
Peer reviewed version

Link to published version (if available): 
10.1201/9780429321559-56

Link to publication record in Explore Bristol Research

PDF-document

This is the accepted author manuscript (AAM). The final published version (version of record) is available online via Taylor & Francis at 10.1201/9780429321559. Please refer to any applicable terms of use of the publisher.
Simplified stress-strain models applied to data from triaxial and pressuremeter tests on London Clay

P.J. Vardanega  
*University of Bristol, Bristol, UK*

M.D. Bolton & S.K. Haigh  
*University of Cambridge, Cambridge, UK*

R.W. Whittle  
*Cambridge Insitu Ltd, Cambridge, UK*

A. Klar  
*Technion – Israel Institute of Technology, Haifa, Israel*

M.G. Williamson  
*Mott MacDonald Group, Singapore*

ABSTRACT: The Mobilisable Strength Design (MSD) philosophy has been used in various applications related to underground construction, e.g. for analysis of deep foundation and retaining wall performance. MSD requires simple models for the stress-strain behaviour of soils. The use of a mobilisation factor on undrained strength to limit soil mobilisation was introduced in BS8002 in 1994. To assist with MSD calculations, the mobilisation strain framework (MSF) has been developed to allow geotechnical engineers to account for the non-linear behaviour of fine-grained soils in routine geotechnical design. In this paper, triaxial and pressuremeter test data from the London Clay deposit are analysed, using the MSF, to study the effects of anisotropy on both the mobilisation strains and non-linearity exponent. The implications for design of underground constructions are also discussed.

1 INTRODUCTION

1.1 Mobilisable Strength Design

This paper presents a summary of some recent advances in simplified soil models that can be used in Mobilisable Strength Design (MSD). Simplified stress-strain models are compared with pressuremeter and triaxial data from the London Clay deposit. The effects of past stress history and shear mode are discussed in this paper. Considerable effort has been undertaken in recent years to characterise the stress-strain behaviour of fine-grained soils in an attempt to improve soil-structure interaction calculations. Characteristic values for soil parameters are required when using Eurocode 7 (CEN, 2004) for the purposes of design. Arguably, most geotechnical engineers have little problem assigning characteristic values for strength parameters e.g. undrained shear strength ($c_u$) or friction angle ($\phi_{\text{peak}}, \phi_{cv}, \phi_{\text{res}}$). Recently the idea of using characteristic stress strain curves to complete simplified soil deformation calculations has entered some design codes and manuals (BSI 2015a, 2015b). The importance of stress-strain (particularly small strain non-linearity) for geotechnical design is well explained in Atkinson (2000). However, it can be argued that the moderate strain region (defined in Vardanega & Bolton (2011a) to be $0.2 \leq \tau_{\text{mob}}/c_u \leq 0.8$ where $\tau_{\text{mob}}$ is the shear stress sufficiently smaller than the undrained shear strength $c_u$) is relevant to many geotechnical constructions and therefore studying the effects of the stress-strain behaviour confined to this region is worthy of continued study.

1.2 London Clay deposit

Considerable tunnelling work has been carried out in London Clay over many years (e.g. Gourvenec et al., 2005 and Mair, 2008). Advanced mechanical testing on London Clay has been performed by many researchers (e.g. Ward et al. 1959, 1965, Parry 1960, Bishop et al. 1965, Skempton et al. 1969, Atkinson 1975, Yimsiri 2001, Gasparre 2005, Nishimura 2005, Hight et al. 2007 and Gasparre et al. 2007). The data from this work has often been used to calibrate numerical models for use in the London clay deposit (e.g. Simpson et al. 1979, Simpson 1992 and Ellison et al. 2012). Vardanega & Bolton (2011b) analysed triaxial compression test data from Gasparre (2005), Yimsiri (2001) and Jardine et al. (1984) using the mobilisation strain framework (MSF) set out below.

1.3 Study aims

This paper has the following aims: (i) study the effects of past stress history (indicated by sample depth) on the MSF parameters of London Clay, taken as a case study; (ii) determine if transformation models...
(e.g. Phoon & Kulhawy 1999a, 1999b) can be determined linking the MSF parameters with depth for the London Clay deposit and (iii) compare the values of the MSF parameters for three test types: triaxial compression (CIUC), triaxial extension (CIUE) and pressuremeter (PMT).

2 SOIL STRESS-STRAIN MODELLING

2.1 Empirical models

Hollomon (1945) proposed a simple equation of the form shown as Equation 1 to model the strain-hardening behaviour of metals:

\[ \sigma = K\varepsilon^n \]  

(1)

where, \(\sigma\) = stress, \(\varepsilon\) = strain, \(K\) and \(n\) are curve fitting parameters with \(n\) representing the ductility of the material.

Jardine et al. (1984) and Jardine et al. (1986) proposed the six-parameter model (Eq. 2) to describe the undrained shear behaviour of some North Sea clays:

\[ \frac{\varepsilon_u}{c_u} = A' + B' \cos \left[ C \left[ \log_{10} \left( \frac{\varepsilon}{\varepsilon_50} \right) \right] \right] \]  

(2)

where, \(A'\) is the strain-dependent undrained Young’s modulus, \(c_u\) is the axial strain in a triaxial test (taken as \(2/3\) the engineering shear strain, \(\gamma\)) and \(A', B', C', \alpha\) and \(I\) are curve-fitting parameters.

For geotechnical work the \(\varepsilon_{50}\) (strain to half the undrained shear strength) was recognised by Matlock (1970) as a useful normalising parameter for soil strains and proposed a power-law model although the non-linearity exponent was assigned a fixed value (Zhang & Anderson, 2017). The use of the \(\varepsilon_{50}\) (axial strain to 50% strength mobilisation) was also discussed in the review of Wroth et al. (1984).

Vardanega and Bolton (2011a) showed that the following power-law expression captures the shear stress-strain behaviour of intact clays and silts in only three parameters (Eq. 3):

\[ \frac{1}{M} = \frac{\tau_{mob}}{c_u} = \frac{0.5}{\gamma_M=2} \left( \frac{\gamma}{\gamma_M=2} \right)^b \quad 0.2 \leq \tau_{mob}/c_u \leq 0.8 \]  

(3)

where, \(\gamma_{M=2}\) is the engineering shear strain required to mobilise 50% of the strength, \(b\) is a curve fitting parameter and \(M\) is a mobilisation factor (see BSI 1994).


Klar and Klein (2014) presented an alternative exponential function (Eq. 4):

\[ c_{mob}(\varepsilon_a) = \left[ 1 - \exp \left( -0.693 \frac{\varepsilon_a}{\varepsilon_{50}} \right) \right] c_u \]  

(4)

which was used to develop an MSD-inspired formulation for volume loss due to tunnelling.

2.2 Influence of past stress history

Undrained shear strength of fine-grained materials is strongly affected by overconsolidation ratio (OCR) (e.g. Henkel 1956, Mayne 1985, Mayne 1988, Mayne & Stewart 1988, Mayne 2001 and Mayne et al. 2009), strain rate (e.g. Kulhawy & Mayne 1990) and shear mode (Chen & Kulhawy, 1990 and Mayne et al. 2009).

Vardanega and Bolton (2011a) presented the analysis of a geo-database of 115 stress-strain curves on 19 clays and silts. An average \(b\) of about 0.6 was determined and a standard deviation (SD) of 0.15 computed. The multiple linear regression analysis yielded the following expression (coefficient of determination, \(R^2 = 0.44\), number of data-points used to generate the correlation, \(n = 97\)):

\[ \gamma_{M=2} = C[I_P]^{0.45} \left[ \frac{c_u}{p'0} \right]^{0.59} \left[ \frac{p'0}{p_{atm}} \right]^{0.28} \]  

(5)

where, \(I_P\) = plasticity index, \(p_{atm}\) = atmospheric pressure, \(p'0\) = confining stress and \(C\) = a regression constant (approximately 0.011). Equation 5 was used by Vardanega & Bolton (2011a) to propose that:

\[ \gamma_{M=2} = f(OCR, I_P, p'0) \]  

(6)

Given that many of the sources of data analysed in Vardanega and Bolton (2011a) did not report the OCR of the materials, a suite of triaxial tests on reconstituted kaolin were conducted. Vardanega et al. (2012b) presented test data for a kaolin and showed that \(\gamma_{M=2}\) correlated strongly with increase in OCR \((R^2 = 0.815, n = 18)\) (Eq. 7) while the \(b\) value was correlated with OCR \((R^2 = 0.591, n = 18)\) (Eq. 8) albeit with a lower \(R^2\) value.

\[ \gamma_{M=2} = 0.0040(OCR)^{0.680} \]  

(7)

\[ b = 0.011(OCR) + 0.371 \]  

(8)

Vardanega and Bolton (2011b) analysed triaxial compression data from samples from the London Clay deposit and reported that a function of \(\gamma_{M=2}\) decreasing with depth (\(d\)) \((R^2 = 0.46, n = 17)\) could be valid (Eq. 9):

\[ 1000 \gamma_{M=2} = -2.84 \ln(d) + 15.42 \]  

(9)

2.3 Influence of K0 and shear mode

Vardanega & Bolton (2011a) showed how Equation 3 could be ‘shifted’ to better account for the ‘K0-effect’. This technique was subsequently used by Li & Bolton (2014) to analyse retaining walls in sand. Later Vardanega (2012), Vardanega & Bolton (2016) (while discussing Casey et al. (2016) and Casey
(2016) presented the following modification of Equation 3 for $K_0$ data:

$$B = \frac{\tau_{mob}}{c_u} = 0.5 \left[ \frac{\gamma}{\gamma_{ref,K_0}} \right]^{b} \quad \text{valid for,} \quad [0.2 \leq \tau_{mob}/c_u \leq 0.8] \quad (10)$$

where, $\tau_0 = \text{initial shear stress}$ and $\gamma_{ref,K_0} = \text{shear strain to mobilise } B = 0.5$.

Beesley & Vardanega (2020a) assembled and analysed a database (named RFG/TXCU-278) of reconstituted fine-grained soils to study in part the effect of shear mode on the $\gamma_{50}$ parameter (referred to as $\gamma_{50}$ with the shear mode denoted as a subscript for the rest of this paper) and the $b$ values from the mobilisation strain framework. Beesley & Vardanega (2020a) found the following correlations (Eq. 11 and Eq. 12) for the $\gamma_{50}$ parameter as related to a comparison of compression and extension testing (also reviewed in Beesley & Vardanega 2020b; for further details on RFG/TXCU-278 see also the thesis of Beesley 2019):

$$\left( \gamma_{50,\text{CIUC}} \right) = 0.749 \left( \gamma_{50,\text{CIUC}} \right) \quad (11)$$

$$R^2 = 0.71, n = 50, \text{standard error, } SE = 0.0031, \text{probability that no correlation exists, } p < 0.001$$

$$\left( \gamma_{50,\text{CKUC}} \right) = 3.76 \left( \gamma_{50,\text{CKUC}} \right) + 0.0054 \quad (12)$$

$$R^2 = 0.46, n = 25, SE = 0.0099, p < 0.001$$

The next section investigates the effect of sample depth in the London Clay deposit on the MSF parameters.

3 ANALYSIS

3.1 Triaxial tests

Vardanega & Bolton (2011b) analysed a collection of CIUC tests ($n=17$) on London clay samples sourced from the literature. In the previous work three London Clay sites were represented: Canon’s Park (Jardine et al. 1984), Kennington Park (Yimsiri 2001) and Heathrow Terminal 5 (Gasparre 2005).

In Mayne’s discussion to Vardanega et al. (2012b) the relevant $c_u$ value needs to be used in the normalisation e.g. if an extension test is performed then $c_u$ (extension) is required (Vardanega et al. 2013). Given the importance of examining the effect of shear mode (Vardanega et al. 2013, Beesley 2019 and Beesley & Vardanega 2020a, 2020b) comparison with triaxial extension test data is warranted.

Figure 1 and Table 1 shows the analysis of extension test data ($n=18$) from Hight et al. (2007), Gasparre (2005) and Nishimura (2005) on London Clay samples from Heathrow Terminal 5 (see also Klar & Klein 2014 where the summary of this analysis was first reported). Figure 1 shows the data on plotted log-log axes and the straight-line fits obtained (for the data generally in the range of $0.2 \tau_{mob}/c_u$ to $0.8 \tau_{mob}/c_u$).

Table 1 shows the $\varepsilon_{50}$ values derived in two ways: from the fit of the function (scaling off the curve) and from the derived log-log relation (the method that was used in Vardanega and Bolton, 2011a, 2011b) – for these tests the results are essentially the same. For the subsequent analysis of the triaxial data in this paper the $\gamma_{50}$ and $b$ values from the fitted log-log curve are used.
Figure 2. (a) $\gamma_{50}^{CIUC}$ versus depth for London Clay; (b) $b_{CIUC}$ versus depth for London Clay (data from Jardine et al. 1984, Yimsiri 2001, Gasparre 2005) (database originally analysed in Vardanega & Bolton 2011b).

Figure 3. (a) $\gamma_{50}^{CIUE}$ versus depth for London Clay; (b) $b_{CIUE}$ versus depth for London Clay (data from Hight et al. 2007, Gasparre 2005, Nishimura 2005).

Figure 4. (a) $\gamma_{50}^{PMT}$ versus depth for London Clay; (b) $b_{PMT}$ versus depth for London Clay (Crossrail boreholes from 1992 (data from Cambridge Insitu Ltd)).
Figure 2a shows that the $\gamma_{50 \, CIUC}$ values range from 0.0043 to 0.0118 with an average value of 0.0075. The computed SD value for this dataset $(n=17)$ is 0.00023 and the COV = 31%. As also reported in Vardanega & Bolton (2011b) a correlation with depth (decreasing OCR) (cf. the data from Gault Clay reported in Butcher & Lord 1993) is found (Eq. 13).

$$d = 0.05(\gamma_{50 \, CIUC})^{-1.17} \quad [R^2=0.43, \ n = 17] \quad (13)$$

This trend follows that observed for reconstituted soils: $\gamma_{50}$ increases with increasing OCR (Vardanega et al. 2012b and Beesley & Vardanega 2020a). It should be noted that the $R^2$ for an exponential fit to the data from Figure 2a does result in a slightly higher $R^2$ value of 0.46 (cf. Eq. 9 and Vardanega & Bolton 2011b): a power fit is shown in this paper.

Figure 2b shows that the $b_{CIUC}$ values range from 0.38 to 0.83 with an average value of 0.57. The computed SD value for this dataset $(n=17)$ is 0.12 and the COV = 21%. As also investigated in Beesley & Vardanega (2020b) a correlation with depth (decreasing OCR) (cf. Butcher & Lord 1993) is found (Eq. 14).

$$d = 42.16(b_{CIUC})^{1.61} \quad [R^2=0.36, \ n = 17] \quad (14)$$

Interestingly this trend is the reverse of Equation 8 (Vardanega et al. 2012b).

Figure 3a shows that the $\gamma_{50 \, CIUC}$ values range from 0.0004 to 0.0064 with an average value of 0.0028. The computed SD value for this dataset $(n=18)$ is 0.00020 and the COV = 71%. No correlation with depth was observed. Equation 11 suggests that on average for reconstituted soils $\gamma_{50 \, CIUC}$ is about 1.3 times that of $\gamma_{50 \, CIUE}$. However, for this natural clay dataset, the difference is approximately a factor of 2.7.

Figure 3b shows that the $b_{CIUE}$ values range from 0.21 to 0.62 with an average value of 0.44. The SD value for this dataset $(n=18)$ is 0.11 and the COV = 25%. A correlation with depth (decreasing OCR) (cf. Butcher & Lord 1993) is found (Eq. 15).

$$d = 42.61(b_{CIUE})^{0.95} \quad [R^2=0.25, \ n = 18] \quad (15)$$

As for Equation 14, this is the reverse of the trend shown as Equation 8 (Vardanega et al. 2012b). It is also observed that the average $b_{CIUE}$ value is lower than the average $b_{CIUC}$ value for the London Clay data analysed here – as similar trend is shown for the RFG/TXCU-278 database (see Beesley 2019 and Beesley & Vardanega 2020b).

3.2 Pressuremeter tests

Pressuremeter tests in London clay have been reported by various researchers (e.g. Wood & Wroth 1977, Marsland & Randolph 1977, and Bolton & Whittle 1999). Figure 4 shows the analogous MSF parameters derived from pressuremeter test information from the 1992 Crossrail boreholes (see Shuttle & Jeffries 1996 who also made use of some of this dataset).

Figure 4a shows that the $\gamma_{50 \, PMT}$ values range from 0.0017 to 0.0057 with an average value of 0.0035. This average value lies between the average value for the $\gamma_{50 \, CIUE}$ and $\gamma_{50 \, CIUC}$ values (and is closer to the $\gamma_{50 \, CIUE}$ value). The computed SD value for this dataset $(n=96)$ is 0.00087 and the COV = 25%. No obvious correlation with depth is observed.

Figure 4b shows that the $b_{PMT}$ value range from 0.49 to 0.75 with an average value of 0.64. This average value lies above the average value for both the $b_{CIUE}$ and $b_{CIUC}$ data (and is closer to the $b_{CIUC}$ value). The computed SD value for this dataset $(n=96)$ is 0.06 and the COV = 9%. No obvious correlation with depth is observed. It is interesting that the COV values of the MSF parameters for the pressuremeter data (Fig. 4) are lower than those for the compression (Fig. 2) and extension test data (Fig. 3).

Tables 2 and 3 show the statistical variation of $\gamma_{50 \, PMT}$ and $b_{PMT}$ for each of the six London clay locations. The COV values for each PMT location are generally lower than for the triaxial locations, especially for the $b$ values. (The two Canon’s Park tests are not considered in the following discussion as the sample size is too small to compute sensible COV values.) The Kennington Park tests $\gamma_{50 \, CIUC}$ values have a similar COV to some of the pressuremeter sites of around 26% while the Heathrow Terminal 5 tests $\gamma_{50 \, CIUC}$ have a COV of around 31% with a COV of 71% for the $\gamma_{50 \, CIUE}$ values. This analysis is interesting as we may postulate that either the pressuremeter causes less disturbance (testing and sampling) than the triaxial testing and sampling regime or the pressuremeter sites are more homogeneous than Heathrow Terminal 5 and Kennington Park especially with respect to soil compliance (ductility).

4 SUMMARY AND CONCLUSIONS

This paper has reviewed the use of the MSF for both triaxial and pressuremeter test data for data from various sites in the London Clay deposit. The following conclusions are made:

(a) The MSF can describe both triaxial and pressuremeter test data from a natural clay deposit (in this case London Clay);

(b) The $\gamma_{50 \, CIUC}$ values are on average three times higher than the $\gamma_{50 \, CIUE}$ values for the triaxial data set analysed in this paper (this difference is much greater that observed for the reconstituted test data from the RFG/TXCU-278 database (Beesley 2019 and Beesley & Vardanega 2020a));

(c) Correlations with depth were found for the $\gamma_{50 \, CIUC}$, $b_{CIUC}$ and $b_{CIUE}$ data (although the computed $R^2$ values are relatively low) but not for the $\gamma_{50 \, CIUE}$, $\gamma_{50 \, PMT}$ and $b_{PMT}$ data;
(d) The COV values of the $\gamma_{50}$ and $b$ values for the pressuremeter sites are generally lower than those for the triaxial sites (especially for $b$). This could be due either to disturbance effects or be an indication of site heterogeneity. In any event, the variation in the MSF values from state-of-the-art triaxial tests on high quality cores is potentially a matter of concern. If such cores can become irretrievably damaged as they are recovered, extruded, trimmed and tested, the greater stiffness of pressuremeter tests may be more relevant to the design of underground facilities. Anisotropy would, however, remain a concern. A future comparison of all three test types on the same site at the same depth(s) would be useful to advance the MSF.

5 ACKNOWLEDGEMENTS

The first author thanks Dr M. Beesley for her helpful comments and work on revising the MSF and Dr A. Gasparre for providing her triaxial test data for analysis.

Data Availability Statement: This research has not generated new experimental data.

6 REFERENCES

Table 2. MSF analysis of Crossrail pressuremeter tests at six locations in London Clay: variation of $\gamma_{50\text{PMT}}$

<table>
<thead>
<tr>
<th>Site</th>
<th>Max.</th>
<th>Average</th>
<th>Min.</th>
<th>SD</th>
<th>COV (%)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davies Street (119.45mTD)</td>
<td>0.00498</td>
<td>0.00355</td>
<td>0.00216</td>
<td>0.00086</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>Farrington (115.4mTD)</td>
<td>0.00341</td>
<td>0.00257</td>
<td>0.00210</td>
<td>0.00043</td>
<td>17</td>
<td>12</td>
</tr>
<tr>
<td>Paddington (1) (126.4mTD)</td>
<td>0.00537</td>
<td>0.00352</td>
<td>0.00169</td>
<td>0.00093</td>
<td>26</td>
<td>21</td>
</tr>
<tr>
<td>Paddington (2) (126.4mTD)</td>
<td>0.00498</td>
<td>0.00362</td>
<td>0.00225</td>
<td>0.00083</td>
<td>23</td>
<td>10</td>
</tr>
<tr>
<td>Royal Oak (121.7mTD)</td>
<td>0.00565</td>
<td>0.00387</td>
<td>0.00230</td>
<td>0.00076</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>Falconberg Court (Tottenham Court Rd) (124.45mTD)</td>
<td>0.00486</td>
<td>0.00340</td>
<td>0.00201</td>
<td>0.00082</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>ALL</td>
<td>0.00565</td>
<td>0.00347</td>
<td>0.00169</td>
<td>0.00087</td>
<td>25</td>
<td>96</td>
</tr>
</tbody>
</table>

Table 3. MSF analysis of Crossrail pressuremeter tests at six locations in London Clay: variation of $b_{\text{PMT}}$

<table>
<thead>
<tr>
<th>Site</th>
<th>Max.</th>
<th>Average</th>
<th>Min.</th>
<th>SD</th>
<th>COV (%)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davies Street (119.45mTD)</td>
<td>0.74</td>
<td>0.65</td>
<td>0.56</td>
<td>0.048</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>Farrington (115.4mTD)</td>
<td>0.68</td>
<td>0.59</td>
<td>0.53</td>
<td>0.051</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>Paddington (1) (126.4mTD)</td>
<td>0.75</td>
<td>0.67</td>
<td>0.56</td>
<td>0.056</td>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>Paddington (2) (126.4mTD)</td>
<td>0.66</td>
<td>0.62</td>
<td>0.58</td>
<td>0.023</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Royal Oak (121.7mTD)</td>
<td>0.71</td>
<td>0.63</td>
<td>0.52</td>
<td>0.058</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>Falconberg Court (Tottenham Court Rd) (124.45mTD)</td>
<td>0.70</td>
<td>0.63</td>
<td>0.49</td>
<td>0.054</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>ALL</td>
<td>0.75</td>
<td>0.64</td>
<td>0.49</td>
<td>0.055</td>
<td>9</td>
<td>96</td>
</tr>
</tbody>
</table>


