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An Investigation of Pattern Correlation and Mutual Coupling in MIMO Arrays

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Introduction

- MIMO systems employ antenna arrays for increased capacity.
- The increased capacity is a result of diversity.
- All antenna systems exhibit mutual coupling between elements.
- The role of this coupling in diversity performance is often controversial.
- The coupling is especially important when elements are closely spaced (e.g. on a user terminal).
- Does the coupling improve diversity performance? or degrade it?
Definition of Pattern Correlation

Following Vaughan and Bach Anderson, the envelope correlation between two radiation patterns $F_1$ and $F_2$ is given by:

$$|\rho|^2 = \left( \frac{\int \mathbf{F}_2 \cdot \mathbf{F}_1^* \, d\Omega}{\left( \int |\mathbf{F}_1|^2 \, d\Omega \right) \left( \int |\mathbf{F}_2|^2 \, d\Omega \right)} \right)^2$$

- This is the measure of similarity between two radiation patterns.
- Considering correlation in 3D, $\Omega$ is the solid angle and the limits of integration are a sphere.
- A rule-of-thumb is that good diversity operation is possible when $|\rho|^2 < 0.5$
- Provided also that the SNR at each antenna is approximately equal (Vaughan & Bach Anderson).
Analytical Pattern Correlation

The correlation between fields at two points (Clarke 1968), separated by a distance $d$:

$$|\rho|^2 = \left[ J_0 \left( \frac{2\pi d}{\lambda} \right) \right]^2$$

This result is also valid for the correlation between identical antenna patterns. It was derived assuming correlation around a circle in the far-field – not a sphere.

Note the more rapid decay
Note the null close to half-wavelength
Experimental Pattern Correlation (1)

Experimental results using an array (or just 2 elements) of monopoles:

- Ideal pattern
- Two elements (Measured)
- Fully populated (Measured)

Patterns are not identical.
Ground plane is finite.
Note again a more rapid decay.
Note some “scatter” on results.
Note correlation at 0.35 similar to 0.7!
Experimental Pattern Correlation (2)

Experimental results using a redesigned array of 2 monopole elements:

Low correlation down to very small spacings
Due purely to mutual coupling distorting the pattern
Alternative Calculation of Pattern Correlation

The pattern correlation result can also be found from the input ports. Derived by Clarke in 1968 in terms of the Z-matrix, and restated by Blanch, Romeu and Corbella in terms of S-parameters (El. Letts. vol 39, May 2003).

\[ |\rho|^2 = \frac{\left( \int_{\Omega} |\mathbf{F}_2 \cdot \mathbf{F}_1^*| d\Omega \right)^2}{\left( \int_{\Omega} |\mathbf{F}_1|^2 d\Omega \right) \left( \int_{\Omega} |\mathbf{F}_2|^2 d\Omega \right)} = \frac{|S_{11}^* S_{12} + S_{21}^* S_{22}|}{\left( 1 - (|S_{11}|^2 + |S_{21}|^2) \right) \left( 1 - (|S_{22}|^2 + |S_{12}|^2) \right)} \]

An important result =>
- The S-parameters, and hence the correlation, may be (almost) arbitrarily altered by the addition of a network at the antenna terminals, regardless of spacing.
- Matching is important (indeed, a perfect match gives zero pattern correlation).
Introducing a 4-Port (1)

- The S-parameters, and hence the correlation, may be (almost) arbitrarily altered by the addition of a network at the antenna terminals, regardless of spacing.
- For a two element array, the network is a 4-port.
Introducing a 4-Port (2)

- For these initial investigations, assume infinitesimally thin dipoles, Richmond's formula is therefore valid (assuming a sinusoidal current distribution).
- Radiation patterns and mutual couplings may be calculated from simple integral expressions.
- The input parameters to the left of the 4 port may then be found.
- These parameters then give the pattern correlation (equivalently the (loaded) patterns may be calculated – the result is the same, but more time-consuming to evaluate).
- 4-port parameters are then optimised to give zero correlation.
Introducing a 4-Port (3): Results

- There is more than one 4-port that will give zero correlation.
- Dipole separations of only $\lambda/20$
Introducing a 4-Port (4): Results

- There is more than one 4-port that will give zero correlation.
- Dipole separations of only $\lambda/20$

Zero correlation
Relatively omni-directional patterns
Note: amplitudes are correlated but phases not shown
Closely-spaced elements

A PDA-sized box
4 identical slot antenna elements (~\(\lambda/2\) spacing)
Different orientations.

\[
\begin{pmatrix}
\left\| \Phi_2 \cdot \Phi_1^* \right\| d\Omega \\
\left\| \Phi_1 \right\| d\Omega \\
\left\| \Phi_2 \right\| d\Omega
\end{pmatrix}^2
\]

\[
\begin{pmatrix}
\left\| F_2 \cdot F_1^* \right\| d\Omega \\
\left\| F_1 \right\| d\Omega \\
\left\| F_2 \right\| d\Omega
\end{pmatrix}^2
\]

<table>
<thead>
<tr>
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<th>Element 1</th>
<th>Element 2</th>
<th>Element 3</th>
<th>Element 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element 1</td>
<td>1.00 (1.00)</td>
<td>0.024 (0.6356)</td>
<td>0.005 (0.5225)</td>
<td>0.024 (0.733)</td>
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<td>0.022 (0.7879)</td>
<td>0.020 (0.5428)</td>
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<td></td>
<td>0.015 (0.6063)</td>
<td>1.00 (1.00)</td>
</tr>
<tr>
<td>Element 4</td>
<td></td>
<td></td>
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</tbody>
</table>

NB: Table (like graphs, gives square root of correlation)
Conclusions

- Pattern correlation gives an indication of diversity performance.
- Element spacing gives low correlation, but this can also be obtained at close spacings.
- Elements spaced at $\lambda/2$ – or even much less - can give decorrelated patterns through:
  - Distortion of embedded patterns due to coupling.
  - Optimisation of S-parameters (see also Morris & Jensen, 2004 URSI EMTS, vol. 1).
  - Dissimilar antenna elements (or differently-orientated but otherwise identical elements)
- While these techniques can give nearly- or exactly-zero correlation (even with $\lambda/20$ spacings), diversity performance will only be maintained if pattern coverage is good (i.e. if the SNR is similar).