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Time-frequency analysis of crowd lateral dynamic forcing from full-scale measurements on the Clifton Suspension Bridge

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Abstract

Pedestrian-induced vibrations have caused significant issues on various footbridges, over the past few decades, as it can lead to an abrupt growth in the amplitude of structural oscillations, i.e. lateral dynamic instability. Measurements were taken during two separate crowd loading events on the Clifton Suspension Bridge, UK, to characterise the human-structure interactions observed. Two lateral modes of the bridge were studied, previously found susceptible to pedestrian-induced excitation. A comparative study is performed to identify the different interactions observed for the differing bridge deck amplitude responses through a novel procedure based on time-frequency analysis. This enables the identification of the fluctuations on the instantaneous modal amplitude and resonant frequency during pedestrian loading. Previous measurements of Clifton Suspension Bridge during crowd loading leading to the onset of large-amplitude vibrations revealed a significant increase in the natural frequency of the two modes considered. The instantaneous frequency, of both modes, appeared to roughly mirror the displacement amplitude response during significant loading, >250 pedestrians. Recent measurements of Clifton Suspension Bridge during crowd loading, illustrated tentative evidence for the presence of human-structure interactions during low-amplitude responses. For high amplitude responses on the bridge the modal frequencies were observed to increase with an increase in amplitude displaying a complex non-linear hardening effect for the two modes of vibration.

1.Introduction

Pedestrian loading of footbridges has caused serious concern for vibration serviceability. Multiple footbridges have observed lateral dynamic instability, resulting in the onset of large amplitude vibrations. The London Millennium Footbridge (LMF) [1] and Clifton Suspension Bridge (CSB) [2] are examples of this phenomenon. Significant multi-modal excitation has been observed on the CSB [2]. Pedestrians have been observed to significantly alter a structure's properties: modal damping and frequency, which is reflected in fluctuations of a bridge's instantaneous amplitude and frequency respectively. Pedestrian lateral dynamic forcing can be characterised as the action of equivalent added mass or damping which may be positive or negative, depending on the pedestrian pacing to bridge forcing frequency ratio.

Decomposing a structure's modal properties may be difficult to successfully approximate, as pedestrian loading contains a frequency spectrum producing multi-modal excitation. This paper presents a time-frequency analysis procedure for evaluating lateral human-structure interactions from full-scale measurements on the Clifton Suspension Bridge, from two separate crowd loading events. The Hilbert transform is utilised to identify the instantaneous amplitude and frequency of two low-frequency lateral resonant modes, modes L2 and L3, previously shown to be susceptible to pedestrian-induced vibrations [2]. The evaluated amplitude and frequency time-histories, of each lateral mode, are quantitatively studied to characterise equivalent added mass and damping of pedestrians, for each crowd loading event. Comparisons are drawn between low- and high amplitude bridge responses. A complex instantaneous amplitude-frequency relationship is observed for the two lateral modes studied, for large amplitude bridge responses. The corresponding 'back-bone' curves are constructed from reduced-order, fifth order, polynomials, for both modes respectively, to characterise the lateral human-structure

interactions. They both display a hardening effect, i.e. an increase in resonant frequency with an increase in amplitude.

2. Experimental Procedure

The Clifton Suspension Bridge spans the River Avon bridging from Clifton, Bristol, to Leigh Woods, North Somerset. It is approximately 2km west of the centre of Bristol. The main span is 214.35m, from centreline to centreline of the towers, with the suspended bridge length spanning 193.85m. The roadway is 6.1m wide between the two longitudinal stiffening girders. These are supported by vertical suspension rods spaced 2.44m apart from each other along the bridge. Total deck width is 9.46m including 1.1m footways either side. A more complete description of the structure is given by Barlow [3]. The bridge layout and rod breakdown are displayed in Figure 1.

The CSB was instrumented with accelerometers and displacement transducers at multiple rod locations: Rod 40LW, Rod 11LW and Rod 40C, as shown in Figure 1. These sensors acquired both lateral and vertical responses during the crowd loading events. Measurements at rod 11LW are used for analysis in this study, as they represent close approximates of the total modal responses of modes' L2 and L3. See reference [4] for full details on the structural health monitoring system used on the CSB.

In 2003, the Clifton Suspension Bridge exhibited two crowd loading events, restricted to the footways, on the nights of Thursday 7th and Saturday 9th August. Data analysis is focused on the measurements acquired from the 7th August for the proposed time-frequency method of analysis. In 2017, the CSB observed a crowd loading event, open to roadway and footways, on the afternoon of Sunday 15th October.

Figure 1 Schematic and layout of the Clifton Suspension Bridge [3] displaying instrumented rod location

2.1 Number of people

During the 2003 crowd loading event, the number of people were monitored by CCTV footage counting the accumulative number in three-minute intervals. The CCTV data was used to approximate the pedestrian counts during the crowd loading event. For the 2017 crowd loading event, the number of people were measured by two GoPro cameras, one either side of the bridge pillars, in conjunction with counting pedestrians on and off the bridge using the phone application, Timestamp, by four volunteers. Figure 2(a) and 2(b) display the number of people during the crowd loading events in 2003 and 2017 respectively. The maximum number of people, at any one given time, was approximated to be 488 and 151 people, during the 2003 and 2017 crowd loading events respectively. This corresponded to approximate pedestrian densities of 1.1m²/ped., per walkway, and 0.13m²/ped., across the roadway, respectively. This suggests that pedestrians, during the crowd loading event witnessing large-amplitude vibrations, displayed significantly small walking speeds corresponding to low pacing frequencies. The combination of pedestrian numbers and prolonged low stepping frequencies may have contributed to the onset of large amplitude vibrations. The 2017 event suggests that pedestrians were more free flowing, promoting larger walking speeds and comfortable pacing frequencies. The 2003 event on the Clifton Suspension bridge was

observed to last approximately two hours in comparison to 20 minutes during the 2017 event. This is reflected in the high and low acceleration amplitude responses measured from each experiment respectively [2, 5].

Figure 2 Number of people during the crowd loading events measured on the Clifton Suspension Bridge (a) 2003; (b) 2017

2.2 Bridge Response

The bridge deck was seen to oscillate in two of its low-frequency lateral modes: L2 (0.52Hz) and L3 (0.75Hz), during the 2003 crowd loading event. This was observed to occur simultaneously displaying multi-modal structural excitation by pedestrians. To be consistent with data processing from the 2003 event, the 2017 measurements were processed in the same manner to compare the two events and corresponding bridge responses. Sixth order bandpass Butterworth filters were applied to extract modes L2 and L3, in their prescribed bandwidths, to mitigate the effects of noise. Figure 3 displays the bridge deck accelerations measured at Rod 11, for modes L2 and L3 for each crowd loading event. Figure 3(a) indicates the 2003 event whilst Figure 3(b) displays the 2017 event. The red indicates mode L2, whilst blue distinguishes mode L3. The amplitude response from the 2003 crowd loading event is observed to be significantly larger than the 2017 event.

Table 1 illustrates the modal properties of lateral modes L2 and L3 of the CSB, based on experimental system identification performed by Macdonald [2]. These two modes have significantly low damping ratios making them susceptible to pedestrian-induced vibrations. It should be clear, lateral modes from L1 through to L4, 0.25Hz – 0.965Hz, have been previously identified. For this study, only modes L2 and L3 are analysed to identify the observed human-structure interactions.

Table 1 Modal properties of lateral modes of the Clifton Suspension Bridge, evaluated from Macdonald [2]

Mode	Natural Frequency [Hz]	Damping Ratio [%]	Modal Mass [Tonnes]
L2	0.524	0.58	538
L3	0.746	0.68	561

Figure 3 Bridge deck acceleration time-histories for lateral modes L2 and L3 on the Clifton Suspension Bridge (a) 2003 crowd loading event; (b) 2017 crowd loading event

3. Time-frequency Analysis

Time-frequency analysis is beneficial in analysing the instantaneous frequency and amplitude of a structure's response to dynamical loading. During crowd loading, pedestrians have been observed to alter a structure's modal instantaneous frequency and amplitude [1,2]. This corresponds to their dynamic forcing producing equivalent added mass and damping, positive or negative, depending on the frequency ratio, human pacing frequency to structural forcing frequency. Pedestrian forcing in-phase with acceleration corresponds to added mass, whilst forcing in-phase with velocity is equated to added damping. The Hilbert transform converts a real continuous, or discrete, signal into an analytic solution [6]. This allows the evaluation of the instantaneous phase and amplitude of a structure's modal response. This time-frequency method only allows unimodal analysis, requiring measured structural responses to be filtered and decomposed into modal components. Lateral bridge acceleration measurements are first filtered to isolate modes L2 and L3 separately. For further understanding of the method, see reference [5].

Assuming a discrete complex signal, $S_q(t)$, for a single mode, of the form:

$$S_q(t) = x_q(t) + iy_q(t) \quad (1)$$

Where $x_q(t)$ is the discrete input signal, $y_q(t)$ is the Hilbert transformed input signal and $i = \sqrt{-1}$. The instantaneous amplitude, $A_q(t)$, and phase, $\theta_q(t)$, of the q th mode, can be evaluated from the modulus and argument respectively, using Pythagorean and trigonometric principles, as shown below:

$$A_q(t) = \sqrt{x^2(t) + y^2(t)}, \quad \theta_q(t) = \tan^{-1} \frac{y(t)}{x(t)} \quad (2)$$

The instantaneous modal frequency, f_q , can then be approximated by differentiating the instantaneous frequency with respect to time, using the finite difference method, shown as:

$$f_q = \frac{1}{2\pi} \frac{d\theta_q}{dt} = \frac{1}{2\pi} \frac{\Delta\theta}{T_{p,s}} = \frac{1}{2\pi} \frac{\theta_i - \theta_{i-1}}{T_{p,s}} \quad (3)$$

Where $T_{p,s}$ is the sampling period which is analogous to the inverse of the sampling frequency, $f_s = 1/T_{p,s}$

This method provides a very good approximation of the response envelope making it very beneficial [6]. This allows the extraction of data from each sample in comparison to the conventional cycle-by-cycle zero-crossing maximum and minimum points of vibrations [7]. Fluctuations in the bridge modal frequency would suggest the presence of equivalent added mass whilst variations in the bridge amplitude would indicate equivalent added damping by pedestrians. There are limitations to this time-frequency analysis method. Noise is attenuated at beginning and end conditions which can cause issues in accurate data extraction and analysis. Inherently noisy signals are amplified through this transform hence, a fourth order low pass Butterworth filter is used to smooth the instantaneous modal frequency responses and identify the underlying trends. To mitigate the end effects a larger window of the signal is used when applying the Hilbert transform which is then isolated about the crowd loading event.

3.1 Human Structure Interactions

During the 2003 crowd loading event, a large-amplitude vibrational response was observed, a sudden onset in diverging large amplitude vibrations. This was a result of the number of people interacting on the bridge over a two-hour window. As detailed by Arup [1], the critical number of people were achieved for the two-low frequency lateral modes on the CSB. This is reflected in the number of people and bridge response in Figure 2a, and 3a. In 2017, the crowd loading event observed low-amplitude oscillations. The number of people interacting with the bridge were considerably less than the critical number of people for the onset of large amplitude vibrations to occur. The loading was observed over a 20-minute window. Figure 4 and 5 are breakdown plots of instantaneous amplitude, (a), and frequency, (b), time-histories for modes L2 and L3 respectively during the 2003 crowd loading event. Figure 5 and 6 are breakdown plots of instantaneous amplitude and frequency time-histories, for modes L2 and L3 respectively, during the 2017 crowd loading event.

In Figure 4, the instantaneous modal frequency roughly mirrors the amplitude response, for mode L2. Interestingly, it reflects the loading, i.e. the number of people in Figure 2(a), reasonably well. The maximum instantaneous acceleration amplitude, 0.13m/s^2 , and frequency, 0.539Hz , are not observed to correspond at the same instance. These correlate to approximately 350 and 400 pedestrians respectively, suggesting different phasing for the pedestrian, self-excited, forces. This mode observes a frequency increase of 2.8% at the maximum instantaneous frequency observed.

Figure 4 Comparison plot from 2003 crowd loading event, Mode L2 (a) Instantaneous amplitude time-history (b) Instantaneous frequency time

Figure 5 follows similar mirroring in amplitude and frequency response during the 2003 crowd loading event, however, not as well correlated to the number of people. Distinct mirroring is evident in three sections of the time histories during large amplitude responses, $A(t) > 0.05 \text{ m/s}^2$. For mode L3, the maximum instantaneous acceleration amplitude, 0.10 m/s^2 , and frequency, 0.775 Hz , are observed to correspond at roughly the same time. This correlates to an approximate pedestrian number of 350. This mode observes a frequency increase of 3.9% at the maximum instantaneous frequency observed. Interestingly, these instantaneous maxima occur at roughly the same instance, for modes L2 and L3, reinforcing that pedestrian synchronisation didn't occur during the 2003 event.

Figure 5 Comparison plot from 2003 crowd loading event, Mode L3 (a) Instantaneous amplitude time-history (b) Instantaneous frequency time-history

Figure 6 and 7 display low-amplitude responses, making it more difficult to identify signs of human-structure interactions, nevertheless, subtle behaviour is observed. In the first 5 minutes of Figure 6, the increase in amplitude is subtly reflected by an increase in frequency of mode L2. This corresponds to the initial rise in the number of people, during 11:30am and 11:34am, in Figure 2(b). Another region, between 11.38am and 11.40am, indicates a slight increase in amplitude and natural frequency for an increase in loading. This suggests that, at low-amplitude bridge responses, lateral human-structure interactions are still observed. In this case, corresponding to equivalent negative added mass and damping.

Figure 7 displays more erratic behaviour in the instantaneous acceleration amplitude of mode L3. The corresponding instantaneous frequency is also observed to be fluctuating considerably. However, it should be noted that the natural frequency is seen to shift by 5% over the 2017 crowd loading event. At instantaneous amplitude peaks of 0.01 m/s^2 , observed just before 11:32am and 11:34am, the corresponding instantaneous frequency is seen to decrease. This may suggest equivalent positive added mass in comparison to mode L2. Interestingly, the equivalent added mass for mode L3, is observed to be positive at low-amplitude, whilst negative at high-amplitude responses. This may indicate that L3 is susceptible to both negative and positive added mass depending on the number of people and bridge deck amplitude.

Figure 6 2017 crowd loading event, Mode L2 (a) Instantaneous amplitude time-history (b) Instantaneous frequency time-history

Figure 7 2017 crowd loading event, Mode L3 (a) Instantaneous amplitude time-history (b) Instantaneous frequency time-history

3.2 Amplitude-frequency relationships

Using the transformed acceleration measurements of modes L2 and L3, as represented in Figures 4-7, the amplitude-frequency relationships are evaluated for both the low and large amplitude bridge responses. Reduced order, fifth-order, polynomials are fitted to the large-amplitude bridge responses, using nonlinear least-squares

approach, to characterise the complex interactions observed. This is displayed in Figure 8. The low-amplitude bridge responses unfortunately do not display a clear relationship. For the 2003 crowd loading event, mode L2, Figure 8(a), is observed to show an obvious increase in frequency with increase in acceleration amplitude. The same effect is observed for mode L3, Figure 8(b), although its instantaneous frequency increases at a steeper rate, above an amplitude of 0.05m/s^2 . The reduced order models are quantified in equations (1) and (2) and displayed in Figure 9 as the approximated backbone curves of modes L2 and L3 respectively. Both modes observe nonlinear ‘hardening’ effects, i.e. an increase in frequency with an increase with amplitude. L3 witnesses a prominent hardening effect whilst L2 is seen to fluctuate around a frequency of 0.53Hz however, still larger than its linear resonant frequency of 0.524Hz .

Figure 8 Instantaneous acceleration amplitude – frequency relationships with reduced-order polynomial fitting (a) mode L2; (b) mode L3

$$f_{L2} = 0.524 + 0.867A_{L2} - 38.6A_{L2}^2 + 751A_{L2}^3 + 6.34 \times 10^3 A_{L2}^4 + 1.92 \times 10^4 A_{L2}^5 \quad (1)$$

$$f_{L3} = 0.756 + 0.307A_{L3} - 3.88A_{L3}^2 + 49.9A_{L3}^3 + 1.66 \times 10^3 A_{L3}^4 + 8.95 \times 10^4 A_{L3}^5 \quad (2)$$

Figure 9 Evaluated backbone curves of lateral modes L2 and L3 during high-amplitude pedestrian loading

Conclusion

This paper has presented a novel application of the time-frequency analysis technique, Hilbert transform, to qualitatively analyse and characterise the lateral human-structure interactions observed on the Clifton Suspension Bridge. The instantaneous acceleration amplitude and frequency are successfully extracted for two low frequency lateral vibration modes, shown to be susceptible to pedestrian-induced vibrations. The instantaneous frequencies, of both modes, are observed to approximately mirror the acceleration amplitudes during significant loading periods of the crowd loading event witnessing large-amplitude bridge responses. Equivalent negative added mass and damping of both modes are simultaneously observed. During the crowd loading event witnessing low-amplitude bridge responses, tentative evidence of human-structure interactions are observed for each mode. An initial increase in loading is mirrored in instantaneous amplitude and frequency, over brief time periods.

Amplitude-frequency relations are evaluated from reduced-order polynomial fitting to identify the backbone curves of each lateral resonant mode during the large-amplitude response crowd loading event. Nonlinear hardening effects are observed, for both modes, indicating an increase in instantaneous modal frequency with an increase in bridge deck acceleration amplitude.

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