder. These results are summarised in Table 5. The high periodicity laminates is of note as it would not be possible to manufacture these cylinders with the AFP process as the steering radius is below the AFP tow-curvature constraint. The small steering radius is a key advantage of CTS; due to the smaller tow-curvature constraint, cylinders with greater tow-steering periodicity, and greater specific imperfect buckling loads, can be manufactured.

By changing the periodicity, it is possible to influence the KDF and $\tilde{P}^{imp}$ of CTS cylinders. In general, the lower the periodicity, the higher the average KDF. Conversely, the lower the periodicity, the lower the specific imperfect buckling load (as the specific perfect buckling load reduces). Periodicity affects the KDF more than $\tilde{P}^{imp}$, shown by the relatively high increase in KDF compared to the small and infrequent increase in $\tilde{P}^{imp}$ when comparing to a QI cylinder.

The current hypothesis for the high KDF of the low periodicity designs is related to the pre-buckling strain field.

By having a small number of embedded hoops and stringers, a well-defined and non-uniform pre-buckling strain field is created that is not significantly influenced by imperfections. This is seen clearly when comparing the perfect and imperfect pre-buckling strain field (e_{11} component) of two cylinders: a high KDF cylinder and a low KDF cylinder, as in shown Figure 19.

The low KDF cylinder of Figure 19a has a defined non-uniform pre-buckling strain field when no imperfections are present. This strain field is also seen in the imperfect low KDF cylinder in Figure 19b but the stochastic influence of the geometric imperfections is clearly visibly by comparing Figures 19a and 19b. Conversely, the ‘imprint’ of the perfect pre-buckling strain field onto the imperfect cylinder is more pronounced in the high KDF cylinder of Figure 19d. Thus, the high KDF cylinder has a more pronounced, non-uniform pre-buckling strain field that is less affected by imperfections than the more uniform pre-buckling strain field of the low KDF cylinder. This discovery is a novel finding: by tailoring the stiffness of a composite cylinder, the KDF can be increased, thereby reducing uncertainty of design. To increase the KDF further, it appears a larger area of non-uniform strain is beneficial. This supports the hypothesis that embedded stiffeners act to concentrate the load and that imperfections in unstrained portions of the cylinder are unlikely to initiate premature buckling, *i.e.* represent a source of imperfection sensitivity.

The influence of a large non-uniform pre-buckling strain field on the sensitivity of cylinders to imperfections is also corroborated by the recent work of Groh and Pirrera [54, 39]. Groh and Pirrera showed that localisations (aperiodic disturbances in the wall geometry) form low energy barriers between the pre-buckling state and post-buckling state of a cylinder. These localisations, due to the rotational invariance of isotropic or quasi-isotropic cylinders, form a large set of available paths for a cylinder to take when traversing from a stable pre-buckling state to post-buckling state. The large non-uniform pre-buckling strain field created by tow shearing significantly reduces the available number of paths by disrupting the axial symmetry of the cylinder, thereby reducing the sensitivity of these cylinders to imperfections. Similar results from Panter *et al.* [55] have shown that a local, circumferential increase in thickness (leading to a 1% increase in mass) can increase the buckling resistance up to 20%, far greater than if the cylinder thickness was uniformly increased for the same mass increase. The exact relationship between tow shearing, symmetry breaking and localisations is yet to be quantified.

The next question then becomes: can the difference between the perfect and imperfect strain field be used as an indicator of the KDF of a cylinder? That is, if the perfect and imperfect pre-buckling strain fields are similar, can this similarity be used as an indication of imperfection (in)sensitivity? If a relationship can be established between these two values then full nonlinear analyses that are computationally expensive would be less necessary in the design process as imperfection sensitivity can be linked to two

Figure 19: Comparison of envelope nominal e_{11} strain values. Each cylinder is compressed 0.5 mm and strain values are recorded. Values of periodicity are: $n_{\phi=0} = n_{\phi=90} = 2$. Subfigures (a) & (b) are design 7: $\langle 0|65 \rangle$ and subfigures (c) & (d) are design 7: $\langle 30|-15 \rangle$. Subfigure (a) is the perfect geometry, low KDF cylinder (KDF = 0.597), (b) is the imperfect geometry, low KDF cylinder (KDF = 0.597) (c) is the perfect geometry, high KDF cylinder (KDF = 0.925) (d) is the imperfect geometry, high KDF cylinder (KDF = 0.925).

single-step nonlinear analyses.

To investigate this relationship, four separate analyses were necessary. The first analysis was a Linear Buckling Analysis with perfect geometry to determine the critical buckling load. As design 7 was already explored, the data could be used again. The next analysis was a nonlinear *STATIC analysis within Abaqus with pseudo-random seeded imperfections. The minimum step size in the full nonlinear analysis was kept small to reduce computational cost and converge on a conservative value of critical buckling load. With these two analyses, a KDF could be calculated. As the KDF is a single value, the difference in strain values must be condensed into a single value. To this end, the difference in element component (e_{11} , e_{12} and e_{22}) strain values in each element in the local material coordinate system was calculated to then derive the root mean squared value of this array. The root mean squared of the difference in strain values is normalised with respect to the highest value of root mean squared strain, as shown in Equation 15 where e_p is the perfect strain value, e_i is the imperfect strain value.

$$\tilde{e} = \text{RMS} \left(\frac{|e_p - e_i|}{\max(e_p - e_i)} \right). \quad (15)$$

The last two analyses were *STATIC nonlinear analyses with perfect and imperfect geometries of the same cylinder. In these analyses the compression value was half the critical buckling load. The small step allowed quick convergence and direct comparison of the nominal strain fields. The imperfection field used is the same as the nonlinear buckling analyses. The *STATIC settings used within Abaqus for the nonlinear imperfect, half-compression perfect and half-compression imperfect analyses are shown in Table 6.

Figure 20 shows the variation in KDF and e_{12} as a function of shearing angle for a $[\pm 0 \langle 0|T_1 \rangle^2, \pm 90 \langle 0|T_1 \rangle^2]_s$ lami-

Table 6

*STATIC and *STEP settings used within Abaqus.

Parameter	Detail
Initial time increment	0.1
Minimum time increment	0.001
NLGEOM	ON

nate. The ‘noisy’ KDF line is related to the random imperfection field having a different effect on each cylinder. For this subset of laminates, the strain component e_{12} provides a good first-order approximation to the KDF. The smaller the RMS of the strain value, the higher the KDF. This conclusion makes intuitive sense, as a smaller difference implies that the strain fields are similar. However, this relationship does not

Figure 20: KDF and e_{12} strain relationship for $[\pm 0 \langle 0|T_1 \rangle^2, \pm 90 \langle 0|T_1 \rangle^2]_s$ where T_1 is the angle sheared from 0.

hold for other strain components e_{11} or e_{22} for this laminate.

Interestingly, e_{11} and e_{22} do not have a wide range of values for this subset, possibly indicating that the key strain component is the one that has the biggest difference in values. This theory holds up for the other subsets ($T_0 = 10^\circ, 20^\circ$, etc.) as shown in Figures 21 and 22. These figures show that for the subsets $T_0 = 30^\circ$ and $T_0 = 80^\circ$ that e_{11} and e_{22} provide a better first-order correlation with KDF, respectively.

Figure 21: KDF and e_{11} strain relationship for $[\pm 0(30|T_1)^2, \pm 90(30|T_1)^2]_s$ where T_1 is the angle sheared from 30.

Figure 22: KDF and e_{22} strain relationship for $[\pm 0(80|T_1)^2, \pm 90(80|T_1)^2]_s$ where T_1 is the angle sheared from 80/

The correlation between a difference in two pre-buckling strain fields and a cylinder KDF is a novel and interesting discovery. More research into this relationship is necessary to quantify and rigorously investigate the extent to which the relationship holds. Further work will include different laminate architectures and using the strain differences as an objective function in an optimisation.

4. Conclusion

Owing to the imperfection sensitivity of thin-walled cylindrical structures, empirical knockdown factors have

been used to account for the geometric imperfections associated with a critical buckling load far lower than theoretical predictions. Many research programs have attempted to characterise new, robust knockdown factors that are better predictions of cylinder buckling as opposed to using outdated lower-bound predictions. The current report has encompassed a preliminary investigation into reducing the imperfection sensitivity from a design perspective, thereby removing the need for conservative empirical knockdown factors.

The novel manufacturing technique CTS was used as a platform for designing these cylinders, as it offered a coupling between fibre angle and wall thickness, enabling the designer to inherently build-in stringers or hoops within the structure. It also offered the benefits of AFP (continuous fibre angle variation) without the downsides of in-plane bending. It was found that coupling increased the overall cylinder mass as the cross-section was kept prismatic. As a by-product, it was also discovered that the mass of a tow-sheared cylinder is invariant to the amount of times a lamina is sheared (called the periodicity), but is primarily related to the maximum shearing angle. Due to the current limitations of the CTS manufacturing method, an idealistic manufacturing variant (where off-axis plies can be sheared) was considered in tandem with the current version. This comparison showed that by opening up the design space to include off-axis plies ($T_0 \neq 0$) without a thickness build up, the CTS cylinders had mechanical behaviour similar to that of a QI in a number of designs.

To quantify the effect of shearing on the imperfection sensitivity of the CTS cylinders, KDFs were calculated. However, to understand the design landscape and to ensure the layups taken forward to imperfection sensitivity analyses were likely to satisfy the inequality of Equation 9 (where it was shown that the product of the buckling load and KDF is the key metric), Linear Buckling Analyses were carried out across 8 designs of varying layups. The buckling load and axial stiffness were normalised by a QI cylinder on a thickness-averaged, mass-specific basis. 90 CTS layups had mechanical properties similar on a mass-specific basis when compare to a QI cylinder. These laminates were taken forward and full nonlinear analyses were carried out on these layups with pseudo-random imperfections seeded into the wall geometry. This procedure was chosen as to ensure a fair comparison between CTS cylinders.

The best performing laminate exhibited an increase in both KDF and specific, imperfect buckling load ($\bar{P}^{imp}$). The laminate has a layup of $[\pm 0(30|20)^2, \pm 90(30|20)^2]_s$ with a KDF of 0.778 and $\bar{P}^{imp}$ of 21.5 kN/kg, a 30% and 4% increase compared to a QI cylinder, respectively. It was found that many of the CTS laminates with similar mechanical properties had a higher KDF than a QI cylinder. To investigate the influence of periodicity on the KDF and buckling load, two further sets of analyses on the 90 CTS layups was carried out. The first analysis reduced the periodicity to $n = 2$ for all plies. The second analysis increased the periodicity to $n_{\phi=0} = 12$ and $n_{\phi=90} = 24$.

As a general trend, it was found that the higher the periodicity, the higher the $\tilde{P}^{imp}$, showing that the embedded stiffeners have a slight stiffening effect. Conversely, the lower the periodicity, the higher the average KDF. The trade-off is that in general the lower periodicity laminates have slightly lower $\tilde{P}^{imp}$. From this study, the largest KDF was in layup $[\pm 0(0|30)^2, \pm 90(0|30)^2]_s$ where the KDF is 0.844, 38% higher than a QI cylinder. However, the critical imperfect buckling load, $\tilde{P}^{imp}$, is 11% lower than a QI, at 18.5 kN/kg. The largest increase in $\tilde{P}^{imp}$ was found in layup $[\pm 90(20|75)^{12}, 0, 90]_s$ with an 8% increase compared to the QI cylinder.

The increase in KDF (*i.e.* a more imperfection insensitive cylinder) is related to the pre-buckling strain field. Tow shearing causes a non-uniform pre-buckling strain field that breaks symmetry and reduces the chaotic nature of localisations. This hypothesis is supported by the work of Groh and Pirrera [54, 39] and Panter *et al* [55]. An investigation into the possibility of using pre-buckling strain fields as an approximation to imperfection sensitivity showed promising preliminary results. By finding the difference between the strain fields of a perfect and imperfect cylinder at half the critical buckling load, a correlation between smaller differences in strain and greater KDFs was found. Across one laminate investigated (design 7) it was found that the strain component that had the largest range of values in a subset provided the best correlation to KDF.

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6. Data Availability

The underlying data for this publication are available through the University of Bristol data repository.

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A. Mass derivation

The mass of a CTS cylinder can be calculated from first principles as follows. Consider a thin-walled cylinder of a given density such that the mass, m , is given by

$$m = \rho \int_V dV, \quad (16)$$

where ρ is the density of the material, V is the entire volume of the thin-walled cylinder and dV is a small volume

element. This volumetric integral can be expanded to give

$$m = \rho \int_0^L \int_0^{t^*} \int_0^{2\pi r} ds \, dt \, dl, \quad (17)$$

where L is the total length of the cylinder, t^* is the thickness function and $2\pi r$ is the circumference of the cylinder, ds is a small circumferential element, dt is a small thickness element and dl is a small axial element. Taking a circumferential shearing direction ($\phi = 90^\circ$), the thickness is a function of the s -ordinate only. Therefore, the first two integrals can be solved to the limits so that

$$m = \rho L \int_0^{2\pi r} t^*(s) \, ds. \quad (18)$$

The limits of this integral are changed to be from $[0, 2\pi r]$ to $[0, \frac{\pi r}{n}]$, which results in $2n$ being removed from the integral such that

$$m = 2n\rho L \int_0^{\frac{\pi r}{n}} t^*(s) \, ds. \quad (19)$$

To solve this new equation, $t^*(s)$ can be found with respect to the shearing angle, θ , so that

$$t^*(s) = \bar{t} \sec(\theta), \quad \theta = \theta(s), \quad (20)$$

where $\bar{t}$ is the nominal thickness of the laminate (*i.e.* no shearing). $\theta(s)$ is assumed to be a j -th order polynomial of the form $\theta = as^j$. As θ is a function of s , this polynomial can be solved with the boundary conditions $\left[\theta = \theta_{max} \text{ at } s = \frac{2\pi r}{2n} = \frac{\pi r}{n} \right]$ to give:

$$\theta = \theta_{max} \left(\frac{n}{\pi r} \right)^j s^j. \quad (21)$$

The new expression for $t^*(s)$ is then

$$t^*(s) = \bar{t} \sec \left(\theta_{max} \left(\frac{n}{\pi r} \right)^j s^j \right), \quad (22)$$

which gives the mass to be

$$m = 2n\rho L \int_0^{\frac{\pi r}{n}} \bar{t} \sec \left(\theta_{max} \left(\frac{n}{\pi r} \right)^j s^j \right) ds. \quad (23)$$

Substitution of

$$u = \sqrt[j]{\theta_{max}} \frac{n}{\pi r} s, \quad (24)$$

into Equation 23 with new limits gives the mass to be

$$m = \frac{2\pi r \bar{t} L \rho}{\sqrt[j]{\theta_{max}}} \int_0^{\sqrt[j]{\theta_{max}}} \sec(u^j) \, du. \quad (25)$$