A nonlinear frequency-dependent spring-mass model for estimating loading caused by rhythmic human jumping.

R. E. White\textsuperscript{a}, J.H.G. Macdonald\textsuperscript{a}, N.A. Alexander\textsuperscript{a}

\textsuperscript{a}Department of Civil Engineering, University of Bristol, Bristol BS8 1TR, UK

*Corresponding Author: rory.white@bristol.ac.uk, 01173 315714

Abstract

An empirical nonlinear, frequency-dependant, spring-mass system is conjectured for modelling human rhythmic jumping. This model is vital for correctly estimating human-structure dynamic interactions. An experimental study was employed to evaluate the leg mechanics and dynamic loading of a human jumper. Testing was performed over a large range of prescribed jumping frequencies. Subjects performed rhythmic jumps on a force plate and they were monitored by a motion capture system from which the displacement of the centre of mass was identified. Least squares system identification was utilised to determine the parameters of the spring-mass model for human rhythmic jumping. A nonlinear stiffness, rather than a conventional linear spring, is proposed to better capture the observed behaviour during periodic jumping. Force-displacement curves of each subject, during the contact phase of rhythmic jumping, were explored. These display an array of both classical Duffing’s type nonlinear softening and hardening spring stiffnesses over the range of jumping frequencies. The coefficients of the Duffing’s type model are observed to be highly sensitive to jumping frequency. A Poincaré section (phase-space) representation is used to visualise the jumping attractor’s topology. Thus, an experimental bifurcation analysis is performed suggesting the presence of both period doubling and fold bifurcations. These describe the transition from observed period-2 to period-1 jumping and coexisting low/high amplitude jumping behaviour. This study presents a framework for characterising the nonlinear loading of a human performing rhythmic jumping from direct measurements of force and displacement.

Keywords: human-induced loading, rhythmic jumping, nonlinear dynamics, biomechanics
1. Introduction

Humans are capable of generating significant loading whilst performing a variety of activities, such as: walking, running, jumping and bobbing [1]. This has proven to cause considerable issues of vibration serviceability for civil structures over recent decades [2-4]. However, before the human-structure-interactions, of induced structural vibrations, can be understood, the human-induced loading must first be rigorously studied and characterised. It is an important step to identify the capabilities of the human body and how this can be summarised conceptually.

A considerable amount of experimentation has been performed to assess the biomechanics and loading potential of a human whilst performing different activities [5-18]. Each loading scenario has been idealised by simple models to try to capture the overall behaviour of the human body. This has led to considerable research and development of linear based models such as, spring-mass-dampers [19-23] and inverted pendula [24-26] in both the biomechanics and human-structure interaction literature. For example, the spring-mass-damper model proposed by Dougill et al. [27] and Alexander [23], describing human rhythmic bobbing on a structure, is adopted in the IStructE guidelines on stadium dynamics [28]. Although a good approximation, this methodology has the potential of over-simplifying the intricate intrinsic behaviour of pedestrian loading. Adopting a nonlinear dynamics-based approach allows a thorough exploration into the human body system to completely capture and quantify the fundamental phenomena.

Previous biomechanics research has indicated that the effective leg stiffness in human hopping is variable and its modulation is primarily achieved by the ankle joint stiffness [29,30]. However, other studies suggest that knee stiffness may dictate this leg stiffness adjustment in some instances [31-34]. This could suggest that multi-body modelling may more accurately capture this interaction in comparison to the conventional linear spring-mass model [35]. There has been significant research into modelling approaches of jumping and the associated dynamics for refinements in differing scenarios [36-38]. These models add complexity by incorporating additional springs, masses and dampers to characterise specific body limbs. Extensions of the spring-mass model have been developed accounting for simple nonlinear behaviour of the human body and for the resulting non-smooth mechanics [39-41]. These models have utilised a phase-space representation to evaluate steady-state behaviour and jumping attractors, as performed in this paper directly from measured kinematic data.

There are two main methods in evaluating the linear leg spring stiffness [36]. The most common method is based on (i) Blickhan’s spring-mass model [35]. The displacement is calculated by double integrating
the body centre of mass acceleration (with all its problems of low frequency errors), which is identified from ground reaction force measurements [42]. The maximum force generated is assumed to coincide with the maximum vertical centre of mass displacement at the midpoint of the contact phase, i.e. the maximum ground reaction force occurring at the maximum leg compression. This is again only universally correct for linear systems. (ii) The second method is based on Cavagna et al. [43] and; McMahon et al. [44], using the test subject’s mass and the period (or frequency) of a jumping cycle to calculate the leg stiffness (which implicitly assumes linearity). These methods assume that the ground reaction force is effectively symmetric and sinusoidal due to linearity. Human-structure interaction models almost universally assume a linear leg stiffness. This has its limitations for ground reaction forces where the peak response is not observed at the mid-point in time of the contact phase of the oscillation which may lead to overestimates in the leg stiffness.

This paper proposes a novel nonlinear spring-mass model and assesses the validity of a linear spring-mass model to capture the pedestrian-induced loading of a human performing rhythmic jumping. Nonlinear regression is employed on force-displacement data, onto a cubic polynomial using direct measurements. Unlike many previous studies in the literature, both force and displacement, are explicitly measured. Thus, the behaviour of the human leg stiffness over a broad prescribed jumping frequency range: 1Hz to 3Hz is rigorously studied. Linear regression analysis is also performed as a direct comparison. Stiffness estimations are performed from entire records of the ground reaction force (force-plate data) and centre of mass measurements (optical motion-capture data), accounting for both compressing and extending stages of the contact phase.

Phase-space and bifurcation analysis are utilised for the first time on this problem to identify the influence of the jumping frequency on the steady-state behaviour of the biomechanical system.

This paper aims to:

1. Characterise a nonlinear leg stiffness function, from direct force and displacement measurements
2. Identify how the estimated linear and nonlinear leg stiffness models vary with jumping frequency
3. Use bifurcation analysis to describe any observed transitions between different jumping styles

2. Experimental Procedure
Biomechanical jumping experiments were performed in the Earthquake & Large Structures (EQUALS) laboratory at the University of Bristol. Eight test subjects were asked to jump at their preferred jumping frequency, i.e. no target frequency (NT), and nine target frequencies, provided by a metronome, from 1 to 3 Hz in steps of 0.25Hz. Their anthropometric data is displayed in Table 1. Subjects were given a 1min rest between each test and a 5min rest every four tests to ensure they would not become fatigued, exhausted or injured. The order of the test frequencies was randomised for each test subject. NT tests were performed for each test subject, one at the beginning and the other at the end of the experimental procedure. This was performed to evaluate the influence of an auditory stimuli to the dynamics of a human’s jumping frequency and the associated dynamics. The subjects provided informed consent for their anonymised data to be used for research purposes. The Chair of the University of Bristol Faculty of Engineering Research Ethics Committee confirmed that good ethical principles were adhered to in the study and that the data collected could be used for publication.

Table 1 Anthropometric data of 8 test subjects performing in rigid surface jumping experiments

<table>
<thead>
<tr>
<th>Test Subject</th>
<th>Height (m)</th>
<th>Mass (kg)</th>
<th>Leg length (m)</th>
<th>Sex</th>
<th>Age (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1.71</td>
<td>76.6</td>
<td>1.03</td>
<td>F</td>
<td>24</td>
</tr>
<tr>
<td>T2</td>
<td>1.97</td>
<td>77.9</td>
<td>1.08</td>
<td>M</td>
<td>44</td>
</tr>
<tr>
<td>T3</td>
<td>1.97</td>
<td>77.9</td>
<td>1.14</td>
<td>M</td>
<td>25</td>
</tr>
<tr>
<td>T4</td>
<td>1.84</td>
<td>83.3</td>
<td>1.06</td>
<td>M</td>
<td>46</td>
</tr>
<tr>
<td>T5</td>
<td>1.65</td>
<td>66.9</td>
<td>0.99</td>
<td>F</td>
<td>20</td>
</tr>
<tr>
<td>T6</td>
<td>1.55</td>
<td>67.6</td>
<td>0.92</td>
<td>F</td>
<td>21</td>
</tr>
<tr>
<td>T7</td>
<td>1.97</td>
<td>89.1</td>
<td>1.19</td>
<td>M</td>
<td>20</td>
</tr>
<tr>
<td>T8</td>
<td>1.81</td>
<td>73.8</td>
<td>0.98</td>
<td>M</td>
<td>20</td>
</tr>
</tbody>
</table>

The experimental setup comprised of an AMTI OR6-7 force plate in parallel with Qualysis™ motion capture software. This provided a complete dataset of combined measurements of displacement and force for each test. Four infrared cameras were used in the configured Qualysis™ vision system. The locations of the cameras were situated in the setup to ensure the system achieved robust tracking of the markers in the jumping tests. The force plate and motion capture system were time-synchronised using an external clock system. Force plate data was acquired at a rate of 5000Hz whilst the vision system was sampled at 100Hz.

Tests were performed over a 50s window where the beginning and end 5s (10s total) recorded the force plate unloaded to retrieve an offset measurement (non-zero value) of the force plate. Subjects were
asked to stand still on the force plate for 5s before and after performing rhythmic jumping to evaluate a separate weight measurement. Subjects were asked to perform rhythmic jumping for a period of 30s.

A 14-marker body system was adopted for the tracking of the human jumping kinematics by the motion capture vision system via propriety software (Qualisys Track Manager, QTM). This is shown in Figure 2. Markers were fastened using double-sided medical (hypo-allergenic) tape. 14mm diameter spherical threaded reflective markers, with a plastic flat-like plate, were used allowing easy application to test subjects. Tight-fitting gym-like attire was worn by test subjects to; (i) allow for placement of markers close to their body, (ii) minimise anomalous dynamics of the clothing. From this marker setup, a nine – segment body model was defined following a procedure illustrated by Winter [45] to determine the human body centre of mass (COM) during the testing procedure. This procedure assumes a kinematic chain model, treating the human body as an assembly of rigid bodies. It was assumed in the analysis that the mass of each segment and location of its COM relative to the ends remain fixed during movement. Markers 3 to 14 were used in the determination of the body COM whilst markers 1 and 2, placed on the fourth metatarsal bones of each foot, determined the take-off and touch-down timings of subjects as additional verification alongside the force plate data. Figure 1 and Table 2 illustrates the markers used to evaluate each body segment. The segment COM is defined relative to the proximal end, i.e. from the closest end of the limb segment.

Table 2 Definition of body segments [45]

<table>
<thead>
<tr>
<th>Segment</th>
<th>Extremities</th>
<th>Defining markers</th>
<th>Segment mass normalised by $m_p$</th>
<th>Position of Segment COM normalised by segment length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper arm</td>
<td>glenohumeral axis</td>
<td>13 and 14</td>
<td>0.028</td>
<td>0.436</td>
</tr>
<tr>
<td></td>
<td>elbow axis</td>
<td>11 and 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>forearm and hand</td>
<td>elbow axis</td>
<td>11 and 12</td>
<td>0.022</td>
<td>0.682</td>
</tr>
<tr>
<td></td>
<td>ulnar styloiod</td>
<td>9 and 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>thigh</td>
<td>greater trochanter</td>
<td>7 and 8</td>
<td>0.1</td>
<td>0.433</td>
</tr>
<tr>
<td></td>
<td>femoral condyles</td>
<td>5 and 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>foot and lower leg</td>
<td>femoral condyles</td>
<td>5 and 6</td>
<td>0.061</td>
<td>0.606</td>
</tr>
<tr>
<td></td>
<td>medial malleous</td>
<td>3 and 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>trunk-head-neck</td>
<td>greater trochanter</td>
<td>7 and 8</td>
<td>0.578</td>
<td>0.66</td>
</tr>
</tbody>
</table>
Each segment’s mass and location of its COM relative to the proximal (nearer to body midline in kinematic chain made of body segments) segment end representing a joint were then identified using anthropometric data (Table 4.1 in Winter [45]), as presented in Table 2. It is noteworthy that the mass distribution model used does not distinguish between men and women. This issue might have introduced some inaccuracies in the results, but the effect is expected to be small and consistent between all tests for any subject. The three-dimensional coordinates of the subject body COM were then calculated according to the formula:

\[
\begin{pmatrix}
X_{\text{COM}} \\
Y_{\text{COM}} \\
Z_{\text{COM}}
\end{pmatrix} = \frac{1}{m_p} \sum_{i=1}^{n_s} m_i \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}
\]

(1)

where \(X_{\text{COM}}, Y_{\text{COM}}, \) and \(Z_{\text{COM}}\) are the three-dimensional coordinates of the position of the COM of the subject’s body, \(m_p\) is the subject’s total body mass, \(n_s\) is the total number of segments, \(i\) is the segment index, \(m_i\) is the mass of the \(i\)-th segment and \(x_i, y_i, \) and \(z_i\) are three-dimensional coordinates of the position of the COM of the \(i\)-th segment.

Figure 1. Locations of fourteen markers attached to the subject’s body, here shown in the frontal plane. Markers 3-14 were used to establish a nine-segment model for determination of the three-dimensional motion of the centre-of-mass. Image from [45] used with permission.
In calculating the body COM the feet were assumed to move vertically with the lower legs. Since the feet represent only approximately 2.9% of the body mass [45], the error in the body COM due to plantarflexion is estimated to be less than 1mm, or less than 1% of typical displacements of the COM during the contact phase. This error is comparable with other estimated errors in the COM.

3. Data Processing

During acquisition of force plate measurements an eighth order anti-aliasing Butterworth filter with a cut-off frequency of 1400Hz was used. Force plate measurements were down sampled and decimated to the vision system’s sampling frequency of 100Hz for data processing and analysis. Finally, a fourth order low-pass zero-phase Butterworth filter with a cut-off frequency of 20Hz was applied to the force plate and motion capture data to mitigate any high-frequency noise effects. Using the 30s jumping record, for each test, a 20s record was extracted for analysis of both displacement and force data measurements.

To characterise the overall dynamic and kinematic parameters of a test subject’s jumping cycle the contact time, $t_{c,i}$, aerial time, $t_{a,i}$, jumping period, $T_i$, and maximum vertical ground reaction force (GRF), $F_{\text{max},i}$, were extracted cycle to cycle from the GRF time-histories measured, where the subscript $i$ represents the jump index. This schematic procedure is illustrated in Figure 2 which is similar to that performed by McDonald & Zivanovic [17] and Chen et al [18]. The blue dashed line represents the bodyweight. These parameters permitted the evaluation of the contact ratio, $\alpha_i$, impact factor, $K_{p,i}$, and achieved jumping frequency, $f_{J,i}$, for the $i$-th jumping cycle. This is indicated below as:

$$\alpha_i = \frac{t_{c,i}}{T_i}, \quad K_{p,i} = \frac{F_{\text{max},i}}{W}, \quad f_{J,i} = \frac{1}{T_i}, \quad W = m_pg$$

(2)

The contact ratio, $\alpha_i$, is defined as the portion of the jumping cycle during the contact phase, $t_{c,i}$, to the total jumping period, $T_i$. This determines how impulsive a jumping cycle is. The impact factor, $K_{p,i}$, is the ground reaction force normalised to the person’s bodyweight. This can be thought as an amplification factor. The contact and flight phases were identified using a threshold force corresponding to a proportion of the test subject’s weight, $m_pg$. This was performed to approximate and define the take-off and touch-down conditions, for each jump cycle, allowing the evaluation of the parameters, defined in Eq. (2). This is shown below as:
Using the described procedure, the cycle-by-cycle force time-history (jump cycles) plots were obtained to quantify the variability (statistics) of each jumping record for each test frequency of each test subject. An example is illustrated in Figure 3. A thick line denotes the mean cyclic force time-history and the thick dashed lines represents the standard deviation above and below the mean to identify the statistical dispersion of the time-history data. The mean evaluated time-histories were used to evaluate the mean, single, force-displacement curves observed over each test frequency which is discussed in detail in Section 4.1. The start and end of each jumping cycle is defined as the touch-down and take-off, per cycle, using the threshold defined in Eq. (3).

At low frequency jumping \( f_{ij} < 1.50 \text{Hz} \) the GRF of a single cycle observes a double maximum. These two ‘peaks’ correspond to prolonged contact time and heel-toe action to accommodate the target frequency. At medium and high frequency jumping only a single maximum is observed. These cycles observe an approximately sinusoidal wave (during the contact phase), behaviour which has been
modelled differently from multiple studies [16, 24, 26]. The no-target frequency test in Figure 4(d) is very similar in contact time and loading as Figure 4(b) indicating a comfortable jumping frequency at approximately 2.0Hz.

The centre-of-mass velocity and acceleration of a jumper were evaluated by differentiating the estimated centre-of-mass (COM) displacement from motion capture measurements using the finite difference method. Figure 4 displays the COM trajectory kinematics for test subject 5 jumping at a test frequency case of 2.75Hz over a 2s period. The sign convention is defined as positive displacements during the contact phase of the jump cycle. The displacement is observed to be approximately sinusoidal whilst the velocity is seen to be ‘saw-tooth’ in shape. During the flight phase, the velocity is observed

![Graphs](image-url)
to be linear indicating a classical parabolic flight-phase motion. During the contact phase, the acceleration mirrors the force measurement shape, in phase and magnitude, whilst during the flight phase, there are slight differences with negative non-zero values. Changes in body geometry, calibration errors (Table 2) and errors in integration (filter thresholds) may account for the differences observed. The acceleration measurement is indicated in red whilst the ground reaction force (GRF) is shown in black.

Figure 4. Exemplar centre of mass jumping kinematics for test subject 5 evaluated from motion capture measurements for a 2.75Hz test case (a) displacement (using motion capture); (b) velocity (using finite difference method); (c) Acceleration, in red, (using finite difference method) and ground reaction force, in black, (using force plate)
4. Force – displacement analysis results

It is common practice in biomechanics to use the force measurements to evaluate the corresponding centre of mass displacement, i.e. the leg compression, for a given jumping test frequency [46-48]. This is performed by dividing the measured force time-history by a test subject’s mass to evaluate the acceleration and double integrating. However, low frequency errors are exaggerated by the double integration. Here instead, using the ground reaction force and centre of mass time-history measurements, it is possible to evaluate the force-displacement relationships directly. To consider only steady state behaviour, a 20s window of usable data was extracted from each 30s jumping test period. Figure 5 displays the corresponding time-histories for test subject 7 at a target jumping frequency of 1.5Hz. The contact phase of the body centre of mass was then identified using the bodyweight threshold defined in Eq. (3). This permitted the identification of the force - displacement relations illustrated in Figure 6 corresponding to test subject 3.

![Figure 5. Example time-history of test subject 7 jumping dynamics for 1.5Hz test case, ground reaction force displayed in red, centre of mass displacement shown in blue.](image)

To quantify the observed force-displacement relations, and the corresponding stiffness-profiles, a cubic model is proposed for jumping frequencies in the 1-3Hz bandwidth. This is compared with a linear fit to identify the complex behaviour of the leg. The model is applied during the contact phase, for the thresholded data using Eq. (3). This assumes that the leg is already in compression by a factor of 5% of the test subject’s bodyweight, for each jumping frequency test case. These are shown below as,

\[ F_{GRF} = b_1 (Z_{COM} - Z_0) + b_3 (Z_{COM} - Z_0)^3 \]  \hspace{1cm} (4)
\[ F_{GRF} = k(Z_{COM} - Z_0) \]  

where \( b_1 \) and \( b_3 \) are the stiffness coefficients of the linear and cubic \((Z_{COM}-Z_0)\) terms for the third order model and \( k \) is the stiffness coefficient of the linear model;

The reference zero force point for the start of each contact phase, \( Z_0 \), was defined to normalise a test subject’s centre of mass displacement, for the force-displacement relationships, to successfully identify the leg stiffness profiles. \( Z_0 \) was defined as the centre of mass displacement at the point at which the force departs from zero, measured relative to the height of the test subject’s COM in the standing posture.

Figure 6 displays the force-displacement relationship for each test case (1-3Hz) performed for test subject 3. The applied linear and cubic models are fitted (using least squares) for each target jumping frequency. These functions do not pass through the origin, but cross the COM axis at \( Z_0 \), which was also found for each test from the fitting. Complex dynamics are exhibited across the frequency ranges explored. Figure 6(a) – 6(c) suggest a nonlinear softening effect corresponding to 1.00Hz, 1.25Hz and 1.50Hz respectively. This is more prominent in the 1.00Hz & 1.25Hz test cases. The cubic model in Figure 6(a) underestimates the peak force, however it still follows the general shape and trend for 1.00Hz jumping. The ground reaction force presented in Figure 3(a) corresponds to the softening behaviour in Figure 6(a), for period-1 jumping (see Section 6 for definitions and discussion of period-1 and period-2 jumping). An explanation for the nonlinear softening behaviour, at low jumping frequencies, could be a result of the knee joint rotation and centre of mass displacement being significantly large. The small angle approximation (the linear case) would no longer hold for joint rotations corresponding to large flexion of the leg. The linear models, of 1.00Hz and 1.25Hz test cases, are applied to the first 200mm of leg compression corresponding to the maximum ground reaction force.

Beyond this point, the force reduces significantly for large deflections indicating the linear model significantly overestimates the force. At 1.75Hz (Figure 6(d)) a switch in nonlinear interactions is observed indicating a subtle hardening effect. Figure 6(d) – 6(f) display a cubic-like relationship between the GRF and COM corresponding to 1.75Hz, 2.00Hz & 2.25Hz. For test frequencies above and including 2.50Hz, the relationship is observed to be linear in nature (Figure 6(g) – 6(i)). At these high frequencies, the knee flexion and centre of mass displacement are considerably smaller corresponding to smaller angles of rotation about the knee.
Eq. (4) and (5) can be represented in non-dimensional format using the test subject’s bodyweight, $W$, simply $W=m_p g$, and leg length, $L$, to directly compare the forcing, displacement and stiffness coefficients between test subjects. This is important in identifying the significance of the coefficients and their influence on the system’s response. Time, $t$, and therefore the jumping frequency, $f_J$, can be non-dimensionalised by scaling accordingly using $L$ and $g$. These are presented as follows,

$$\frac{F_{GRF}}{W} = \beta_1 \frac{(Z_{COM} - Z_0)}{L} + \beta_3 \frac{(Z_{COM} - Z_0)^3}{L^3} \quad (6)$$

$$\frac{F_{GRF}}{W} = \kappa \frac{(Z_{COM} - Z_0)}{L} \quad (7)$$

$$\tau = t\lambda, \quad \lambda = \sqrt{\frac{g}{L}} \quad (8)$$

where $\beta_1$ and $\beta_3$ are the normalised (non-dimensional) linear and cubic stiffness coefficients, $\kappa$ is the normalised (non-dimensional) linear stiffness coefficient, $\lambda$ is the time scaling factor and $\tau$ is non-dimensional time. The jumping frequency can be non-dimensionalised using $\lambda$. 
The mean force-displacement curves were constructed using the procedure presented for the ground reaction forces displayed in Figure 3. The mean centre of mass displacement was evaluated using all the processed jumping cycles from a test record, for a target jumping frequency, and averaging them. This procedure allows simple qualitative analysis to identify changes in the dynamics of the relationship and stiffness profiles over a broad range of jumping frequencies. Figure 7 displays the mean force-displacement curve for the contact phase of the jumping cycle for low to high jumping frequencies, 1-3Hz test cases for test subject 3. This is presented in a non-dimensional format as described in Eq. (6) - Eq. (8). The initial rise in both force and displacement, on the force-displacement curves, presented in Figure 6, is a result of the pre-tensioning of the leg muscles in priming for the contraction cycle [49].

Figure 6. Force displacement curves for each test frequency case for test subject 3 (shown in grey) with model fits, red displays the linear fit whilst blue shows the cubic fit; (a) 1.00Hz; (b) 1.25Hz; (c) 1.50 Hz; (d) 1.75Hz; (e) 2.00Hz; (f) 2.25Hz; (g) 2.50Hz; (h) 2.75Hz; (i) 3.00Hz
Low frequency jumping is observed to undergo a nonlinear softening effect whilst medium jumping frequencies observe a hardening effect. At higher jumping frequencies, the relationship is observed to be linear-like, or a steep cubic polynomial. A qualitative change in topology and behaviour, is observed between 1.50Hz (0.51) and 1.75Hz (0.60), i.e. describing the transition from spring softening to hardening.

Figure 8 shows the mean-force displacement curves for test subject 5 jumping at frequencies from 1 to 3Hz. At low frequency jumping, 1.00Hz ($f_J/\lambda = 0.32$) and 1.25Hz ($f_J/\lambda = 0.40$), test subject 5 is performing a jump-bob behaviour, a type of period-2 jumping style. This is discussed and explored in detail in Section 6. This subject displays a transition point in spring behaviour, from nonlinear softening to hardening springs, between 1.75Hz ($f_J/\lambda = 0.55$) and 2.00Hz ($f_J/\lambda = 0.63$). The transition point in Figure 7, for test subject 3, is observed at a slightly lower jumping frequency suggesting that this point of transition could be dependent on physiological factors such as, height, leg length, or mass.

The curves displayed in Figures 7 and 8, for test subject 3 and 5, display hysteresis loops. This is more prominent in the 1Hz ($f_J/\lambda = 0.34$) and 2.75Hz ($f_J/\lambda = 0.87$) test cases, in Figure 7 and 8, respectively. The path integral describing the leg shortening and lengthening phases, for the contact phase of human jumping, are observed to be very similar throughout the cycle. This indicates a slight change in mechanical energy, which is replenished by the muscle input energy.
Figure 8. Comparison of mean force-displacement curves of test subject 5 for target jumping frequencies, 1-3Hz, indicating a transition of softening to hardening springs between 1.75Hz and 2.00Hz

5 Apparent frequency-dependency of leg spring

Figures 6, 7 and 8, in Section 4, indicate the dynamics of human rhythmic jumping are heavily dependent on the jumping frequency. This elaborate frequency-dependent behaviour must therefore be characterised to provide engineers with the dynamic loading capabilities of human jumping. The kinematics and dynamic relations are illustrated and explored in this section. The leg stiffness frequency dependency has been explored in biomechanics literature however, only for the linear spring-mass case [29, 32, 35, 37]. In this paper, the frequency dependency of both linear and cubic leg stiffness models are explored and characterised including low frequency jumping which hasn’t been rigorously studied in the literature thus far. It is indicative of the variability in the linear and nonlinear leg stiffnesses evaluated in modelling the loading capabilities of humans in a variety of applications.

Table 3 characterises the mean peak ground reaction force (normalised to bodyweight) – impact factor, mean peak centre of mass displacement (normalised to leg length), linear leg stiffness, contact time, achieved jumping frequency and contact ratio over all test subjects for each target jumping frequency. Brackets indicate the standard deviation for each evaluated mean value. This data is used to evaluate the coefficient of variation (relative standard deviation), for each parameter, which is displayed in Figure 9, against the mean achieved jumping frequencies for all target and no-targeted tests. The coefficient of variation estimates are displayed in black with blue lines displaying indicative trends. The no target frequency tests NT1 and NT2 are overlaid in blue and red circles respectively. For achieved jumping frequency (Figure 9(a)) and mean peak COM displacement (Figure 9(c)), the variability is observed to be larger at low \( f_J < 1.50 \text{Hz} \) and high \( f_J > 2.50 \text{Hz} \) jumping frequencies than in the
intermediate frequency range. The distribution of the impact factor, in Figure 9(b), however displays a
maximum at 1.5Hz suggesting the variation is significant at this jumping frequency. The linear leg
stiffness, using Eq (5), shown in Figure 9(d), is identified to be more variable at 1.00Hz and 1.25Hz
jumping frequencies whilst the contact time distribution, displayed in Figure 9(e), is reasonably
distributed with a maximum at 1.25Hz. The variation in the contact ratio, shown in Figure 9(f), indicates
a local maxima and minima at 1.25Hz and 2.5Hz respectively. The high variation observed in some of
the parameters at 1.25Hz and 1.5Hz jumping could be a consequence of the jumping style employed by
different subjects. The frequency at which steady-state jumping behaviour transitioned from period-2
(jump-bob) to period-1 differed for each individual test subject. This is discussed and explored in detail
in Section 6. The NT tests indicate more variability, as portrayed in Figure 9(a), (b), (d) and (e), which
may be due to a combination of lack of auditory stimulus and variability between the preferred jumping
frequencies of different subjects. Some of the data represented in Table 3 are presented in Figure 10
and 11 to aid visual understanding of the presented dynamics.

Table 3 Test subject kinematic & dynamic data for test frequencies performed on a rigid surface (bracketed
values indicate the standard deviation while non-bracketed values represent mean values over all test subjects)
Figure 10 displays the human dynamic frequency relations for the non-dimensional mean peak ground reaction force and centre of mass displacement in 10(a) and 10(b) respectively. The mean peak GRF is observed to increase and reach a maximum around 2.25Hz and begins to decrease above 2.5Hz. The corresponding mean centre of mass displacement amplitude is seen to decrease significantly with an increase in jumping frequency. A reason for this could be due to the nonlinear frequency dependency of the contact time for this system. This would describe the reduction in maximum displacement and an increase in maximum force with an increase in jumping frequency up until 2.5Hz. These coincide and agree with previous findings [15,16].
Exploring linear leg spring stiffness formulation results

The common convention adopted in the biomechanics literature is to identify the peak ground reaction force, of a given jumping cycle, and divide this value by the corresponding maximum leg compression (from double-integration of force measurements [42]) to estimate the linear leg stiffness [46]. This assumes that the peak GRF coincides with the maximum centre-of-mass displacement. However, Figure 5 demonstrates that this is not always the case. Figure 3(a) further indicates that at significantly low jumping frequencies the maximum leg compression corresponds to a GRF that is smaller than the maximum. This illustrates the nonlinear softening behaviour in Figure 6(a). The method of analysis employed to evaluate the linear leg stiffness uses linear regression, applying Eq. (5) and Eq. (7), as shown in Figure 6. This is averaged over all test subjects to identify a robust mean linear leg stiffness-jumping frequency dependency. This is displayed in Figure 11(a). Black points denote the mean data, dashed blue lines indicate the standard deviation and the thick blue line displays the cubic regression (least-squares) fit. A third-order polynomial was chosen as it was the lowest order that gave curves lying within ±1 standard deviation of the mean values. This procedure was applied to the cubic regression fits describing the jumping frequency dependencies of the nonlinear stiffness coefficients discussed in Section 5.3. Figure 11(b) illustrates the non-dimensional mean leg stiffness jumping frequency relationship, for all test subjects.

At low jumping frequencies the stiffness is observed to be fairly constant which transitions into a linear region in the frequency bandwidth 1.75-2.50Hz. At high frequencies the stiffness increases at a slower
rate suggesting a potential plateau at considerably higher frequencies. Eq. (5) indicates this model as a linear spring but with an intricate nonlinear frequency dependency. The dimensional and non-dimensional linear leg stiffness-frequency relationships are quantified below as,

\[
\begin{align*}
\{k(f_J)\} = \{61.2 - 94.3f_J + 53.1f_J^2 - 8.36f_J^3\} \\
\kappa(f_J/\lambda) = \{86.5 - 410(f_J/\lambda) + 710(f_J/\lambda)^2 - 344(f_J/\lambda)^3\}
\end{align*}
\]

The R-squared value, for both fits, is found to be 0.993. The stiffness coefficient unit corresponds to newton per metre, with \(f_J\) expressed in hertz.

\[\text{(9)}\]

5.3 Exploring cubic leg spring stiffness formulation results

Although the application of a cubic stiffness model for large deformation structures (such as [50]), this utilisation is novel in capturing and characterising the nonlinear loading behaviour of human rhythmic jumping. This formulation has not been previously adopted for leg compression mechanics. The reduced-order cubic model, presented in Eq. (4) and Eq. (6), is applied using nonlinear regression to all force-displacement curves to identify a mean linear and cubic coefficient for each target jumping frequency. Table 4 illustrates the non-dimensional coefficients which are displayed in Figure 12 and Figure 13; brackets indicate the standard deviation. The dimensional linear coefficient frequency dependency is displayed in Figure 12(a), using Eq. (4), whilst the non-dimensional form is presented in
Table 4 Best-fit cubic stiffness formulation coefficients, Eq. (6)  
(bracketed values indicate the standard deviation while non-bracketed values represent mean values)

<table>
<thead>
<tr>
<th>Target jumping frequency</th>
<th>Achieved jumping frequency, $f_j$</th>
<th>Normalised achieved jumping frequency, $f_j/\lambda$</th>
<th>Eq (6) Coeffs.</th>
<th>$\beta_1$</th>
<th>$\beta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NT1</td>
<td>1.97 (0.28)</td>
<td>0.61 (0.11)</td>
<td>19.0 (6.64)</td>
<td>105</td>
<td>120</td>
</tr>
<tr>
<td>1.00</td>
<td>1.01 (0.13)</td>
<td>0.32 (0.01)</td>
<td>15.5 (3.82)</td>
<td>-79.3</td>
<td>41.4</td>
</tr>
<tr>
<td>1.25</td>
<td>1.26 (0.09)</td>
<td>0.41 (0.01)</td>
<td>18.8(3.16)</td>
<td>-150</td>
<td>60.4</td>
</tr>
<tr>
<td>1.50</td>
<td>1.51 (0.09)</td>
<td>0.49 (0)</td>
<td>18.2 (1.65)</td>
<td>-123</td>
<td>55.5</td>
</tr>
<tr>
<td>1.75</td>
<td>1.76 (0.08)</td>
<td>0.56 (0.024)</td>
<td>19.1 (2.24)</td>
<td>-7.22</td>
<td>70.6</td>
</tr>
<tr>
<td>2.00</td>
<td>2.00 (0.08)</td>
<td>0.64 (0.024)</td>
<td>21.5 (3.30)</td>
<td>135</td>
<td>106</td>
</tr>
<tr>
<td>2.25</td>
<td>2.25 (0.09)</td>
<td>0.73 (0.024)</td>
<td>23.7(1.32)</td>
<td>340</td>
<td>159</td>
</tr>
<tr>
<td>2.50</td>
<td>2.50 (0.09)</td>
<td>0.81 (0)</td>
<td>29.0 (1.95)</td>
<td>482</td>
<td>209</td>
</tr>
<tr>
<td>2.75</td>
<td>2.72 (0.13)</td>
<td>0.89 (0.02)</td>
<td>36.1 (4.39)</td>
<td>364</td>
<td>433</td>
</tr>
<tr>
<td>3.00</td>
<td>2.95 (0.17)</td>
<td>0.94 (0.08)</td>
<td>38.7 (7.13)</td>
<td>500</td>
<td>522</td>
</tr>
<tr>
<td>NT2</td>
<td>2.00 (0.22)</td>
<td>0.65 (0.09)</td>
<td>23.2 (3.52)</td>
<td>132</td>
<td>164</td>
</tr>
</tbody>
</table>

The coefficient ($\beta_1$, Eq. (6)) displays a broadly similar relationship to Figure 10 giving good agreement in the applied model. The cubic coefficient ($\beta_3$, Eq. (6)), shown in Figure 13, is observed to shift between a negative and positive value between 1.50Hz and 1.75Hz corresponding to the transition point discussed in Figure 8 and 9; the change from a softening to hardening spring. This transition point did differ slightly for each test subject. This suggests that the leg is able to actively control the stiffness to sustain rhythmic jumping over a broad frequency bandwidth on rigid ground. At high jumping frequencies, the standard deviation is larger. This would suggest that the linear coefficient, and model, dominates at considerably large jumping frequencies where the cubic nonlinearity diminishes. The
order of magnitude of the non-dimensional cubic coefficient is significantly large implying that this nonlinearity cannot be disregarded when modelling the dynamics of the leg during rhythmic jumping.

The dimensional and non-dimensional empirical linear coefficients relationships for Figure 12 are quantified below as,

\[
\begin{align*}
\{b_1(f_j)\} &= \left\{ 0.32 + 20.8f_j - 12.9f_j^2 + 3.12f_j^3 \right\} \\
\beta_1\left(\frac{f_j}{\lambda}\right) &= \left\{ 0.45 - 90.5\left(\frac{f_j}{\lambda}\right) + 173\left(\frac{f_j}{\lambda}\right)^2 + 129\left(\frac{f_j}{\lambda}\right)^3 \right\}
\end{align*}
\]

Figure 12. Frequency dependency of the linear coefficient of the cubic model using mean data for all test subjects (in black) with standard deviation (in dashed blue) and third-order polynomial fit (in solid blue) for 1-3Hz jumping, (a) dimensional format using Eq. (4); (b) non-dimensional format using Eq. (6) and Eq. (8)

The R-squared values of cubit fit for the linear stiffness coefficient frequency relationships were identified as 0.989 for both fits. These coefficients are the averaged values for all test subjects.
The dimensional and non-dimensional empirical cubic coefficients relationships for Figure 13 are quantified below as,

\[
\begin{align*}
\left\{ b_3(f_j) \right\} &= \left\{ 1.460 \times 10^3 - 2.90 \times 10^3 f_j + 1.67 \times 10^3 f_j^2 + 0.28 \times 10^3 f_j^3 \right\} \\
\left\{ \beta_3 \left( \frac{f_j}{\lambda} \right) \right\} &= \left\{ 2.22 \times 10^3 - 13.6 \times 10^3 \left( \frac{f_j}{\lambda} \right) - 24.0 \times 10^3 \left( \frac{f_j}{\lambda} \right)^2 + 12.3 \times 10^3 \left( \frac{f_j}{\lambda} \right)^3 \right\}
\end{align*}
\]

The R-squared values, of the cubic fits, for the cubic stiffness coefficient’s frequency dependencies were identified as 0.966 for both fits. These coefficients are the averaged values for all test subjects.

6. Characterising jumping styles

Humans are capable of jumping in various styles which can produce complex dynamics. Different body structures, somatotypes, favour different techniques where people can increase their contact or aerial time accordingly to accommodate different frequencies. Characterising different jumping styles aids the identification and understanding of the steady-state jumping behaviour and associated dynamic loading. This paper proposes a method to characterise these jumping techniques utilising the phase-space representation. This approach is widely used in nonlinear dynamics [51]. This is also referred to as the state-space, representing all the possible states, position and velocity of a dynamical system [51].
For the system of a human performing rhythmic jumping the centre of mass displacement and velocity are used to characterise the jumping attractors for a range of target/prescribed jumping frequencies. The periodicity of each attractor is evaluated by applying the Poincaré section [51]. This stroboscopic sampling, at the jumping period, of the person’s displacement and velocity time-history enables a simple evaluation of the true periodicity of the steady-state. If one Poincaré point is observed we have a period-1 attractor, if two Poincaré points are observed then we have a period-2 attractor, and so on and so forth. This approach has not been previously used for the case of human jumping experiments. It enables identification of jumping attractors directly from experimental kinematic data. The Poincaré section and points were evaluated using the cycle-by-cycle local maxima of the centre-of-mass time-history and averaging to identify the mean Poincaré estimates.

At low jumping frequencies test subjects displayed a range of strategies to perform rhythmic jumping. Figure 14 and 15 displays the time-histories of test subjects 1 and 2, respectively, jumping at a target frequency of 1Hz. The blue indicates the centre of mass displacement measurement and red displays the ground reaction force for both Figure 14(a) and 15(a). Two different jumping styles are observed. Test subject 1 employs a period-2 jump whilst test subject 1 sustains period-1 jumping. This is clearly observed in the phase-space portraits in Figure 14(b) and 15(b). The period-2 jumping phenomenon was observed to be a ‘jump-bob’ behaviour. Interestingly, test subjects 5, 6 and 7 displayed this ‘jump-bob’ strategy to sustain 1Hz jumping. These are shown in Figure 16. Interestingly, five out of the eight test subjects adopted this jumping style with similar structure and shape in their attractors, suggesting a general trend in low frequency jumping.

![Figure 14](image)

**Figure 14.** Example of period-2 jumping at 1Hz target frequency for test subject 1 (jump-bob), (a) Centre-of-mass displacement, blue, and ground reaction force, red, time-histories; (b) Phase-space portrait with Poincaré points in black.
Figure 17 illustrates the phase-space jumping attractors observed over the studied jumping frequencies, 1-3Hz, for test subject 4. The circular dots denote the Poincaré points for the attractor. The figure shows a parameter sweep indicating the dependency of the periodicity on the jumping frequency. At low frequency jumping, Figure 17(a) (1.00Hz) and 17(b) (1.25Hz) display period-2 jumping attractors. At frequencies greater than 1.5Hz, period-1 solutions are observed with an oval-like shape in the attractor. Each test subject’s attractors may subtly differ over the frequency range due to the different jumping styles performed to accommodate the target frequency.

Figure 16. Example of jump-bob, period-2, phenomena at 1.00Hz target jumping with Poincaré points in black indicating periodicity, (a) Test subject 5; (b) Test subject 6; (c) Test subject 7
Using the information from the Poincaré sections displayed in Figure 17, for each jumping frequency, the bifurcation diagram is constructed to estimate when the steady state responses bifurcate from period-2 to period-1 jumping. This describes the jumping frequency as a controlling parameter of the oscillating biological system. The bifurcation diagram is displayed in Figure 18. The Poincaré solution of the centre of mass displacement and velocity are displayed in Figure 18(a) and 18(b) respectively. The grey lines show a suggested underlying trend in the bifurcation plots. Period-halving bifurcations can be observed indicating the transition from period-2 to period-1 jumping. Interestingly for both Figure 18(a) and 18(b), there is a significant jump in the Poincaré solution at 1.5Hz suggesting a second bifurcation, a
This phenomenon is very common in the resonance curves of nonlinear dynamical systems. This behaviour could explain the transition from the jump-bob phenomena into clear jumping as the amplitude of oscillation significantly increases. Figure 18 suggests a nonlinear softening effect for test subject 6’s resonance curve, as this shows an increase in amplitude with a decrease in frequency. This agrees with the findings with the nonlinear cubic stiffness coefficient frequency dependency, in Figure 12, indicating a negative coefficient for jumping frequencies 1.75Hz and less. This is the first time that bifurcation diagrams have been constructed from experimental human rhythmic jumping data.

![Figure 18. Bifurcation diagram of test subject 4 jumping in frequency bandwidth 1-3Hz, grey lines show trend (a) centre of mass displacement Poincare solution; (b) centre of mass velocity Poincaré solution](image)

7. Conclusions

Human rhythmic jumping has been experimentally explored and analysed over the frequency bandwidth 1-3Hz in intervals of 0.25Hz. The leg stiffness is characterised utilising regression analysis, from force-plate and motion-capture measurements over full records during the contact phase. The leg stiffness is observed to be extremely sensitive to changes in jumping frequency indicating active control to sustain the induced dynamic loading. A conventional linear leg-spring model displays a complex frequency dependency. This required proposing a variation of the linear leg-spring stiffness coefficient with frequency. In addition, the preferableility of using a nonlinear leg-spring model is demonstrated. The experimentally observed nonlinear behaviour of the leg-spring is better characterised by a cubic leg-stiffness model which also requires frequency dependent coefficients. The advantage of the cubic stiffness (Duffing’s) model is that it is able to characterise both hardening and softening leg-spring stiffnesses over the investigated jumping frequency range. The linear spring-mass model is incapable
of capturing the intricate leg behaviour well at moderately low jumping frequencies. Thus, the
limitations of the conventional linear stiffness model is highlighted in making sense of the experimental
observations.

At low frequency jumping (<1.50Hz), a jump-bob phenomenon is observed in approximately half of
the test subjects as a strategy to sustain rhythmic jumping. Application of the phase-space portrait and
Poincaré section has enabled the characterisation of complex jumping attractors indicating a multitude
of jumping approaches over a range of jumping frequencies. Multiple jumping attractors are observed
over the explored jumping frequency range indicating that humans can employ a variety of strategies
to adapt to different auditory stimuli. Period-2 jumping is observed during jump-bobbing for 1Hz and
1.25Hz jumping for 63% of test subjects. Bifurcation analysis indicates period-halving and fold local
bifurcations as the target jumping frequency increases. Multiple coexisting attractors are observed to
occur in this range, including period-1 (jump), period-2 (jump-bob) and possibly high/low amplitude
period-1 jumping, due to the presence of a fold bifurcation.

Acknowledgements
The authors greatly acknowledge the assistance of Alex Nunn in performing the experimental study
and preliminary data processing, Dr Matt Dietz for consultation of data processing techniques, Dr Luiza
Dihoru for assistance in use of Qualysis Motioncapture software and Dr Tony Horseman for
clarification of electrical sensors and time synchronisation. Rory White is funded by an EPSRC
Doctoral Training Partnership studentship.

Data Access
To be confirmed.

References
Conference on Structural Dynamics EURODYN, Trondheim, Norway, June 1993.


