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The effects of in-transit inventory financing on the capital-constrained supply chain

Bangdong Zhi (First author)
School of Management,
University of Bristol, Howard House, Queen’s Avenue, BS8 1SD, Bristol, UK
Email: bangdong.zhi@bristol.ac.uk; Tel: +44 07724809466

Xiaojun Wang (Second and corresponding author)
School of Management,
University of Bristol, Howard House, Queen’s Avenue, BS8 1SD, Bristol, UK
Email: xiaojun.wang@bristol.ac.uk; Tel: (0117) 39 40532

Fangming Xu (Third author)
School of Accounting and Finance,
University of Bristol, 15-19 Tyndalls Park Road, BS8 1TU, Bristol, UK
Email: fangming.xu@bristol.ac.uk; Tel: (0117) 39 41503
The effects of in-transit inventory financing on the capital-constrained supply chain

Abstract
In-transit inventory financing is gaining popularity as an alternative way to access financing. However, compared with other financing means such as trade credit and bank loans, the effects on the supply chain of a third-party logistics provider (TPL) providing in-transit inventory financing are seldom investigated. This study adopts a channel competition model to examine the impact of such financing on the supply chain. Through comparative analysis with conventional financing approaches, we find that the availability of in-transit inventory financing allows retailers to take advantage of a lower financing fee for trade credit, even when the demand for trade credit decreases. In addition, the TPL-provided financing service can lead to a reduction in financing fee for both trade credit and in-transit inventory financing when both the supplier and retailers have financing demand or the TPL is subject to a high degree of risk aversion. Finally, when the TPL has the first-mover advantage in setting its financing fee, the supplier with a financing demand pays a lower financing fee to the TPL due to the decreased demand for in-transit inventory financing. In this case, we find that the low logistics cost, the low unit product price without trade credit and a high financing ratio can make in-transit inventory financing more attractive than trade credit. Our results have important implications for the implementation of in-transit inventory financing in supply chains.

Keywords: in-transit inventory financing; logistics providers; capital constraint; trade credit; channel competition.

1. Introduction
Trade credit has been used widely as a financing scheme. It is particularly useful for small and medium-sized enterprises (SMEs) without an established credit history who have difficulty accessing loans from traditional financial institutions (Yang and Birge, 2018). According to the Federal Reserve Board (2019), trade credit in the United States (US) increased by 17.77% in 2018, amounting to 15.35% of total assets on the aggregated balance sheets of non-financial businesses. In recent years, third-party logistics providers (TPLs), such as UPS, SF Express and Eternal Asia, have been cultivating their financing activities by providing in-transit inventory financing—a financing service facilitated by their superior information on supply chain transactions and strong cash reserves. Through the use of in-transit inventory financing, retailers can obtain external financing more easily and receive price discounts from suppliers. For example, WD Music, a leading US retailer of guitar parts, was able to order components from its suppliers and expand its manufacturing capacity by relying on UPS’s financing service (UPS, 2018). Another example is the financing service from SF Express, through which retailers pay the supplier at a discounted wholesale price, while SF Express controls the products until the retailers sell them off and repay the loan plus a financing fee.
While missing out on early payment discounts is expensive for retailers, an alternative source of credit can motivate retailers to take advantage of such discounts (Petersen and Rajan, 1997). In-transit inventory financing enables retailers to pay the supplier in advance and benefit from discounted wholesale prices (Yang and Birge, 2018). However, an alternative mean of financing also brings competition to the existing financing approach in the supply chain. The increasing use of in-transit inventory financing reduces retailers’ demand for trade credit—an important revenue stream for cash-abundant suppliers. The existing studies on the supply chain financing have addressed several factors, including the ways in which suppliers can finance capital-constrained retailers through trade credit (Haley and Higgins, 1973; Wu et al., 2018; Yang and Birge, 2018; Feng and Chan, 2019), and the potential for capital-constrained retailers to access financing services from peer-to-peer (P2P) lending platforms (Gao et al., 2018). Moreover, several studies have explored the role of TPLs in supply chain financing, such as the effect of TPL-provided credit on capital-constrained supply chains (Chen and Cai, 2011; Huang et al., 2019), the cash flow advantages enjoyed by firms that use TPLs for financing (Chen et al., 2019), and the impawn rate optimisation of TPL-provided inventory financing (Zhi et al., 2020). However, despite the increasing popularity of the integrated logistics and financial services offered by TPLs, to our knowledge, no research has yet systematically examined the effects of TPL-provided in-transit inventory financing on the supply chain while also considering the competition between alternative financing channels. In response to this oversight, this study examines how TPL-provided in-transit inventory financing affects the supply chain when retailers need external financing. In many cases, the supplier also faces financing demands despite providing trade credit service to retailers and may seek financing services from TPLs (e.g., UPS and DHL). Therefore, what are the effects of TPL-provided in-transit inventory financing on the supply chain when both the supplier and retailers have financing demands? Additionally, the TPL may be risk sensitive when providing financing service to the supplier and retailers. It is, therefore, interesting to examine whether and how the TPL’s risk-aversion can affect the financing decisions of the supply chain members. Furthermore, TPLs can gain more power by providing financing service and becoming dominant players in the supply chain. Consequently, how does the power relationship in the supply chain influence the effects of TPL-provided in-transit inventory financing on the supply chain?

To answer the above questions, we formulate a financing channel competition model by considering the substitution effects of financing schemes provided by a supplier and a TPL. Our work contributes to the existing literature in four ways. First, we explore the effects of TPL-provided in-transit inventory financing on the supply chain with financing demands while taking into consideration the competition between trade credit and in-transit inventory financing. This differentiates our study from previous studies that have largely analysed the use of either trade credit or inventory financing by downstream retailers separately (Chen and Cai, 2011; Chen et al., 2019; Huang et al., 2019). Our analysis also shows how logistics fees and costs affect the financing costs and the demand for trade credit and in-transit inventory financing, thereby providing important insights for TPLs. Second, we systematically examine the effects of in-transit inventory financing on both partially (in which the retailer has a financing demand) and fully capital-constrained supply chains (in which both the retailer and the supplier have financing demands). To the best of our knowledge, most existing studies only consider the financing demands of retailers (Chen and Cai, 2011; Chen et al., 2019; Huang et al., 2019).
Third, we investigate how the degree of TPL’s risk-aversion can affect the impact of the TPL-provided in-transit inventory financing on the capital-constrained supply chain, which contributes to the existing literature that mostly assumes all relevant parties in the capital-constrained supply chain are risk-neutral (Chen and Cai, 2011; Yang and Birge, 2018). Finally, we take into consideration the supply chain power relationship by studying how power relationships and other financial and operational factors influence the effects of TPL-provided in-transit inventory financing on the supply chain. This consideration complements existing studies that assume the supply chain is dominated by one specific member (e.g., the finance-sufficient supplier) (Chen and Cai, 2011; Huang et al., 2019).

The remainder of this paper is organised as follows: Section 2 reviews prior research in related areas, Section 3 describes the financing channel competition models, and Section 4 presents a benchmark model in which retailers can obtain commercial loans from conventional financial institutions. Through a comparison with the benchmark model, we reveal the optimal strategy and the advantages of TPL-provided in-transit inventory financing to retailers who have financing demands. Section 5 extends the analysis to a supply chain setting where both the retailers and the supplier have financing demands. Section 6 studies the impact of TPL’s risk preference and supply chain power relationships on the effect of TPL-provided in-transit inventory financing on the supply chain. A numerical analysis is provided in Section 7, while Section 8 discusses the key findings, managerial implications and recommendations for future research. All proofs are presented in the Appendix.

2. Literature Review

With its focus on examining the effect of in-transit inventory financing on the capital-constrained supply chain, our research is closely related to three streams of inquiry found in the literature: the capital-constrained supply chain, in-transit inventory financing and channel competition.

The first stream investigates decisions made in capital-constrained supply chains with trade credit. There are two types of commonly used trade credit terms: net terms and two-part terms (Wilson and Summers, 2002; Klapper et al., 2012; Seifert et al., 2013; Yang and Birge, 2018). Net terms are interest-free loans provided by a supplier to a retailer. For instance, ‘net 30’ requires the retailer to make a payment to the supplier within 30 days of invoice issuance (Yang and Birge, 2018). Two-part terms allow the retailer to benefit from a discounted price if they pay within a certain period (Klapper et al., 2012). For instance, 2/10 Net 30, a widely used two-part term, means a buyer can obtain a 2% discount by paying within ten days or less. Otherwise, the undiscounted payment is required within 30 days of invoice issuance (Wilson and Summers, 2002). In operations management, early studies on trade credit can be traced back to Haley and Higgins (1973), who studied how members in a supply chain make decisions regarding optimal order quantity and payment with trade credit under an economic order quantity (EOQ) model. Based on this seminal work, numerous studies have investigated how various factors affect a firm’s optimal decision-making strategies under trade credit conditions. For instance, Zhang et al. (2018) explored the effect of customer balking in a two-level supply chain under trade credit. They demonstrated that the wholesale price and order quantity at equilibrium increase with balking probability when the production cost is relatively low. By considering the effects of both in-transit
and retail deterioration on trade credit, Lin et al. (2019) established more feasible solutions for the joint economic lot-sizing problem. Furthermore, based on the assumption that the manufacturer’s production cost follows a learning curve effect, Feng and Chan (2019) constructed an inventory model to determine optimal lot-sizing and pricing strategies for both upstream and downstream trade credit. They concluded that the learning curve effect significantly lowers the selling price but increases both demand and profit. While the above-mentioned papers analyse the effects of various factors on trade credit policies, however, the impact of other financing schemes on such policies is seldom investigated. By focusing on the substitution effect of in-transit inventory financing on trade credit, this research provides some new insights into how the competition between in-transit inventory financing services provided by TPLs and trade credit provided by suppliers affects supply chain decisions and performance.

A second relevant research stream focuses on the financing role of TPLs in the supply chain. Hofmann (2009) first attempted to develop the concept of inventory financing, offering initial insights on the role of TPLs in the field. He demonstrated that the value and quantity of goods can have a strong effect on the profit yielded by inventory financing businesses. Li and Chen (2018) presented multiple case studies to show how TPLs use financial services to generate a sustainable competitive advantage, while, in contrast, other studies have explored this problem using analytical approaches. Specifically, Chen et al. (2019) investigated the advantages of TPL-provided financial assistance for capital-constrained retailers and demonstrated that supply chain profit can be higher under the leadership of TPLs than under that of manufacturers. Chen and Cai (2011) built an extended supply chain model with a supplier, a budget-constrained retailer, a bank and a TPL, and compared the different roles of the TPL in providing financial services. They demonstrated that the entire supply chain performs better in a control role model where the TPL integrates logistics with financial services. The papers discussed above mainly investigated the advantages of TPL-provided financial services in the supply chain. However, there has been little research into strategies that help TPLs to develop in-transit inventory financing services. By adopting the financing channel competition model, we are able to identify optimal strategies that can be adopted by TPLs to develop in-transit inventory financing when facing competition from trade credit provided by suppliers.

Our research is also closely related to channel competition. The existing literature on channel competition mainly concentrates on vertical competition and horizontal competition. The former focuses on the channel competition between manufacturers and retailers. For example, Giri and Roy (2016) studied how a single manufacturer competes with multiple retailers on trading channels and found that the pricing strategies and effort levels of supply chain entities were heavily affected by the direct and retail channels’ market shares. The latter focuses on two areas: competition among manufacturers (Choi, 1996; McGuire and Staelin, 2008) and competition among retailers (Brynjolfsson et al., 2009). Choi (1996) modelled price competition among manufacturers with multiple common retailers and demonstrated that product differentiation is beneficial to manufacturers while store differentiation is beneficial to retailers. McGuire and Staelin (2008) investigated the effect of product substitutability on a Nash-equilibrium distribution structure in a duopoly where two manufacturers distribute their products through their own channels. Brynjolfsson et al. (2009) empirically
investigated the competition levels among e-commerce and brick-and-mortar retailers, and found that the competition structure depends on the characteristics of the products. Specifically, e-commerce retailers faced strong competition from brick-and-mortar retailers when selling mainstream products. The studies mentioned above mainly investigated retail and distribution channel competition, rather than the competition among financing channels. In contrast to those studies, we introduce channel competition into the financing services and study the substitution effect between trade credit and in-transit inventory financing.

Among the relevant studies, this research is closely related to Chen and Cai (2011), Chen et al. (2019) and Huang et al. (2019), all of which explored the role of TPLs in providing financing in the supply chain. However, this study differs from them in several respects. First, both Chen et al. (2019) and Huang et al. (2019) did not consider the scenario where retailers can also access trade credit, while Chen and Cai (2011) separately considered the cases where a retailer can access either trade credit or financing from a TPL. This study explores the case where retailers can access trade credit from a supplier as well as inventory financing from a TPL and examines the substitution effects between these two financing schemes. In addition, we take retailers’ preference for trade credit into consideration and examine the effect of in-transit inventory financing on the capital-constrained supply chain under different levels of preference for trade credit. Second, in comparison to studies that assume the supplier has no financing demand, we further consider the case where a supplier has a capital constraint and investigate what impact the financing service provided by the TPL has on the capital-constrained supply chain. Finally, in contrast to Chen and Cai (2011) and Chen et al. (2019), we consider TPL’s risk preference by examining how the degree of its risk-aversion affects the decision equilibrium in the capital-constrained supply chain. Furthermore, we investigate how different supply chain power structures influence the effects of TPL-provided in-transit inventory financing on the supply chain.

3. Model Framework

Our model consists of multiple retailers, one supplier and one TPL. We assume that all supply chain members are risk-neutral. Retailers have financing demands and have limited access to commercial loans from banks. They may not have sufficient capital to make an immediate payment when placing an order with their supplier. Unable to make payments immediately, retailers may seek trade credit from the supplier but will have to pay a higher unit product price. This is in line with the two-part terms discussed in the literature review, through which the retailer can enjoy a discounted price by making an early payment (Klapper et al., 2012). Existing empirical studies (Petersen and Rajan, 1997; Klapper et al., 2012) show that the supplier would offer discounts to high-risk buyers in return for early payment. In this study, retailers are assumed to have a high-risk profile and difficulty accessing credit from banks. Alternatively, retailers can seek a short-term loan from the TPL in the form of in-transit inventory financing. In this way, retailers are able to buy products from the supplier at a discounted price but have to pay a fee (i.e., interest) for the inventory financing service from the TPL. The two alternative funding sources result in channel competition, as illustrated in Figure 1. While the supplier and the TPL separately set the financing fee for trade credit and in-transit inventory financing, respectively, the fee level for each financing channel is influenced by the fee level of the competing channel.
The key variables and parameters of this study are described in Table 1. There are two decision variables: \( w_{S1} \) (denoted as the unit product price with trade credit set by the supplier) and \( f_T \) (denoted as the unit financing fee for in-transit inventory financing set by the TPL). The supplier allows retailers who have a financing demand to buy products without making immediate payments, but the retailers must pay a higher unit product price when the receivables are due. With in-transit inventory financing, retailers are charged a low unit product price without trade credit—\( w_{S2} \) (\( w_{S2} < w_{S1} \))—but have to pay the TPL a unit financing fee \( f_T \) for the inventory financing service. The maximum demand for products from retailers who have a financing demand is \( D \), which can be financed through one of two channels: trade credit or in-transit inventory financing. Demand for trade credit \( q_S \) is mainly affected by the unit product price with trade credit and the difference between the financing fee of the two financing channels \( f_T - (w_{S1} - w_{S2}) \), in which \( w_{S1} - w_{S2} \) is the financing fee for trade credit. The sensitivity of retailers to the fee difference between the two financing schemes is represented by \( \theta \). Similar to the existing channel competition literature (Chen and Iyer, 2002; Cai et al., 2013; Chen et al., 2014), we assume that the demand for trade credit is also affected by retailers’ preference for trade credit \( \mu \), which is influenced by retailers’ previous experience and funding cycle of trade credit. Based on the transaction cost theory, trade credit reduces the transaction cost of paying bills (Ferris, 1981; Fisman and Love, 2003; Seifert et al., 2013). For example, retailers can cumulate obligations and pay them only monthly or quarterly, which can separate the payment cycle from the delivery schedule (Petersen and Rajan, 1997). Therefore, the demand for products financed by trade credit can be represented as

\[
q_S = D[\mu - w_{S1} + \theta(f_T - (w_{S1} - w_{S2}))]
\]  

This type of demand function is commonly used in marketing and operations management (McGuire and Staelin, 2008; Chen and Chang, 2013; Mitra, 2016). Similarly, the quantity of products financed with in-transit inventory financing can be represented as
\[ q_T = D[(1 - \mu) - w_{S2} - f_T + \theta((w_{S1} - w_{S2}) - f_T)] \]  

(2)

The logistics market is assumed to be competitive, and the logistics fee is externally given and is undertaken by the retailers. To focus on the channel competition between two financing schemes, we assume that the demand for each financing scheme is mainly affected by the fee associated with each financing scheme. Correspondingly, the profit functions of the supplier and the TPL are represented by Eqs. (3) and (4), respectively.

\[
\pi^N_S = (w_{S1} - c_s)q_s + (w_{S2} - c_s)q_T
\]

(3)

\[
\pi^N_T = f_T q_T + (w_L - c_L)q_T
\]

(4)

Specifically, \(f_T q_T\) is the profit that the TPL gains from its in-transit inventory financing service, \((w_L - c_L)q_T\) is the profit from its logistics service, and \(N\) denotes a case where the supplier has no capital constraints. Furthermore, the profit of retailers can be represented as

\[
\pi^N_R = (p - w_{S1} - w_L)q_s + (p - w_{S2} - f_T - w_L)q_T
\]

(5)

Similar to Yang and Birge (2018), we assume that the retail price is externally given and consistent due to the market competition.

**Table 1. Parameters and variables.**

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{S1})</td>
<td>Unit product price with trade credit</td>
</tr>
<tr>
<td>(f_T)</td>
<td>Unit financing fee for in-transit inventory financing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D)</td>
<td>Maximum demand from retailers who have financing demand</td>
</tr>
<tr>
<td>(w_{S2})</td>
<td>Unit product price without trade credit</td>
</tr>
<tr>
<td>(q_s)</td>
<td>Demand for trade credit</td>
</tr>
<tr>
<td>(w_{S1} - w_{S2})</td>
<td>Unit financing fee for trade credit</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Sensitivity of retailers to the difference in financing fees between the two financing schemes</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Retailers’ preference for trade credit</td>
</tr>
<tr>
<td>(q_T)</td>
<td>Demand for in-transit inventory financing</td>
</tr>
<tr>
<td>(c_s)</td>
<td>Unit product cost</td>
</tr>
<tr>
<td>(w_L)</td>
<td>Unit logistics fee</td>
</tr>
<tr>
<td>(c_L)</td>
<td>Unit logistics cost</td>
</tr>
<tr>
<td>(p)</td>
<td>Retail price of unit product</td>
</tr>
</tbody>
</table>

We formulated a Nash game model to examine the effect of in-transit inventory financing on the capital-constrained supply chain. The sequence of events is illustrated in Figure 2. Based on the predetermined unit product price without trade credit \(w_{S2}\) and unit logistics fee \(w_L\), the supplier and the TPL simultaneously decide the unit product price with trade credit and the unit fee for in-transit inventory financing, respectively.
When the receivables are due, the supplier receives payments from retailers who selected trade credit, while the TPL receives payments from retailers who selected in-transit inventory financing.

**Figure 2.** Sequence of events when the TPL provides in-transit inventory financing.

4. Effect of In-transit Inventory Financing on the Partially Capital-constrained Supply Chain

4.1 Benchmark Model

To investigate the effects of the TPL-provided in-transit inventory financing service, we first established a benchmark model in which retailers who have financing demands can access commercial loans from a financial institution. In this case, the demand for conventional finance services can be expressed by

\[ q_F = D[(1 - \mu) - w_{S2} - f_F + \theta((w_{S1} - w_{S2}) - f_F)] \]

while the financial institution’s profit from conventional financing is \( \pi_F^N = f_F q_F \). The equilibrium decisions for the supplier and the financial institution for this case are shown in Table A.1, where BN means that the financial institution provides a financing service in the partially capital-constrained supply chain. In Table A.1, \( w_{1S} \) and \( f_{1F} \) are the thresholds of unit product price with trade credit and the financing fee charged by the financial institution. The relevant proof is provided in Appendix A.1.

Through a comparison of equilibriums in Table A.1, we derive the following lemma:

**Lemma 1:** (1) For trade credit, (i) the unit financing fee \( w_{S2}^{BN} - w_{S2} \) decreases with the unit product price without trade credit \( w_{S2} \); (ii) if \( \mu_{S1}^{BN} \leq \mu < \mu_{S2}^{BN} \), the demand for trade credit increases with the unit product price without trade credit \( w_{S2} \); if \( \mu_{S2}^{BN} \leq \mu \leq \mu_F^{BN} \), the demand for trade credit decreases with the unit product price without trade credit \( w_{S2} \).

(2) For conventional financing, (i) the unit financing fee \( f_{1F}^{BN} \) decreases with the unit product price without trade credit \( w_{S2} \); (ii) if \( 0 \leq \mu \leq \mu_F^{BN} \), the demand for conventional financing decreases with the unit product price without trade credit \( w_{S2} \).

Lemma 1 indicates that the unit financing fees of both trade credit and conventional financing decrease with the unit product price without trade credit \( w_{S2} \). As the financing cost contributes to the product price, a higher unit product price will drive down the fees of the two alternative financing services in order to compete.

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1 In reality, this is a special case because retailers often lack the fixed assets necessary to obtain a loan from the financial institution. The reason we provide this benchmark model is to identify the effectiveness of in-transit inventory financing.
for demand. Consequently, a lower fee for trade credit will increase its demand when retailers’ preference for trade credit is within the threshold range $\mu_{S_1} \leq \mu \leq \mu_{S_2}^{RN}$. Significantly, a low level of retailers’ preference for trade credit ($0 \leq \mu \leq \mu_{S_1}^{RN}$) will incentivise the supplier to reduce the unit financing fee ($w_{S_1}^{RN} - w_{S_2}$) of trade credit further, and, as a result, the demand for conventional financing decreases.

Based on the optimal solutions in Table A.1, we can obtain the optimal profit gained by the financial institution:

$$
\pi^F = \begin{cases} 
\frac{\beta[(1 - \mu) - (\theta + 1)w_{L} + \theta w_2]^2}{4(\theta + 1)} & 0 \leq \mu < \mu_{S_2}^{RN} \\
\frac{\beta (1 + \theta)[2(1 - \mu) - 2(\theta + 1)w_{L} + \theta w_2 + \theta(2 - \mu)]^2}{(3\theta^2 + \theta + 4)^2} & \mu_{S_2}^{RN} \leq \mu < \mu_{B_1}^{EN} \\
0 & \mu_{B_1}^{EN} \leq \mu < 1 
\end{cases} 
$$

Eq. (5) suggests that when retailers’ preference for trade credit is lower than the threshold $\mu_{S_1}^{RN}$, the profit gained by the financial institution is negatively affected by the unit product price without trade credit ($w_{S_2}$). This is intuitive. If the unit product price without trade credit is high, the retailer will be less motivated to pay the supplier in advance and thus keener to use the trade credit provided by the supplier. Otherwise, the retailer is more willing to use the conventional financing provided by the financial institution, benefitting from the low unit product price without trade credit.

### 4.2 In-transit Inventory Financing Model

In in-transit inventory financing, the TPL provides both financing and logistics services to retailers. The optimal strategies devised by the supplier and the TPL are shown in Table A.1, where $EN$ means that the TPL provides in-transit inventory financing to a partially capital-constrained supply chain. In Table A.1, $w_{L}$ and $f_{IT}$ are the thresholds of the unit product price with trade credit and the financing fee of the TPL. Based on the results obtained from Table A.1, we derive Lemma 2 to show how logistics-related factors (i.e., the unit logistics fees and costs) affect the equilibrium decision under each financing scheme.

**Lemma 2:** (1) For trade credit, (i) if $\hat{\mu}_S^{EN} \leq \mu \leq \hat{\mu}_T^{EN}$, the unit financing fee ($w_{S_1}^{RN} - w_{S_2}$) decreases with $w_L$; (ii) if $\hat{\mu}_S^{EN} \leq \mu \leq \hat{\mu}_T^{EN}$, the demand for trade credit increases with $c_L$.

(2) For in-transit inventory financing, (i) if $0 \leq \mu \leq \hat{\mu}_T^{EN}$, the unit financing fee $f_{IT}^{EN}$ decreases with $w_L$; (ii) if $0 \leq \mu \leq \hat{\mu}_T^{EN}$, the demand for in-transit inventory financing decreases with $c_L$.

This lemma indicates that the unit financing fees of both trade credit and in-transit inventory financing are decreasing functions of the unit logistics fee $w_L$. With the increase in the logistics fee, the TPL is motivated to charge a lower financing fee, as it is able to gain more revenue from his logistics service. This intensifies the competition between the two financing services and makes the supplier decrease the financing fee of trade credit. Regarding the effect of logistics cost on the demand for each financing scheme, as the logistics cost increases, the TPL is less willing to decrease the financing fee of in-transit inventory financing, resulting in an increased demand for trade credit. In contrast, as the logistics cost decreases, the TPL can charge a lower financing fee, which will increase the demand for in-transit inventory financing. This lemma shows that major
logistics service providers such as UPS and DHL have an advantage in providing in-transit inventory financing because of their relatively low logistics cost and their economies of scale. Accordingly, the optimal profit gained by the TPL is given by

$$
\pi^* = \begin{cases} 
\frac{D[(1-\mu) - (\theta + 1)w_c + \theta w_o + (\theta + 1)(w_o - c_o)]}{K \theta + 1} & 0 \leq \mu < \mu^R \\
\frac{D(1+\theta)[-2w_o(2\theta + 1) + c_o \theta + (2-\mu)(\theta + 2(1-\mu)) + (\theta^2 + 4 + 2)(w_o - c_o)]}{2(\theta + 1)} & \mu^R \leq \mu < \mu^L \\
\frac{D[w_o \gamma - (\theta + 4 + 2)\gamma w_o + \beta_o + (2-\mu)\gamma + (2-\mu)](w_o - c_o)}{2(\theta + 1)} & \mu^L \leq \mu < 1 \\
0 & \mu \geq 1
\end{cases}
$$

(7)

Eq. (6) suggests that when retailers’ preference for trade credit is lower than the threshold $\hat{\mu}_T$, as the logistics cost $c_L$ decreases, the TPL can make a larger profit from in-transit inventory financing. This is intuitive because when the logistics cost is low, the TPL can gain more profit from its traditional logistics services. Therefore, it will charge a lower fee for in-transit inventory financing, which can increase the demand for both logistics services and in-transit inventory financing (see the proof in Appendix A.4).

Next, we examine the effect of in-transit inventory financing on the financing fee of trade credit and derive the following proposition:

**Proposition 1:** (1) In the EN model, if $w_L = \frac{1+w_{S2} - 2\mu - c_S}{1 + \theta}$, the financing fee of in-transit inventory financing is lower than the financing fee of trade credit, i.e., $f_{T}^{EN} < w_{S1}^{EN} - w_{S2}$; if $w_L > \frac{1+w_{S2} - 2\mu - c_S}{1 + \theta}$, the financing fee of in-transit inventory financing is higher than the financing fee of trade credit, i.e., $f_{T}^{EN} > w_{S1}^{EN} - w_{S2}$.

(2) When the TPL replaces the financial institution by providing in-transit inventory financing, if $0 < \mu < \hat{\mu}_F^{BN}$, compared with conventional financing, the fee for in-transit inventory financing is lower, i.e., $f_{T}^{EN} < f_{F}^{BN}$. If $\hat{\mu}_S^{BN} < \mu < \hat{\mu}_F^{BN}$, the supplier will decrease the financing fee of the trade credit, i.e., $w_{S1}^{EN} - w_{S2} < w_{S1}^{BN} - w_{S2}$.

This proposition implies that the TPL charges a lower financing fee than the supplier when its logistics cost is below a certain threshold. Intuitively, when the logistics cost is relatively low, the TPL has the motivation to lower its financing fee to attract more retailers to adopt in-transit inventory financing. Higher usage of in-transit inventory financing will increase the revenue from the TPL’s logistics service. In addition, the threshold is negatively affected by the unit price without trade credit. This means that a lower unit price without trade credit makes it easier for TPL to set a lower financing fee. This finding provides some practical insights into the operation of TPLs. TPLs who are able to control logistics costs will have a greater competitive advantage in providing financing service. This advantage becomes more evident when the unit price without trade credit is low.

If the TPL charges a lower financing fee than the financial institution when retailers’ preference for trade credit is below a certain threshold ($\hat{\mu}_F^{BN}$), the difference disappears when retailers’ preference for trade credit is relatively high ($\hat{\mu}_F^{BN} \leq \mu \leq 1$). This happens because when retailers’ preference for trade credit is relatively high, the financing service provided by a financial institution or a TPL becomes less competitive despite the
low financing fee. In addition, although the TPL can set a lower financing fee than that of the financial institution, there is a limited range in which to set the optimal financing fee. The TPL only represents an optimal decision when the retailers’ preference is between 0 and $\hat{\mu}_{EN}$ ($\hat{\mu}_{EN} < \hat{\mu}_{FN}$), whereas the financial institution represents the optimal decision when retailers’ preference for trade credit is between 0 and $\hat{\mu}_{FN}$. In addition, when retailers’ preference for trade credit is between $\hat{\mu}_{EN}$ and $\hat{\mu}_{FN}$, as it increases, the difference between the financing fees charged by the financial institution and the TPL decreases. This shows that the advantage of in-transit inventory financing is more evident in the industry when retailers’ preference for trade credit is relatively low.

Furthermore, when retailers’ preference for trade credit is between $\hat{\mu}_{SN}$ and $\hat{\mu}_{FN}$, the in-transit inventory financing provided by the TPL decreases the trade credit financing fee. This indicates that the in-transit inventory financing service is beneficial to retailers who have financing demands as it intensifies the competition and pushes down the financing fees of the two competing financing schemes. However, when retailers’ preference for trade credit is relatively low ($0 < \mu \leq \hat{\mu}_{SN}$), trade credit is not affected by in-transit inventory financing. In this case, the supplier will set a low unit product price with trade credit to compete with the TPL’s in-transit inventory financing service. When retailers’ preference for trade credit is relatively high ($\hat{\mu}_{FN} \leq \mu < 1$), in-transit inventory financing also has little impact on trade credit. In this case, the financing fees set by the financial institution and the TPL do not affect retailers’ financing decisions because of their previous experience with trade credit. Therefore, a supplier is likely to adopt the same pricing strategies. For instance, Haier, a leading manufacturer of consumer white goods in China, established a financing platform known as ‘hairongyi’ that facilitates the provision of trade credit services to retailers with flexible funding cycles (Hairongyi, 2018). This enables Haier to adopt stable pricing strategies in competition with other financing services.

We now explore the effect of in-transit inventory financing on the demand for each financing scheme and derive the following proposition:

**Proposition 2:** If $0 \leq \mu < \hat{\mu}_{FN}$, the demand for in-transit inventory financing will be higher than the demand for conventional financing, i.e., $q_{EN}^{FN} > q_{FN}^{BN}$. If $\hat{\mu}_{SN} < \mu < \hat{\mu}_{BN}$, the demand for trade credit decreases, i.e., $q_{SN}^{EN} < q_{SN}^{BN}$.

This proposition reveals that when retailers’ preference for trade credit is between 0 and $\hat{\mu}_{FN}$, the lower financing fee charged by the TPL makes in-transit inventory financing more popular than conventional financing. When retailers’ preference is between $\hat{\mu}_{EN}$ and $\hat{\mu}_{FN}$, as retailers’ preference for trade credit increases, the difference between the demand for conventional financing and the demand for in-transit inventory financing decreases. This happens because when retailers’ preference for trade credit is above $\hat{\mu}_{EN}$, the TPL’s advantage over the conventional finance institution in setting a lower financing fee diminishes. This advantage does not make any difference when retailers’ preference for trade credit is above $\hat{\mu}_{FN}$, so the TPL and financial institution would set the lowest fixed financing fee. Significantly, when retailers’ preference for
trade credit is between $\hat{\mu}_{SN}^{BN}$ and $\hat{\mu}_{FN}^{BN}$, although the supplier reduces the financing fee of trade credit, its demand decreases. Compared with the case of the conventional financing service, the TPL-provided in-transit inventory financing service can set a more competitive financing fee because of the multiple income sources from logistics and financial services, which will force the supplier to make trade credit more attractive to retailers. When retailers’ preference for trade credit is above $\hat{\mu}_T^{EN}$, the TPL would charge the lowest financing fee to ensure market competitiveness. As retailers’ preference for trade credit increases further, the demand for trade credit will increase swiftly until retailers’ preference for trade credit reaches the point where the financial institution sets the lowest fixed financing fee.

Based on the above two propositions, we can further analyse the performance of the financial institution and the TPL and derive the following proposition:

**Proposition 3**: Compared with the case where the supplier is competing with the financial institution to provide financial services in the supply chain, (i) if $0 \leq \mu < \hat{\mu}_{FN}^{BN}$, the TPL can make a larger total profit than the financial institution, i.e., $\pi_T^{EN} > \pi_F^{BN}$; (ii) if $0 \leq \mu < \hat{\mu}_T^{BN}$, the TPL earns less profit from the in-transit inventory financing than the bank from the conventional financing service, i.e., $\pi_T^{BN} > \pi_T^{EN}$; if $\hat{\mu}_T^{BN} \leq \mu < \hat{\mu}_{FN}^{BN}$, the TPL and the bank earn the same profit from their financing service. Here, $\pi_T^{EN} = f_T^{EN} q_T^{EN}$.

Proposition 3 shows that when retailers’ preference for trade credit is between 0 and $\hat{\mu}_{FN}^{BN}$, the TPL earns a greater total profit than the financial institution even when it charges a lower financing fee. This is mainly due to the TPL’s dual revenue streams: financing services and logistics. Taking an extreme case as an example, if the TPL cannot earn a profit from its logistics service (i.e., $w_L = c_L$), its profit margin will be the same as the financial institution’s; thus, it would follow the financial institution in charging the same financing fee to retailers. Although the TPL can gain extra profit from its logistics service, the difference in profit between the two financial service providers diminishes marginally when retailers’ preference for trade credit is relatively high ($\hat{\mu}_T^{EN} \leq \mu < \hat{\mu}_{FN}^{BN}$). However, when the profit of logistics service is deducted from the TPL’s total profit, the TPL does not earn a greater profit than the bank. More specifically, when retailers’ preference for trade credit is extremely high ($\hat{\mu}_T^{BN} \leq \mu < \hat{\mu}_T^{EN}$), both the financial institution and the TPL become less competitive despite the lowest financing fee being set by them. In this case, they earn the same profit from their financing service. When retailers’ preference is between 0 and $\hat{\mu}_T^{BN}$, the TPL earns less profit than the bank. This result indicates an interactive effect between the logistics service and in-transit inventory financing service. The advantage of in-transit inventory financing disappears if the TPL separates its logistics and financing services. This result is echoed by industry practices. WD Music, who adopted the in-transit inventory financing service provided by UPS, claimed that it had formed a long-term relationship with USP regarding the usage of logistics service (UPS, 2018).

**Proposition 4**: Compared with the case where the supplier is competing with the financial institution to provide financial services in the supply chain, (i) $\hat{\mu}_T^{BN} \leq \mu \leq 1$, retailers gain the same total profit, i.e., $\pi_R^{EN} =$
\( \pi^B_{R} \); (ii) if \( 0 \leq \mu < \hat{\mu}^B_{F} \) and \( p > \max(\hat{p}_1, \hat{p}_2, \hat{p}_3) \), the total profit gained by retailers is higher, i.e., \( \pi^E_{R} > \pi^B_{R} \).

Proposition 4 indicates that when retailers’ preference for trade credit is between \( \hat{\mu}^B_{F} \) and 1, retailers gain same profit. This happens because both the TPL and financial institution charge lowest financing fee in this case. The fierce competition forces the supplier to charge the same unit financing fee for trade credit. Correspondingly, the unchanged financing fee and the demand for each financing scheme results in equal profit of retailers. When retailers’ preference for trade credit is between 0 and \( \hat{\mu}^B_{F} \), there is no demand for trade credit in both the BN and EN models. However, the TPL charges a lower financing fee than the financial institution, and the demand for in-transit inventory financing is higher than that for the conventional financing fee. Therefore, retailers earn a greater profit after adopting in-transit inventory financing. Furthermore, when retailers’ preference for trade credit is between \( \hat{\mu}^B_{S_1} \) and \( \hat{\mu}^B_{F} \), the TPL charges lower financing fee than the financial institution and the demand for trade credit decreases, resulting greater total profit for retailers.

5. Effect of In-transit Inventory Financing on the Capital-constrained Supply Chain

In many cases, the supplier also has a financing demand despite providing trade credit to retailers. To satisfy the financing demand from the supplier, leading TPLs like UPS also provide financing services to suppliers based on supply chain transaction information. This section investigates how the extra financing service provided by a TPL affects the supply chain when both the supplier and the retailers have financing demands.

5.1 Sequence of Events

The sequence of events for this case is illustrated in Figure 3. Initially, based on the given unit product price without trade credit \( (w_{S_2}) \) and unit logistics fee \( (w_L) \), the supplier and the TPL simultaneously determine the unit product price with trade credit and the fee for in-transit inventory financing, respectively. To satisfy the orders from the retailers, based on the interest rate charged by the TPL, the supplier requests the capital required to produce the products. The retailers who adopt in-transit inventory financing then pay the supplier at the discounted price. When the receivables are due, the supplier and the TPL receive payment from retailers who adopted their financing schemes.

![Figure 3](image-url)

**Figure 3.** Sequence of events when the TPL provides extra financing service to the supplier.

The following equations describe the profit functions of the supplier and the TPL in the case where the TPL provides extra financing service to the supplier. \( r \) is the interest rate. Based on the transaction amount \( q_T \), the TPL provides capital \( \rho c_5 q_T \) to the supplier to help it fulfil the order from the retailers. \( \rho \) (0 \( \leq \rho \leq 1 \)) is the
financing ratio determined by the supplier. \( \rho = 0 \) means the supplier has no financing demand, while \( \rho = 1 \) means that supplier exhausts the credit and asks the TPL to provide full financing support.

\[
\pi^C_S = (w_{S1} - c_S)q_S + (w_{S2} - c_S)q_T - \rho c_S q_T r \\
\pi^C_T = f_T q_T + (w_L - c_L)q_T + \rho c_S q_T r
\]

Here, \( C \) is the case where both the supplier and the retailer have financing demands. \( \pi^C_T = f_T q_T + (w_L - c_L)q_T \) is the profit from the in-transit inventory financing service, and \( \pi^C_T = \rho c_S q_T r \) is the profit from the extra financing service to the supplier.

5.2 Equilibrium Analysis

The equilibrium solutions are displayed in Table A.2, which shows the optimal decisions for the supplier and the TPL when the TPL provides the extra financing service to the supplier. In Table A.2, EC represents the case where the TPL provides in-transit inventory financing to the capital-constrained supply chain. In Table A.2, \( w_{S1} \) and \( f_T \) are the thresholds of unit product price with trade credit and the financing fee of the TPL. To understand how the financing decision is influenced by relevant factors, we derive the following lemma:

**Lemma 3:** (1) For in-transit inventory financing, (i) if \( 0 \leq \mu < \hat{\mu}^T \), the unit financing fee \( f_T^E \) decreases with \( r \); (ii) if \( 0 \leq \mu < \hat{\mu}^S \), its demand \( q^E_T \) increases with \( r \); if \( \hat{\mu}^T \leq \mu < \hat{\mu}^S \), its demand decreases with \( r \).

(2) For trade credit, (i) if \( \hat{\mu}^S \leq \mu \leq 1 \), the unit financing fee \( (w^E_{S1} - w_{S2}) \) decreases with \( r \); (ii) if \( \hat{\mu}^S \leq \mu < \hat{\mu}^E \), its demand \( q^E_S \) decreases with \( r \); if \( \hat{\mu}^E \leq \mu \leq 1 \), its demand increases with \( r \).

This lemma shows that the fee for in-transit inventory financing decreases with the increase in the interest rate charged by the TPL when retailers’ preference for trade credit is between 0 and \( \hat{\mu}^T \). The explanation for this is intuitive. On the one hand, a higher interest rate improves the profitability of the TPL’s extra financing service to the supplier, while, on the other, it makes the financing fee of the supplier’s trade credit service less competitive compared to the TPL’s in-transit inventory financing service. However, the situation changes when retailers’ preference for trade credit is relatively high \( (\hat{\mu}^T \leq \mu < \hat{\mu}^S) \). In this case, the TPL charges the lowest fixed financing fee to protect the profit gained from its in-transit inventory financing service. However, the higher interest rate continually lowers the trade credit financing fee, reducing the demand for in-transit inventory financing.

When retailers’ preference for trade credit is high \( (\hat{\mu}^S \leq \mu \leq 1) \), as the interest rate of the extra financing service provided by the TPL increases, the financing fee for trade credit decreases. This is due to the fact that a higher interest rate increases the profit margin of the TPL’s extra financing service to the supplier, which enables the TPL to set a more competitive financing fee for its in-transit inventory financing service. In response, the supplier has to reduce the trade credit financing fee to compete with the TPL’s inventory financing service. Therefore, a lower fee will increase the demand for trade credit, and the supplier can still ensure the profit margin of its trade credit service when retailers’ preference for trade credit is high enough \( (\mu \geq \hat{\mu}^E) \). However, the situation changes when retailers’ preference for trade credit is relatively low \( (\hat{\mu}^S \leq \mu \leq 1) \), as the interest rate charged by the TPL decreases with the increase in the interest rate, the demand for trade credit increases. This is due to the fact that a higher interest rate improves the profitability of the TPL’s extra financing service to the supplier, which enables the TPL to set a more competitive financing fee for its in-transit inventory financing service. In response, the supplier has to reduce the trade credit financing fee to compete with the TPL’s inventory financing service. Therefore, a lower fee will increase the demand for trade credit, and the supplier can still ensure the profit margin of its trade credit service when retailers’ preference for trade credit is high enough \( (\mu \geq \hat{\mu}^E) \). However, the situation changes when retailers’ preference for trade credit is relatively low \( (\hat{\mu}^S \leq \mu \leq 1) \), as the interest rate charged by the TPL decreases with the increase in the interest rate, the demand for trade credit increases. This is due to the fact that a higher interest rate improves the profitability of the TPL’s extra financing service to the supplier, which enables the TPL to set a more competitive financing fee for its in-transit inventory financing service. In response, the supplier has to reduce the trade credit financing fee to compete with the TPL’s inventory financing service. Therefore, a lower fee will increase the demand for trade credit, and the supplier can still ensure the profit margin of its trade credit service when retailers’ preference for trade credit is high enough \( (\mu \geq \hat{\mu}^E) \).
In this case, as the interest rate of the extra financing service increases, the demand for trade credit decreases. This happens because, on the one hand, the TPL would charge a lower financing fee to increase the demand for in-transit inventory financing, and, on the other, the supplier can no longer afford to further reduce the financing fee for trade credit while maintaining profitability.

When the TPL provides the extra financing service to the supplier, the optimal profit gained by the TPL is given by

\[
\pi_{EC} = \begin{cases} 
\frac{0(1-\mu)-\theta(1+1)w_2+r(\theta+1)(w_1-c_s)+(\theta+1)w_2r}{4(\theta+1)} & 0 \leq \mu < \hat{\mu}_S^{EC} \\
\frac{\mu(1+\theta)-2w_2(2\theta+1)+c_s(1-\mu)}{2(\theta+1)} & \mu \leq \hat{\mu}_S^{EC} \leq \mu < \hat{\mu}_G^{EC} \\
\frac{\mu}{2(\theta+1)} & \mu \geq \hat{\mu}_G^{EC}
\end{cases}
\]

and

\[
M_1 = \frac{\partial \pi_{EC}}{\partial \mu} = \frac{0(1-\mu)-\theta(1+1)w_2+r(\theta+1)(w_1-c_s)+(\theta+1)w_2r}{4(\theta+1)} + M_1
\]

\[
M_2 = \frac{\partial \pi_{EC}}{\partial r} = \frac{\mu(1+\theta)-2w_2(2\theta+1)+c_s(1-\mu)}{2(\theta+1)}
\]

Lemma 4: When \(0 < \mu < \hat{\mu}_1^{EC}\), \(\pi_{EC}^{\mu} \) increases with \(\rho\) and \(r\). When \(\hat{\mu}_1^{EC} \leq \mu < \hat{\mu}_G^{EC}\), \(\pi_{EC}^{\mu} \) decreases with \(\rho\) and \(r\).

Lemma 4 suggests that when retailers’ preference for trade credit is lower than \(\hat{\mu}_1^{EC}\), as the financing ratio \(\rho\) and interest rate \(r\) increase, the TPL can gain more profit from the extra financing service. Therefore, when retailers’ preference for trade credit is between \(0\) and \(\hat{\mu}_1^{EC}\), as the financing ratio \(\rho\) and interest rate \(r\) increase, the TPL can earn more profit from its extra financing service. The situation changes when retailers’ preference of trade credit further increases to the range between \(\hat{\mu}_1^{EC}\) and \(\hat{\mu}_G^{EC}\). In this case, when the financing ratio \(\rho\) and interest rate \(r\) increase, the profit of the extra financing service decreases. This happens because when the financing ratio \(\rho\) and interest rate \(r\) increase, the demand for in-transit inventory financing decreases. Thus, the profit from the extra financing service decreases. When the sensitivity to the difference in financing fees is high, this effect becomes particularly evident (see the proof in Appendix A.11).

5.3 Comparison Analysis

Now we examine the effects of TPL-provided in-transit inventory financing on the supply chain when both the supplier and the retailers have financing demand and derive the following proposition:

Proposition 5: (1) In the EC model, if \(\rho > \frac{1+w_2-2\mu-c_s-1}{w_1-c_s}\), the financing fee of in-transit inventory financing is lower than the financing fee of trade credit, i.e., \(f_T^{EC} < w_{S1}^{EC} - w_{S2}\); if \(\rho < \frac{1+w_2-2\mu-c_s-1}{w_1-c_s}\), the financing fee of in-transit inventory financing is higher than the financing fee of trade credit, i.e., \(f_T^{EC} > w_{S1}^{EC} - w_{S2}\).

(2) If \(0 \leq \mu < \hat{\mu}_T^{EN}\), the TPL will charge a lower in-transit inventory financing fee to retailers, i.e., \(f_T^{EC} < f_T^{EN}\). If \(\hat{\mu}_S^{EN} \leq \mu \leq 1\), the supplier will charge retailers a lower fee for trade credit, i.e., \(w_{S1}^{EC} - w_{S2} < w_{S1}^{EN} - w_{S2}\).
This proposition implies that the TPL charges lower a financing fee when the financing ratio is above a certain threshold. Intuitively, a high value of the financing ratio $\rho$ will motivate the TPL to reduce its financing fee for in-transit inventory financing to attract more retailers to adopt inventory financing, thereby increasing its revenue. This means that a lower unit price without trade credit makes it easier for the TPL to set a lower financing fee. The TPL will have a greater competitive advantage in providing financing service when the supplier is extremely capital-constrained. The advantage becomes more evident when the unit price without trade credit is low.

This proposition also shows that if retailers’ preference for trade credit is below a certain threshold ($\hat{\mu}_T^{EN}$), the TPL’s extra financing service to the supplier will lead to a lower financing fee for in-transit inventory financing. As the extra financing service provides the TPL with an additional revenue stream, it enables the TPL to reduce its fee for in-transit inventory financing to increase demand when retailers’ preference for trade credit is low. However, when retailers’ preference for trade credit increases to a high threshold ($\hat{\mu}_T^{EC} < \mu \leq 1$), the TPL would charge the lowest fixed fee for in-transit inventory financing. For trade credit, when retailers’ preference is between $\hat{\mu}_T^{EN}$ and $\hat{\mu}_T^{EC}$, the supplier will reduce the financing fee for trade credit. This happens because the decreased fee for in-transit inventory financing intensifies the competition between the two financing schemes, which makes the supplier charge a lower financing fee for trade credit. This situation continues even when retailers’ preference is high ($\hat{\mu}_T^{EN} \leq \mu \leq 1$), because the supplier’s financing demand reflects the favourable position due to retailers’ preference for trade credit. In contrast, when retailers’ preference for trade credit is extremely low ($0 \leq \mu \leq \hat{\mu}_S^{EC}$), the supplier consistently charges a fixed unit product price with trade credit to ensure its profit margin.

Accordingly, Proposition 6 shows the effect of the extra financing service on the demand for each financing scheme.

**Proposition 6:** (1) For trade credit, if $\hat{\mu}_T^{EN} < \mu \leq 1$, there exists $\hat{\mu}_T^{EC}$ between $\hat{\mu}_T^{EN}$ and $\hat{\mu}_T^{EC}$. If $\hat{\mu}_S^{EN} < \mu < \hat{\mu}_S^{EC}$, the extra financing service provided by the TPL decreases the demand for trade credit, i.e., $q_S^{EC} < q_S^{EN}$. If $\hat{\mu}_S^{EC} < \mu \leq 1$, it increases the demand for trade credit, i.e., $q_S^{EC} > q_S^{EN}$.

(2) For in-transit inventory financing, if $0 \leq \mu < \hat{\mu}_T^{EN}$, the extra financing service provided by the TPL increases the demand for in-transit inventory financing, i.e., $q_T^{EC} > q_T^{EN}$.

This proposition reveals that when retailers’ preference for trade credit is between 0 and $\hat{\mu}_T^{EN}$, the lower financing fee charged by the TPL results in an increased demand for in-transit inventory financing. When retailers’ preference for trade credit is between $\hat{\mu}_T^{EC}$ and $\hat{\mu}_T^{EN}$, as it increases, the change in the demand brought by the extra financing decreases. This happens because when the TPL provides the extra financing service to the supplier and retailers’ preference for trade credit is above $\hat{\mu}_T^{EC}$, the TPL would set the lowest fee for in-transit inventory financing to compete with the supplier. However, in the case where the TPL does not provide the extra financing service, when retailers’ preference for trade credit is between $\hat{\mu}_T^{EC}$ and $\hat{\mu}_T^{EN}$, the TPL can continually choose an optimal financing fee. This decreases the difference in the demand for in-transit
inventory financing between the case where the supplier has a financing demand and the case where the supplier has no financing demand. When retailers’ preference for trade credit reaches $\mu_{EN}$, the difference disappears. For trade credit, when retailers’ preference is between $\mu_{S1}$ and $\mu_{S2}$, the lower financing fee charged by the TPL reduces the demand for trade credit. The low retailers’ preference for trade credit ($\mu_{S1} \leq \mu < \mu_{S2}$) makes the supplier set the lowest fixed unit product price with trade credit. By providing the extra financing service, the TPL can set a lower fee for in-transit inventory financing, which improves its competitiveness and decreases the demand for trade credit. However, this situation changes when the degree of retailers’ preference for trade credit is above $\mu_{S2}$. In this case, when the supplier charges retailers a lower financing fee, the demand for trade credit increases.

Furthermore, we formulate Proposition 7 to show whether it is beneficial for the TPL to provide the extra financing service to the supplier.

**Proposition 7:** If $\mu_{EN}^EC \leq \mu \leq 1$, the TPL earns no profit from in-transit inventory financing. If $0 \leq \mu < \mu_{EN}^EC$, the TPL can earn more profit by providing the extra financing service to the supplier, i.e., $\pi_{T}^{EC} > \pi_{T}^{EN}$.

Proposition 7 shows that when retailers’ preference for trade credit is extremely high, there is no demand for in-transit inventory financing. Correspondingly, the TPL cannot earn a profit from its in-transit inventory financing. When retailers’ preference for trade credit is between 0 and $\mu_{EN}^EC$, although the TPL charges a lower fee for in-transit inventory financing, the TPL still earns more profit. The extra profit gained by the TPL mainly comes from three sources: increased demand for in-transit inventory financing service to retailers, logistics service and extra financing service to the supplier. This is quite intuitive. Take an extreme case as an example: if the TPL cannot earn a profit from its extra financing service (i.e., $\rho = 0$ or $r = 0$), the TPL’s source of profit will be the same as in the case of the partially capital-constrained supply chain. Even when the TPL can gain more profit from its extra financing service, the profit difference between the cases—whether or not the extra financing service is provided by the TPL—is marginally decreased when retailers’ preference for trade credit is relatively high ($\mu_{EN}^EC \leq \mu < \mu_{EN}^EC$). Specifically, when retailers’ preference for trade credit is between $\mu_{EN}^EC$ and $\mu_{EN}^EC$, the TPL that provides the extra financing service will set the lowest fixed financing fee, and in the case of not providing extra financing service, it still charges a higher fee for inventory financing. However, as retailers’ preference for trade credit increases, the TPL that does not provide the extra financing service charges a lower financing fee to increase the demand for in-transit inventory financing. However, when retailers’ preference for trade credit is between $\mu_{EN}^EC$ and $\mu_{EN}^EC$, although the TPL continues to charge the lowest financing fee, the extra financing service can still generate more profit.

**Proposition 8:** If $0 \leq \mu \leq 1$ and $p > \max(\hat{p}_4, \hat{p}_5)$, $\pi_{R}^{EC} > \pi_{R}^{EN}$.

It means that when the retail price is higher than the threshold, i.e., $p > \max(\hat{p}_4, \hat{p}_5)$, retailers consistently earn a greater total profit in the EC model than that in the EN model. More specifically, when retailers’ preference for trade credit is extremely low, i.e., $0 \leq \mu \leq \mu_{S2}$, although the supplier charges same financing fee in both the EN and EC models, the TPL charges lower financing fee in the EC model than it is in the EN.
model, resulting greater profit for the retailers. When retailers’ preference for trade credit is between \( \hat{\mu}^{EN}_S \) and \( \hat{\mu}^{EN}_T \) i.e., \( \hat{\mu}^{EN}_S < \mu \leq \hat{\mu}^{EN}_T \), the lower financing fee charged by both the TPL and supplier makes retailers more profitable. When retailers’ preference for trade credit is extremely high, i.e., \( \mu > \hat{\mu}^{EN}_T \), although the TPL charges same financing fee in both the EN and EC models, the supplier charges lower unit financing fee for trade credit in the EC model than that in the EN model. In this case, the retailers continually gain greater profit in the EC model.

6. Extended Analysis

6.1 The Effect of TPL’s Risk Preference

In previous sections, we assume all agents are risk-neutral. The TPL may be sensitive to risk when providing financing services to both the supplier and retailers. Therefore, it is interesting to explore how supply chain members’ decisions and profits can be affected by the degree of TPL’s risk-aversion. When considering the risk preference of a TPL, the demand and utility function of TPLs can be represented by Eq. (11) and Eq. (12).

\[ q_T = D\left[ (1 - \mu) - w_{S2} - f_T + \theta\left( w_{S1} - w_{S2} \right) - f_T + \delta_T \right] \]  \hspace{1cm} (11)

\[ E[\pi_T^C] = f_T q_T + (w_L - c_L)q_T + \rho c_S q_T \]  \hspace{1cm} (12)

Similar to Choi et al. (2019), here \( \delta_T \) is a random variable following the distribution with zero mean and variance of \( \sigma_T^2 \), whereas \( \lambda \) is the level of the TPL’s risk-aversion. Table A.2 identifies the equilibrium decisions for the supplier and the risk-averse TPL (represented as ER in Table A.2). By taking variance and then square root, the standard deviation of profits for the TPL can be represented as:

\[ SD[\pi_T^C] = D\left[ f_T \sigma_T + (w_L - c_L)\sigma_T + \rho \rho c_S \sigma_T \right] \]  \hspace{1cm} (13)

The mean-risk optimisation objective function for the TPL can be shown as

\[ U_T = E[\pi_T^C] - \lambda SD[\pi_T^C], \]  \hspace{1cm} (14)

where \( \lambda \geq 0 \) is the degree of risk aversion of the TPL.

**Lemma 5:** (1) For trade credit, (i) if \( \hat{\mu}^{ER}_S \leq \mu \leq \hat{\mu}^{ER}_T \), the unit financing fee \( (w_{S1}^{ER} - w_{S2}) \) decreases with \( \lambda \); (ii) if \( \hat{\mu}^{ER}_S \leq \mu \leq \hat{\mu}^{ER}_T \), the demand for trade credit decreases with \( \lambda \).

(2) For in-transit inventory financing, (i) if \( 0 \leq \mu \leq \hat{\mu}_T^{ER} \), the unit financing fee \( f_T^{ER} \) decreases with \( \lambda \); (ii) if \( 0 \leq \mu \leq \hat{\mu}_T^{ER} \), the demand for in-transit inventory financing increases with \( \lambda \).

This lemma indicates that the unit financing fees of both trade credit and in-transit inventory financing are decreasing functions of the degree of TPL’s risk aversion. It means that the TPL is motivated to charge a lower financing fee with an increase in its risk aversion level, as it can gain more utility from lower risk due to the demand uncertainty. This intensifies the competition between the two financing services and makes the supplier further decrease the financing fee of trade credit. Regarding the effect of the TPL’s risk aversion on the needs for each financing scheme, as the degree of risk aversion increases, the TPL is willing to decrease
the financing fee of in-transit inventory financing, resulting in a decreased demand for trade credit. In contrast, as the degree of TPL’s risk aversion increases, the TPL charges a lower financing fee, which subsequently increases the demand for in-transit inventory financing. This lemma shows that the supplier has a disadvantage in providing trade credit when the TPL is risk-averse.

**Proposition 9:** Compared with the case where the TPL is risk-neutral, when the TPL is risk-averse,

1. If $0 \leq \mu < \hat{\mu}_{EC}^{T}$, the TPL charges lower financing fee, i.e., $f_{T}^{ER} < f_{T}^{EC}$. If $\mu < \hat{\mu}_{T}^{EC}$, the supplier will decrease the financing fee of trade credit, i.e., $w_{S1}^{ER} - w_{S2}^{EC} < w_{S1}^{EC} - w_{S2}^{EC}$.

2. If $0 \leq \mu < \hat{\mu}_{EC}^{T}$, the demand for in-transit inventory financing increases, i.e., $q_{T}^{ER} > q_{T}^{EC}$. If $\hat{\mu}_{S1}^{EC} < \mu < \hat{\mu}_{T}^{EC}$, the demand for trade credit decreases, i.e., $q_{S}^{ER} < q_{S}^{EC}$.

As shown in Proposition 9, the TPL charges a low financing fee when retailers’ preference for trade credit is below a certain threshold ($\hat{\mu}_{EC}^{T}$). When retailers’ preference for trade credit is relatively high ($\hat{\mu}_{EC}^{T} \leq \mu \leq 1$), in-transit inventory financing becomes less competitive despite the low financing fee. When $\mu < \hat{\mu}_{EC}^{T}$, there is a limited range to set the optimal financing fee. The TPL only makes an optimal decision when the retailers’ preference is between 0 and $\hat{\mu}_{EC}^{T}$ ($\hat{\mu}_{T}^{ER} < \hat{\mu}_{EC}^{T}$), whereas the risk-neutral TPL represents the optimal decision when retailers’ preference for trade credit is between 0 and $\hat{\mu}_{EC}^{T}$. In addition, when retailers’ preference for trade credit is between $\hat{\mu}_{EC}^{ER}$ and $\hat{\mu}_{EC}^{T}$, the difference between the financing fees in the ER and EC models decreases as $\mu$ increases. This shows that the risk-averse TPL has more advantage when retailers’ preference for trade credit is relatively low. Furthermore, when retailers’ preference for trade credit is between $\hat{\mu}_{S}^{EC}$ and $\hat{\mu}_{EC}^{T}$, the in-transit inventory financing provided by the risk-averse TPL decreases the financing fee of trade credit, suggesting that retailers gain more benefit when the TPL is risk-averse as the competition between two financing schemes intensifies. However, when retailers’ preference for trade credit is relatively low ($0 < \mu \leq \hat{\mu}_{S}^{EC}$), trade credit is not affected by the degree of risk aversion of the TPL. In this case, the same low unit product price with trade credit will be set by the supplier. When retailers’ preference for trade credit is high ($\hat{\mu}_{EC}^{T} \leq \mu < 1$), the degree of risk aversion of the TPL has no impact on trade credit.

Regarding the demand for the financing schemes in the ER and EC models, when retailers’ preference for trade credit is between 0 and $\hat{\mu}_{EC}^{T}$, the low financing fee charged by the TPL makes in-transit inventory financing more popular. When retailers’ preference is between $\hat{\mu}_{EC}^{ER}$ and $\hat{\mu}_{EC}^{T}$, as retailers’ preference for trade credit increases, the difference between the demand for in-transit inventory financing in the ER and EC models decreases. This happens because when retailers’ preference for trade credit is above $\hat{\mu}_{EC}^{T}$, the TPL’s motivation to set low financing fee diminishes. Significantly, when retailers’ preference for trade credit is between $\hat{\mu}_{S}^{EC}$ and $\hat{\mu}_{EC}^{T}$, although the supplier reduces the financing fee of trade credit, its demand decreases. Compared with the case where the TPL is risk-neutral, the risk-averse TPL sets a more competitive financing fee because of reducing the risk brought by the demand uncertainty, which will force the supplier to make trade credit more attractive to retailers. When retailers’ preference for trade credit is above $\hat{\mu}_{EC}^{T}$, the TPL would charge the lowest financing fee. As retailers’ preference for trade credit increases further, the demand for trade credit will
continuously increase until retailers’ preference for trade credit reaches the point where the risk-neutral TPL sets the lowest fixed financing fee.

**Proposition 10:** Compared with the case where the TPL is risk-neutral, when the TPL is risk-averse, (i) $\hat{\mu}_{EC}^T \leq \mu \leq 1$, retailers gain the same total profit, i.e., $\pi_{ER}^T = \pi_{RE}^T$; (ii) if $0 \leq \mu < \hat{\mu}_{EC}^T$ and $p > \max(\hat{p}_6, \hat{p}_7, \hat{p}_8)$, the total profit gained by retailers is higher, i.e., $\pi_{RE}^T > \pi_{EC}^T$.

Proposition 10 indicates that when retailers’ preference for trade credit is between $\hat{\mu}_{EC}^T$ and 1, the total profit gained by retailers becomes the same as the case of the risk-neutral TPL. This is intuitive because the TPL either risk-neutral or risk-averse charges lowest financing fee. Due to the intense competition, the supplier charges the same unit fee for trade credit. In response, retailers gain the same profit due to the unchanged financing fee and the demand for each financing scheme. When retailers’ preference for trade credit is between 0 and $\hat{\mu}_{EC}^T$, there is no demand for trade credit in both the ER and EC models. However, the risk-averse TPL charges lower financing fee, and the demand for in-transit inventory financing increases. Therefore, retailers earn a greater profit when the TPL is risk-averse. Furthermore, when retailers’ preference for trade credit is between $\hat{\mu}_{EC}^T$ and $\hat{\mu}_{SI}^T$, the risk-averse TPL charges low financing fee and the demand for in-transit inventory financing increases, resulting in greater total profits gained by retailers. Therefore, when the TPL is risk-averse, retailers are more likely to gain extra benefit when their trade credit preference is not extremely high.

**6.2 The Effect of TPL’s Power**

In the previous sections, we assumed that the supplier and the TPL have a symmetrical power relationship and simultaneously decide the unit product price with trade credit and the fee for in-transit inventory financing, respectively. However, when the TPL provides a financing service to a supplier who has a financing demand, the TPL is in a dominant position and has a first-mover advantage in setting the financing fee. This section further examines the impact that the supply chain power relationship has on the effects of in-transit inventory financing.

The sequence of events in this scenario is similar to the sequence of events shown in Figure 3. The main difference is that the supplier decides the financing fee for trade credit later than the TPL sets the fee for in-transit inventory financing. Table A.3 identifies the equilibrium decisions made by the supplier and the TPL, where the TPL is the Stackelberg leader. In Table A.3, $TC$ represents the case where the TPL is the Stackelberg leader.

Based on the optimal solutions in Table A.3, when the TPL is the Stackelberg leader, the optimal profit of the TPL can be expressed as:

$$
\pi_{TC}^T = \begin{cases} 
  \frac{f_{TC}^T q_{TC}^T + (w_L - c_L)q_{TC}^T + pc_{q_{TC}^T}}{\sqrt{(\theta^2 + 4\theta + 2)(\theta_1 - \theta_2))}} & 0 \leq \mu < \hat{\mu}_{TC}^T \\
  \frac{f_{TC}^T q_{TC}^T + (w_L - c_L)q_{TC}^T + pc_{q_{TC}^T}}{\sqrt{(\theta^2 + 4\theta + 2)(\theta_1 + 1))}} & \hat{\mu}_{TC}^T \leq \mu < \hat{\mu}_{SI}^T \\
  0 & \hat{\mu}_{SI}^T \leq \mu < 1 
\end{cases}
$$

(55)
An extremely low \((0 \leq \mu < \hat{\mu}_1^{TC})\) or extremely high \((\hat{\mu}_1^{TC} \leq \mu < 1)\) retailers’ preference for trade credit is rare, so we consider a more general case in which there is a medium preference for trade credit \((\hat{\mu}_1^{TC} \leq \mu < \hat{\mu}_1^{TC})\) and investigate it in detail. The profit of the TPL (as the Stackelberg leader) has similar characteristics to those in the case where the TPL and the supplier have symmetrical power. As the logistics cost decreases and the interest rate increases, the TPL, as Stackelberg leader, can gain more profit.

The following proposition describes the effect on the fee for in-transit inventory financing when the TPL has the first-mover advantage:

**Proposition 11:** If \(\max\{\mu_1^{EC}, \hat{\mu}_1^{TC}\} \leq \mu \leq \min\{\hat{\mu}_1^{EC}, \mu_1^{TC}\}\), the TPL can earn more profit from its in-transit inventory financing service (i.e., \(\pi_{TC}^{EC} > \pi_{TC}^{EC}\)) but gains less profit from its extra financing service (i.e., \(\pi_{TC}^{EC} < \pi_{TC}^{EC}\)). Generally, the TPL, as Stackelberg leader, earns a greater total profit than in a symmetrical supply chain power structure (i.e., \(\pi_{TC}^{EC} > \pi_{TC}^{EC}\)).

This proposition shows that when retailers’ preference for trade credit falls into the interval of \(\max\{\hat{\mu}_1^{EC}, \mu_1^{TC}\} \leq \mu \leq \min\{\hat{\mu}_1^{EC}, \mu_1^{TC}\}\), the TPL can benefit from a greater overall profit, i.e., \(\pi_{TC}^{EC} > \pi_{TC}^{EC}\). This differs from the case where the TPL and the supplier have a symmetrical power relationship. This finding is consistent with the existing literature (Shi et al., 2013; Chen and Wang, 2015; Chen et al., 2017), which posits that a dominant power position in the supply chain enables firms to earn a higher profit. Interestingly, the effect of the power relationship on in-transit inventory financing and extra financing services is different. When the TPL is the Stackelberg leader, it can earn more profit from in-transit inventory financing but less profit from the extra financing service. This happens because although the TPL’s first-mover advantage allows it to set a higher financing fee to gain extra profit on in-transit inventory financing, the retailers’ demand for trade credit increases. The decreased demand for in-transit inventory financing reduces the profit that TPL can gain from the extra financing service.

The following proposition further investigates the popularity of each financing scheme when the TPL is more powerful:

**Proposition 12:** The demand for each financing scheme is governed by the following inequalities: (i) if \(w_L > G^1(c_L), q_t^{TC} > q_s^{TC}\), otherwise \(q_t^{TC} \leq q_s^{TC}\); (ii) if \(\rho > G^2(r), q_t^{TC} > q_s^{TC}\), otherwise \(q_t^{TC} \leq q_s^{TC}\); (iii) if \(\mu < G^3(w_{S2}), q_t^{TC} > q_s^{TC}\), otherwise \(q_t^{TC} \leq q_s^{TC}\) \((G^1(c_L), G^2(r)\) and \(G^3(w_{S2})\) are presented in Appendix A.21).

This proposition shows the preferred financing scheme under different conditions. We examine how three factors—logistics fee, financial ratio and the unit product price without trade credit—affect the popularity of each financing scheme. Figures 4, 5 and 6 are provided to illustrate the results. In line with Huang et al. (2019) and Feng and Chan (2019), the following parameters are used. For Figure 4, the parameters are \(D = 100, \mu = 0.5, w_{S2} = 0.22, c_S = 0.15, \theta = 0.1, \rho = 1\) and \(r = 0.05\). For Figure 5, the parameters are \(D = 100, \mu = 0.5, w_{S2} = 0.22, c_S = 0.15, \theta = 0.1, w_L = 0.08\) and \(c_L = 0.02\). For Figure 6, the parameters are as follows: \(D = 100, c_S = 0.2, \theta = 0.6, w_L = 0.08, c_L = 0.02, \rho = 1\) and \(r = 0.1\). Note that the results continue to hold for other parameters as long as they satisfy the assumption in the model.
Regarding the logistics fee, when the fee charged by the TPL is above a threshold (i.e., $G^1(c_L)$), the demand for in-transit inventory financing will be higher than that for trade credit (see the area shaded in blue in Figure 4). Since $G^1(c_L)$ is positively affected by the logistics cost, a lower unit logistics cost will make in-transit inventory financing more popular than trade credit. This indicates that for leading logistics service providers who have a competitive advantage in logistics services owing to economies of scale, it is easier to gain market share in the emerging supply chain financing market. This is intuitive. On the one hand, the provision of in-transit inventory financing helps the TPL attract more customers and therefore increases its market share of logistics services, while, on the other, multiple revenue streams from logistics and inventory financing services also enable the TPL to set competitive financing fees to compete against other financing service providers, such as banks and capital-adequate suppliers.

Additionally, the financing ratio and interest rate also directly affect the popularity of each financing scheme. In-transit inventory financing is more popular than trade credit only when the financing ratio of extra financing service exceeds a threshold (see the area shaded in blue in Figure 5). Therefore, when the supplier asks for a high financing ratio, the TPL can set a lower financing fee and improve the competitiveness of its in-transit inventory financing. Furthermore, the unit product price without trade credit also has a direct effect on the popularity of each financing scheme. Specifically, when retailers’ preference for trade credit is low, a low unit product price without trade credit can make in-transit inventory financing more attractive (see the left part of the area shaded in blue in Figure 6). From Figure 6, it is intuitive that when retailers’ preference for trade credit is below a certain threshold, in-transit inventory financing is more popular than trade credit. This threshold is negatively affected by the unit product price without trade credit, which also indicates that a lower unit product price without trade credit can make in-transit inventory financing more popular (see Figure 6).

Therefore, those suppliers who would like to develop their trade credit business need to improve retailers’ preference for trade credit by reducing punitive action, lengthening the funding cycle or setting a relatively high unit product price without trade credit.
7. Numerical Analysis

The analysis results reported in the previous sections highlight several important factors that influence firms’ decisions on in-transit inventory financing and performance. In this section, numerical analysis is carried out to examine how the retailers’ preference for trade credit affects the financing fee and demand of each financing scheme. In line with Choi et al. (2019), Feng and Chan (2019) and Huang et al. (2019), the following parameters are used: $w_l = 0.24$, $f_{lt} = f_{lt'} = 0.02$, $w_{s2} = 0.22$, $c = 0.2$, $\theta = 0.6$, $w_L = 0.08$, $c_L = 0.02$, $\rho = 1$, $r = 0.1$, $\sigma_T = 1$ and $\lambda = 0.05$.

We first look at how retailers’ preference for trade credit affects the financing fee of each financing scheme. From Figure 7, we found that when retailers’ preference for trade credit is above a certain threshold, as it increases, the financing fee for trade credit increases. In contrast, when retailers’ preference for trade credit is below a certain threshold, as it increases, the financing fee for other financing schemes decreases. Our results also show that the effects of retailers’ preference for trade credit vary between different models.
Figure 7. The comparison of financing fee among different financing models.

Figure 7(a) shows how retailers’ preference for trade credit affects the difference between the financing fee in the BN model and that in the EN model. When retailers’ preference for trade credit is between $\hat{\mu}^T_{EN}$ and $\hat{\mu}^F_{BN}$, as retailers’ preference for trade credit increases, the difference between the financing fees for conventional financing in the BN model and in-transit inventory financing in the EN model decreases. When retailers’ preference for trade credit is extremely high, this difference disappears. However, Figure 7(b) shows that the difference between the financing fee in the EN model and that in the EC model still exists even when the retailers’ preference for trade credit is extremely high. Figure 7(c) presents how retailers’ preference for trade credit affects the difference between the ER and EC models in terms of the financing fee. When retailers’ preference for trade credit is between $\hat{\mu}^T_{ER}$ and $\hat{\mu}^T_{EC}$, as retailers’ preference for trade credit increases, the difference in the financing fees between the ER and EC models decreases. Figure 7(d) shows the effect of TPL’s first-mover advantage on the financing fee for each financing scheme. Interestingly, the effect of TPL’s first-mover advantage has more impact on in-transit inventory financing than trade credit. Its effect is related to retailers’ preference for trade credit; when this is very low, the financing fee of in-transit inventory financing in the TC model could be lower than that in the EC model (see red and blue solid lines in Figure 7(c)). This indicates that a TPL with dominant power tends to set lower financing fees when retailers have had a negative experience of using trade credit. Correspondingly, it would be more beneficial for those retailers to use in-transit inventory financing.

We then examine how retailers’ preference for trade credit affects the demand for each financing scheme. The result is displayed in Figure 8, which is consistent with the finding of Figure 7. Lower financing fee will lead to higher financing demand. When retailers’ preference for trade credit is medium, as it increases, the demand for trade credit increases and the demand for in-transit inventory financing decreases.

Figure 8(a) shows that when the TPL replaces the financial institution to provide financing service, the demand for trade credit decreases (See dashed blue line in Figure 8(a)). In addition, the in-transit inventory financing is more popular than the conventional financing service (See solid blue line in Figure 8(a)).
8(b) shows that when the TPL provides extra financing service to the supplier, the demand for in-transit inventory financing increases. In contrast, the situation for the trade credit is more complex. When retailers’ preference for trade credit is low ($\mu^{EC}_{S1} < \mu \leq \mu^{EC}_{S2}$), the extra financing service provided by the TPL will decrease the demand of trade credit. When retailers’ preference for trade credit is higher than $\mu^{EC}_{S2}$, extra financing service provided by the TPL will increase the demand of trade credit (See dashed blue line in Figure 8(b)). Figure 8(c) shows that when the TPL is risk-averse, the demand for trade credit decreases (See dashed blue line in Figure 8(c)). Figure 8(d) reveals how the TPL’s first-mover advantage affects the demand for each financing scheme. Figure 8(d) shows that the TPL’s first-mover advantage has more impact on the demand for in-transit inventory financing than the demand for trade credit. When retailers’ preference of trade credit is low, the TPL can make in-transit inventory financing more popular.

Figure 8. The comparison of financing demand among different financing schemes.

8. Conclusions
This paper examines the effects of TPL-provided in-transit inventory financing on the capital-constrained supply chain. We begin with the case where retailers have financing demands and then extend the analysis to
a supply chain setting where both suppliers and retailers have financing demands. Furthermore, we investigate how the risk preference and power relationship influences the effects of in-transit inventory financing on the supply chain. The main findings are summarised below:

- For capital-constrained retailers, a lower unit product price without trade credit increases the financing fee for trade credit. This means that the increased demand for financing due to a lower unit product price without trade credit will increase the financing burden on retailers. Different from the financing fee, the relationship between the demand for trade credit and the unit product price without trade credit is not so straightforward, since it is affected by the degree of retailers’ preference for trade credit. This finding extends the work of Yang and Birge (2018), which identified the mechanism by which retailers’ default rate and market power affect the unit product price without trade credit.

- TPL-provided in-transit inventory financing drives down the financing fee for trade credit despite a negative effect on the demand for trade credit. With in-transit inventory financing, the TPL’s logistics fee and cost also have an impact on the financing fee and demand for each financing scheme. For instance, a higher logistics fee decreases the financing fees of trade credit and in-transit inventory financing, and a higher logistics cost increases the demand for trade credit but decreases the demand for in-transit inventory financing. This finding is partially supported by Chen and Cai (2011), who claimed that a higher logistics fee increases the probability of a low financing fee for trade credit.

- When the TPL provides the extra financing service to the supplier, the financing fees for both trade credit and in-transit inventory financing decrease, while the demand for in-transit inventory financing increases. The effect of the extra financing service on the demand for trade credit is more complex and dependent on retailers’ preference for trade credit and its relationship with the relevant critical threshold. In addition, retailers’ preference for trade credit influences the effect of interest rates on the financing fee and the demand for each financing scheme. We find that an increase in the interest rate charged by the TPL decreases the financing fee of each financing scheme.

- The risk-averse TPL pushes down the financing fee of both trade credit and in-transit inventory financing. However, the demand for trade credit decreases despite decreased financing fee. This finding is partially supported by Choi et al. (2019), who demonstrated that when the upstream is more risk-averse (i.e., \( \lambda \) increases), the upstream firm will set a lower wholesale price. Furthermore, with a higher degree of risk aversion of the TPL, both the financing fees of trade credit and in-transit inventory financing and the demand for trade credit decreases, but the demand for in-transit inventory financing increases.

- Finally, when the TPL is the Stackelberg leader, it can gain more profit from in-transit inventory financing. This result is in line with Chen et al. (2019), who indicate that a TPL can gain more economic benefit as the supply chain leader. In this case, the supplier asks for less financing support from the TPL because of the decreased demand for in-transit inventory financing. Furthermore, the popularity of each financing scheme is also influenced by financing and logistics-related factors such as logistics cost, unit product price and financing ratio.

8.1 Management insights
The research findings provide some valuable management insights to supply chain firms. First, for suppliers, the product price affects the demand for trade credit, and retailers’ preference for trade credit influences its effect. When suppliers set up the optimal financing fee for trade credit, they should take into account the unit product price, the financing fee for in-transit inventory financing, and retailers’ preference for different financing channels. Second, for TPLs, the provision of financing services can increase the demand for their traditional logistics services and their operating profits. Therefore, TPLs should actively extend their service boundary to provide more in-transit inventory financing, which tends to be more beneficial for retailers who adopt trade credit. In addition, TPLs should also provide extra financing services to suppliers in order to boost the demand for in-transit inventory financing services and increase profit gain. Third, when the TPL is risk sensitive, a higher degree of risk aversion decreases financing fees of both trade credit and in-transit inventory financing. Therefore, for the retailers who would like to adopt in-transit inventory financing, they can select risk-averse TPLs as service providers to reduce their financing cost. Finally, when the competition between financing schemes is considered, a lower logistics cost will make the TPL more willing to set a lower financing fee, thereby making trade credit less attractive. Therefore, retailers who are seeking alternative financing sources may choose the in-transit inventory financing service provided by leading logistics service providers because the TPLs’ economies of scale in their logistics operations enable them to charge lower financing fees.

8.2 Future research

This paper is the first study to examine in detail how TPL-provided in-transit inventory financing affects the capital-constrained supply chain. The study does, though, have some limitations, suggesting several promising future research avenues. First, this paper uses one parameter ($\mu$) to represent retailers’ preference for trade credit. Generalising the usage of this parameter could omit the heterogeneity among retailers’ preference regarding trade credit. Further consideration of the heterogeneity among retailers’ preference on the trade credit and study how it affects the competition between financing schemes would represent a significant enhancement. Second, although the linear demand function used in our study is widely employed for studying channel competition (Choi, 1996; McGuire and Staelin, 2008; Mitra, 2016), it would be valuable to evaluate demand using stochastic methods and explore how the demand with different distribution affects optimal decisions about trade credit and in-transit inventory financing. Third, we assume that the logistics market is competitive and externalise the logistics fee, as is commonly done in the existing literature (Chen and Cai, 2011). However, the logistics service provider could change the logistics fee when providing in-transit inventory financing, which consequently influences the demand for each financing scheme. One future research avenue would be to consider the logistics fee as an endogenous variable and examine how the TPL coordinates financing and logistics services to improve their competitive advantage.

9. References


Appendix A

Appendix A.1 The Proof of Equilibrium in Table A.1

According to Eq. (2) and the profit function of the financial institution, taking the first-order and the second-order derivative of $\pi^N$ and $\pi^F$ with the respect to $w_{S1}$ and $f_F$, we have

$$\frac{d\pi^N}{dw_{S1}} = D[(f_F - 2w_{S1} + 2w_{S2})\theta + \mu + c_S - 2w_{S1}]$$

$$\frac{d^2\pi^N}{dw_{S1}^2} = -2D(\theta + 1)$$

$$\frac{d\pi^N}{df_F} = D[1 - \mu - (2f_F - w_{S1} + w_{S2})\theta - 2f_F - w_{S2}]$$

$$\frac{d^2\pi^N}{df_F^2} = -2D(\theta + 1)$$

It is straightforward that $\frac{d^2\pi^N}{dw_{S1}^2} < 0$ and $\frac{d^2\pi^N}{df_F^2} < 0$. Therefore, $\pi^N_S$ and $\pi^N_F$ are concave in $w_{S1}$ and $f_F$.

According to the first-order optimality condition (F.Q.C), we have the best response of the supplier and financial institution as

$$\frac{(\theta+1)(3\theta w_{S2}+2c_S)\theta + \mu}{(\theta+2)(3\theta+2)} + \frac{\mu}{3\theta+2}$$

and

$$\frac{2(\theta+1)+\theta c_S-2(2\theta+1)w_{S2}}{(\theta+2)(3\theta+2)} - \frac{\mu}{3\theta+2}.$$  

In reality, the unit product price with trade credit has a threshold ($w_{lS}$), which is higher than the unit product price without trade credit ($w_{S2}$). Similarly, the unit financing fee for conventional financing also has a threshold ($f_{lF}$). Therefore, there exists a $\mu^{BN}_{S}$. When $0 \leq \mu < \mu^{BN}_{S}$, the supplier would set lowest unit product price with trade credit ($w_{lS}$). When $\mu^{BN}_{F} \leq \mu \leq 1$, the financial institution would like to charge lowest financing fee for conventional financing $f_{lF}$. Then

$$\mu^{BN}_{S} = \frac{(\theta-2)(3\theta-2)w_{lS}-3\theta(\theta+1)w_{S2}+2c_S(\theta+1)-\theta}{\theta+2}$$

and

$$\mu^{BN}_{F} = \frac{(3\theta^2+4\theta+2)w_{S2}-(3\theta+5\theta+4)w_{lF}+\theta c_S+2(\theta+1)}{\theta+2}.$$  

Appendix A.2 The Equilibrium for BN and EN model
Table A.1. Equilibrium decisions for the supplier, financial institution and TPL.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Conditions ($i = BN$)</th>
<th>Optimal Value ($i = BN$)</th>
<th>Conditions ($i = EN$)</th>
<th>Optimal Value ($i = EN$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{i1}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_S$</td>
<td>$w_{iS}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_S$</td>
<td>$w_{iS}$</td>
</tr>
<tr>
<td>$\tilde{\mu}_j \leq \mu \leq \hat{\mu}_j$</td>
<td>$\frac{(\theta+1)(\theta+2)+\theta}{(\theta+2)(\theta+2)}w_{iS}+\frac{\mu}{\theta+2}$</td>
<td>$\tilde{\mu}_j \leq \mu &lt; \hat{\mu}_j$</td>
<td>$f_{FN} - \frac{\theta(w_L-c_L)(\theta+1)}{(\theta+2)(3\theta+2)}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\mu}_j \leq 1$</td>
<td>$f_{iF}$</td>
<td>$\hat{\mu}_j \leq 1$</td>
<td>$f_{iF}$</td>
<td></td>
</tr>
<tr>
<td>$f_{iF/T}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_S$</td>
<td>$\frac{1-(\theta+1)}{\theta+1}w_{iS}+\theta w_{iS}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_S$</td>
<td>$f_{iF} - \frac{w^L-c^L}{2}$</td>
</tr>
<tr>
<td>$\tilde{\mu}_j \leq \mu \leq \hat{\mu}_j$</td>
<td>$\frac{2(\theta+1)+\theta}{(\theta+2)(\theta+2)}w_{iS}+\frac{\mu}{\theta+2}$</td>
<td>$\tilde{\mu}_j \leq \mu &lt; \hat{\mu}_j$</td>
<td>$f_{iF} - \frac{(1+\theta)^2(w_L-c_L)}{(\theta+2)(3\theta+2)}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\mu}_j \leq 1$</td>
<td>$\tilde{\mu}_j \leq 1$</td>
<td>$\tilde{\mu}_j \leq 1$</td>
<td>$\tilde{\mu}_j \leq 1$</td>
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<tr>
<td>$q_{i1}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_{i1}$</td>
<td>$0$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_{i1}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\tilde{\mu}<em>{i1} \leq \mu \leq \hat{\mu}</em>{i1}$</td>
<td>$\frac{D<a href="%5Ctheta+1">\theta(\theta+1)</a>w_{iS}+\theta(\theta+1)\mu+\theta]}{(\theta+1)}$</td>
<td>$\tilde{\mu}<em>{i1} \leq \mu &lt; \hat{\mu}</em>{i1}$</td>
<td>$\frac{D<a href="%5Ctheta+1">\theta(\theta+1)</a>w_{iS}+\theta(\theta+1)\mu+\theta]}{(\theta+1)}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\mu}_{i1} \leq 1$</td>
<td>$\tilde{\mu}_{i1} \leq 1$</td>
<td>$\tilde{\mu}_{i1} \leq 1$</td>
<td>$\tilde{\mu}_{i1} \leq 1$</td>
<td></td>
</tr>
<tr>
<td>$q_{iF/T}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_S$</td>
<td>$\frac{D[\theta(w_{iS}-\mu)]}{2}$</td>
<td>$0 \leq \mu &lt; \hat{\mu}_S$</td>
<td>$\frac{D[\theta(w_{iS}-\mu)]}{2}$</td>
</tr>
<tr>
<td>$\tilde{\mu}_j \leq \mu \leq \hat{\mu}_j$</td>
<td>$\frac{D<a href="%5Ctheta+1">\theta(\theta+1)</a>w_{iS}+\theta(\theta+1)\mu+\theta]}{(\theta+1)}$</td>
<td>$\tilde{\mu}_j \leq \mu &lt; \hat{\mu}_j$</td>
<td>$\frac{D<a href="%5Ctheta+1">\theta(\theta+1)</a>w_{iS}+\theta(\theta+1)\mu+\theta]}{(\theta+1)}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{\mu}_j \leq 1$</td>
<td>$\tilde{\mu}_j \leq 1$</td>
<td>$\tilde{\mu}_j \leq 1$</td>
<td>$\tilde{\mu}_j \leq 1$</td>
<td></td>
</tr>
</tbody>
</table>

Note: (1) $\mu_{S1} = \frac{w_{iS}}{\theta+2} - \frac{(\theta+2)(\theta+1)w_{iS}+\theta(\theta+1)\mu-\theta}{(\theta+1)}$ and $\mu_{F1} = \frac{w_{iS}}{\theta+2} - \frac{(\theta+2)(\theta+1)w_{iS}+\theta(\theta+1)\mu-\theta}{(\theta+1)}$

(2) $\mu_{F1} = \mu_{F1}^w + \frac{w_{iS}}{\theta+2} + \frac{w_{iS}}{\theta+2} - 2(\theta+1)^3(w_{iS}-\mu)$ and $\mu_{F1} = \mu_{F1}^w + \frac{w_{iS}}{\theta+2} + \frac{w_{iS}}{\theta+2} - 2(\theta+1)^3(w_{iS}-\mu)$.
Appendix A.3 The Proof of Lemma 1

(i) For trade credit, when $0 \leq \mu < \mu_S^{BN}$, the first-order derivative of $w_S^{BN} - w_S$ regarding $w_S$ is $w_S - 1$. When $\mu_S^{BN} \leq \mu < \mu_F^{BN}$, the first-order derivative of $w_S^{BN} - w_S$ regarding $w_S$ is $-\frac{5\theta - 4}{(\theta + 2)(3\theta + 2)}$. When $\mu_S^{BN} \leq \mu \leq 1$, the first-order derivative of $w_S^{BN} - w_S$ regarding $w_S$ is $-\frac{1}{2}$. Obviously, the first-order derivative of $w_S^{BN} - w_S$ is less than 0. Therefore, the unit financing fee for trade credit decreases in $w_S$. Regarding the demand for trade credit, if $\mu_S^{BN} \leq \mu < \mu_F^{BN}$, the first-order derivative of $q_S^{BN}$ is $\frac{D\theta(\theta + 1)}{2(\theta + 1)} > 0$. Therefore, the demand for trade credit increases with the unit product price without trade credit ($w_S$); if $\mu_S^{BN} \leq \mu \leq \mu_F^{BN}$, the first-order derivative of $q_S^{BN}$ is $-\frac{D\theta(\theta + 1)}{2(\theta + 1)} < 0$. Therefore, the demand for trade credit decreases with the unit product price without trade credit ($w_S$).

(ii) For the conventional financing, when $0 \leq \mu < \mu_S^{BN}$, the first-order derivative of $f_F^{BN}$ regarding $w_S$ is $-\frac{1}{2}$. When $\mu_S^{BN} \leq \mu < \mu_F^{BN}$, the first-order derivative of $f_F^{BN}$ regarding $w_S$ is $-\frac{1}{2}$. Obviously, the first-order derivative of $f_F^{BN}$ is less than 0. Therefore, the unit financing fee for conventional financing also decreases in $w_S$. Regarding the demand for conventional financing, if $0 \leq \mu < \mu_S^{BN}$, the first-order derivative of $q_F^{BN}$ is $-\frac{D\theta(\theta + 1)}{2} < 0$; if $\mu_S^{BN} \leq \mu < \mu_F^{BN}$, the first-order derivative of $q_F^{BN}$ is $-\frac{D\theta(\theta + 1)(\theta + 2)}{2(\theta + 2)(\theta + 2)} < 0$; if $\mu_F^{BN} \leq \mu \leq \mu_F^{1}$, the first-order derivative of $q_F^{BN}$ is $-\frac{D\theta(\theta + 1)(\theta + 2)}{2(\theta + 1)(\theta + 2)} < 0$; if $\mu_F^{1} \leq \mu \leq \mu_F^{1}$, the demand for conventional financing decreases with the unit product price without trade credit ($w_S$).

Appendix A.4 The Proof of Lemma 2

Regarding the financing fee for trade credit, when $\mu_S^{EN} \leq \mu < \mu_T^{EN}$, the first-order derivative of $w_T^{EN} - w_L$ regarding $w_L$ is $-\frac{\theta(\theta + 1)}{(\theta + 2)(3\theta + 2)} < 0$. Therefore, when $\mu_S^{EN} \leq \mu < \mu_T^{EN}$, the financing fee for trade credit decreases in $w_L$. Regarding the financing fee for in-transit inventory financing, when $0 \leq \mu < \mu_S^{EN}$, the first-order derivative of $f_T^{EN}$ regarding $w_L$ is $-\frac{1}{2} < 0$. When $\mu_S^{EN} \leq \mu \leq \mu_T^{EN}$, the first-order derivative of $f_T^{EN}$ regarding $w_L$ is $-\frac{2(1 + \theta)^2}{(\theta + 2)(3\theta + 2)} < 0$. Therefore, when $0 \leq \mu < \mu_T^{EN}$, the financing fee for in-transit inventory financing decreases in $w_L$.

Regarding the financing demand for trade credit, when $\mu_S^{EN} \leq \mu < \mu_S^{EN}$, the first-order derivative of $q_S^{EN}$ regarding $c_L$ is $\frac{D\theta}{2} > 0$. When $\mu_S^{EN} \leq \mu < \mu_T^{EN}$, the first-order derivative of $q_S^{EN}$ regarding $c_L$ is $\frac{D\theta(\theta + 1)^2}{(\theta + 2)(3\theta + 2)} > 0$. Therefore, when $\mu_S^{EN} \leq \mu < \mu_T^{EN}$, the financing demand for trade credit increases in $c_L$. 


Regarding the financing demand for in-transit inventory financing service, when $0 \leq \mu < \bar{\mu}_S^{EN}$, the first-order derivative of $q_T^{EN}$ regarding $c_L$ is $-\frac{1}{2} < 0$. When $\bar{\mu}_S^{EN} \leq \mu < \bar{\mu}_F^{EN}$, the first-order derivative of $q_T^{EN}$ regarding $c_L$ is $-\frac{D(\theta+1)(\theta^2+4\theta+2)}{2(\theta+1)(3\theta+2)} < 0$. Therefore, when $0 \leq \mu < \bar{\mu}_F^{EN}$, the financing demand for in-transit inventory financing decreases in $c_L$.

**Appendix A.5 The Proof of Proposition 1**

(1) Given $\bar{\mu}_S^{EN} \leq \mu < \bar{\mu}_F^{EN}$, $w_{S1}^{EN} - w_{S2} = \frac{(\theta+1)(3\theta w_{S2}+2c_s)+\theta}{(\theta+2)(3\theta+2)} + \frac{\mu}{3\theta+2} - \frac{\theta(w_L-c_L)(\theta+1)}{(\theta+2)(3\theta+2)} - w_{S2}$ and $f_T^{EN} = \frac{2(\theta+1)+\theta c_s-2(\theta+1)w_{S2}}{(\theta+2)(3\theta+2)} - \frac{\mu}{3\theta+2} - \frac{2(1+\theta)^2(w_L-c_L)}{(\theta+2)(3\theta+2)}$. When $f_T^{EN} < w_{S1}^{EN} - w_{S2}$, we have $c_L < w_L - \frac{1+w_{S2}-2\mu-c_s}{1+\theta}$. When $f_T^{EN} > w_{S1}^{EN} - w_{S2}$, we have $c_L > w_L - \frac{1+w_{S2}-2\mu-c_s}{1+\theta}$.

(2) Regarding the financing fee for conventional and in-transit inventory financing, when $\bar{\mu}_S^{EN} \leq \mu \leq \bar{\mu}_F^{EN}$, $f_T^{EN} - f_F^{BN} = -\frac{2(1+\theta)^2(w_L-c_L)}{(3\theta+2)(\theta+2)} < 0$. When $\mu = \bar{\mu}_S^{EN}$, $f_T^{EN} < f_F^{BN}$. When $\mu = \bar{\mu}_F^{EN}$, $f_T^{EN} < f_F^{BN}$. Owing to the first-order derivative of $f_T^{EN} - f_F^{BN} = \frac{(\theta+1)(3\theta w_{S2}+2c_s)+\theta}{(\theta+2)(3\theta+2)} + \frac{\mu}{3\theta+2} - \frac{(f_T^{EN}+2w_{S2})\theta+\mu+c_s}{(\theta+1)}$ regarding $\mu$ is $\frac{\theta}{2(3\theta+2)(\theta+1)}$ and $\theta > 0$, thus $f_T^{EN} - f_F^{BN} < 0$ in the interval $(\bar{\mu}_S^{EN}, \bar{\mu}_F^{EN})$. When $\mu = \bar{\mu}_F^{EN}$, $f_T^{EN} = f_F^{BN} = f_T^{EN}$.

Regarding the unit financing fee for trade credit, when $\bar{\mu}_S^{EN} \leq \mu \leq \bar{\mu}_F^{EN}$, $w_{S1}^{EN} - w_{S1}^{BN} = \frac{\theta(\theta+1)(w_L-c_L)}{(3\theta+2)(\theta+2)} < 0$. When $\mu = \bar{\mu}_S^{BN}$, $w_{S1}^{EN} = w_{S1}^{BN} = w_{S1}^{IS}$. When $\mu = \bar{\mu}_F^{BN}$, $w_{S1}^{EN} < w_{S1}^{BN}$.

**Appendix A.6 The Proof of Proposition 2**

Regarding the financing demand for conventional financing and in-transit inventory financing, when $\bar{\mu}_S^{EN} \leq \mu \leq \bar{\mu}_F^{EN}$, $q_T^{EN} - q_F^{BN} = \frac{D(\theta+1)(w_L-c_L)(\theta^2+4\theta+2)}{(\theta+2)(3\theta+2)} > 0$. When $\mu = \bar{\mu}_F^{BN}$, $q_T^{EN} = q_F^{BN}$. When $\mu \leq \bar{\mu}_F^{EN}$, $q_T^{EN} - q_F^{BN}$ regarding $\mu$ is $-\frac{D(\theta^2+4\theta+2)}{2(\theta+1)(3\theta+2)}$ and $\theta > 0$, thus $q_T^{EN} - q_F^{BN} > 0$ in the...
interval \([\mu_{T}^{EN}, \mu_{F}^{BN}]\). When \(\mu = \hat{\mu}_{S}^{BN} \), \(q_{T}^{EN} - q_{F}^{BN} = \frac{W_{L}-C_{L}}{2} > 0\). When \(\mu_{S}^{BN} \leq \mu \leq \mu_{S}^{EN}\), the first-order derivative of \(q_{T}^{EN} - q_{F}^{BN}\) regarding \(\mu\) is \(-\frac{D(5(\theta + 4))}{2(3\theta + 2)}\). When \(0 \leq \mu \leq \mu_{S}^{BN} \), \(q_{T}^{EN} - q_{F}^{BN} = \frac{W_{L}-C_{L}}{2} > 0\). Therefore, \(q_{T}^{EN} - q_{F}^{BN} > 0\), when \(0 \leq \mu \leq \mu_{F}^{BN}\).

Regarding the financing demand for trade credit, when \(\hat{\mu}_{S}^{EN} \leq \mu \leq \hat{\mu}_{T}^{EN}\), \(q_{S}^{EN} - q_{S}^{BN} = -\frac{DB(\theta + 1)^{2}(W_{L}-C_{L})}{(\theta + 2)(3\theta + 2)} < 0\). When \(\mu = \hat{\mu}_{S}^{BN} \), \(q_{S}^{EN} - q_{S}^{BN} = -\frac{DB(\theta^{2}+4\theta+2)}{2(\theta + 1)(3\theta + 2)} < 0\). When \(\mu_{S}^{BN} \leq \mu \leq \mu_{S}^{EN}\), the first-order derivative of \(q_{S}^{EN} - q_{S}^{BN}\) regarding \(\mu\) is \(-\frac{D(\theta^{2}+4\theta+2)}{2(\theta + 1)(3\theta + 2)}\) and \(\theta > 0\), thus \(q_{S}^{EN} - q_{S}^{BN} < 0\) in the interval \([\hat{\mu}_{S}^{BN}, \hat{\mu}_{S}^{EN}]\). When \(\hat{\mu}_{S}^{EN} \leq \mu \leq \hat{\mu}_{F}^{EN}\), \(q_{S}^{EN} - q_{S}^{BN} = -\frac{DB(\theta^{2}-4\theta+2)}{2(\theta + 1)(3\theta + 2)} < 0\). When \(\mu_{S}^{EN} \leq \mu \leq \mu_{F}^{EN}\), the first-order derivative of \(q_{S}^{EN} - q_{S}^{BN}\) regarding \(\mu\) is \(-\frac{D(\theta^{2}+4\theta+2)}{2(\theta + 1)(3\theta + 2)}\). Therefore, when \(\hat{\mu}_{F}^{EN} \leq \mu \leq \hat{\mu}_{F}^{BN}\), \(q_{S}^{EN} - q_{S}^{BN} < 0\).

### Appendix A.7 The Proof of Proposition 3

1. When \(\hat{\mu}_{S}^{EN} \leq \mu \leq \hat{\mu}_{T}^{EN}\), \(\pi_{F}^{BN} = \frac{D(1+\theta)[2(1-\mu)-2(\theta+1)w_{S2}+\theta c_{S}+\theta(2-\mu)]^{2}}{(3\theta^{2}+8\theta+4)^{2}}\) and \(\pi_{T}^{EN} = \frac{D(1+\theta)[2(1-\mu)-2(\theta+1)w_{S2}+\theta c_{S}+\theta(2-\mu)]^{2}}{(3\theta^{2}+8\theta+4)^{2}}\). We have \(\frac{\pi_{EN}^{EN}}{\pi_{F}^{BN}} = \frac{(2(1-\mu)-2(\theta+1)w_{S2}+\theta c_{S}+\theta(2-\mu))^{2}}{(3\theta^{2}+8\theta+4)^{2}}\). Owing to \(2(1-\mu) - 2(\theta + 1)w_{S2} + \theta c_{S} + \theta(2 - \mu) > 0\) and \((\theta^{2} + 4\theta + 2)(w_{L} - c_{L}) > 0\), \(\pi_{EN}^{EN} > \pi_{F}^{BN}\). When \(0 \leq \mu \leq \mu_{S}^{BN}\), \(\pi_{T}^{EN} = \frac{D(1-\mu)-(\theta+1)w_{S2}+\theta w_{IS}}{4(\theta + 1)}\). We have \(\frac{\pi_{EN}^{EN}}{\pi_{F}^{BN}} = \frac{(1-\mu)-(\theta+1)w_{S2}+\theta w_{IS}}{(1-\mu)-(\theta+1)w_{S2}+\theta w_{IS}}\).

When \(\hat{\mu}_{S}^{BN} \leq \mu \leq \hat{\mu}_{F}^{EN}\), the first-order derivative of \(\pi_{T}^{EN}\) and \(\pi_{F}^{EN}\) regarding \(\mu\) are \(-\frac{D((1-\mu)-(\theta+1)w_{S2}+\theta w_{IS})}{2(\theta + 1)}\) and \(-\frac{2D(1+\theta)(-2w_{S2}(\theta+1)+\theta c_{S}+(2-\mu)(\theta^{2}+4\theta+2))}{2(3\theta^{2}+8\theta+4)^{2}}\), both of which are linearly and negatively affected by \(\mu\). Thus, when \(\hat{\mu}_{S}^{BN} \leq \mu \leq \hat{\mu}_{F}^{EN}\), \(\pi_{T}^{EN} > \pi_{F}^{BN}\). When \(\mu = \hat{\mu}_{F}^{EN}\), \(\pi_{T}^{EN} - \pi_{F}^{BN} = \frac{D(\theta^{2})w_{S2}-(\theta^{2}+4\theta+2)(f_{IT}+w_{S2})+\theta c_{S}+(2-\mu)(\theta^{2}+4\theta+2)}{2(\theta + 1)} > 0\). When \(\mu_{T}^{EN} \leq \mu \leq \hat{\mu}_{F}^{BN}\), the first-order derivative of \(\pi_{T}^{EN}\) and \(\pi_{F}^{EN}\) regarding \(\mu\) are \(-\frac{D((1-\mu)-(\theta+1)w_{S2}+\theta w_{IS})}{2(\theta + 1)} < 0\) and \(-\frac{D((1-\mu)-(\theta+1)w_{S2}+\theta w_{IS})}{2(\theta + 1)} < 0\). Thus, when \(\hat{\mu}_{T}^{EN} \leq \mu \leq \hat{\mu}_{F}^{BN}\), \(\pi_{T}^{EN} > \pi_{F}^{EN}\). When \(\mu_{F}^{EN} \leq \mu < \hat{\mu}_{F}^{EN}\), \(\pi_{T}^{EN} = \frac{D(\theta^{2})w_{S2}-(\theta^{2}+4\theta+2)(f_{IT}+w_{S2})+\theta c_{S}+(2-\mu)(\theta^{2}+4\theta+2)}{2(\theta + 1)} > 0\). Therefore, when \(0 \leq \mu < \hat{\mu}_{F}^{EN}\), \(\pi_{T}^{EN} > \pi_{F}^{EN}\).

2. When \(\hat{\mu}_{S}^{EN} \leq \mu \leq \hat{\mu}_{T}^{EN}\), \(\pi_{F}^{BN} = \frac{D(1+\theta)[2(1-\mu)-2(\theta+1)w_{S2}+\theta c_{S}+\theta(2-\mu)]^{2}}{(3\theta^{2}+8\theta+4)^{2}}\) and \(\pi_{T}^{EN} = \frac{(1-\mu)\theta[(c_{L}-w_{L})(\theta^{2}+4\theta+2)-M_{1}][c_{L}-w_{L})(\theta^{2}+4\theta+2)-M_{1}]}{(3\theta^{2}+8\theta+4)^{2}}\).
\[\pi_{EN}^{BN} - \pi_{TF}^{EN} = \frac{(c_l-w_l)(\theta+1)D[2(c_l-w_l)(\theta^4+6\theta^3+11\theta^2+8\theta+2)+\theta^2M_2]}{(3\theta^2+8\theta+4)^2}. \quad M_2 = (\mu - c_s + 4w_{S2} - 2)\theta + (2\mu + 2w_{S2} - 2). \]

When \(\hat{\mu}_S^{EN} \leq \mu \leq \hat{\mu}_T^{EN}, q_{EN}^B = D[(\theta+1)[(2-\mu+c_s-4w_{S2})\theta^2+2-2\mu-2w_{S2}]] \) o wing to \( q_{EN}^B > 0, (2 - \mu + c_s - 4w_{S2})\theta + 2 - 2\mu - 2w_{S2} > 0. \) Therefore, when \(\hat{\mu}_S^{EN} \leq \mu \leq \hat{\mu}_T^{EN}, M_2 < 0. \) Owing to \(2(c_l-w_l)(\theta^4+6\theta^3+11\theta^2+8\theta+2) < 0 \) and \((c_l-w_l)(\theta+1)D < 0, \quad \pi_{EN}^{BN} - \pi_{TF}^{EN} \) is defined as:

\[\pi_{EN}^{BN} - \pi_{TF}^{EN} = \frac{(c_l-w_l)(\theta+1)D[2(c_l-w_l)(\theta^4+6\theta^3+11\theta^2+8\theta+2)+\theta^2M_2]}{(3\theta^2+8\theta+4)^2} < 0. \]

When \(0 \leq \mu \leq \hat{\mu}_S^{BN}, \pi_{EN}^{BN} = \frac{D[(\theta+1)(c_l-w_l)\theta^2+2-2\mu-2w_{S2}]}{4(\theta+1)} \) and \(\pi_{TF}^{EN} = \frac{D[(\theta+1)(c_l-w_l)\theta^2+2-2\mu-2w_{S2}]}{4(\theta+1)} \). Owing to \(2 - \mu + c_s - 4w_{S2} > 0, \) \((c_l-w_l)\theta + 2 - 2\mu - 2w_{S2} - 1 < 0, \) \(\pi_{EN}^{BN} \) and \(\pi_{TF}^{EN} \) are linearly and negatively affected by \(\mu. \) Therefore, when \(\hat{\mu}_S^{BN} \leq \mu \leq \hat{\mu}_T^{EN}, \pi_{EN}^{BN} - \pi_{TF}^{EN} > 0. \)

When \(\hat{\mu}_F^{BN} \leq \mu < \hat{\mu}_T^{EN}, \pi_{TF}^{BN} - \pi_{TF}^{EN} = 0. \) When \(\hat{\mu}_S^{EN} \leq \mu < \hat{\mu}_F^{BN}, \) the first-order derivative of \(\pi_{EN}^{BN} \) and \(\pi_{TF}^{EN} \) regarding \(\mu \) are defined as:

\[\frac{2D(\theta+1)[(\mu-c_s+4w_{S2}-2)\theta^2+2\mu+2w_{S2}]}{(3\theta^2+8\theta+4)^2} \quad \text{and} \quad \frac{D[(\theta+1)(c_l-w_l)\theta^2+2-2\mu-2w_{S2}]}{2(\theta+1)}. \]

Owing to \((2 - \mu + c_s - 4w_{S2})\theta + 2 - 2\mu - 2w_{S2} > 0 \) and \(f_{IR} > 0, \pi_{EN}^{BN} \) and \(\pi_{TF}^{EN} \) are linearly and negatively affected by \(\mu \) when \(\hat{\mu}_T^{EN} \leq \mu < \hat{\mu}_F^{BN}. \)

Therefore, when \(\hat{\mu}_T^{EN} \leq \mu < \hat{\mu}_F^{EN}, \pi_{TF}^{EN} > \pi_{TF}^{EN} \). To sum up, when \(0 \leq \mu < \hat{\mu}_F^{BN}, \pi_{EN}^{BN} - \pi_{TF}^{EN} > 0. \) When \(\hat{\mu}_F^{BN} \leq \mu < \hat{\mu}_F^{EN}, \pi_{TF}^{EN} = \pi_{TF}^{EN}. \)

**Appendix A.8 The Proof of Proposition 4**

(1) From Proposition 1, we know when \(\hat{\mu}_F^{BN} \leq \mu \leq 1, w_{S1}^{EN} = w_{S1}^{BN} \) and \(f_{TF}^{EN} = f_{TF}^{BN}. \) From Proposition 2, we know when \(\hat{\mu}_F^{BN} \leq \mu \leq 1, q_s^{EN} = q_s^{BN} \) and \(q_{TF}^{EN} = q_{TF}^{BN}. \)

(2) From Proposition 1, we know when \(0 \leq \mu \leq \hat{\mu}_S^{BN}, w_{S1}^{EN} = w_{S1}^{EN} \) and \(f_{TF}^{EN} < f_{TF}^{BN}. \) From Proposition 2, we know when \(0 \leq \mu \leq \hat{\mu}_S^{BN}, q_s^{EN} = q_s^{BN} = 0 \) and \(q_{TF}^{EN} > q_{TF}^{BN}. \)

When \(\hat{\mu}_S^{EN} \leq \mu \leq \hat{\mu}_S^{BN}, \pi_{EN}^{EN} - \pi_{TF}^{EN} = \frac{D[(\theta+1)(c_l-w_l)\theta^2+2-2\mu-2w_{S2}]}{4}. \) Therefore, when \(\hat{\mu}_S^{BN} \leq \mu \leq \hat{\mu}_S^{EN} \) and \(p > (c_l-w_l)\theta^4+4Q_1+Q_2+(4\mu+16\theta+12\theta+32\theta^2)\theta^2+(4\theta^4+8\theta^2+4\theta+2)\theta = \hat{p}_1, \pi_{EN}^{EN} > \pi_{EN}^{BN}. \) \(Q_1 = (-\mu + 10\theta - 2c_s - w_l + 20w_{S2} - 4)\theta^2. \)

When \(\hat{\mu}_S^{BN} < \mu < \hat{\mu}_S^{BN} \), the derivative of \(\pi_{EN}^{EN} - \pi_{EN}^{BN} \) regarding \(\mu \) is defined as:

\[\frac{D[(\theta+1)(c_l-w_l)\theta^2+2-2\mu-2w_{S2}]}{2(\theta+1)}. \]

Therefore, when \(\hat{\mu}_S^{BN} < \mu < \hat{\mu}_S^{EN} \), \(\pi_{EN}^{EN} > \pi_{EN}^{BN}. \) When \(\hat{\mu}_S^{BN} < \mu < \hat{\mu}_S^{EN} \) and \(p > (\mu+2\theta+9\theta-2w_{S2}+18\theta+5)^2+(4\theta+12\theta+2w_{S2}+24\theta-4)^2\theta=\hat{p}_2, \) the derivative of \(\pi_{EN}^{EN} - \pi_{EN}^{BN} \) regarding \(\mu \) is defined as:

\[q_{EN}^{BN} < \mu < q_{EN}^{EN} \quad \text{and} \quad p > (\mu+2\theta+9\theta-2w_{S2}+18\theta+5)^2+(4\theta+12\theta+2w_{S2}+24\theta-4)^2\theta=\hat{p}_2, \) the derivative of \(\pi_{EN}^{EN} - \pi_{EN}^{BN} \) regarding \(\mu \) is defined as:**
Regarding the financing fee for in-transit inventory financing, when \(0 \leq \mu < \hat{\mu}^{EC}_S\), the first-order derivative of \(f_T^{EC}\) regarding \(r\) is \(-\frac{pc_S}{2} < 0\). When \(\hat{\mu}^{EC}_S \leq \mu < \hat{\mu}^{EC}_T\), the first-order derivative of \(f_T^{EN}\) regarding \(r\) is \(-\frac{(3\theta^2+4\theta+2)pc_S}{(\theta+2)(3\theta+2)} < 0\). Therefore, when \(0 \leq \mu < \hat{\mu}^{EC}_T\), the financing fee for in-transit inventory financing decreases in \(r\).
Regarding the demand for in-transit inventory financing, when \(0 \leq \mu \leq \hat{\mu}^{EC}S\), the first-order derivative of \(q^{EC}_T\) regarding \(r\) is \(\frac{(\theta + 1)D_{pcs}}{2} > 0\). When \(\hat{\mu}^{EC}_S \leq \mu < \hat{\mu}^{EC}T\), the first-order derivative of \(q^{EC}_T\) regarding \(r\) is \(\frac{2(\theta + 1)D_{pcs}}{(\theta + 2)(3\theta + 2)} > 0\). When \(\hat{\mu}^{EC}_T \leq \mu < \hat{\mu}^{EC}T\), the first-order derivative of \(q^{EC}_T\) regarding \(r\) is \(-\frac{\theta D_{pcs}}{2(\theta + 1)} < 0\). Therefore, when \(0 \leq \mu < \hat{\mu}^{EC}_T\), the demand for in-transit inventory financing increases in \(r\). When \(\hat{\mu}^{EC}_T \leq \mu < \hat{\mu}^{EC}T\), the demand for in-transit inventory financing decreases in \(r\).

Regarding the financing fee for trade credit, when \(\hat{\mu}^{EC}_S \leq \mu < \hat{\mu}^{EC}T\), the first-order derivative of \(w^{EC}_{S1} - w_{S2}\) regarding \(r\) is \(-\frac{3\theta(\theta + 1)D_{pcs}}{(\theta + 2)(3\theta + 2)} < 0\). When \(\hat{\mu}^{EC}T \leq \mu \leq 1\), the first-order derivative of \(w^{EC}_{S1} - w_{S2}\) regarding \(r\) is \(\frac{\theta D_{pcs}}{2(\theta + 1)} > 0\). Therefore, when \(\hat{\mu}^{EC}_S \leq \mu < \hat{\mu}^{EC}T\), the demand for trade credit decreases in \(r\). When \(\hat{\mu}^{EC}_S \leq \mu < 1\), the demand for trade credit increases in \(r\).

Appendix A.11 The Proof of Lemma 4

If \(0 < \mu < \hat{\mu}_S^{EC}\), \(q^{EC}_T = q^{EN}_T + \frac{(\theta + 1)D_{pcs}}{2}\), the first-order derivative of \(q^{EC}_T\) regarding \(\rho\) is \(\frac{(\theta + 1)D_{pcs}}{2} > 0\) and the first-order derivative of \(q^{EC}_T\) regarding \(r\) is \(\frac{(\theta + 1)D_{pcs}}{2} > 0\). If \(\hat{\mu}_S^{EC} \leq \mu < \hat{\mu}^{EC}T\), \(q_T^{EC} = q_T^{EN} + \frac{2(\theta + 1)(\theta + 1)D_{pcs}}{(3\theta + 2)(\theta + 2)}\), the first-order derivative of \(q^{EC}_T\) regarding \(\rho\) is \(\frac{2(\theta + 1)(\theta + 1)D_{pcs}}{(3\theta + 2)(\theta + 2)} > 0\) and the first-order derivative of \(q^{EC}_T\) regarding \(r\) is \(\frac{2(\theta + 1)(\theta + 1)D_{pcs}}{(3\theta + 2)(\theta + 2)} > 0\). Therefore, if \(0 < \mu < \hat{\mu}^{EC}T\), the demand of in-transit inventory financing increases with the increase of \(\rho\) and \(r\). If \(\hat{\mu}^{EC}_T \leq \mu < \hat{\mu}^{EC}T\), \(q_T^{EC} = q_T^{EN} - \frac{\theta D_{pcs}}{2(\theta + 1)}\), the first-order derivative of \(q^{EC}_T\) regarding \(\rho\) is \(-\frac{\theta D_{pcs}}{2(\theta + 1)} < 0\) and the first-order derivative of \(q^{EC}_T\) regarding \(r\) is \(-\frac{\theta D_{pcs}}{2(\theta + 1)} < 0\). Therefore, the demand of in-transit inventory financing decreases with the increase of \(\rho\) and \(r\).

The first-order derivative of \(\pi_T^{EC} = \rho c_sq_tr\) regarding \(q_T\) is \(\rho c_s r > 0\). Obviously, \(\pi_T^{EC}\) is positively related to \(q_T\). Therefore, when \(0 < \mu < \hat{\mu}^{EC}_T\), \(\pi_T^{EC}\) increases with \(\rho\) and \(r\). When \(\hat{\mu}^{EC}_T \leq \mu < \hat{\mu}^{EC}T\), \(\pi_T^{EC}\) decreases with \(\rho\) and \(r\).

Appendix A.12 The Proof of Proposition 5

(1) Given \(\hat{\mu}^{EC}_S \leq \mu < \hat{\mu}^{EC}_T\), \(w_{S1}^{EC} - w_{S2} = \frac{(\theta + 1)(3\theta w_{S2} + 2c) + \theta}{(\theta + 2)(3\theta + 2)} + \frac{\theta(w_l - c_l)(\theta + 1)}{3\theta + 2} - \frac{3\theta(\theta + 1)D_{pcs}}{(3\theta + 2)(\theta + 2)} - w_{S2} \), and \(f_T^{EC} = \frac{2(\theta + 1) + \theta c_s - 2(\theta + 1)w_{S2}}{(\theta + 2)(3\theta + 2)} - \frac{\mu (1 + \theta)^2(w_l - c_l)}{(\theta + 2)(3\theta + 2)} - \frac{(3\theta^2 + 4\theta + 2)c_s r}{(3\theta + 2)(\theta + 2)}\). When \(f_T^{EC} < w_{S1}^{EC} - w_{S2}\), we have \(\rho < \frac{1 + w_{S2} - 2c_s - (1 + \theta)(w_l - c_l)}{r c_s}\). When \(f_T^{EC} > w_{S1}^{EC} - w_{S2}\), we have \(\rho < \frac{1 + w_{S2} - 2\mu c_s - (1 + \theta)(w_l - c_l)}{r c_s}\).
(2) Regarding the unit financing fee for trade credit, when \( \hat{\mu}^EC \leq \mu \leq \hat{\mu}^EC_T \), \( w^EC \)- \( w^EN \) = \( \frac{3\theta \rho c_s r (\theta+1)}{(3\theta+2)(\theta+2)} < 0 \).

When \( \mu = \hat{\mu}^EN_S \), \( w^EN_S = w^EC_S = w_I \). When \( \mu = \hat{\mu}^EC_S \), \( w^EC_S < w^EN_S \). Owing to the first-order derivative of \( w^EN_S - w_I = \frac{\theta(wl-cl)}{(\theta+2)(3\theta+2)} + \frac{\theta}{3\theta+2} \) regarding \( \mu \) is \( \frac{1}{3\theta+2} \) and \( \theta > 0 \), therefore \( w^EC_S - w^EN_S < 0 \) in the interval of \((\hat{\mu}^EN_S, \hat{\mu}^EC_S)\). In addition, when \( \mu = \hat{\mu}^EN_T \), \( w^EC_S - w^EN_S = -\frac{\theta \rho c_s r}{2(\theta+1)} \). When \( \mu = \hat{\mu}^EC_T \), \( w^EC_S < w^EN_S \). Owing to the first-order derivative of \( w^EN_S - w_I = \frac{\theta(wl-cl)}{(\theta+2)(3\theta+2)} \) regarding \( \mu \) is \( \frac{\theta}{2(3\theta+3)(\theta+1)} \) and \( \theta > 0 \), thus \( w^EC_S - w^EN_S < 0 \) in the interval \([\hat{\mu}^EC_T, \hat{\mu}^EN_T]\). In addition, when \( \hat{\mu}^EN_T < \mu \leq 1 \), \( w^EC_S - w^EN_S = -\frac{\theta \rho c_s r}{2(\theta+1)} < 0 \). Therefore, \( w^EC_S - w^EN_S < 0 \) when \( \hat{\mu}^EN_T < \mu \leq 1 \).

Regarding the unit financing fee for in-transit inventory financing, when \( \hat{\mu}^EC \leq \mu \leq \hat{\mu}^EC_T \), \( f^EC_T - f^EN_T = \frac{3\theta + 4(\theta + 2)\rho c_s r}{(3\theta + 2)(\theta + 2)} < 0 \). When \( \mu = \hat{\mu}^EC_T \), \( f^EC_T - f^EN_T = -\frac{\theta \rho c_s r}{2} < 0 \). Owing to the first-order derivative of \( f^EC_T - f^EN_T = \frac{\theta(wl-cl)}{(\theta+2)(3\theta+2)} + \frac{\theta}{3\theta+2} \) regarding \( \mu \) is \( \frac{\theta}{2(3\theta + 3)(\theta+1)} \) and \( \theta > 0 \), thus \( f^EC_T - f^EN_T < 0 \) in the interval \((\hat{\mu}^EN_T, \hat{\mu}^EC_T)\). In addition, when \( 0 \leq \mu \leq \hat{\mu}^EN_T \), \( f^EC_T - f^EN_T = \frac{\theta \rho c_s r}{2} < 0 \). When \( \mu = \hat{\mu}^EN_T \), \( f^EC_T = f^EN_T = f^IT \). When \( \mu = \hat{\mu}^EC_T \), \( f^EC_T < f^EN_T \). Owing to the first-order derivative of \( f^EC_T - f^EN_T = \frac{\theta(wl-cl)}{(\theta+2)(3\theta+2)} - \frac{\theta}{3\theta+2} < 0 \) and \( \theta > 0 \), therefore \( f^EC_T - f^EN_T < 0 \) in the interval \((\hat{\mu}^EC_T, \hat{\mu}^EN_T)\).

**Appendix A.13 The Proof of Proposition 6**

Regarding the demand for trade credit, when \( \hat{\mu}^EC \leq \mu \leq \hat{\mu}^EC_T \), \( q^EC - q^EN = \frac{\theta \rho c_s r}{(3\theta + 2)(\theta + 2)} > 0 \). When \( \mu = \hat{\mu}^EN_S \), \( q^EC - q^EN = -\frac{\theta \rho c_s r}{2} < 0 \). When \( \hat{\mu}^EN_S \leq \mu \leq \hat{\mu}^EC \), the first-order derivative of \( q^EN = q^BN \) regarding \( \mu \) is \( -\frac{\theta(\theta + 4\theta + 2)}{2(\theta + 1)^2} \) and \( \theta > 0 \), thus there exists \( \hat{\mu}^EC_S \) in the interval \([\hat{\mu}^EN_S, \hat{\mu}^EC_S]\). When \( \hat{\mu}^EN_S, \hat{\mu}^EC_S \), \( q^EC - q^EN < 0 \). When \( \hat{\mu}^EC_S < q^EC < q^EN \). When \( \mu = \hat{\mu}^EN_S \), \( q^EC - q^EN = \frac{\theta \rho c_s r}{(3\theta + 2)(\theta + 2)} > 0 \). When \( \mu = \hat{\mu}^EC_S \), \( q^EC - q^EN = -\frac{\theta \rho c_s r}{2} < 0 \). When \( \hat{\mu}^EC_S < q^EC < q^EN \).

When \( \hat{\mu}^EN_T < \mu \leq 1 \), \( q^EC - q^EN = \frac{\theta \rho c_s r}{2} > 0 \). Therefore, when \( \hat{\mu}^EN_T < \mu < \hat{\mu}^EC \), \( q^EC < q^EN \). When \( \hat{\mu}^EC < \mu \leq 1 \), \( q^EC > q^EN \).

Regarding the demand for in-transit inventory financing, when \( \hat{\mu}^EC \leq \mu \leq \hat{\mu}^EC_T \), \( q^EC - q^EN = \frac{2(\theta+1)(2\theta+1)\rho c_s r}{(3\theta + 2)(\theta + 2)} > 0 \). When \( \mu = \hat{\mu}^EN_T \), \( q^EC = q^EN \). When \( \hat{\mu}^EC \leq \mu \leq \hat{\mu}^EN_T \), the first-order derivative of
\( q^{EC}_T - q^{EN}_T \) regarding \( \mu \) is \( -\frac{D(\theta^2+4+2\theta + z)}{2(\theta+1)(3\theta+z)} \) and \( \theta > 0 \), thus \( q^{EC}_T - q^{EN}_T > 0 \) in the interval \([\hat{\mu}^{EC}_T, \hat{\mu}^{EN}_T]\). When

\( \mu = \hat{\mu}^{EC}_S \), \( q^{EC}_T - q^{EN}_T = \frac{(\theta+1)D\rho c s r}{2} > 0 \). When \( \hat{\mu}^{EN}_S \leq \mu \leq \hat{\mu}^{EC}_S \), the first-order derivative of \( q^{EC}_T - q^{EN}_T \) regarding \( \mu \) is \( -\frac{D\theta}{2(3\theta+z)} \). Thus, \( q^{EC}_T - q^{EN}_T > 0 \), when \( \hat{\mu}^{EN}_S \leq \mu \leq \hat{\mu}^{EC}_S \). When \( 0 \leq \mu \leq \hat{\mu}^{EN}_S \), \( q^{EC}_T - q^{EN}_T = \frac{(\theta+1)D\rho c s r}{2} > 0 \). Therefore, \( q^{EC}_T - q^{EN}_T > 0 \), when \( 0 \leq \mu < \hat{\mu}^{EN}_T \).

**Appendix A.14 The Proof of Proposition 7**

When \( \hat{\mu}^{EC}_T \leq \mu \leq \hat{\mu}^{EC}_T \), \( \pi^{EC}_T = \frac{D(1+\theta)(-2w_{S2}(2\theta+1)+\omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2}{(3\theta^2+4\theta+4)^2} \) and \( \pi^{EN}_T = \frac{D[1+(1+\theta)(-2w_{S2}(2\theta+1)+\omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2]}{(3\theta^2+4\theta+4)^2} \). We have \( \frac{\pi^{EC}_T}{\pi^{EN}_T} = \frac{(1+\theta)(-2w_{S2}(2\theta+1)+\omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2)}{(1+\theta)(-2w_{S2}(2\theta+1)+\omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2)} \). Owing to \( 2(1-\mu) - 2(2\theta+1)w_{S2} + \omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2 > 0 \) and \( \omega(2+\theta+z)(\theta^2+z)((w_l-c_l))^2 > 0 \), then we have \( \pi^{EC}_T - \pi^{EN}_T > 0 \). Owing to \( \pi^{EC}_T - \pi^{EN}_T > 0 \), we have \( \frac{\pi^{EC}_T - M_1}{\pi^{EN}_T} > 1 \) and \( \pi^{EC}_T - M_1 > \pi^{EN}_T \). When \( \hat{\mu}^{EN}_S \leq \mu \leq \hat{\mu}^{EC}_S \), the first-order derivative of \( \pi^{EC}_T \) and \( \pi^{EN}_T \) regarding \( \mu \) are

\[ \frac{D(1+\theta)(-2w_{S2}(2\theta+1)+\omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2}{2(\theta+1)} \] and

\[ \frac{D[(1+\theta)(-2w_{S2}(2\theta+1)+\omega(2-\mu)(2+\theta+z)(\theta^2+z)((w_l-c_l))^2]}{2(\theta+1)} \] \( < 0 \). Thus, \( \hat{\mu}^{EN}_S \leq \mu \leq \hat{\mu}^{EC}_S \) \( \pi^{EC}_T > \pi^{EN}_T \).

When \( \mu = \hat{\mu}^{EC}_T \), \( \pi^{EC}_T - \pi^{EN}_T = (f l_{r}+w_{L}+c_{L}-c_{L})/2(\theta+1) + M_2 \). When \( \hat{\mu}^{EC}_T \leq \mu \leq \hat{\mu}^{EN}_T \), the first-order derivative of \( \pi^{EC}_T \) and \( \pi^{EN}_T \) regarding \( \mu \) are

\[ \frac{D(\theta+2)(f l_{r}+w_{L}+c_{L})}{2(\theta+1)} - \frac{D\rho c s r}{2(\theta+1)} \] \( < 0 \) and

\[ \frac{2D(\theta+2)(f l_{r}+w_{L}+c_{L})}{(3\theta^2+4\theta+4)(\theta+2)} \] \( < 0 \). Thus, \( \hat{\mu}^{EC}_T \leq \mu \leq \hat{\mu}^{EN}_T \) \( \pi^{EC}_T > \pi^{EN}_T \).

Therefore, when \( 0 \leq \mu < \hat{\mu}^{EN}_T \), \( \pi^{EC}_T > \pi^{EN}_T \).

**Appendix A.15 The Proof of Proposition 8**

(1) From Proposition 5, we know when \( 0 \leq \mu \leq \hat{\mu}^{EN}_S \), \( w^{EC}_{S1} = w^{EN}_{S1} \) and \( f^{EC}_T < f^{EN}_T \). From Proposition 6, we know when \( 0 \leq \mu \leq \hat{\mu}^{EN}_S \), \( q^{EC}_S = q^{EN}_S = 0 \) and \( q^{EC}_T > q^{EN}_T \). Therefore, when \( 0 \leq \mu \leq \hat{\mu}^{EN}_S \), \( \pi^{EC}_T > \pi^{EN}_T \).

From Proposition 5, we know when \( \hat{\mu}^{EC}_S \leq \mu \leq \hat{\mu}^{EC}_T \), \( w^{EC}_{S1} < w^{EN}_{S1} \) and \( f^{EC}_T < f^{EN}_T \). From Proposition 6, we know when \( \hat{\mu}^{EC}_S \leq \mu \leq \hat{\mu}^{EC}_T \), \( q^{EC}_S > q^{EN}_S \) and \( q^{EC}_T > q^{EN}_T \). Therefore, when \( \hat{\mu}^{EC}_S \leq \mu \leq \hat{\mu}^{EC}_T \), \( \pi^{EC}_T > \pi^{EN}_T \).

From Proposition 5, we know when \( \hat{\mu}^{EN}_T \leq \mu \leq 1 \), \( w^{EC}_{S1} < w^{EN}_{S1} \) and \( f^{EC}_T = f^{EN}_T \). From Proposition 6, we know when \( \hat{\mu}^{EN}_T \leq \mu \leq 1 \), \( q^{EC}_S > q^{EN}_S \) and \( q^{EC}_T = q^{EN}_T = 0 \). Therefore, when \( \hat{\mu}^{EN}_T \leq \mu \leq 1 \), \( \pi^{EC}_T > \pi^{EN}_T \).
When $\hat{\mu}_{S}^{EC} \leq \mu < \hat{\mu}_{S}^{EN}$, $\pi_{F}^{EC} - \pi_{F}^{EN} = \frac{D\varphi_{C}\varphi_{T}(\varphi c_{C}r + 2w_{1}L - 2c_{C}L + 2w_{2}S - 2c_{C}L - 2w_{2}S)}{4} > 0$. Therefore, when $\hat{\mu}_{S}^{EC} \leq \mu < \hat{\mu}_{S}^{EN}$, $\pi_{F}^{EC} - \pi_{F}^{EN} > 0$. Then $\hat{\mu}_{S}^{EN} < \mu < \hat{\mu}_{S}^{EC}$, the derivative of $\pi_{F}^{EC} - \pi_{F}^{EN}$ regarding $\mu$ is
\[
\frac{D((w_{1}L + w_{2}L - p)^{2} + 2w_{1}L + 2w_{2}L - 2p)}{2(\theta + 1)} < 0.
\]
When $\hat{\mu}_{S}^{EN} < \mu < \hat{\mu}_{S}^{EC}$ and $p > \frac{2w_{1}L + w_{2}L + 2w_{2}S + (4w_{L} - 2S)(\theta + 1)}{(4\theta + 2) + 2(\theta + 1)} = \hat{p}_{4}$, the derivative of $\pi_{F}^{EC} - \pi_{F}^{EN}$ regarding $\mu$ is less than 0. $H_{1} = (-3\mu + 4c_{L} + 2c_{S} + 9w_{L} + 6w_{S} - 6w_{L} + 7)\theta^{2}, H_{2} = (-8\mu + 2c_{L} + 2c_{S} + 10w_{L} + 2w_{S} - 16w_{L} + 16)\theta$. When $\mu = \hat{\mu}_{S}^{EN}$, $\pi_{F}^{EC} > \pi_{F}^{EN}$. When $\mu = \hat{\mu}_{S}^{EC}, \pi_{F}^{EC} > \pi_{F}^{EN}$. Therefore, when $\hat{\mu}_{S}^{EN} < \mu < \hat{\mu}_{S}^{EC}$ and $p > \hat{p}_{4}, \pi_{F}^{EC} > \pi_{F}^{EN}$. When $\mu = \hat{\mu}_{T}^{EN}, \pi_{F}^{EC} > \pi_{F}^{EN} = \frac{D\varphi_{C}\varphi_{T}(\varphi c_{C}r + 2w_{1}L + 2w_{2}S + 2f_{IT} - 1)}{4} > 0$. When $\hat{\mu}_{T}^{EC} < \mu < \hat{\mu}_{T}^{EN}$ and $p > \hat{p}_{5}, \pi_{F}^{EC} > \pi_{F}^{EN}$.

To sum up, when $0 \leq \mu \leq 1$ and $p > \max(\hat{p}_{4}, \hat{p}_{5}), \pi_{F}^{EC} > \pi_{F}^{EN}$.

**Appendix A.16 The Proof of Lemma 5**

Regarding the financing fee for trade credit, when $\hat{\mu}_{S}^{ER} \leq \mu < \hat{\mu}_{T}^{ER}$, the first-order derivative of $w_{2}^{ER} - w_{2}$ regarding $\lambda$ is $-\frac{\theta_{C}r}{(\theta + 2)(3\theta + 2)} < 0$. Therefore, when $\hat{\mu}_{S}^{ER} \leq \mu < \hat{\mu}_{T}^{ER}$, the financing fee for trade credit decreases in $\lambda$. Regarding the financing fee for in-transit inventory financing, when $0 \leq \mu < \hat{\mu}_{S}^{ER}$, the first-order derivative of $f_{IT}^{ER}$ regarding $\lambda$ is $-\frac{\sigma_{T}r}{2(\theta + 1)} < 0$. When $\hat{\mu}_{S}^{ER} \leq \mu < \hat{\mu}_{T}^{ER}$, the first-order derivative of $f_{IT}^{ER}$ regarding $\lambda$ is $-\frac{2(\theta + 1)}{3(\theta + 2)} < 0$. Therefore, when $0 \leq \mu < \hat{\mu}_{T}^{ER}$, the financing fee for in-transit inventory financing decreases in $\lambda$.

Regarding the financing demand for trade credit, when $\hat{\mu}_{S}^{ER} \leq \mu < \hat{\mu}_{T}^{ER}$, the first-order derivative of $q_{S}^{ER}$ regarding $\lambda$ is $-\frac{2(\theta + 1)}{(3\theta + 2)(\theta + 2)} < 0$. Therefore, when $\hat{\mu}_{S}^{ER} \leq \mu < \hat{\mu}_{T}^{ER}$, the financing demand for trade credit decreases in $\lambda$.

Regarding the financing demand for in-transit inventory financing service, when $0 \leq \mu < \hat{\mu}_{S}^{ER}$, the first-order derivative of $q_{IT}^{ER}$ regarding $\lambda$ is $\frac{D\sigma_{T}r}{2} > 0$. When $\hat{\mu}_{S}^{ER} \leq \mu < \hat{\mu}_{T}^{ER}$, the first-order derivative of $q_{IT}^{ER}$ regarding $\lambda$ is $\frac{D\sigma_{T}r}{3(\theta + 2)} > 0$. Therefore, when $0 \leq \mu < \hat{\mu}_{T}^{ER}$, the financing demand for in-transit inventory financing increases in $\lambda$.

**Appendix A.17 The Proof of Proposition 9**
(1) Regarding the financing fee for conventional and in-transit inventory financing, when \( \hat{\mu}^{ER} \leq \mu \leq \hat{\mu}^{ER} \), \( f^{ER}_I - f^{EC}_I = -\frac{2\lambda \sigma_T}{3(\theta+2)} \theta < 0 \). When \( \mu = \hat{\mu}^{ER} \), \( f^{ER}_I < f^{EC}_I \). When \( \mu = \hat{\mu}^{EC} \), \( f^{ER}_I < f^{EC}_I \). Owing to the first-order derivative of \( f^{ER}_I - f^{EC}_I \) regarding \( \mu \) is \( \frac{\theta}{(3\theta+2)(\theta+1)} \) and \( \theta > 0 \), therefore \( f^{ER}_I - f^{EC}_I < 0 \) in the interval \( (\hat{\mu}^{EC}, \hat{\mu}^{ER}) \). When \( \mu = \hat{\mu}^{EC} \), \( f^{ER}_I = f^{EC}_I \). When \( \mu = \hat{\mu}^{ER} \), \( f^{ER}_I < f^{EC}_I \). Owing to the first-order derivative of \( f^{ER}_I - f^{EC}_I \) regarding \( \mu \) is \( -\frac{1}{3\theta+2} \) and \( \theta > 0 \), therefore \( f^{ER}_I - f^{EC}_I < 0 \) in the interval of \( (\hat{\mu}^{ER}, \hat{\mu}^{EC}) \). In addition, when \( 0 \leq \mu \leq \hat{\mu}^{EC} \), \( f^{ER}_I - f^{EC}_I = -\frac{\lambda \sigma_T}{2(\theta+1)} < 0 \). Therefore, \( f^{ER}_I - f^{EC}_I < 0 \), when \( 0 < \mu < \hat{\mu}^{EC} \).

Regarding the unit financing fee for trade credit, when \( \hat{\mu}^{ER} \leq \mu \leq \hat{\mu}^{EC} \), \( w^{ER}_{S1} - w^{EC}_{S1} = w_{IS} \). When \( \mu = \hat{\mu}^{EC} \), \( w^{ER}_{S1} < w^{EC}_{S1} \). Owing to the first-order derivative of \( w^{EC}_{S1} - w^{ER}_{S1} \) regarding \( \mu \) is \( \frac{1}{3\theta+2} \) and \( \theta > 0 \), therefore \( w^{ER}_{S1} - w^{EC}_{S1} < 0 \) in the interval of \( (\hat{\mu}^{EC}, \hat{\mu}^{ER}) \). In addition, when \( \mu = \hat{\mu}^{EC} \), \( w^{ER}_{S1} = w^{EC}_{S1} \). When \( \mu = \hat{\mu}^{ER} \), \( w^{ER}_{S1} < w^{EC}_{S1} \). Owing to the first-order derivative of \( w^{EC}_{S1} - w^{ER}_{S1} \) regarding \( \mu \) is \( -\frac{\theta}{(3\theta+2)(\theta+1)} \) and \( \theta > 0 \), thus \( w^{ER}_{S1} - w^{EC}_{S1} < 0 \) in the interval \( [\hat{\mu}^{ER}, \hat{\mu}^{EC}] \). Therefore, \( w^{ER}_{S1} - w^{EC}_{S1} < 0 \) when \( \hat{\mu}^{EC} < \mu < \hat{\mu}^{EC} \).

(2) Regarding the financing demand for conventional financing and in-transit inventory financing, when \( \hat{\mu}^{ER} \leq \mu \leq \hat{\mu}^{EC} \), \( q^{ER}_I - q^{EC}_I = \frac{D\lambda \sigma_T}{(3\theta+2)(\theta+2)} > 0 \). When \( \mu = \hat{\mu}^{EC} \), \( q^{ER}_I = q^{EC}_I \). When \( \mu = \hat{\mu}^{ER} \leq \mu \leq \hat{\mu}^{EC} \), the first-order derivative of \( q^{ER}_I - q^{EC}_I \) regarding \( \mu \) is \( -\frac{D(\theta^2+4\theta+2)}{2(\theta+1)(3\theta+2)} \) and \( \theta > 0 \), thus \( q^{ER}_I - q^{EC}_I > 0 \) in the interval \( [\hat{\mu}^{ER}, \hat{\mu}^{EC}] \). When \( \mu = \hat{\mu}^{EC} \), \( q^{ER}_I - q^{EC}_I = \frac{\lambda \sigma_T}{2} > 0 \). When \( \mu = \hat{\mu}^{EC} \), \( q^{ER}_I - q^{EC}_I = \frac{\lambda \sigma_T}{2} > 0 \). Therefore, \( q^{ER}_I - q^{EC}_I > 0 \), when \( 0 \leq \mu < \hat{\mu}^{EC} \).

Regarding the financing demand for trade credit, when \( \hat{\mu}^{ER} \leq \mu \leq \hat{\mu}^{EC} \), \( q^{ER}_I - q^{EC}_I = -\frac{D\lambda \sigma_T}{(3\theta+2)(\theta+2)} < 0 \). When \( \mu = \hat{\mu}^{EC} \), \( q^{ER}_I < q^{EC}_I \). When \( \mu = \hat{\mu}^{EC} \), the first-order derivative of \( q^{ER}_I - q^{EC}_I \) regarding \( \mu \) is \( -\frac{D(\theta^2+4\theta+2)}{2(\theta+1)(3\theta+2)} \) and \( \theta > 0 \), thus \( q^{ER}_I - q^{EC}_I < 0 \) in the interval \( [\hat{\mu}^{EC}, \hat{\mu}^{ER}] \). When \( \mu = \hat{\mu}^{EC} \), \( q^{ER}_I - q^{EC}_I = \frac{\lambda \sigma_T}{2} > 0 \). When \( \mu = \hat{\mu}^{EC} \), \( q^{ER}_I - q^{EC}_I = \frac{\lambda \sigma_T}{2} > 0 \). Therefore, \( q^{ER}_I - q^{EC}_I > 0 \), when \( 0 \leq \mu < \hat{\mu}^{EC} \).

Appendix A.18 The Proof of Proposition 10
(1) From Proposition 9, we know when \( \hat{\mu}^E_T \leq \mu \leq 1 \), \( W_{S1}^{ER} = W_{S1}^{EC} \) and \( f_T^{ER} = f_T^{EC} \). From Proposition 9, we also know when \( \hat{\mu}^E_T \leq \mu \leq 1 \), \( q_S^E = q_S^E \) and \( q_T^E = q_T^E \). Therefore, when \( \hat{\mu}^E_T \leq \mu \leq 1 \), \( \pi_R^{ER} = \pi_R^{EC} \).

(2) From Proposition 9, we know when \( 0 \leq \mu \leq \hat{\mu}^E_{S1} \), \( W_{S1}^{ER} = W_{S1}^{EC} \) and \( f_T^{ER} < f_T^{EC} \). From Proposition 9, we know when \( 0 \leq \mu \leq \hat{\mu}^E_{S1} \), \( q_S^E = q_S^E = 0 \) and \( q_T^E > q_T^E \). Therefore, when \( 0 \leq \mu \leq \hat{\mu}^E_{S1} \), \( \pi_R^{ER} > \pi_R^{EC} \).

When \( \hat{\mu}^E_{S1} \leq \mu \leq \hat{\mu}^E_S \), \( \pi_R^{EN} - \pi_R^{BN} = \lambda \sigma_T D(2 \theta (pcsr - cl + wL - wS2 + wL5) + 2pcsr + \lambda \sigma_T - 2wS2 + 2p - 2cl) > 0 \). Therefore, when

\[
\hat{\mu}^E_{S1} \leq \mu \leq \hat{\mu}^E_S, \quad \pi_R^{ER} > \pi_R^{EC}.
\]

When \( \hat{\mu}^E_{S1} < \mu < \hat{\mu}^E_S \), the derivative of \( \pi_R^{ER} - \pi_R^{EC} \) regarding \( \mu \) is

\[
\frac{D((p - wL - wL)\theta + 2p - 2wL - 2wL)}{p(\theta + 1)} < 0.
\]

Therefore, when \( \hat{\mu}^E_{S1} < \mu < \hat{\mu}^E_S \) and \( p > (2wL - cL)\theta^2 + ((2p + 2) cS - 4cS + 12cL + 13wL - 2wS2 + 18wL5 - 5)\theta^2 + ((2p + 2) cS - 4cL + 12cL + 14wL - 2wS2 + 24wL5 - 4)\theta - 4(\mu - wL - wL5) = \hat{\rho}_7 \), the derivative of \( \pi_R^{ER} - \pi_R^{EC} \) regarding \( \mu \) is bigger than 0. Therefore, when \( \hat{\mu}^E_{S1} < \mu < \hat{\mu}^E_S \) and \( p >\max(\hat{\rho}_6, \hat{\rho}_7) \), \( \pi_R^{ER} > \pi_R^{EC} \). When \( \hat{\mu}^E_T < \mu < \hat{\mu}^E_S \) and \( p > (2wL - cL)\theta^2 + (2p + 2) cS - 4cS + 12cL + 14wL - 2wS2 + 24wL5 - 4)\theta - 4(\mu - wL - wL5) = \hat{\rho}_7 \), the derivative of \( \pi_R^{ER} - \pi_R^{EC} \) regarding \( \mu \) is

\[
\frac{D((p - wL - wL)\theta + 2p - 2wL - 2wL)}{p(\theta + 1)} < 0.
\]

Therefore, when \( \hat{\mu}^E_T < \mu < \hat{\mu}^E_S \) and \( p >\max(\hat{\rho}_6, \hat{\rho}_7) \), \( \pi_R^{ER} > \pi_R^{EC} \).

To sum up, when \( 0 \leq \mu < \hat{\mu}^E_T \) and \( p >\max(\hat{\rho}_6, \hat{\rho}_7) \), \( \pi_R^{ER} > \pi_R^{EC} \).

**Appendix A.19 The Equilibrium for TC Model**

**Table A. 3.** Equilibrium decision when the TPL is more powerful.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Conditions</th>
<th>Optimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_{S1}^{EC} )</td>
<td>( 0 \leq \mu &lt; \hat{\mu}_1^T ) ( \hat{\mu}_1^S \leq \mu &lt; \hat{\mu}_1^T ) ( \hat{\mu}_1^T \leq \mu \leq 1 )</td>
<td>( W_{S1} ) ( K_1 - \frac{(wL - cL)\theta}{4(\theta + 1)} + \frac{(2\theta + 6\theta + 4)pcsr}{2(2\theta + 4\theta + 2)(\theta + 1)} + \frac{2wS2}{(2\theta + 4\theta + 2)} ) ( \frac{2(\theta + 1)}{2(\theta + 1)} ) ( \frac{2(\theta + 1)}{2(\theta + 1)} )</td>
</tr>
<tr>
<td>( f_T^{EC} )</td>
<td>( 0 \leq \mu &lt; \hat{\mu}_1^T ) ( \hat{\mu}_1^S \leq \mu &lt; \hat{\mu}_1^T ) ( \hat{\mu}_1^T \leq \mu \leq 1 )</td>
<td>( f_T ) ( K_2 - \frac{\theta^2 + 4\theta + 2}{2(\theta + 1)}(wL - cL) ) ( \frac{(\theta + 1)^2 pcsr}{\theta^2 + 4\theta + 2} ) ( \frac{2(\theta + 1)}{2(\theta + 1)} )</td>
</tr>
<tr>
<td>( q_S^{EC} )</td>
<td>( 0 \leq \mu &lt; \hat{\mu}_1^S ) ( \hat{\mu}_1^{S1} \leq \mu &lt; \hat{\mu}_1^S ) ( \hat{\mu}_1^S \leq \mu &lt; \hat{\mu}_1^T ) ( \hat{\mu}_1^T \leq \mu \leq 1 )</td>
<td>( K_3 - \frac{DB(wL - cL)}{4} + \frac{DB(\theta + 1)}{2(\theta + 1)}pcsr ) ( \frac{D[f_T(\theta + 1)pcsr]}{2} )</td>
</tr>
</tbody>
</table>
\[
q_{\alpha}^{TC} = \begin{cases} 
0 & \mu \leq \bar{\mu}_{S} \\
\frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)} & \mu > \bar{\mu}_{S} 
\end{cases}
\]

\[
\bar{\hat{\mu}}_{S} \leq \mu < \bar{\mu}_{S} \\
\hat{\mu}_{S} \leq \mu < \bar{\mu}_{S} \\
\hat{\mu}_{T} \leq \mu < \bar{\mu}_{T} 
\]

\[
\hat{\mu}_{S} \leq \mu \leq 1 
\]

Note: \(\hat{\mu}_{S} = \frac{(\theta^2 + 2\theta + 2)w_{S2} + 2(\theta^2 + \theta + 1)\rho c_{S}r_{T} + \theta c_{S}r_{T} + 4w_{S2} - 2(\theta + 1)\rho + 2(\theta + 1))w_{S2} - 2(\theta + 1)\theta c_{S}r_{T}}{\theta^2 + 4\theta + 2}\)

\[
\hat{\mu}_{T} = \frac{(\theta^2 + 4\theta + 2)(w_{L} - c_{L}) + 2(\theta(\theta + 1)\rho c_{S}r_{T} + \theta c_{S}r_{T} - 4w_{S2} - 2(\theta + 1)\rho + 2(\theta + 1))w_{S2} - 2(\theta + 1)\theta c_{S}r_{T}}{\theta^2 + 4\theta + 2}
\]

\[
K_{1} = \frac{(\theta^2 + 2\theta + 2)w_{S2} + 2(\theta^2 + \theta + 1)\rho c_{S}r_{T} + \theta c_{S}r_{T} + 4w_{S2} - 2(\theta + 1)\rho + 2(\theta + 1))w_{S2} - 2(\theta + 1)\theta c_{S}r_{T}}{\theta^2 + 4\theta + 2} \\
K_{2} = \frac{(\theta^2 + 4\theta + 2)(w_{L} - c_{L}) + 2(\theta(\theta + 1)\rho c_{S}r_{T} + \theta c_{S}r_{T} - 4w_{S2} - 2(\theta + 1)\rho + 2(\theta + 1))w_{S2} - 2(\theta + 1)\theta c_{S}r_{T}}{\theta^2 + 4\theta + 2}
\]

Appendix A.2 The Proof of Proposition 10

When max \(\{\bar{\mu}_{S}^{EC}, \bar{\mu}_{T}^{TC}\} \leq \mu \leq \min \{\bar{\mu}_{S}^{EC}, \bar{\mu}_{T}^{TC}\}, \)

\[
\pi_{T}^{EC} = \frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)} \\
\pi_{T}^{TC} = \frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)} \\
\pi_{E}^{EC} = \frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)} \\
\pi_{T}^{EC} - \pi_{E}^{EC} = \frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)}
\]

For the extra financing fee paid by the supplier,

\[
\pi_{T}^{EC} = \frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)} \\
\pi_{E}^{EC} = \frac{D[2(1+2\theta) - (\theta^2 + 5\theta + 2)\mu - \theta(\theta+1)c_{S} - 2(\theta^2 + 3\theta + 2(\theta+1))w_{S2} + k_{1} + k_{2}]}{2(\theta^2 + 4\theta + 2)}
\]

Adopting proof by contradiction, we assume \((w_{L} - c_{L})\theta^2 + (4w_{L} - 4c_{L} + (1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2)\rho + 2pc_{S}r_{T} - 2\mu + 2(w_{L} - c_{L} - 2w_{S2} + 2 \leq 0$. Owing to $2pc_{S}r_{T} - 2\mu + 2(w_{L} - c_{L} - 2w_{S2} + 2 = 2pc_{S}r_{T} + 2(w_{L} - c_{L}) + [(1 - \mu) - w_{S2}] > 0$ and $(w_{L} - c_{L})\theta^2 > 0$, $(4w_{L} - 4c_{L} + (1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2)\theta < (w_{L} - c_{L})\theta^2 + (4w_{L} - 4c_{L} + (1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2)\theta + 2pc_{S}r_{T} - 2\mu + 2(w_{L} - c_{L}) - 2w_{S2} + 2 \leq 0$. Owing to $4w_{L} - 4c_{L} > 0$, we have $(1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2 < 0$. Therefore, we have $(1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2 = 4pc_{S}r_{T} + (1 - \mu) - w_{S2} + 1 < 0$. Owing to $c_{S} + w_{S2} - 4w_{S2} + 1 > 0$ and $w_{S2} > w_{S2}, c_{S} + w_{S2} - 4w_{S2} + 1 > c_{S} + w_{S2} - 4w_{S2} + 1 = c_{S} - 3w_{S2} + 1 > 0$. In addition, $1 - \mu - w_{S2} > 0$ and $4pc_{S}r_{T} > 0$. $(1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2 < 0$ is invalid. Therefore, the null hypothesis is invalid. Therefore, we have $(w_{L} - c_{L})\theta^2 + (4w_{L} - 4c_{L} + (1 + 4p)\rho c_{S}r_{T} - \mu - 4w_{S2} + 2)\theta + 2pc_{S}r_{T} - 2\mu + 2(w_{L} - c_{L}) - 2w_{S2} + 2 > 0$. Owing to $4(\theta + 1)(\theta + 2)(3\theta + 2) > 0$ and $Dpc_{S}r\theta^2 > 0$, we have $\pi_{T}^{EC} - \pi_{E}^{EC} < 0$. Therefore, $\pi_{T}^{EC} < \pi_{E}^{EC}$. 

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Owing to $\pi_{\text{TC}} > \pi_{\text{EC}}$, we have $\pi_{\text{TC}} + \pi_{\text{EC}} > \pi_{\text{EC}} + \pi_{\text{TC}}$ and $\pi_{\text{TC}} - \pi_{\text{EC}} > \pi_{\text{EC}} - \pi_{\text{TC}}$. Owing to $0 < \pi_{\text{EC}} - \pi_{\text{TC}}$, we have $\pi_{\text{TC}} - \pi_{\text{EC}} > \pi_{\text{EC}} - \pi_{\text{TC}} > 0$. Therefore, $\pi_{\text{TC}} > \pi_{\text{EC}}$.

### Appendix A.21 The Proof of Proposition 11

Given $\tilde{\mu}_S^{\text{TC}} \leq \mu \leq \mu_\text{L}$, $q_\text{SC}^{\text{TC}} = \frac{D[(\theta^2+6\theta+4)\mu-(\theta^2+8\theta+4)c_S-2\theta(\theta+1)w_{S_2}+2\theta(\theta+1)]}{4(\theta^2+4\theta+2)}$ and $q_\text{T}^{\text{TC}} = \frac{D[2(\theta+1)-(\theta+2)(w_{L}-c_L)+(2\theta+2)(w_{L}-c_L)]}{4(\theta+1)} + \frac{D w_{L} c_S}{2(\theta+1)}$. When $q_\text{T}^{\text{TC}} > q_\text{SC}^{\text{TC}}$, we have

(i) $w_L > G^1(c_L)$,

$$G^1(c_L) = \frac{-2(6\theta^2+7\theta+2)pc_Sr+(2\theta^4+13\theta^3+26\theta^2+18\theta+4)(c_L-w_L)-(2\theta^3+13\theta^2+14\theta+4)c_S+2(6\theta^2+7\theta+2)w_{S_2}+(2\theta^3+13\theta^2+20\theta+8)\mu-2(\theta+1)(3\theta+2)}{(\theta^2+4\theta+2)(\theta+2)(\theta+1)}$$

(ii) $\rho > G^2(r)$,

$$G^2(r) = \frac{(2\theta^4+13\theta^3+26\theta^2+18\theta+4)(c_L-w_L)+(2\theta^3+13\theta^2+20\theta+8)\mu-(2\theta^3+13\theta^2+14\theta+4)c_S+2(6\theta^2+7\theta+2)w_{S_2}+2(\theta+1)(3\theta+2)}{2rc_S(\theta+1)(3\theta+2)}$$

(iii) $\mu < G^3(w_{S_2})$,

$$G^3(w_{S_2}) = \frac{2(6\theta^2+7\theta+2)pc_S+(2\theta^4+26\theta^2+13\theta^3+18\theta+4)(w_L-c_L)+(2\theta^3+13\theta^2+14\theta+4)c_S-2(6\theta^2+7\theta+2)w_{S_2}+2(\theta+1)(3\theta+2)}{2\theta^3+13\theta^2+20\theta+8}$$

The above equations show the constraint for each condition in Proposition 11.