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Efficient Implementation of The Spectral Domain Method Including Pre-calculated Corner Basis Functions

S.A. Meade and C.J. Railton
Centre for Communications Research,
Faculty of Engineering, University of Bristol, U.K.

Abstract

A general implementation of the spectral domain method, formulated for planar microstrip circuits of arbitrary metallisation pattern is presented. The inclusion of *a priori* knowledge of the edge and corner singularities in the set of basis functions results in a large decrease in the order of the problem to be solved. Libraries of basis functions allow the rapid rigorous analysis of realistically complex circuits. Calculated S-parameters are given for three microstrip lowpass filters and compared to results from both measured and other techniques.

Introduction

The Spectral Domain Method (SDM) has been widely implemented for the rigorous analysis of microwave passive planar circuits. It has been shown [1] that the use of rooftop current basis functions allows an irregular metallisation pattern to be defined. As with other techniques [2, 3] the wide application of the method is limited by the requirement for a large amount of computer resources.

In [4], Railton and Meade proposed an efficient algorithm based on the SDM (with rooftop basis functions) which exploited the asymptotic behaviour both of the Green's function and of the current distribution, thus dramatically reducing the CPU time required for a given problem. Pre-computed basis functions were limited to isolated microstrip resonators. In this contribution the basic algorithm has been expanded to allow the treatment of more complex structures, modelled by pre-computed current basis functions.

It is envisaged that a library of such functions will allow the rapid rigorous analysis of complex metallisa-

tion patterns, with improvements in run-times of the order of 100 to 1000 over that of a rooftop basis function algorithm [1]. With the number of basis functions reduced, approximately, from 1000 to 100 the need to implement an iterative solution to the resulting matrix equation (e.g. Conjugate Gradient Method [5]) has been eliminated; at the present level of circuit complexity. Moreover to the authors' knowledge this is the first time that corner singularities have been included, using pre-calculated basis functions, in such a general implementation of SDM.

Equations to be solved

As in [4] a Method of Moments solution, formulated in the spectral domain, applied to a planar structure leads to the following set of equations:

$$\sum_s a_s Z_{st} = V_t \quad (1)$$

where the elements of the impedance matrix are

$$Z_{st} = \sum_{n,m} \tilde{w}_t(n,m) (\tilde{G}(n,m,w) - \tilde{G}^\infty(n,m)) \tilde{J}_s(n,m) + \tilde{G}_1^\infty \tilde{Z}_{st}^\infty \quad (2)$$

and

$J_s(x,y)$ is the set of current basis functions
 $w_t(x,y)$ is the set of weighting functions
 $G(w)$ is the dyadic Green's function
 $\sim$ indicates the Fourier transform
 ∞ indicates the asymptotic part
 V_t is the excitation vector

A Fast Fourier Transform algorithm is used to efficiently calculate the asymptotic part of the impedance matrix (Z_{st}^∞ in equation 2) as described in [4].

OF2

Pre-computed basis functions

Railton *et al*[4] introduce the concept of pre-computed basis functions for the modes of microstrip resonators. This is now expanded to allow a set of arbitrary basis functions to be defined:

$$J(r) = \sum_{p=1}^P b_p \psi_p(r) \quad (3)$$

where $\psi_p(r)$ the current distribution of the p^{th} basis function is

$$\psi_p(r) = \sum_{q=1}^Q a_{pq} R_q(r) \quad (4)$$

Thus an algorithm has been defined which allows current basis functions (ψ_p) to be expressed as a linear combination of rooftop functions (R_q). It is now proposed that such a set of basis functions can be derived which fully describe the response of relatively complex metallisation patterns.

Simple Lowpass Filter

In order to illustrate the manner in which the basis functions are derived, the application to a simple lowpass filter [2], shown in Figure 1, will be described. The analysis involves dividing the metallisation into

Figure 1: Lowpass Filter Layout

regions. In each region we define a set of basis functions which we will call region basis functions. With reference to Figure 1 the highlighted region 1 is modelled by the set of basis functions shown in Figure 2.

Two corners are present in region 1 (Figure 1). The functions Figure 2(a)-(c) are derived assuming a simple straight line model (i.e. no corners). It is proposed that only two extra region basis functions are required

to describe the perturbation of the current distribution from a simple straight line to region 1. Such a region basis function is illustrated in Figure 2(d), note the inclusion of the corner singularity.

Figure 2: Set of Pre-computed Basis Functions

Thus a function has been derived which fully describes the perturbation of the current due to a corner over the full frequency band of interest. Moreover the function depends only on the geometry of the corner itself, not on the surrounding circuitry. Thus reduces the necessity to define new functions for new circuits. Similarly functions are derived for the other regions of the filter metallisation.

Results using this method are compared to measured S-parameters [2]. A fully shielded structure is assumed in this implementation therefore comparison

to the open measured data [2] is affected by the proximity of the shield walls. For example, Figures 3 and 4 show results for the identical models of the filter except the latter is housed in a box twice the length of the former. Comparison clearly indicates convergence to measured open response, for the larger box. Note, this is limited by the introduction of box modes into the frequency band of interest.

Figure 3: Plot of S-parameters magnitude for Sheen filter model : Box 26.096mm x 6.7733mm x 12mm

Figure 4: Plot of S-parameters magnitude for Sheen filter model : Box 26.096mm x 13.5466mm x 12mm

Both models only required $P=44$ (equation 3) basis functions in total. An equivalent basic rooftop model requires $P=305$ rooftop basis functions. This results in a large decrease in CPU time as this is approximately proportional to N^3 . The run-time on a HP9000series720 workstation, using non-optimised code, was 3 seconds to calculate S-parameters at each spot frequency. The corresponding time for a basic rooftop algorithm was 400 seconds per spot frequency. Moreover a fine grid of rooftops could be defined with

a resulting improvement in modelling the current singularities without an increase in the number of region basis functions required.

Multi-element Lowpass Filters

To illustrate the efficiency of the region basis functions the analysis of the multi-element lowpass filter in Figure 5 is outlined.

The technique assumes a fully shielded structure; shielded measured S-parameters are available for the lowpass filters of the form shown in Figure 5. The structures are such that use of the basic rooftop technique is limited by the size of the matrix created by the large number of rooftops required. With reference to Figure 5 and Table 1 the relative dimensions of the feedlines to the input lines and stubs results in a fine grid on the latter. A possible solution is to use different size rooftops for the feedlines and the stubs/input; but it is proposed that a more efficient use of region basis functions allows the full definition of the fine grid to be utilised, resulting in improved modeling of current singularities; with fewer functions than the former.

Figure 5: Multi-element Lowpass Filter Layout

A set of region basis functions is used to model two different versions of the lowpass filter in Figure 5. Only $P=77$ (equation 3) basis functions are needed resulting in a run-time on a HP9000series720 of 12 seconds per spot frequency to calculate S-parameters.

The S-parameters for the two filters are compared to the measured response in Figures 6 and 7 respectively. A close match to the measured data is evident. S11 for the filters is modeled accurately both in the pass-band as well as the stop band. The difference between S21 predicted and measured in the passband is

due to the lossless nature of the present model. An anomaly in the measured response is evident in S21 at approximately 12GHz to 16GHz for the second filter. Figure 7, which is believed to be an inaccuracy in the measurements (note: S21 approx. -30dB). This assumption is justified by the model accurately predicting the prominent feature at 18GHz. Further comparison with an FDTD[3] model is currently being undertaken.

Figure 6: Plot of S-parameters magnitude for Multi-element lowpass filter 1 : Box $a=3.2\text{mm}$, $b=12.8\text{mm}$, $h=6\text{mm}$

Figure 7: Plot of S-parameters magnitude for Multi-element lowpass filter 2 : Box $a=3.2\text{mm}$, $b=25.6\text{mm}$, $h=6\text{mm}$

filter	Dimensions (mm)							
	L1	L2	L3	L4	L5	W1	W2	W3
1	2.6	1.0	1.2	1.0	1.6	0.05	0.8	0.35
2	6.4	1.3	1.3	2.0	3.2	0.05	0.8	0.35

Table 1: Dimension of lowpass filters

Conclusion

A general implementation of the SDM has been presented which efficiently characterizes planar microstrip circuits of arbitrary metallisation pattern. We have shown that sets of pre-computed current basis functions can be defined which include *a priori* knowledge of the edge and corner singularities. Thus a significant reduction in the order of the problem to be solved has been achieved. A library of such functions allows the rapid rigorous analysis of realistically complex circuits.

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