
Peer reviewed version

Link to published version (if available):
10.1109/APS.2004.1330410

Link to publication record in Explore Bristol Research
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
http://www.bristol.ac.uk/pure/about/ebr-terms
An Investigation of Pattern Correlation and Mutual Coupling in MIMO Arrays

Ian J. Craddock, G.S. Hilton and P. Urwin-Wright
Centre for Communications Research, University of Bristol, UK. (ian.craddock@bristol.ac.uk)

Abstract
Measured results for pattern correlation in antenna arrays are compared to idealised values. It is shown that pattern correlation at small element separations is lower than the ideal values (a beneficial effect of mutual coupling). Observations are made regarding the various decorrelating effects in the array, including spacing, ground plane size and mutual coupling.

Introduction
Interest in the use of multi-element antennas at both the transmitter and receiver of a communications link has grown dramatically in recent years. These "Multiple-Input Multiple-Output" (MIMO) systems offer considerably improved performance (e.g. capacity) in a multipath propagation environment.

The propagation characteristics of the environment, together with the characteristics of the antennas, determine the performance of the MIMO system in a particular scenario. The antennas themselves should ideally have radiation patterns with low (or zero) correlation over the possible angles of arrival of multipath components.

Following [1] the envelope correlation between two radiation patterns $F_1$ and $F_2$ is given by:

$$\rho^2 = \frac{\int F_1 \cdot F_2^* \, d\Omega}{\sqrt{\int F_1^2 \, d\Omega \cdot \int F_2^2 \, d\Omega}}$$

- a rule-of-thumb is that good diversity operation is possible when $\rho^2$ is <0.5 (provided also that the SNR at each antenna is approximately equal [1]).

In the literature $\rho$ is usually derived from simulations. Herein practical results for the correlation parameter are presented along with some related observations.

Results for Pattern Correlation
Theoretical Values
A result often quoted for pattern correlation may be found in [2]:

$$\rho^2 = \left[ J_0 \left( \frac{2\pi d}{\lambda} \right) \right]^2$$

- where $J_0$ is a zeroth-order Bessel function of the first kind and $d$ is the separation between elements. This result is plotted in Figure 1.

While this result is often quoted in the literature, it should be noted that it was derived under various assumptions, notably that the patterns are identical and that the correlation over azimuth is the quantity of interest. If, however, the antennas are on a handheld terminal (and hence not vertical), if they are not collinear (perhaps they have different orientations) and - importantly - if scattering from the indoor environment is likely to include large signals from the ceiling and floor, correlation over the full 3D...
space is the more relevant measure. The 3D correlation does not reduce to a Bessel
function, but it is straightforward to perform the correlation integral numerically.

Plotting the 3D correlation, as in Figure 1, demonstrates that (apart from at small
separations, for which these idealised results are of little value, as explained below) the
correlation actually drops more quickly with element separation than would be
expected from equation (3) [2].

![Figure 1 - Analytical and Numerical pattern correlations](image)

Also notable from Figure 1, especially from the 3D-correlation curve, is that
correlation decays quickly with separation. It is the angle-dependent phase term,
arising from the antenna separation, which is entirely responsible for this, since the
patterns are assumed identical.

![Figure 2 - Measured and ideal pattern correlations](image)

**Measured Values**

Measured results, using 3D patterns from a fully populated linear array of 10
monopoles, spaced by $\frac{\lambda}{3}$ over a 43 x 93 ground plane are presented in Figure 2
(figures for isolated pairs are also included). By considering pairs of elements at
identical spacings within the fully populated array, a number of pattern correlation
values are calculated for each spacing. The slight variation in embedded patterns,
caused by the different local environment of each pair, causes the correlation values to
"scatter" somewhat. For example the "stray" value obtained with nearest neighbour
correlations ($\frac{\lambda}{3}$ separation) corresponds to a pair involving an end-element.
The measured correlation follows the trend expected reasonably closely. It can be noted that the reduction in correlation with element separation is far from monotonic - the correlations at 2λ/3 are only a little lower than those at λ/3 - as expected from the idealised curve. The more rapid than expected drop in correlation as the antennas are separated is thought to be due to the finite ground plane introducing extra decorrelation effects.

Figure 3: Correlation at small separations.

Figure 3 presents correlation results for a 2 element monopole array, with elements at much smaller separations. Here it is particularly noticeable that the correlation is smaller than expected and that even separations of a quarter of a wavelength (or perhaps even one-eighth) yield acceptably low values of correlation. This effect is readily explained by the embedded pattern of each element being distorted by its neighbours and, for a traditional array consisting of otherwise identical antenna elements, the influence of the neighbours on the pattern is clearly a beneficial effect as far as diversity is concerned.

The Role of Mutual Coupling

This distortion of the element pattern by the proximity of other elements is associated with mutual coupling, and, viewed in this light, it is not entirely obvious how it could be a beneficial effect. Indeed, recent publications often disagree in terms of whether mutual coupling is advantageous [3, 4] or disadvantageous [5, 6].

It is illustrative to examine the argument put forth in [5], by means of which it was concluded that mutual coupling decreased capacity of a MIMO system. In [5] the expression for the feed voltage of the m-th array element, arising from a plane wave incident at an angle θ on the array from (a result originally derived in [7]) is:

\[ v_m = C_{pm} E_0 f(u) e^{jkd_m} + \sum_{m \neq n} C_{pm} E_0 f(u) e^{jkd_n} \] (3)

where \( f \) is the isolated element pattern, the exponential represents the phase shift introduced by the position \( d_m \) of the element in the array and \( u \) is the direction cosine of the plane wave.

This expression implies that the embedded pattern of any element in the array (i.e. with the other elements present) may be expressed as a weighted sum of the isolated element patterns \( (C_{pm} \text{ are the weights}) \). Data in [5], and elsewhere, indicates that, to within a certain level of approximation, this is indeed the case.

The physical basis for this hypothesis is that the incoming plane wave sets up a current distribution on each element according to its normal isolated pattern, this current then couples into neighbouring elements - thereby altering the feed voltage. Clearly the interaction is rather complex (involving multiple scatterings between elements), however the outcome can only be that each element receives a weighted sum of the
information incident at the other elements. This argument suggests that mutual coupling cannot introduce extra diversity into the system, and [5] uses data derived from (3) to demonstrate this quite clearly.

On more detailed consideration however, it can be seen that there is a flaw in this argument if the antennas are multi-moded. In this case the shape of the current distribution induced on the receiving antenna (not merely its amplitude and phase) will depend on the angle of reception. Hence the coupling factors themselves (the $C_{mn}$ above) will depend on the variation in coupling exhibited by different modes of the antenna and hence the angle of arrival. Indeed, inspection of the original reference [7] reveals that (3) was intended for single-moded antennas - application of this theory to pattern diversity for practical, multi-moded, antennas is therefore misleading.

Conclusions
1) The oft-quoted correlation result from [2] is not appropriate if multipath components may be received from 3D space (e.g. indoor scenarios).
2) Even with identical elements, low correlations are achieved in practice at small spacings through coupling to adjacent elements. These effects are beneficial.
3) The finite ground plane seems (as would be expected) to reduce pattern correlation between elements.
4) At spacings of $\lambda/2$, low correlation is achieved purely though the phase separation (since even identical patterns yield low correlation in this scenario).
5) While array theory often makes use of the result of [7] (i.e. that the embedded pattern is a weighted sum of isolated patterns) this leads to a misleading picture of the mutual coupling effects.

This contribution is very much a work-in-progress and further measurements and analysis of pattern correlation and mutual coupling are under way.

Acknowledgement
The authors would like to thank Dr P. N. Fletcher and Prof. C. J. Railton for helpful discussions on this subject.

References