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Use of Linear Transverse Equalisers and Channel State Information in Combined OFDM-Equalisation

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ABSTRACT

The efficiency of a Coded Orthogonal Frequency Division Multiplexing (COFDM) receiver can be improved by use of a Pre-FFT equaliser (PFE). This technique is known as combined OFDM-equalisation. This paper considers the noise amplification effect of the PFE and its effect on the performance of a combined COFDM-equalisation modem and proposes a method to improve this performance.

This paper first reviews and compares the conventional COFDM and combined COFDM-equalisation techniques. The PFE is then described and the conditions required to allow it to take the form of a Decision Feedback Equaliser (DFE) are discussed. It is shown that these conditions often cannot be met. In these cases the PFE must take the form of a Linear Transversal Equaliser (LTE).

The performance of the LTE is inferior to that of the DFE due to significant noise amplification. Hence, if the performance of combined COFDM-equalisation is to match that of conventional COFDM, a method for mitigating the noise amplifying effect of an LTE type PFE is required. One such method is proposed in this paper.

This technique exploits Channel State Information (CSI) available in a COFDM receiver in order to enhance the performance of a Viterbi convolutional decoder. This 'CSI modified' Viterbi algorithm is capable of mitigating the noise amplification occurring in the equaliser. To demonstrate this, the performance of conventional COFDM and combined COFDM-equalisation are compared by means of software simulation using both standard and CSI modified Viterbi decoding. The results for the standard Viterbi decoding illustrate the performance penalty due to noise amplification. The results for the CSI modified Viterbi algorithm demonstrate the effective mitigation of noise amplification and the comparable performance of conventional COFDM and combined COFDM-equalisation.

INTRODUCTION

The technique of Coded Orthogonal Frequency Division Multiplexing (COFDM) offers a robust method for digital radio transmission. Conventionally, COFDM employs a guard interval to combat delay spread of the radio signal that will otherwise result in inter-carrier interference (ICI) of the OFDM modulated data [1]. The use of a guard interval reduces transmission efficiency according to the ratio of un-extended and extended OFDM symbol periods [1].

A combined COFDM-equalisation technique incorporating a pre-FFT Equaliser has been proposed previously [2][3]. The performance of this pre-FFT Equaliser has been analyzed in terms of its performance under additive noise in [4] and time variant channel conditions in [5]. This paper reviews the combined COFDM-Equalisation receiver and pre-FFT Equaliser designs and summarizes the implications of their use by comparison with the conventional COFDM method.

As has been widely considered in the literature [6][7], the Decision Feedback Equaliser (DFE) offers improved performance under additive noise conditions in comparison to the Linear Transverse Equaliser (LTE) due to reduced noise amplification. Thus, it is desirable to implement the pre-FFT Equaliser in the form of a DFE. In reality the best that can be achieved is a compromised form of the DFE [4]. The closeness of this approximation to an actual DFE that can be achieved is dependent upon the OFDM symbol period, channel delay spread and any delays that occur in the feedback loop of the combined COFDM-equalisation receiver. In those cases where a sufficiently close approximation to the DFE cannot be achieved, it is preferable to implement the pre-FFT Equaliser as an LTE.

When the pre-FFT Equaliser is implemented as an LTE, the frequency response of the Equaliser defines the frequency spectrum of the noise output by the equaliser (assuming white noise). Thus, the noise spectrum can be determined and this information can be exploited in the Viterbi decoder.

Figure 1. OFDM Modulation and Transmission.

The CSI information can be exploited as a per-sub-band estimate of the SNR. By providing this information to the Viterbi algorithm as an estimate of the reliability of the input information, a greater weighting can be given to data with high SNR than to data with low SNR.

This paper reviews the conventional COFDM technique in section II and the combined COFDM-equalisation receiver in section III. The pre-FFT Equaliser is reviewed in section IV. Section V considers the conditions required to implement the pre-FFT equaliser as a good approximation to a DFE. Section VI describes and compares the standard and CSI modified Viterbi decoding algorithms. Software simulation results are presented in VII and conclusions are drawn in section VIII.

II. CONVENTIONAL OFDM TRANSMISSION AND RECEPTION

A conventional COFDM modulation process [1] as illustrated in Figure 1 is employed to generate the transmitted signal for reception by both conventional OFDM and combined COFDM-equalisation. A frequency domain input data vector, $X(k,l)$, consisting of N convolutionally encoded data symbols, is input to an IFFT to produce a time domain vector, $x(n,l)$. (k indexes the OFDM sub-band, n indexes the transmission symbol and l indexes the OFDM symbol). The data symbol period is T_s and the OFDM symbol period is NT_s . The time sequence is cyclically extended by M symbols to produce the transmission vector $x'(n,l)$. This signal is then transmitted via the channel.

A conventional COFDM receiver is shown in Figure 2. The receiver takes input $y'(n,l)$, where:

$$y'(n,l) = (x'(n,l) * h(n)) + \eta'(n,l) \quad (1)$$

Where $h(n)$ is the impulse response of the radio channel and $\eta'(n,l)$ represents additive noise with a spectral density N_0 .

The receiver removes the guard interval from $y'(n,l)$ and an FFT is applied to generate $Y(k,l)$. A channel estimation process exploits pilot signals inserted in the transmitted signal to generate $S(k,l)$ which is an estimate of the channel's frequency response. The information in $S(k,l)$ is used to compensate $Y(k,l)$ for the frequency selective fading that occurs in the radio channel. Thus:

$$V_{k,l} = \frac{Z_{k,l}}{S_{k,l}} \quad (2)$$

A measure of SNR for the individual sub-bands (and hence the reliability of the corresponding data) is determined from:

$$SNR_{k,l} = \frac{S_{k,l}}{N_0 f_{sb}} \quad (3)$$

Where f_{sb} is the sub-band bandwidth.

In practice, the noise spectral density is unknown. However, the reliability of each sub-band is proportional to the magnitude of the corresponding element of the CSI vector $S(k,l)$. Thus:

$$SNR_{k,l} \propto S_{k,l} \quad (4)$$

This reliability measure can be exploited in the Viterbi decoder.

Figure 2. Conventional OFDM Receiver

Figure 3. Combined OFDM-Equalisation Receiver

III. THE COMBINED OFDM-EQUALISATION RECEIVER

The structure of an OFDM receiver employing a pre-FFT Equaliser is shown in Figure 3. Its function can be seen to be that of a conventional OFDM receiver with the addition of the adaptive equalising filter, a feedback loop and an optional process to generate the CSI vector $S(k,l)$. The CSI information can be generated either from the equaliser or from a conventional channel estimation process. Also, no channel compensation is required since the function of the pre-FFT equaliser is to reciprocate the frequency response of the channel.

The feedback path in the receiver generates $w'(n,l)$ which is an estimate of the transmitted OFDM symbol, $x'(n,l)$ based on the post-decision output data symbols.

IV. THE PRE-FFT EQUALISER

The structure of the pre-FFT Equaliser is shown in Figure 4. The feedforward filter operates on the received signal $y'(n,l)$. Where possible, a feedback filter is also implemented. The feedback filter operates on one of three possible input signals at different times. During Equaliser training, the training sequence $x'(n,l)$ is input to the feedback section. During decision directed adaptation, the feedback section operates on either the feedback vector $w'(n,l)$ or the equaliser's output $z'(n,l)$. $z'(n,l)$ is a noise bearing estimate of the transmitted signal. $w'(n,l)$ is a noise free estimate of the transmitted signal and is thus superior to $z'(n,l)$. $x'(n,l)$ is a perfect estimate of the transmitted signal provided by

a priori knowledge of the transmitted signal and is thus superior to both $w'(n,l)$ and $z'(n,l)$.

Thus, under certain conditions, the pre-FFT Equaliser can be implemented in a form that approximates a DFE. This type of pre-FFT Equaliser will cancel out delay spread without significant noise amplification. The equaliser's frequency response and its output noise spectrum will not be directly related.

If the pre-FFT equaliser cannot be made to operate as an approximate DFE, it should be implemented as an LTE. In this case, the feedback filter section and its inputs are omitted from the filter design shown in Figure 4. The equaliser will still adapt to an inverse frequency response to that of the radio channel but greater noise amplification will occur. The frequency response of the equaliser and the noise spectrum are proportional.

Figure 4. Pre-FFT Equaliser

V. REQUIREMENTS FOR IMPLEMENTATION OF THE PFE IN DFE FORM

The pre-FFT equaliser can only be implemented as a DFE type structure if it is possible to generate the feedback vector quickly enough to replace enough noisy symbols in the feedback section such that the majority of symbols in the feedback section at any one time are noise free. If this cannot be achieved, the advantages of the DFE structure are lost and the pre-FFT equaliser is best implemented as an LTE. For a DFE type pre-FFT equaliser structure with J_2 feedback taps spaced at intervals T_s and a combined OFDM Equalisation receiver which generates $w'(n, l)$ after a feedback delay of $d_{fb}T_s$ seconds:

1. The maximum number of noise free samples in the feedback section will occur immediately after $w'(n, l)$ becomes available and will be equal to $J_2 - d_{fb}$.
2. The minimum number of noise free samples in the feedback section will occur immediately before $w'(n, l)$ becomes available and will be equal to $J_2 - d_{fb} - N$
3. The average fraction of Equaliser taps in the feedback section that hold noise free samples, J_{NFAV} will be given by:

$$J_{2NFAV} = 1 - \frac{d_{fb}}{J_2} - \frac{N}{2J_2} \quad (5)$$

Thus, for example, to maintain an average of 50% of taps noise free:

$$J_2 > N + 2d_{fb} \quad (6)$$

Thus, even with zero feedback delay, a high average number of noise free taps can only be maintained provided that the feedback section is longer than the OFDM symbol. As the feedback delay increases, the feedback section length must increase by twice as much to maintain the average number of noise free taps. Clearly, in most cases (where the delay spread of the radio channel is not significantly longer than the OFDM symbol period), it is not effective to implement the PFE in the form of a DFE.

VI. EXPLOITING CSI IN THE VITERBI DECODER

In the cases of both conventional OFDM and combined OFDM-equalisation, CSI information can be exploited to improve the performance of the Viterbi decoding algorithm. In the case of conventional OFDM, CSI is necessarily generated from the channel estimation process.

A similar technique can also be used in Combined OFDM-equalisation. The advantage of implementing the PFE in a Combined OFDM-equalisation receiver

as an LTE is that the equaliser's tap coefficient vector can also be used to generate the CSI vector. The CSI vector can be conveniently derived from the Equaliser tap coefficients by application of an FFT. However, this requires that the input and output vectors have size 2^X . Thus it is convenient if the number of equaliser taps is a power of two.

Whichever method is used to generate the CSI, the method for exploiting it in the Viterbi decoder remains the same. As has been widely described in the literature [8][9], the Viterbi algorithm is a maximum likelihood sequence estimation algorithm. When applied to the decoding of convolutional codes, the Viterbi algorithm selects its output sequence by choosing the path through the code trellis which has the lowest distance (metric) from the received sequence.

In the case of hard decision decoding, the metric employed is the Hamming distance between the hard limited received sequence and the various trellis paths. Thus:

$$m_H = \sum_{k,l} D(Z_{k,l}, X_{k,l}) \quad (7)$$

Where the function $D(\cdot)$ defines the Hamming distance.

In the case of soft decision decoding, the metric employed is the squared Euclidean distance between the soft received sequence and the trellis paths. Thus:

$$m_S = \sum_{k,l} (Z_{k,l} - X_{k,l})^2 \quad (8)$$

In an OFDM system, where a CSI modified Viterbi algorithm can be employed, the metric is modified to:

$$m_C = \sum_{k,l} S_{k,l}^2 (Z_{k,l} - X_{k,l})^2 \quad (9)$$

VII. RESULTS

In order to demonstrate the effects of noise amplification in the PFE and its mitigation by means of the CSI modified Viterbi algorithm, software simulations have been performed to determine the BER versus E_b/N_0 performance of an OFDM modem in a fixed wideband Rayleigh channel. Simulations have been performed for the cases of both conventional OFDM and combined OFDM-equalisation. The simulated modem employed a 64 point FFT, QPSK modulation and 1/2 rate convolutional code. A symbol rate of 20MHz was simulated with a 50ns channel RMS delay spread. This required the use of either an 800ns guard interval or a 9-tap LTE type PFE to prevent ISI.

The results for standard hard decision decoding are shown in figure 5. In this case it can be seen that the conventional OFDM system outperforms the combined OFDM-equalisation system for $BER < 10^{-2}$

and that the performance difference increases as E_b/N_0 increases. It should also be noted that conventional OFDM suffers a 0.97dB E_b/N_0 penalty in comparison to the combined OFDM-equalisation method due to the additional energy required for the guard interval. The performance penalty suffered by the combined OFDM-equalisation technique can be attributed to noise amplification in the equaliser.

Figure 5. BER versus E_b/N_0 for standard Viterbi Decoding

Results for the case of CSI modified Viterbi decoding are shown in figure 6. In this case it can be seen that the performance of the combined OFDM-equalisation system is superior to that of conventional OFDM even for $BER < 10^{-2}$. However, allowing for the 0.97dB for the guard interval, the performance of the two systems is approximately equal.

Figure 6. BER versus E_b/N_0 for CSI enhanced Viterbi Decoding

VIII. CONCLUSIONS

The combined OFDM-equalisation reception technique offers superior transmission efficiency to the conventional OFDM reception technique. Where

possible, it is desired to implement the pre-FFT Equaliser as a DFE to avoid noise amplification in the Equalisation process. However, it is often not possible to achieve an effective DFE within the combined OFDM-Equalisation receiver and in these cases an LTE should be used. The frequency response of the radio channel can be determined from the LTE's tap coefficients and used as an estimate of the reliability of the individual sub-bands. This allows for the effects of noise amplification to be mitigated by use of a CSI modified Viterbi decoding algorithm.

Thus, when using either a DFE type pre-FFT equaliser or an LTE type pre-FFT equaliser in combination with a CSI modified Viterbi algorithm, the combined OFDM-equalisation method can achieve comparable BER versus SNR performance to conventional OFDM but without the loss in efficiency due to the use of a guard interval.

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