The plastic limit of clays

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The plastic limit of soils was first described by Atterberg in 1911. The thread-rolling test was standardised at the US Public Roads Bureau in the 1920s and 1930s, and has subsequently become one of the standard tests of soil mechanics. This paper reviews the original definitions of plastic limit as proposed by Atterberg, and proposes that the brittle failure observed in the plastic limit test is caused by either air entry or cavitation in the clay. Critical state soil mechanics is used to show that the observed range of undrained shear strengths of soils at plastic limit is consistent with this hypothesis. The fallacy that strength at plastic limit is a constant is highlighted, and the implications for geotechnical practice are discussed.

KEYWORDS: clays; laboratory tests; soil classification; suction

**INTRODUCTION**

A hundred years after Atterberg first described die ausrollgrenze, it is timely to review the original definition, and study the mechanical interpretation of one of the most basic tests in soil mechanics. Atterberg (1911) qualitatively described seven limits that governed the behaviour of cohesive soils at varying water contents. Of these limits, only the liquid and plastic limits, which determine the range of plastic behaviour of the soil, and the shrinkage limit, have remained in common usage.

It is widely assumed that both the liquid and plastic limits of soil can be found by measuring soil strength. While the liquid limit fundamentally relates to clay strength (Haigh, 2012), the plastic limit instead relates to the capillary suction at which the water phase ceases to act as a continuum. This can be caused either by air entry into the soil or by heterogeneous cavitation of the pore water. In this paper the critical state framework (Schofield & Wroth, 1968) is used to show that the undrained shear strengths observed at plastic limit are consistent with this hypothesis.

**Liquid limit**

Atterberg (1911) proposed a method for measuring the liquid limit of soils based on the number of blows required to cause a groove in a clay bed to collapse when the soil container was struck on the hand. As slope stability is a strength-based phenomenon, it would seem rational to assume that soil at the liquid limit exhibits a fixed soil strength that could in principle be measured by more repeatable methods. The fall cone test now used in BS 1377 (BSI, 1990) is a measurement of soil strength, as shown via plasticity analysis by Houlshy (1982).

**Plastic limit test**

The plastic limit, given by the rolling test, has a long history in geotechnics. In this paper it will be shown that while the test appears unscientific, the test is actually well designed to test the onset of brittleness in a way that is not the case with the alternatives proposed to date.

Atterberg (1911) stated that (translated from German):

> [roll] ... until the thread shears both longitudinally and transversely when it has been rolled to about 3 mm diameter, as gauged by the rod. [A standard 3 mm rod is used to guide the operator as to the approximate thickness of their clay sample.] Do not gather the pieces of soil together after they have crumbled in order to reform a thread and continue rolling; the first crumbling point is the plastic limit.

Casagrande (1932) suggests that the diameter at which the clay thread breaks is important, citing Terzaghi (1926a, 1926b). These papers, however, do not contain any data, and subsequent testing by the present authors and by Prakash et al. (2009) has shown that there is no statistically significant trend of varying water content with thread diameter at failure.

The test has its drawbacks; it is a test that is highly reliant on judgement, and could be considered to be operator specific. Sherwood (1970) showed that identical soil samples, when sent to 45 independent UK soil-testing laboratories, showed a standard deviation on plastic limit values of around 3%.

If an alternative test for plastic limit is to be developed, it is vital to understand what mechanical processes are leading to soil failure within the thread. Schofield & Wroth (1968) contended that the ‘crumbling’ of soil in the plastic limit test implies a tensile failure, similar to that observed in split-cylinder tests on concrete. While this may well explain the eventual failure of the soil thread, examination of the test method shows that this cannot be a test of soil strength, tensile or otherwise.

In any material strength test, some stress must be controlled or measured. This may occur either using a load cell,
as in a split-cylinder concrete strength test, or using dead weight, as in the fall cone test for liquid limit. In the plastic limit test no stresses are controlled directly; enough vertical stress is applied using the hand to cause the soil thread to yield and elongate, but this stress is never measured.

It is the contention of this paper that what is actually occurring in the plastic limit test is that the soil is being dried until it reaches a brittle transition point. The continuous ductile failure of the thread during its plastic elongation allows this point of brittle transition to be observed. This change coincides with the point at which the fluid phase ceases to behave as a continuum, with suction in the fluid allowing the soil to behave in a plastic manner. This sudden change in behaviour can be brought about either by air entry, as suggested by Bolton & Cheng (2002), or by cavitation of the pore water.

Water at pressures lower than the vapour pressure at the current temperature is in a metastable state (Apfel, 1971). If a vapour bubble greater than a critical diameter exists within the water, this will grow in an unstable manner, as described by Or & Tuller (2002), leading to tensile fracture of the water. This unstable expansion process is termed ‘cavitation’ in this paper.

THE STRENGTH FALLACY

Figure 1 shows data for the variation of undrained shear strength with liquidity index reported by Skempton & Northey (1952), on which Schofield & Wroth (1968) observed that experimental results with four different clays give similar variation of strength with liquidity index … From these data it appears that the liquid limit and plastic limit do correspond approximately to fixed strengths which are in the proposed ratio of 1:100.

This observation of an approximately hundredfold increase in strength has been used by many authors as justification for the use of a fixed strength at plastic limit. Extrusion (e.g. Whyte, 1982) and cone test (e.g. Stone & Phan, 1995) methods for the determination of the plastic limit rely on this assumption that the plastic limit corresponds to a defined shear strength, but a closer inspection of Fig. 1 shows that the plastic and liquid limits do not occur at fixed strengths – and, more importantly, that the ratio is not always a factor of 100. This was implicitly recognised by Houston & Mitchell (1969), who proposed the bounds on soil strengths also shown in Fig. 1. Table 1 summarises data from seven publications that report undrained shear strengths at plastic limit. The data cover the range from 17 to 530 kPa, making the use of a strength test to find the plastic limit of a soil highly unreliable.

Figure 2 shows the cumulative distribution of shear strengths at plastic limit for the tests detailed in Table 1. The median strength is 132 kPa, and the mean and standard deviation are 152 kPa and 89 kPa respectively. The assumption of a fixed shear strength at the plastic limit is thus invalid.

While the strength at the Atterberg plastic limit may be variable, there will exist a water content at which the soil strength is 100 times that at liquid limit. This could be termed the plastic strength limit, PL100. Several methods have been proposed for the measurement of plastic limit based on strength, which implies that these methods are in fact measuring the plastic strength limit. Campbell (1976) suggested that fall cone tests should be carried out at water contents close to the plastic limit. As the preparation of a homogeneous sample close to the plastic limit can be difficult, owing to the high strength of the soil, several authors have proposed extrapolation methods from fall cone tests carried out at higher water contents. This may involve cones of two different weights, as proposed by Wood & Wroth (1978), or merely extrapolation from the data for a single cone (Harison, 1988). It should be remembered in these cases that the choice of regression line will have a major effect on the value of the plastic strength limit so determined.

Stone & Phan (1995) recognised that the very small

![Fig. 1. Variation of strength with liquidity index](image1)

![Fig. 2. Cumulative distribution of shear strengths of soil at plastic limit](image2)

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penetration of a standard cone at the plastic limit resulted in a very small volume of soil being tested, and hence in the tests being sensitive to inhomogeneity. As the use of a very heavy cone to achieve adequate penetration was problematic, they instead proposed the use of an instrumented cone penetrometer driven at quasi-static rates into a soil sample with the load increased until it reached 80 N. At this point a penetration of 20 mm corresponded to the plastic limit.

There has been discussion as to whether the plastic limit from the thread-rolling test correlates well with the plastic strength limit. Harison (1988) showed for Indonesian Bandung clays and Sharma & Bora (2003) showed for bentonite samples that there was good agreement between the two approaches. Wijeyakulasuriya (1990) showed for some Sri Lankan residual clays of kaolinitic origin that the thread-rolling test gave considerably lower values of $w_p$ than the fall cone test. When comparing the behaviour of residual and sedimentary soils, it is worth remembering that the same correlations may not apply (Wesley, 2003), but it can be seen from these three studies that the plastic limits determined by the two methodologies do not always coincide.

**BRITTLE FAILURE IN THE PLASTIC LIMIT TEST**

In order for a soil thread to remain intact during the plastic limit test, tensile failure of the soil must be prevented in the longitudinal and horizontal directions. In these directions no total stress is applied to the thread, so failure can be prevented only by soil suction. This suction will increase during the rolling test owing to evaporation of pore fluid, caused by the heat of the operator’s hands, until such point as the pore suction cannot be sustained.

For any soil there exists a value of pore suction above which the soil will begin to desaturate. At greater suction values than this the soil will exhibit continually decreasing water content, with the strength of the soil progressively departing from that which might be expected of a saturated soil. Marinho & Oliveira (2012) demonstrate for a residual soil that air entry occurring around the plastic limit results in a change in the strength regime of the soil; strength no longer increases rapidly with decreasing water content. This desaturation of the soil results in its becoming brittle, as the soil strength is governed by the behaviour of the menisci in water droplets at the particle contacts. Failure of the soil thus tends to occur through crumbling of the thread, which meets the criterion for plastic limit in the thread-rolling test.

From examination of desaturation curves available in the literature, it is clear that this mechanism is insufficient to explain the brittle behaviour of high-air-entry clays at the plastic limit. Fig. 3 shows data from Fleureau et al. (2002) for the desaturation and compression behaviour of three soils, each prepared by consolidation from slurry: FoCa clay, Jossigny silt and La Verne dam material. While the soil suction at the plastic limit corresponds reasonably well with air entry for the coarser Jossigny silt and La Verne dam material, for the finer FoCa clay the plastic limit corresponds to a suction two orders of magnitude lower than that required for air entry into the clay. Similar behaviour is also observed in other published data, including Fleureau et al. (1993). An alternative mechanism must thus be postulated in order to account for this behaviour.

A possible mechanism for the breakdown of suction in the soil thread is heterogeneous cavitation of the pore water, which may result in the pore-water phase within the still-saturated soil becoming discontinuous. Within a thread of soil subject to tension on a vertical plane, owing to compression from the hand of the operator, this could result in the clay thread fracturing into two saturated pieces – a failure scenario different from that caused by air entry, but still one that would register as the plastic limit in a thread-rolling test.

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**Fig. 3. Desaturation and compression behaviour of saturated slurries (replotted from Fleureau et al., 2002): (a) FoCa clay; (b) Jossigny silt; (c) La Verne dam material**
Cavitation results in the unstable expansion of small, vapour-filled cavities within a fluid, owing to extremely low pressures. As the cavity propagates through the soil, the pore suction is immediately replaced by a small positive pressure – the vapour pressure of the fluid. Wherever the vapour-filled cavity extends, the intergranular effective stresses must equally, and suddenly, drop. The pore pressures remote from the propagating crack will remain highly negative, and may decrease further with time if no cavitation nuclei larger than the critical size exist in the remainder of the specimen. If heterogeneous cavitation occurs in a soil thread that is under vertical compression, there will be a tendency for unstable tensile fracture to propagate on vertical planes, splitting the thread. There is an obvious analogy with fast fracture propagating from microcracks in a tensile test specimen of cast iron.

If no cavitation nuclei are present from which cavitation can propagate, cavitation can, in theory, be suppressed at suctions of up to 160 MPa (Speedy, 1982). The pore water of real soils, even those prepared under laboratory conditions, will, however, contain cavitation nuclei, owing to the non-ideal nature of the pore fluid, so suctions smaller than the theoretical maximum will cause cavitation. Experimental data for cavitation within pore-suction transducers (e.g. Tarantino & Mongiovi, 2001) shows that once cavitation initiates in a water-filled cavity, the pore suction falls rapidly to −100 kPa. The mechanism of cavitation within a soil matrix may differ from that within free water, but its effects have been observed experimentally in a variety of materials.

Considerable evidence has been published of cavitation in coarse soils: for example, McManus & Davis (1997) found that, following dilation of sands during undrained shear tests, pore pressures tended towards a value of approximately −80 kPa, with gas bubbles being observed in the water. While the cavitation tension in clays may be greater than that in coarser materials, researchers including Bishop et al. (1975) have concluded that cavitation can occur in clays at high suctions. Fig. 4 shows data from Bishop et al. (1975), in which it is shown that samples of London clay tested in confined and unconfined compression show markedly different behaviour. The brittle behaviour shown in an unconfined compression test is stated to be due to ‘propagation of “passages” from the external boundaries or from internal nuclei’, a process described both by Bishop et al., and in this paper, as cavitation.

Based on a review of available experimental data on cavitation in clays, Baker & Frydman (2009) stated that ‘there is no doubt that [cavitation] exists, even in clays,’ and concluded that an upper bound on the cavitation suction in clays is at a gauge pressure of around −400 kPa. While this ‘maximum’ value will be used in the analysis presented later, increases in the maximum cavitation tension will merely increase the maximum possible value of undrained shear strength at the plastic limit, rather than change the main assertions of this paper.

The fabric of the soil during a plastic limit test may also have a significant effect on the value of suction at which cavitation occurs. Remoulding of the soil thread once it reaches a diameter of 3 mm will almost inevitably result in the soil’s becoming inhomogeneous; cavitation and cracking will occur at the weakest point, but the suction will be retained in the remainder of the thread. The cavitation tension within the thread may hence be significantly lower than that in a triaxial sample whose fabric is well controlled.

**CRITICAL STATE BASIS FOR PLASTIC LIMIT**

During the thread-rolling test for plastic limit the soil is subjected to a complicated stress path, as discussed by Whyte (1982). This can, however, be simplified by assuming that the shear stresses applied to the soil by the hand are small in relation to the vertical stress. This leads to the schematic representation of the total and effective stresses on a soil element shown in Fig. 5, where $u$ is the pore pressure and is negative.

During the rolling test, the soil is continually remoulded, and hence its stress state lies on the critical state line, as shown in Fig. 6. It can hence be shown that

$$p' = \frac{\sigma_v}{3} - u \quad (1)$$

$$q = \sigma_v - Mp' \quad (2)$$

Assuming Tresca’s criterion for yield

$$c_u = \frac{q}{2} \quad (3)$$

**Fig. 5. Stresses within soil thread during plastic limit test:** (a) total stress; (b) effective stress

![Fig. 4. Comparison of confined and unconfined compression tests on London Clay (replotted from Bishop et al., 1975)](image-url)
It can hence be shown that
\[ 2c_u = M (2c_u - u) \]  \quad (4)
and hence
\[ c_u = \frac{3M}{6 - 2M} (-u) \]  \quad (5)

The shear strength of the soil therefore increases linearly with increasing pore suction. The maximum pore suction that can be achieved without desaturation of the soil is limited by the lower of the air-entry value of the soil and the cavitation tension that can be sustained by the pore water.

Taking a range of values of \( M \) for clays as 0.9–1.2 and a range of cavitation tensions of 100–400 kPa, as suggested by Baker & Frydman (2009), equation (5) gives a range of undrained shear strengths at the plastic limit of 65 kPa to 400 kPa. This corresponds to the range between the ninth and 98th percentiles of the data shown in Fig. 2. Values of undrained strength at the plastic limit below 65 kPa may be caused by air entry prior to cavitation. The samples within the database exhibiting the lowest strengths are those from the Bracklesham Beds reported by Demehy (1978) and an outlying value from the boulder clay on the Hyde bypass, reported by Arrowsmith (1978), both of which are silty materials.

Fracture due to cavitation

During the process of cavitation, water vapour bubbles within the pore water become unstable when their surface tension is no longer able to support the differential pressure between the vapour and fluid phases. Once this occurs, the bubble grows, the differential pressure that can be maintained falls, and a fracture is formed across the soil thread under the action of the fluid tension. Equating the surface energy created in cracking the soil thread to the elastic energy released by a drop in suction in the water phase, it can be shown that
\[ 2TA = Al \left( \frac{\mu \Delta u}{K} \frac{\Delta u^2}{2K} \right) \]  \quad (6)

where \( T \) is the surface tension of water (0.072 J/m²), \( K \) is the bulk modulus of water (2-2 GPa), \( u \) is the soil suction prior to cavitation, \( \Delta u \) is the change in soil suction during fracture, \( A \) is the cross-sectional area of the soil thread, and \( l \) is its length. It can hence be shown that, if the suction prior to cavitation is assumed to be 400 kPa and the length of the soil thread to be 50 mm, the change in pore pressure required to release enough energy to fracture the soil thread is only 16 kPa. The process will thus result in fracture of the soil thread into two pieces, each of which will retain considerable suction.

IMPLICATIONS FOR GEOTECHNICAL PRACTICE

A major use of the plasticity index and hence the plastic limit in geotechnical design is as a proxy for undrained strength. This can be derived by assuming a logarithmic dependence of undrained strength on plasticity index such that there is an increase in strength by a factor of 100 between the liquid and plastic limits. The conventional plastic limit test, measuring the onset of brittleness, has been shown in this paper not to imply any particular fixed strength, and hence the plasticity index may not give an accurate prediction of soil strength.

For correlations with strength and stiffness, the plastic strength limit or the plasticity index (PL_{100}) defined using equation (7) would be the favourable choice, as it is linked to the variation of strength with water content.

\[ PL_{100} = w_L - PL_{100} \]  \quad (7)

However, if the brittle transition point is required, then a more conventional thread-rolling test in the spirit of Atterberg, Terzaghi and Casagrande would be recommended. An example of the applicability of this would be in the design of clay-landfill liners, or in the evaluation of road pavement subgrades for arid areas.

CONCLUSIONS

This paper has focused on the fundamentals of the plastic limit test, and has shown the following.

(a) The plastic limit as defined by Atterberg (1911) is a measure of soil brittleness, and does not correspond to a fixed soil strength.
(b) The plastic limit is the result of either cavitation or air entry, resulting in the water phase ceasing to act as a continuum within the soil thread. Soil type, especially the presence of a significant silt fraction, will determine which of these mechanisms governs.
(c) The plastic limit as measured by a fall cone or static cone apparatus relies on an assigned ratio of strengths between the plastic and liquid limits, and hence is not the plastic limit described by Atterberg (1911).
(d) A quantity termed the plastic strength limit, PL_{100}, is suggested for correlations with strength properties, but not for analysis of the water content at which the soil becomes brittle.

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NOTATION

\( A \) cross-sectional area of soil thread
\( c_u \) undrained shear strength
\( K \) bulk modulus of water
\( l \) length of soil thread
\( M \) slope of critical state line
\( PL \) plastic limit

\( PL_{100} \) plastic strength index; defined by equation (7)
PL_{100} plastic strength limit; water content at 100 times \( c_u \) at the liquid limit.

\( p' \) mean effective stress

\( q \) deviatoric stress

\( T \) surface tension of water

\( u \) pore pressure; soil suction

\( \Delta u \) change in soil suction during fracture

\( w_L \) water content at liquid limit

\( w_P \) water content at plastic limit

\( a_v \) vertical total stress

\( a_s \) triaxial cell pressure

REFERENCES


