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Electromagnetic Analysis of Complex 3D PEC Structures using a Stabilised CPFDTD Algorithm

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Introduction

The electromagnetic analysis of complex, curved metal structures using the Finite Difference Time Domain (FDTD) technique has proved a difficult challenge and one which has not yet been satisfactorily resolved. Such problems do, however, occur in a wide variety of application areas ranging from propagation in waveguides to scattering from aircraft fuselages. A number of techniques have been suggested to address this type of analysis such as the use of globally distorted meshes [1] and the incorporation of Static Field Solutions (SFS) into the standard Cartesian mesh [2]. The former requires approximately three times the computer resources of the standard Cartesian FDTD at the same number of points per wavelength, while the latter, in its present state of development, is prone to late time instability. A third approach to the problem is to use a locally distorted mesh where the basic Cartesian grid is modified only in the vicinity of the metal boundaries such as the Contour Path Finite Difference Time Domain (CPFDTD) method [3]. This approach is also, however, prone to numerical instability. In this contribution, a modification to the CPFDTD scheme is presented which is numerically stable, accurate and which requires only marginally more computation than the standard FDTD. The effectiveness of the technique is demonstrated for the analysis of a circular cylindrical resonant cavity which is tilted with respect to the Cartesian FDTD mesh.

A major advantage of the CPFDTD approach when compared with other conformal techniques is that the simplicity and efficiency of the Cartesian mesh is retained throughout the majority of the problem space and only those nodes which are adjacent to the curved surface need be given special attention. In addition, the algorithms for absorbing boundaries, near-to-far field transformations and Huygens' sources, which are well developed for the standard FDTD method, can be applied without change. Despite the fact that this type of algorithm appears to allow the efficient analysis of very complex structures, comparatively little use of the method has been reported. Some researchers have called into question the stability of the original CPFDTD scheme since it employs a non-causal and non-reciprocal "nearest neighbour" approximation [4]. For lossless resonant structures, especially, for which there is no mechanism for dissipating spuriously generated energy, meaningful results are not usually obtainable [4]. Moreover, the generation of the three-dimensional distorted grid is itself a non-trivial task [5]. These difficulties prevent the scheme being used in general purpose codes which are intended to be used by non-specialists in FDTD.

In [6], a modification to stabilise the basic three-dimensional CPFDTD algorithm was presented. This modification recasts the "nearest neighbour" approximation, employed in standard CPFDTD, such that reciprocal interaction of nodes is obtained. This yields a scheme whose update equations are identical to those representing a passive electrical network consisting only of capacitors and gyrators which must necessarily conserve energy [7]. Moreover, the generation of the distorted mesh is simplified and readily automated.

The philosophy of the modification can be seen by considering Figure 1 which shows a cross-section of the CPFDTD mesh for a cylinder in the plane $z=z_0$. The dots at the centre of each undistorted cell represent the H_z nodes, the crosses represent E nodes which can be calculated using the standard FDTD update equations and the arrows represent E nodes whose values are "borrowed" from the nearest available collinear neighbour. The lengths of each contour edge and the area of each cell are shown normalised such that a value of 100 corresponds to an unmodified cell. For this example, the update equation for $H_z(1.5,0.5,z_0)$ in standard CPFDTD will include a term containing $E_x(2.5,1,z_0)$ while the update equation for $E_x(2.5,1,z_0)$ does not involve $H_z(1.5,0.5,z_0)$. It has been found that it is this non-reciprocity which causes numerical instability. By altering the update equation for $E_x(2.5,1,z_0)$ a reciprocal system can be obtained which is formally equivalent to the passive electrical circuit shown in Figure 2. Here, the voltages on each node correspond to the field amplitudes, the capacitors at each node represent energy storage and the gyrators represent the transfer of energy between neighbouring nodes. By ensuring that any modified CPFDTD scheme can be expressed as an equivalent passive electrical circuit, numerical instability has been found **never** to occur even though the time step has not been reduced below that used for standard FDTD.

In order to demonstrate the accuracy and robustness of the modified scheme a cylindrical resonator, tilted with respect to the Cartesian grid is analysed and the convergence properties of the algorithm are shown. This structure, containing tilted planes, smooth curved surfaces and right angle bends is considered to be a severe test of the algorithm.

Application to a tilted circular cylindrical cavity resonator

The circular cylindrical cavity resonator to be analysed has a height of 30cm, and a radius of 19cm. It is centrally placed in a cubic computational domain having sides of length 50cm. All the results presented here were obtained using 5000 time steps. Additional runs were done using 32000 time steps and no instability was ever observed. In all cases the mesh was generated automatically using a simple extension of the procedure described in [8]. Figures 3 - 6 show the error in the calculated resonant frequencies, plotted versus angle of tilt, for the first two TE and TM modes of the cylinder, using unit cell sizes of 5cm, 2.5cm and 1.25cm. It can be seen that substantial improvements in accuracy over that achieved using the staircase approximation have been obtained over the whole range of tilt angles. Although attempts to use the staircase approximation with a mesh as coarse as 5cm were not successful, in this case the modified CPFDTD scheme gave results within better than 3% of the exact values.

We see that, with few exceptions, the error reduces as the mesh is made finer. Those cases where the error does not vary in this way can be explained by the fact that the relationship between the discretised system and the real system leads to the error varying discontinuously as the geometry is gradually altered.

Conclusions

It has been shown that, with a simple modification, the CPFDTD scheme can be stabilised while still being accurate for structures with a complex geometry containing tilted planes, smooth curved surfaces and right-angle bends. This has been demonstrated for the example of a circular cylindrical resonator which is tilted with respect to the FDTD mesh. It is anticipated that the added robustness which the modification provides will facilitate more widespread use of the three dimensional CPFDTD algorithm.

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Figure 1 - The CPFDTD method applied to a cylinder.

Figure 2 - Electrical circuit analogous to the modified CPFDTD scheme for the cylinder.

Figure 3 - The resonant frequency of the tilted cylinder - first TE mode

Figure 4 - The resonant frequency of the tilted cylinder - first TM mode

Figure 5 - The resonant frequency of the tilted cylinder - second TE mode

Figure 6 - The resonant frequency of the tilted cylinder - second TM mode