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Rateless Distributed Source Code Design

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MobiMedia 2009
Outline

1. Fountain codes: state of the art
2. Rateless coding with side info
3. Fountain coding with multiple source nodes
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1. Fountain codes: state of the art
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Multicast transmission in a lossy packet network

- Receivers experience different and dynamically changing packet loss rates.
- **Wireless erasure networks** / mobile environments.
- ARQ/Feedback implosion.
Erasure coding

- Erasure codes (MDS - Reed-Solomon)?
  - low operational complexity (mobile devices: computational resources and battery power)
  - Sparse graph codes coupled with belief propagation (BP) algorithm: LDPC, Turbo, LDGM, IRA...

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Topics in Fountain Coding
Erasure coding

- Erasure codes (MDS - Reed-Solomon)?
  - support for a wide range of (and dynamically changing) packet loss rates
    - code rate= ??
# encoding packets=$\infty$, code rate = 0
Fountain codes are:

- **rateless** - a potentially limitless amount of encoding packets.
- **computationally efficient and scalable** - fast and parallelizable algorithms.
- **nearly optimal** - reliable data reconstruction from any set of encoding packets only slightly greater than the size of the original message.
Digital Fountain’s Raptor FEC has been adopted by:

- **3GPP** Multimedia Broadcast/Multicast
- **DVB-h** IP datacast to handheld devices
- **IETF** Reliable Multicast Transport (RMT)
(Luby 2002), $LT(k, \Omega(x))$ code ensemble: $k$- size of the message, $\Omega(x) = \sum_{d=1}^{k} \Omega_d x^d$ probability distribution on $\{1, 2, \ldots, k\}$ (gen. poly.)

- Sample an output degree $d$ with probability $\Omega_d$.
- Sample $d$ distinct data packets uniformly at random and XOR them.
LT codes achieve capacity

Fact

There exist sequences of LT code ensembles $LT(k, \Omega^{(k)}(x))$ which achieve capacity regardless of the erasure probability of the channel (universality) with computational cost of $\mathcal{O}(k \log k)$.

- $\Omega^{(k)}(x)$ converges pointwise to limiting soliton distribution:

$$\Psi_\infty(x) = \sum_{d=2}^{\infty} \frac{x^d}{d(d-1)}$$

- Small perturbations suffice at finite lengths: robust soliton.
Soliton distributions

![Graph showing soliton distributions with probability on the y-axis and degree d on the x-axis.]

- **Ideal soliton**
- **Robust soliton**
Raptor codes

(Shokrollahi 2006), \( \Omega(x) \) capped at a max. degree \( d_{\text{max}} \) as \( k \to \infty \) (lowers computational cost to \( \mathcal{O}(k) \) but introduces an error floor) - decode fraction \( 1 - \delta \).

Error floor is removed by an outer very high rate LDPC code - sufficient redundancy to finish off decoding.
Decoding graph

\[ \Lambda(x) \quad k \text{ source packets} \]

\[ \Omega(x) \quad k(1+\varepsilon) \text{ encoded packets} \]

\( \varepsilon \): code overhead
BP decoding asymptotic analysis

- (Luby, Mitzenmacher, Shokrollahi 1998) AND-OR tree evaluation
- Generalized to density evolution techniques (Richardson, Urbanke, MCT, 2008)
- Recipe for fountain code design:
  - formulate a particular version of density evolution - set of recursive equations
  - generate an optimization procedure based on the density evolution equations (typically LP).
Optimisation of $\Omega(x)$

Fix $d_{\text{max}}$ and $\delta$ and minimise $\varepsilon$:

$$\text{LP : } \min \sum_{d}^{d_{\text{max}}} \frac{\omega_d}{d} (\sim 1 + \varepsilon)$$

$$\sum_{d=1}^{d_{\text{max}}} \omega_d (1 - y_i)^{d-1} \geq -\ln y_i, \ i \in \{1, 2, \ldots, m\},$$

$$\omega_d \geq 0, \ d \in \{1, 2, \ldots, d_{\text{max}}\}.$$

- $1 = y_1 > y_2 > \cdots > y_m = \delta$ are $m$ equidistant points on $[\delta, 1]$, $\delta$ is the desired error rate, and $d_{\text{max}}$ is the max. degree.
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The admissible rate region for the pairs of rates \((R_X, R_Y)\) is given by:

\[
R_X \geq H(X|Y) \\
R_Y \geq H(Y|X) \\
R_X + R_Y \geq H(X, Y).
\]
Slepian-Wolf Coding (SWC)

$H(X,Y)$

$H(Y)$

$H(Y|X)$

$H(X|Y)$

$H(X)$

$H(X,Y)$

$R_x$

$R_y$

asymmetric SWC

symmetric SWC

admissible rate region
Scalable video multicast

- scalable video over loss-prone wireless networks
- Single channel code for both:
  - video compression (Slepian-Wolf coding)
  - packet loss protection

Rateless Asymmetric SWC

$$m \geq kH(X \mid Y)$$
Y is the output of an erasure channel when X is the input.

Receivers have a priori knowledge of a number of data packets (transmission from other sources ?).
Asymmetric SWC - side information

- Systematic Raptor - *Fresia, Vandendorpe (Globecom 2007)*.
- Non-systematic LT (shifted robust soliton) - *Agarwal, Hagedorn, Trachtenberg (ITA Workshop 2008)*.
- IR-HARQ with LDPC/Fountain codes - *Sejdinovic, Ponnampalam, Piechocki, Doufexi (IEEE WCNC 2008)*.
Asymptotic code design with side info

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Topics in Fountain Coding
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Multiple source nodes

Fountain codes: state of the art
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Topics in Fountain Coding
Rateless Symmetric SWC

\[
X^k \quad \text{encoder 1} \quad Z^\infty_X \quad \bar{Z}_X^m \quad \text{decoder} \quad \bar{X} \\
Y^k \quad \text{encoder 2} \quad Z^\infty_Y \quad \bar{Z}_Y^m \quad \bar{Y}
\]

\[
m \geq \frac{kH(X,Y)}{2}
\]
Obstacles:
- No cooperation or centralized controller
- Each source node produces localized encoding packets.

Questions:
- How to perform (small) decentralized encoding tasks such that the resulting decoding problem is well-behaved?
- Can relay help by combining data from multiple sources?
DE: General case

- Packets dispersed across $s$ source nodes.
- Sets of packets available at different nodes are not necessarily disjoint nor of equal size.
- Each source node oblivious of which packets are available at other source nodes.
- *IEEE Commun. Letters 2009* (under review, with Piechocki, Doufexi, Ismail)
DE: General case

- Rigorous asymptotic analysis for many data dissemination scenarios.
- Generalized DE leads to a simple asymptotic code design yielding both multiterminal source coding and channel coding gains.
- Easy modification to include the case of informed collector node, i.e., decoder side information.
- Amenable to extension for noisy channels and general belief propagation algorithm (channels like BSC and BIAWGNC).
Example

Source nodes $S_1$ and $S_2$ are trying to multicast $k$ packets. Each source node contains $t > k/2$ packets, but is oblivious of which $t$ packets are available at other node. Source nodes use $LT(t, \Omega(x))$ ensemble and receiver obtains $n/2$ encoding packets from each source node.
Symmetric SWC - DDLT

\[
\begin{align*}
A_1 & : \frac{1-p}{2} k \\
B_1 & : \frac{1}{2} n \\
A_2 & : \frac{2p}{1+p} k \\
B_2 & : \frac{1}{2} n \\
A_3 & : \frac{1-p}{2} k \\
B_2 & : \frac{1}{2} n
\end{align*}
\]
Optimization of $\Omega(x)$, $p = 1/3$

LP: $\min \sum_{d=1}^{d_{\text{max}}} \frac{\omega_d}{d}$

\[
\frac{1}{1 + p} \omega (1 - \frac{1 - p}{1 + p} y_i - \frac{2p}{1 + p} y_i^2) \geq - \ln y_i, \quad i \in \{1, 2, \ldots, m\},
\]

$\omega_d \geq 0, \quad d \in \{1, 2, \ldots, d_{\text{max}}\}$. 
Intermediate performance

\[
\rho = \frac{\text{(number of received symbols)}}{k}
\]

\[
\zeta = \frac{\text{(number of reconstructed symbols)}}{k}
\]

\[
\Omega(x) = x^2
\]

\[
\Omega(x) = \Psi_\infty(x)
\]

distributions obtained by LP

\[
\Omega(x) = x
\]
Numerical results

![Graph showing packet error rate versus overhead ε. The graph compares different scenarios with varying parameters such as k values and Ωraptor.]
Limiting distribution

- Perturbation of the Limiting soliton for correlated data.

\[ \Omega(x) = -\frac{2(1+p)}{1+3p} \int_0^x \ln \frac{\sqrt{t(p^2 - 1) + 1 - p}}{1-p} dt, \quad x \in [0, 1). \] (1)

Fact

*When two terminals contain erasure correlated data, fountain coding can still achieve information theoretic limits, provided that the code design is appropriately modified.*
Summary

- Overview of fountain coding - LT, Raptor codes
- DSC with fountain codes
- Generic setting with multiple source nodes
- Symmetric SWC - perturbation of soliton distribution achieves SW limit.