Manzini, P., Mariotti, M., & Petri, H. (2019). Corrigendum to “Dual random utility maximisation” [J. Econ. Theory 177 (2018) 162–182]. *Journal of Economic Theory*, 184, [104944].
<https://doi.org/10.1016/j.jet.2019.104944>

Peer reviewed version

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Link to published version (if available):
[10.1016/j.jet.2019.104944](https://doi.org/10.1016/j.jet.2019.104944)

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Dual Random Utility Maximisation: Corrigendum

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This version: September 2019

Abstract

Lemma 2 (and hence the statement of Theorem 1) in Manzini and Mariotti [3] is incorrect as stated. For the claim to hold an additional axiom is required. We correct the mistake in the proof in Manzini and Mariotti [3].

J.E.L. codes: D03, D01

Keywords: Stochastic Choice, Attraction Effect, Similarity Effect.

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[§]We thank the Editor Marciano Siniscalchi and Sean Horan for helpful comments and suggestions.

The Random Utility Model (RUM) is a model of stochastic choice. When seen as depending on a probability distribution over rankings of alternatives, the choice probability of every element in a set is given by the overall probability of being ranked top in that set. Manzini and Mariotti [3] (henceforth MM) characterise a variation of the Random Utility Model (RUM) called Dual Random Utility Maximisation (dRUM), in which utility depends on only two states, and state probabilities are allowed to depend on the menu. MM also characterises the special case in which state probabilities are constant across menus (idRUM), but this latter result is incorrect as stated. In this note we correct the mistakes in the statement of Theorem 1 and the proof of Lemma 2 in MM. The main result (Theorem 2) is unaffected.

Let X be a finite set of alternatives, define a ranking as a bijection $r : \{1, \dots, n\} \rightarrow X$, and let $\mathcal{R}$ be the set of all rankings, and $\mathcal{R}(a, A)$ denote the collection of rankings that rank alternative a as top in set A (that is $\mathcal{R}(a, A) = \{r \in \mathcal{R} : r^{-1}(b) < r^{-1}(a) \Rightarrow b \notin A\}$). A *stochastic choice rule* is a map $p : X \times \mathcal{D} \rightarrow [0, 1]$ such that: $\sum_{a \in A} p(a, A) = 1$ for all $A \in \mathcal{D}$; $p(a, A) = 0$ for all $a \notin A$; and $p(a, A) \in [0, 1]$ for all $a \in A$, for all $A \in \mathcal{D}$. The value $p(a, A)$ may be interpreted either as the probability with which an individual agent (on whose behaviour we focus) chooses a from the menu A , or as the fraction of a population choosing a from A . Let μ be a probability distribution on $\mathcal{R}$. A *RUM* is a stochastic choice rule p such that $p(a, A) = \mu(\mathcal{R}(a, A))$. We can then re-state the definitions of dRUM and idRUM from MM:

Definition 1. A *dual RUM (dRUM)* is a stochastic choice rule p for which there exists a triple $(r_1, r_2, \tilde{\alpha})$, where r_1 and r_2 are rankings on X and $\tilde{\alpha} : 2^X \setminus \emptyset \rightarrow (0, 1)$ is a function that for each menu A assigns probabilities $\tilde{\alpha}(A)$ to r_1 and $1 - \tilde{\alpha}(A)$ to r_2 , such that for all A :

$$p(a, A) = \tilde{\alpha}(A) \mathbf{1}_{\mathcal{R}(a, A)}(r_1) + (1 - \tilde{\alpha}(A)) \mathbf{1}_{\mathcal{R}(a, A)}(r_2)$$

where $\mathbf{1}_K$ denotes the indicator function for a set K and $\mathcal{R}(a, A)$ denotes the collection of rankings that rank alternative a as top in set A .

A *Menu-independent dRUM (idRUM)* is a dRUM for which there exists an α such that $\tilde{\alpha}(A) = \alpha$ for all menus A . An idRUM p is thus identified by a triple (r_1, r_2, α) of

two rankings and a number $\alpha \in (0, 1)$.

The Axioms used in MM to characterise an idRUM are below - the interested reader can refer to MM for their discussion. Say that b *impacts* a in A whenever $p(a, A) > p(a, A \cup \{b\})$ (Manzini and Mariotti [3]). The following three conditions are observable implications of idRUM:

Monotonicity: If $a \in A \subset B$ then $p(a, A) \geq p(a, B)$.

Contraction Consistency: If b impacts a in A and $B \subset A$, then b impacts a in B .

Negative Expansion: If $a \in A$ and $p(a, A) < p(a, B) < 1$ then $p(a, A \cup B) = 0$.

Theorem 1 in MM stated that an idRUM is characterised by the three axioms above. In fact, an additional property is needed. The reason being that there are stochastic choice functions that satisfy Monotonicity, Negative Expansion and Contraction Consistency but are not idRUMs. Example 1 illustrates.

Example 1 A stochastic choice function satisfying Monotonicity, Negative Expansion and Contraction Consistency which is not idRUM

	a	b	c	d
$\{a, b, c, d\}$	0	0	$\frac{2}{3}$	$\frac{1}{3}$
$\{a, b, c\}$	0	0	1	—
$\{a, b, d\}$	0	0	—	1
$\{a, c, d\}$	0	—	$\frac{2}{3}$	$\frac{1}{3}$
$\{b, c, d\}$	—	0	$\frac{2}{3}$	$\frac{1}{3}$
$\{a, b\}$	$\frac{1}{6}$	$\frac{5}{6}$	—	—
$\{a, c\}$	0	—	1	—
$\{a, d\}$	0	—	—	1
$\{b, c\}$	—	0	1	—
$\{b, d\}$	—	0	—	1
$\{c, d\}$	—	—	$\frac{2}{3}$	$\frac{1}{3}$

This stochastic choice function is generated by the rankings r_1, r_2 and r_3 with $\mu(r_1) = \frac{1}{3}$, $\mu(r_2) = \frac{1}{2}$ and $\mu(r_3) = \frac{1}{6}$ where:

r_1	r_2	r_3
d	c	c
c	d	d
b	b	a
a	a	b

It is immediate to see that the stochastic choice function of example 1 cannot be an idRUM, as each probability can take any of four interior values (note that an idRUM can only take two interior values α and $1 - \alpha$). In fact, any counterexample to MM Theorem 1 has a structure similar to example 1. Define a graph by letting X be the vertex set and by drawing an edge between $a, b \in X$ if and only if $p(a, \{a, b\}) \in (0, 1)$. One can show that Monotonicity, Negative Expansion and Contraction Consistency imply that p uses a constant α when restricted to a connected component of the graph.¹ However, as example 1 illustrates α is not necessarily constant across components. In example 1 p is an idRUM with $\alpha = \frac{1}{3}$ when restricted to $\{c, d\}$ and $\alpha = \frac{5}{6}$ when restricted to $\{a, b\}$. Essentially, our next axiom assures that p uses the same α on each component of the graph.

For any $a, b \in X$, b dominates a if $p(a, A) = 0$ for all A for which $b \in A$. Then consider the following Axiom:

Alignment: Suppose that c, d dominate a, b and that $1 > p(a, \{a, b\}) > p(b, \{a, b\}) > 0$, $1 > p(c, \{c, d\}) > p(d, \{c, d\}) > 0$. Then $\frac{p(a, \{a, b\})}{p(b, \{a, b\})} = \frac{p(c, \{c, d\})}{p(d, \{c, d\})}$.

First note that the stochastic choice function of example 1 violates Alignment since while the premise holds (i.e. c, d dominate a, b), the conclusion is violated, as $\frac{p(a, \{a, b\})}{p(b, \{a, b\})} = \frac{1}{5} \neq 2 = \frac{p(c, \{c, d\})}{p(d, \{c, d\})}$.

Moreover, Alignment is clearly necessary for an idRUM. To see this, let the premise hold for a an idRUM p with (r_1, r_2, α) . W.l.o.g. let $\alpha \geq \frac{1}{2}$, so that $\frac{p(a, \{a, b\})}{p(b, \{a, b\})} = \frac{\alpha}{1-\alpha}$. Then since $1 > p(c, \{c, d\}) > p(d, \{c, d\}) > 0$ it can only be that $p(c, \{c, d\}) = \alpha$ so that c maximises r_1 in $\{c, d\}$ while d maximises r_2 in $\{c, d\}$. But then $\frac{p(c, \{c, d\})}{p(d, \{c, d\})} = \frac{\alpha}{1-\alpha}$. Note that a much stronger property than Alignment holds where A replaces $\{a, b\}$ and B replaces $\{c, d\}$, as this same substitution can be made in the argument just provided.

The explanation of this property is as follows. In the population interpretation, if exactly one group ranks a over b and exactly one group ranks c above d (which is expressed

¹This result is part of ongoing work and a proof is available upon request. An immediate corollary to this result is that if X is path connected (i.e. has a single path component) then Monotonicity, Negative Expansion and Contraction Consistency implies Lemma 2 of MM. Thus the characterisation in MM holds for stochastic choice functions on path connected domains. This domain restriction is quite natural and appears as richness in an early version of Echenique and Saito [1] and as linked domain in Horan [2].

in terms of behaviour by the inequalities in the axiom), given the fixed proportions of the groups it must be the case that the pairwise choice probability ratio between a, b coincides with that for c, d . The axiom asks for this alignment property to hold only in the special case in which, in addition, both groups rank a pair of alternatives above the other pair (which is expressed in terms of behaviour by the dominance condition in the axiom).

We next show how to modify the proof of Lemma 2. First we recall the statement of Lemma 1, which is used in the proof of Lemma 2:

Lemma 1. (*Manzini and Mariotti [3]*) *Let p be a stochastic choice rule that satisfies Monotonicity and Negative Expansion. Then for any menu A , if $p(a, A) \in (0, 1)$ for some $a \in A$ there exists $b \in A$ for which $p(b, A) = 1 - p(a, A)$.*

Then we can state and prove a correct version of Lemma 2 - the corrections are from *** onwards.

Lemma 2. (*corrected*) *Let p be a stochastic choice rule that satisfies Monotonicity, Negative Expansion and Alignment. Then there exists $\alpha \in (0, 1)$ such that, for any menu A and all $a \in A$, $p(a, A) \in \{0, \alpha, 1 - \alpha, 1\}$.*

Proof: The case $p(a, A) = 1$ for all A for some a is trivial. Then take A for which $p(a, A) = \alpha \in (0, 1)$ for some a . By Lemma 1 there exists exactly one alternative $b \in A$ for which $p(b, A) = 1 - \alpha$. Note that by Monotonicity $p(a, \{a, c\}) \geq \alpha$ for all $c \in A \setminus \{a\}$ and $p(b, \{b, c\}) \geq 1 - \alpha$ for all $c \in A \setminus \{b\}$.

Now suppose by contradiction that there exists a menu B and a $c \in B$ for which $p(c, B) = \beta \notin \{0, \alpha, 1 - \alpha, 1\}$. By Lemma 1 there exists exactly one alternative $d \in B$ for which $p(d, B) = 1 - \beta$.

Consider the menu $A \cup B$. By Monotonicity $p(e, A \cup B) = 0$ for all $e \in (A \cup B) \setminus \{a, b, c, d\}$.

*** By Lemma 1 and since $\{a, b, c, d\}$ contains at least three distinct alternatives² it follows that $p(e, A \cup B) = 0$ for some $e \in \{a, b, c, d\}$. W.l.o.g., let $p(a, A \cup B) = 0$.

²To see this assume that $|\{a, b, c, d\}| = 2$. Then w.l.o.g. $a = c$ and $b = d$. If $\rho(a, A \cup B) > 0$ and $\rho(b, A \cup B) > 0$ then Monotonicity implies that $1 = \rho(a, A \cup B) + \rho(b, A \cup B) \leq \rho(a, A) + \rho(b, A) = 1$. Thus $\rho(a, A \cup B) = \rho(a, A) = \alpha$ and $\rho(b, A \cup B) = \rho(b, A) = 1 - \alpha$. Applying Monotonicity again yields $\rho(c, A \cup B) = \rho(c, B) = \beta$ and $\rho(d, A \cup B) = \rho(d, B) = 1 - \beta$. It follows that $\alpha = \rho(a, A \cup B) = \rho(c, A \cup B) = \beta$. A contradiction to $\alpha \neq \beta$.

If $p(b, A \cup B) > 0$ then by Monotonicity and Negative Expansion $p(b, A \cup B) = 1 - \alpha$ (if it were $p(b, A \cup B) < 1 - \alpha$, then $p(b, A \cup B) \neq p(b, A)$ with $p(b, A \cup B), p(b, A) < 1$, so that Negative Expansion would imply the contradiction $p(b, A \cup B) = 0$). It follows from Lemma 1 that either $p(c, A \cup B) = \alpha$ or $p(d, A \cup B) = \alpha$. In the former case, since $\alpha > 0$ a reasoning analogous to the one followed so far implies that $p(c, A \cup B) = p(c, B) = \beta$, contradicting $\alpha \neq \beta$; and in the latter case we obtain $p(d, A \cup B) = p(d, B) = 1 - \beta$, contradicting $\alpha \neq 1 - \beta$. Consequently, it can only be that $p(b, A \cup B) = 0$. W.l.o.g. let $p(c, A \cup B) > 0$, $\alpha > 1 - \alpha$ and $\beta > 1 - \beta$. By Monotonicity and Negative Expansion $p(c, A \cup B) = \beta$, so that it must be that $p(d, A \cup B) = 1 - \beta$. (To see this, since $p(c, B) = \beta$ and $p(d, B) = 1 - \beta$, then by Monotonicity $p(c, A \cup B) \leq \beta$ and $p(d, A \cup B) \leq 1 - \beta$. If $p(c, A \cup B) < \beta$, then by Negative Expansion $p(c, A \cup B) = 0$. But then given that $p(a, A \cup B) = 0 = p(b, A \cup B)$, we have $p(d, A \cup B) = 1$, contradicting $p(d, A \cup B) \leq 1 - \beta$. Therefore, $p(c, A \cup B) = \beta$, and it follows that $p(d, A \cup B) = 1 - \beta$.)

We now want to show that c dominates a . Consider $p(c, \{a, c\})$. By Monotonicity, $p(c, \{a, c\}) \geq \beta$, and then by Negative Expansion it can only be either $p(c, \{a, c\}) = 1$ or $p(c, \{a, c\}) = \beta$. In the former case we have $p(a, \{a, c\}) = 0$ and the dominance claim follows by Monotonicity. Suppose then that $p(c, \{a, c\}) = \beta$, so that $p(a, \{a, c\}) = 1 - \beta$. Since $\alpha \neq 1 - \beta$, by Negative Expansion we have $p(a, \{a, b, c\}) = 0$. By Monotonicity and Negative Expansion it must be either $p(c, \{a, b, c\}) = 1$ or $p(c, \{a, b, c\}) = \beta$. In the former case, by Monotonicity $p(c, \{a, c\}) = 1$, which is a contradiction to $p(c, \{a, c\}) = \beta < 1$. If on the other hand $p(c, \{a, b, c\}) = \beta$, since $p(a, \{a, b, c\}) = 0$ we have $p(b, \{a, b, c\}) = 1 - \beta$. Since $1 - \alpha \neq 1 - \beta$ and $p(b, \{a, b\}) = 1 - \alpha$, Negative Expansion implies $p(b, \{a, b, c\}) = 0$, a contradiction. The same reasoning shows that c dominates b and that d dominates a and b . Since by assumption $\frac{p(a, \{a, b\})}{p(b, \{a, b\})} \neq \frac{p(c, \{c, d\})}{p(d, \{c, d\})}$, we contradict Alignment. $\square$

As a consequence, we can state the corrected version of Theorem 1 in Manzini and Mariotti [3] as:

Theorem 1. (corrected) *A stochastic choice rule is an idRUM if and only if it satisfies Monotonicity, Negative Expansion, Contraction Consistency and Alignment.*

The proof of this Theorem stays unchanged.

References

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