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CFD Aerodynamic Models for Separated Flow on High Aspect Ratio Flexible Wings

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In this paper the process of determining semi-empirical coefficients for three dynamic stall models is demonstrated using Computational Fluid Dynamics (CFD). Results of the models are then evaluated against two-dimensional Unsteady Reynolds Averaged Navier-Stokes (URANS) and it is seen that the models can capture unsteady aerodynamic non-linearities. This work is then extended to model separated flow and dynamic stall on a finite wing. In order to do this, a correction is applied to the Unsteady Lifting Line Theory (ULLT) using two-dimensional viscous data, which can be obtained either from two-dimensional URANS or a dynamic stall model. The ability to couple ULLT to a dynamic stall model allows for fast unsteady, three-dimensional and nonlinear load predictions, without relying on a vast URANS database. For this reason it is also possible to use the model within aeroelastic calculations.

Nomenclature

\( \alpha \) Angle of Attack
\( M \) Mach number
\( C_p \) Pressure coefficient
\( U_\infty \) Freestream velocity
\( \alpha_{ss} \) Static stall Angle of Attack
\( \alpha_{0L} \) Static zero lift Angle of Attack
\( \alpha_0 \) Mean Angle of Attack
\( \alpha_1 \) Angle of Attack Amplitude
\( \omega \) Circular frequency
\( k \) Reduced frequency \( \frac{\omega c}{U_\infty} \)
\( c \) Chord Length
\( b \) Wing semi-span
\( f \) Trailing-edge separation parameter
\( \Delta \) Deviation from static force curve
\( a, e, \sigma, r \) Empirical ONERA coefficients
\( \lambda, s \) Empirical ONERA coefficients
\( F_1 \) Linear ONERA load
\( F_2 \) Non-linear ONERA load supplement
\( F_l \) Linear extrapolation of static force curve
\( S_1, S_2 \) Beddoes-Leishman separation point fit parameters
\( C_N \) Normal force coefficient
\( C_N' \) Normal force coefficient with first order lag
\( T_P, T_V, T_F \) Beddoes-Leishman time constants
\( \tau_1 \) Goman-Khrabrov relaxation time constant

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τ_2 \quad \text{Goman-Khrabrov dynamic stall vortex lag}

ℜ \quad \text{Real part of a quantity}

ℑ \quad \text{Imaginary part of a quantity}

I. Introduction

Dynamic stall at high angles of attack has a significant impact on the forces experienced by an aircraft in flight. There have been numerous experiments that have demonstrated dynamic stall\(^1\).\(^8\).\(^9\) which show that the characterising feature is the formation and convection of a strong Dynamic Stall Vortex (DSV). This DSV has the potential to cause abrupt and severe changes in the unsteady force coefficients compared to equivalent static behaviour. In order to model the aeroelastic instabilities that result from this phenomenon accurate models of the aerodynamics are needed. In principle using computational fluid dynamics (CFD) and solving the Reynolds-averaged Navier-Stokes (RANS) equations within aeroelastic calculations could be employed to investigate such flows, however to date this approach is still of extremely limited value due to the infeasibly high computational costs. Thus a number of compact models have been developed over the years to approximate the aerodynamic responses to motions exceeding the static stall incidence. The models have the ability to quickly provide results, with acceptable levels of accuracy, but all share a reliance on semi-empirical coefficients that must be evaluated from experimental data or from higher fidelity simulations. This paper will implement and compare three popular dynamic stall models, namely the ONERA, Beddoes-Leishman and the Goman-Khrabrov models. In this study the semi-empirical coefficients are derived from CFD. The models are all profile and Reynolds number dependent, which has an implication for the semi-empirical coefficients that must be re-evaluated as the conditions vary. The goal of this research is to produce models of the unsteady spanwise forces and moments on a highly flexible finite wing where predictions of separated flow behaviour are needed. To reduce the cost of model production this work investigates how the 2D nonlinear aerodynamic models can be used to update 3D unsteady inviscid potential flow models in order to provide a viable tool for the early design stages of such highly flexible wings. Before this can be done the process of understanding the process of updating the potential flow with CFD data is investigated. This approach allows an understanding of the problem of going from 2D to 3D in such cases.

II. Background

A. Beddoes-Leishman Model

The Beddoes-Leishman (B-L) model\(^6\) is a semi-empirical model that is formulated to calculate the unsteady lift, moment and drag on an aerofoil undergoing dynamic stall. Dynamic stall is evaluated by modelling three different behaviours, starting with the attached flow. This is achieved using indicial response functions and these are assumed to be made of two parts, one to model the non-circulatory loading and the other for the circulatory component. Secondly the behaviour of separation is accounted for. In the B-L model, both leading edge and trailing edge separations are introduced. The model predicts the onset of leading edge separation by determining the leading edge velocity field. However, many aerofoils, especially thicker aerofoils\(^10\) do not initially stall at the leading edge. For this reason, a trailing edge separation mechanism is also implemented. A developing trailing edge separation creates non-linearities in the lift and moment response due to the reduction in circulation. Therefore a relationship between the effective separation point and Angle of Attack (AoA) is implemented from static aerofoil data. The vortex induced lift that the DSV provides is modelled as an accumulation of circulation, which is held on the upper surface of the aerofoil until leading edge separation occurs. After the separation, the vortex is allowed to convect over the aerofoil. Leishman and Beddoes state in their description of the model that the convection speed should be less than half of the free-stream velocity, in the initial formulation of the model the vortex convects at 0.45\(U_\infty\).\(^6\) The vortex lift provided from DSV lasts from the instant when the vortex begins to detach from the leading edge until it is shed at the trailing edge. No vortex lift is generated during the formation of the DSV at the leading edge.

An advantage of the B-L model when compared to the ONERA model is fewer dynamic cases are required to determine the semi-empirical coefficients. Initially a separation point, which is dimensionalized by the aerofoil chord is calculated using static data and theory from Kirchhoff flow, this means that the trailing edge separation point is a function of \(C_N\) such that

\[
C_N = 2\pi \left(\frac{1 + \sqrt{f}}{2}\right)^2 (\alpha - \alpha_{0L}) \tag{1}
\]
where $2\pi$ is the force-curve-slope, derived from thin-aerofoil theory, which can be substituted for the empirically derived force-curve slope. The parameter $f$ represents the trailing-edge separation point, where $0 \leq f \leq 1$. When $f = 1$ the flow is modelled as being fully attached, when $f = 0$, the flow is fully separated. Rearranging Eq. 1, the trailing-edge separation point can be deduced for static $C_N$ data. A relationship between $f$ and $\alpha$ can be deduced from static data and by the use of coefficients $S_1$ and $S_2$ be represented as

$$f = \begin{cases} 
1 - 0.3\exp((\alpha - \alpha_{ss})/S_1), & \text{if } \alpha \leq \alpha_{ss} \\
0.04 + 0.66\exp((\alpha_{ss} - \alpha)/S_2), & \text{if } \alpha > \alpha_{ss}
\end{cases}$$

(2)

Figure 1  Variation of trailing edge separation parameter $f$ with angle of attack for a NACA 0012 at Mach = 0.3. $\alpha_1 = 15.66, S_1 = 0.0402, S_2 = 0.0378$

Another lag is also introduced through $C'_N$. The trailing edge separation point is modified as a response to unsteady flow. Generally speaking, higher motion frequencies will result in the flow remaining attached at higher angles of attack than the static response, equally once separation has occurred, reattachment is usually delayed. By modifying the trailing edge separation condition Eq. 1 can be used to evaluate non-linear responses when subjected to dynamic conditions. This is introduced by calculating an effective AoA, which maps the unsteady leading edge pressures to the steady counterparts so that

$$\alpha_f = \frac{C'_N}{C_{Na}}$$

(3)

where $\alpha_f$ is the effective AoA and $C_{Na}$ is the normal force curve slope at the Mach number under consideration. Now $\alpha_f$ can be substituted into Eq. 2 to generate an effective separation point, denoted in$^6$ as $f'$. After this, an
Figure 2  Variation of trailing edge separation parameter $f$ with angle of attack for a NACA 0012 at Mach = 0.3. $\alpha_1 = 15.66, S_1 = 0.0402, S_2 = 0.0378$
additional first order lag is placed on $f'$ culminating in a final unsteady trailing edge separation parameter $f''$. This lag accounts for effects arising for the unsteady boundary layer. The final step of the model concerns determining the response to the formation of a dynamic stall vortex at the leading edge of the aerofoil and the convection of the vortex along the upper surface of the aerofoil. The vortex lift in the B-L model is treated as a build up of circulation. The excess circulation is held until a critical normal force coefficient is met, at which point the vortex lift will convect over the upper surface of the aerofoil. The effect of the vortex is terminated once it surpasses the trailing edge, in order to keep track of the convection, a vortex time parameter is included in the model. This phenomenon is illustrated in Figure 3, it can be seen that the AoA at which stall occurs is delayed with the vortex convection active. This is then followed by a more severe loss of normal force once stall does occur.

![Figure 3](image.png)

**Figure 3** Beddoes-Leishman vortex convection contribution. NACA 0012, $M=0.3$, $k=0.0756$

### B. Goman and Khrabrov Model

The Goman and Khrabrov model\(^4\) (G-K) is a state-space model that uses an internal dynamic variable, $f$. Similarly to the B-L model, $f$ describes the trailing edge separation position. As before, fully attached flow is associated with $f=1$, whilst fully separated flow results in $f=0$. A key advantage of the G-K model is the few semi-empirical coefficients needed to model unsteady aerodynamic forces. The G-K model requires just two coefficients, $\tau_1$ and $\tau_2$. The coefficient $\tau_2$ is a time lag that physically represents the time taken for a dynamic stall vortex to form and convect, whereas $\tau_1$ is a relaxation time constant. In order to evaluate an unsteady response, the trailing edge separation parameter for static conditions, $f_0$, must be known. It should be noted that physically $f_0$ may well not represent the actual separation location. Instead it is a function that is derived from the lift curve slope obtained using CFD. The dynamic trailing edge separation point can then be evaluated by solving the first order differential equation

$$
\tau_1 \frac{df}{dt} + f = f_0(\alpha - \tau_2 \dot{\alpha})
$$

The difference in behaviour between $f$ and $f_0$ is illustrated in Figure 4. Once $f$ has been found, it can then be substituted into Eq. 1 to determine the force coefficient.

### C. ONERA Model

The ONERA model describes dynamic stall by representing the moment and lift coefficients of an aerofoil using non-linear differential equations. The model was introduced by Tran and Petot in 1981\(^1\) and this original version of the model is considered here. The ONERA model has been updated by Peters\(^1\) and later by Petot.\(^1\) The model requires
semi-empirical coefficients that delineate the motion of the aerofoil. Classically, in order to obtain the coefficients, an experiment is required in which the aerofoil under consideration is pitched at a range of frequencies

around a number of mean angles. The amplitude of the oscillation is generally very small, this idea comes from the fact that a non-linear system will predominately behave linearly for small variation in the input parameters. This behaviour is not always the case, however for stall configurations the lift and moment will remain linear for changes in AoA of around 0.5°. Harmonic variations in AoA can be written as

$$\alpha = \alpha_m + \Re\left(\alpha_v e^{ik\tau}\right)$$

(5)

In general the resulting force response will be a summation of harmonic terms, however if the amplitude of the oscillation is small then the force response can frequently be approximated using just a first order approximation as

$$F = \tilde{F} + \Re\left(\tilde{F} e^{ik\tau}\right)$$

(6)

For this approximation to be accurate, the higher order terms must be small so that they may be neglected. This approximation holds especially well at lower angles of attack, but often less well at higher angles of attack. This is illustrated using URANS to simulate a NACA0012 aerofoil subject to a harmonic excitation of amplitude 0.5° about two mean angles of attack in Figure 5. When the mean AoA=0°, the response is elliptical which means that the first harmonic representation of Eq. (2) is a reasonably accurate representation of the force, however when the mean AoA=21° it can be seen that the higher order harmonics in the force are no longer negligible and therefore Eq. (6) gives a less accurate prediction of the force. Hence one of the fundamental assumptions of the ONERA model is that the force response to a small harmonic input can be approximated by the first harmonic at all flow conditions is shown to be incorrect at higher angles of attack.

The ONERA model expresses the aerodynamic loads as the sum of two components; a linear component which describes attached flow and a non-linear component which models the separated flow. The total aerodynamic load, $F$, can be calculated as the sum of the linear unsteady loads where stall is absent, $F_1$, and the non-linear unsteady loads, $F_2$, where the model equations for the force are given by.

Figure 4  Variation of static and dynamic trailing edge separation parameter $f$ and $f_0$ with angle of attack for a NACA 0012 at Mach = 0.3

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Figure 5  Lift coefficient response to small amplitude oscillations.

(a) Mean angle of attack = $0^\circ$, $k = 0.8$

(b) Mean angle of attack = $21^\circ$, $k = 0.1$
\[ F = F_1 + F_2 \] (7)
\[ \dot{F}_1 + \lambda F_1 = \lambda F_1 + (\lambda s + \sigma)\dot{\alpha} + s\ddot{\alpha} \] (8)
\[ F_2 + aF_2 + rF_2 = -(r\Delta F + e\dot{\alpha}) \] (9)

Here \( F_l \) is the linear extrapolation of the static curves and \( \Delta \) represents the difference between the extrapolation and the static value. These static coefficients are illustrated in Figure 6. The six coefficients for the ONERA model, \( \lambda, s, \sigma, a, r \) and \( e \) are functions of the AoA and the velocity of the free-stream flow.\(^5\) It is common practice to keep the coefficients \( \lambda \) and \( s \) constant. The value for \( \lambda \) can be approximated from the real component response of the small amplitude oscillation when the mean AoA=0°, shown in Figures 7, 8, 9 and 10. Furthermore, \( s \) represents the asymptotic slope of the imaginary response at high reduced frequencies. The remaining four coefficients, \( \sigma, a, r \) and \( e \), need to be considered simultaneously. In order to evaluate them a non-linear least squares optimisation routine is used to find the solution to Eqns [10 and 11]. It should also be noted that since these coefficients vary at angles of attack post static stall, the four coefficients are presented as a function of \( \Delta \) in Eqns [10 and 11]. Figure 11 shows the result of the optimisation routine.

\[ \Re{} \left( \frac{\ddot{F}}{\alpha_l} \right) = \frac{dF_1}{d\alpha} + \frac{k^2}{(\lambda^2 + k^2)} \left( \sigma - \frac{dF_l}{d\alpha} \right) + \frac{k^2(r - ae) - r^2}{(k^2 - r)^2 + (ak)^2} \frac{d\Delta}{d\alpha} \] (10)
\[ \Im{} \left( \frac{\ddot{F}}{\alpha_l} \right) = ks + \frac{k\lambda}{(\lambda^2 + k^2)} \left( \sigma - \frac{dF_1}{d\alpha} \right) + \frac{ek(k^2 - r) + akr}{(k^2 - r)^2 + (ak)^2} \frac{d\Delta}{d\alpha} \] (11)

![Figure 6 Static Coefficients generated using CFD for a NACA 0012](image-url)
Figures 12-13 show prediction of the lift coefficient using the ONERA model. Included in the figures are the linear and non-linear contribution to the load that are summed to produce the overall load, labelled $F_1$ and $F_2$ respectively. Note that the semi-empirical coefficients used for the ONERA results were created using experimental data for a Boeing-Vertol VR-7, which was used by McAlister, Lambert and Petot.\textsuperscript{5}
Figure 11  Optimisation of real and imaginary lift response to obtain $r, \sigma, a$ and $e$

Figure 12  Response for $\alpha = 7^\circ + 5^\circ \sin(\omega t)$, $k = 0.12$

Figure 13  Response for $\alpha = 10^\circ + 7^\circ \sin(\omega t)$, $k = 0.10$
III. Two-Dimensional Results

This section presents comparisons between the B-L model and the G-K model which have already been investigated. In order to obtain model coefficients and to evaluate the performance of the dynamic stall models, URANS CFD simulations were run using the compressible, unstructured DLR Tau-Code.\textsuperscript{14} The turbulence model used is the one equation negative Spalart-Allmaras turbulence model.\textsuperscript{2} The mesh used for the NACA 0012 aerofoil is generated using a structured conformal mapping approach and can be seen in Figures 14-15. The NACA 0012 has a chord length of 1 m, with 701 points on the aerofoil surface and there are a total of $2.0 \times 10^5$ cells in the mesh. All the cases were run at sea level conditions with a Mach number of 0.3, giving a Reynolds number of $7.29 \times 10^6$.

![Figure 14](image1.png) Whole domain view, showing mesh density variation

![Figure 15](image2.png) Leading edge of NACA 0012 mesh

<table>
<thead>
<tr>
<th>Case</th>
<th>Mean $\alpha$</th>
<th>Amplitude of Oscillation</th>
<th>Reduced frequency $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12^\circ$</td>
<td>$\pm 6^\circ$</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>$10^\circ$</td>
<td>$\pm 8^\circ$</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>$10^\circ$</td>
<td>$\pm 8^\circ$</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>$13^\circ$</td>
<td>$\pm 4^\circ$</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>$6^\circ$</td>
<td>$\pm 8^\circ$</td>
<td>0.13</td>
</tr>
<tr>
<td>6</td>
<td>$8^\circ$</td>
<td>$\pm 8^\circ$</td>
<td>0.06</td>
</tr>
</tbody>
</table>

In figure 16, the mean AOA is set close to the static stall AoA at $12^\circ$, with an oscillation amplitude of $6^\circ$, in order to capture a strong dynamic stall. The maximum $C_N$ is captured reasonably well by both models, however the G-K model stalls slightly earlier than the B-L, which in turn stalls before the CFD. The B-L model gives a closer representation of the CFD than the G-K model during pitch up, with the G-K model underestimating the $C_N$ until the AoA is approximately $14^\circ$. Neither the B-L or G-K model captured the sudden loss in $C_N$ that the CFD predicts very well, however there is a similarity between the models. The CFD also shows a peak at the maximum AOA where the vortex leaves the trailing edge, this is seen in all the deep stall cases (Figures 16 to 19) and isn’t reproduced by either model. However, the B-L model in both Case 2 and 3 very closely matches the $C_N$ immediately before the peaks seen in the CFD. Figure 20 shows the results from a case in which the CFD doesn’t predict stall will occur. This is a particularly difficult case, since the maximum AOA falls very close to the AoA where static stall occurs. Both the B-L and G-K models predict that a light stall will occur here, causing a reduction in $C_N$ which doesn’t occur in the CFD solution. Figure 21 presents a similar scenario to the previous, however the mean AOA is increased to $8^\circ$, allowing the aerofoil to pitch above the static
stall AoA. It appears here that the DSV is not shed from the trailing edge before the maximum AoA is reached. Initially as the aerofoil begins to pitch down, the edge before the maximum AoA is reached. Initially as the aerofoil begins to pitch down, the $C_N$ reduces slowly, until at approximately $15^{\circ}$ where a sudden drop in $C_N$ occurs. As in the previous case, the B-L and G-K model predicts a light dynamic stall to occur sooner than the CFD.

Table 2  Prediction of normal force during harmonic oscillations

<table>
<thead>
<tr>
<th>Figure 16  Case 1</th>
<th>Figure 17  Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>Figure 18  Case 3</td>
<td>Figure 19  Case 4</td>
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<tr>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
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<tr>
<td>Figure 20  Case 5</td>
<td>Figure 21  Case 6</td>
</tr>
<tr>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
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</tbody>
</table>

IV. Unsteady Lifting Line Model

The choice of unsteady three-dimensional aerodynamic model for this application is an unsteady lifting line theory (ULLT). An unsteady vortex lattice method (UVLM) may also be used, however the current correction only requires the ability to change the local angle of attack at spanwise stations which can be accomplished in ULLT. Another benefit to
using ULLT over UVLM is that it is computationally less expensive.

The classic Prandtl lifting line theory (LLT) was introduced in 1914. LLT models a finite wing as a bound spanwise vortex line, in which the strength of the vortex line, $\Gamma(y)$, varies along the span. Trailing edge vortices are shed down stream, with the strength of each vortex equal to $\frac{d\Gamma}{dy}$. The trailing edge vortices impose a downwash on the local aerofoil section, which changes the effective angle of attack, $\alpha_e$, at the section by an angle $\alpha_i$. Therefore the effective angle at an aerofoil section can be computed as:

$$\alpha_e = \alpha - \alpha_i$$  \hspace{1cm} (12)

The induced angle of attack can calculated by integrating the circulation distribution across the wing span.

$$\alpha_i(y) = \frac{1}{4\pi U_\infty} \int_{-b/2}^{b/2} \frac{d\Gamma(y_0)}{y-y_0}$$  \hspace{1cm} (13)

Eq. 13 forms the basis of Prandtl’s classical lifting line theory. The unsteady lifting line implementation used in this paper is described in.\(^3\) The underlying concept is the combination of steady lifting line theory and the Wagner function through the use of the unsteady Kutta-Joukowsky theory. This represents the evolution of the circulation build-up on a finite wing and ultimately allows for a first order state-space representation as given in Eqs 14 and 15, the full derivation can be found in.\(^3\)

$$[M_A] [\dot{A}_n] + [F_A][A_n] = C [\dot{q}] + D[q] + [E][z]$$  \hspace{1cm} (14)

$$I [z] = [W][z] + G[A_n] + F[q]$$  \hspace{1cm} (15)

Assembling Eqs 14 and 15 into matrix form allows the system of ODEs to be solved and the Fourier coefficients $A_n$ to be evaluated.

In order to validate the ULLT, lift predictions are compared to those obtained using UVLM. A rectangular wing of aspect ratio 12.5 with a root twist of $3^\circ$ linearly reducing to $0^\circ$ at the tip is chosen. In the case shown the wing with a mean angle of $0^\circ$ is pitched around the mid-chord with an amplitude and reduced frequency of $5^\circ$ and 0.1 respectively. The ULLT model shows good agreement with the UVLM, Figures 22a and 22b show the unsteady lift distribution at the maximum and minimum angle of attack.

![Figure 22](image)

Figure 22  ULLT and UVLM comparison for a pitching motion defined by $\alpha = 0^\circ + 5^\circ \sin \omega t$, $k = 0.1$

It is important that the ULLT is able to replicate the results from steady LLT when the reduced frequency and amplitude of motions are very small. Figure 23 demonstrates that as $k \to 0$ and $\alpha_1 \to 0$, the unsteady sectional lift coefficient tends to the steady lift coefficient.
V. Modelling Dynamic Stall on Finite Wings

The aforementioned stall models are only capable of modelling unsteady non-linear aerodynamics in two dimensions. The G-K model has been previously applied to a delta wing and a wing-body configuration with an aspect ratio of 9.7. However this did not achieve spanwise force coefficients, instead it provided an overall force coefficient for the whole body. The ONERA model has been experimentally investigated for three-dimensional pitching oscillations by Tang and Dowell. In their study three sections of a finite NACA 0012 were evaluated for the ONERA semi-empirical coefficients, however this was carried out only for a low Mach number on a very simple geometry and very few results were presented at the wing tip, where three dimensional effects are strongest. In order to model the non-linear unsteady dynamic stall the current procedure needs to be modified. The wing configuration used is the so-called UAV wing, which is shown in Figure 24. The UAV wing is a straight wing with an aspect ratio of 12.5 and a root twist of 2°, linearly decaying to 0° at the wing tip.

A. Unsteady Lifting Line and 2D Dynamic Stall Model Coupling

Attempts have been made to incorporate a viscous correction to Lifting Line Theory (LLT), thus creating a steady, 3D, non-linear aerodynamic model. Here two-dimensional viscous data (either from computational simulations or experiments) can be used to alter the local angle of attack at varying spanwise sections. LLT can then be reapplied with the updated incidences, and this process is repeated until $|C_{L,LLT} - C_{L,visc}| < \epsilon$ where $\epsilon$ is typically of the order 0.01. More recently an unsteady coupling method was proposed to couple the Unsteady Vortex Lattice Method (UVLM) with 2D RANS data. This method is very similar to the steady method of Van Dam. However in order to incorporate unsteady aerofoil motions, the unsteady thin aerofoil theory from Theodorsen is included in the formulation. The algorithm for the unsteady coupling is given as
Algorithm 1 Unsteady Coupling Algorithm

Solve the unsteady lifting line method to calculate the initial inviscid $C_L$ distribution $C_{L_{us}}^{3D}$

for Every strip $i$ do

Calculate the 2D unsteady lift coefficient $C_{L_{us}}^{2D}$

Calculate the effective angle of attack $\alpha_e$: $\alpha_e(i) = \frac{C_{L_{us}}^{2D}(i) - C_{L_{us}}^{3D}}{2\pi} + \alpha(i)$

Using viscous data, interpolate the lift coefficient at $\alpha_e$ to get $C_{L_{visc}}(i)$

Calculate the correction to the angle of attack: $\alpha_{cor}(i) = \alpha_{cor}(i) + \frac{C_{L_{visc}}(i) - C_{L_{us}}^{3D}(i)}{C_{L_{us}}}$

Update the angle at each station by $\alpha_{cor}$

end for

The steps above are repeated until $|C_{L_{visc}} - C_{L_{us}}^{3D}| < \epsilon$

This approach makes it possible to correct unsteady 2D sectional data using lifting line theory and an inviscid 2D unsteady model. In this work, the 2D unsteady model used is identical to the attached flow subsystem of the Beddoes-Leishman dynamic stall model.6 Figures 25 and 26 illustrate the 2D URANS data needed to correct the inviscid data for a pitching wing at one combination of mean angle of attack and reduced frequency. In order to effectively interpolate the data must be put into the time domain, then the data can be interpolated between the different mean amplitudes, this is shown in Figures 27 and 28. Therefore even though the necessity of obtaining 3D, unsteady viscous data is avoided, a substantial amount of sectional data still needs to be generated. However, replacing the expensive 2D URANS computations with a dynamic stall model leads to a great reduction in computational expense, whilst retaining the ability to capture unsteady aerodynamic nonlinearities.

Figure 25  2D URANS data for UAV wing

Figure 26  2D URANS Data in the time domain
VI. Results

The unsteady lifting line correction has tested against three-dimensional URANS using the unstructured DLR Tau-Code. The cases are performed using the UAV wing described above at Mach 0.3. Firstly the model is tested at a reduced frequency of $k = 0.05$, with a periodic pitching motion of $\alpha = \alpha_0 + 0.5^\circ \sin \omega t$. At smaller angles of attack, where the flow remains attached and linear, both the corrected and uncorrected lifting line models are able to closely match the three dimensional URANS. Figures 29a and 29b show the first and last iteration of the correction cycle, where the wing is in mid cycle (the wing is at the mean incidence and has a negative pitch rate), the model quickly converges with very good agreement to the three dimensional CFD.

The model has also been tested at a higher reduced frequency of $k = 0.1$, with a periodic pitching motion of a larger amplitude, defined by $\alpha = \alpha_0 + 6^\circ \sin \omega t$. For a mean angle of attack of $14^\circ$, the nonlinear model is able to reduce the lift at the spanwise sections due to flow separation. Figure 30 illustrates how the nonlinear lifting line model gives improved results compared to the linear model. As the mean angle of attack is increased further to $18^\circ$, the correction again is able modify the inviscid lift due to the effects of flow separation as seen in Figure 31. Figure 31 shows the lift distribution at the point in the cycle where the angle of attack is at a minimum. Generally the model is able to reduce the sectional lift coefficient to provide much improved distributions compared to the linear model, however the full effect of the three dimensional flow separation is not accounted for. The root has the largest angle of attack and therefore is subject to more complex flow separation which the nonlinear model is unable to fully predict and a more detailed model is needed.
Figure 29  Corrected ULLT result, $\alpha_1 = 4.0$, $k = 0.05$

Figure 30  Corrected ULLT result, $\alpha_1 = 14.0$, $k = 0.1$
Figure 31  Corrected ULLT result, $\alpha_1 = 18.0$, $k = 0.1$
A. Kriging

Another method that is being considered in order to model nonlinear aerodynamics around a finite wing is Kriging. The inputs to the Kriging model are similar to the unsteady coupling method. The Kriging model is defined as follows:

\[
C_{L_{CFD}}^{3D} = f_{krig} \left( C_{L_{LLT}}, C_{L_{uns}}^{2D}, \alpha, \dot{\alpha}, \ddot{\alpha}, y \right)
\]  \hspace{1cm} (16)

Eq. 16 shows that both the 2D unsteady model, \( C_{L_{uns}}^{2D} \) and lifting line theory, \( C_{L_{LLT}} \) is required as before. However for the Kriging model to be built, 3D RANS data is needed, therefore the full wing configuration needs to be simulated and sectioned into spanwise strips in order to train the model. The input \( \alpha \) represents the angle of attack, with \( \dot{\alpha} \) used to determine if the wing is on an up-stroke or down-stroke. The input \( \ddot{\alpha} \) is required in order to dictate whether the wing is at a maximum or minimum angle of attack.

The Kriging model has been initially trained for one strip on the UAV wing, where the strip spans from 0 to 0.08y, which is the strip located closest to the wing root. The training data is built from pitching the wing defined by \( \alpha = \alpha_0 + 2.5^\circ \sin k t^* \). Where \( \alpha_0 \) and \( k \) for the two training sets are given in the Table 3. The time, \( t^* \) is time in semi-chords, \( \frac{2Vt}{c} \).

<table>
<thead>
<tr>
<th>DB#</th>
<th>Mean Amplitude ( \alpha_0 )</th>
<th>Reduced frequency ( k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5°, 7.5°, 10.0°</td>
<td>0.01, 0.04, 0.06</td>
</tr>
<tr>
<td>2</td>
<td>3.0°, 5.0°, 6.0°</td>
<td>0.02, 0.04, 0.06, 0.08</td>
</tr>
<tr>
<td>3</td>
<td>3.0°, 5.0°, 6.0°, 10.0°</td>
<td>0.02, 0.04, 0.06, 0.08</td>
</tr>
</tbody>
</table>

Figures 32 and 33 shows the output from the Kriging model compared to full 3D CFD when the reduced frequency is varied from the training data. For small reduced frequency variations from the training data, the Kriging model is able to accurately reproduce the 3D CFD, even in more complicated hysteresis loops which format higher angles of attack where the lift curve intersects with itself, see Figure 33. However it should be noted that currently the test data has been restricted to only small variations from what was used to train the model. In Figure 34 the mean angle of attack has been extrapolated, so that the training data no longer encompasses the test data. Here the mean angle of attack is increased to 8.5° and the CFD data shows an intersection in the lift hysteresis loop. However since the Kriging model has not been trained with any data that features the intersection, it is unable to reproduce this phenomenon. This shows that it is vital for the training data to cover a diverse number of cases. It is however difficult to select training cases in a complete and efficient manner. Figure 36 demonstrates the identical case to Figure 34 with dataset #3, which features the addition of data generated at a mean amplitude of 10.0°. With just the incorporation of this dataset, the Kriging model is able to accurately capture the intersection of the hysteresis loop and provide a significantly better representation of the three-dimensional CFD data. Figure 35 presents an example where both the reduced frequency and mean angle of attack are extrapolated from the training data. Here the results are not as good to the similar case illustrated in 32 where only the reduced frequency was varied (and not extrapolated). However the Kriging model is still able to capture a hysteresis loop of similar shape and magnitude to the CFD data.
VII. Conclusions

In this paper the methodology for implementing three stall models capable of predicting the unsteady loads on an aerofoil undergoing dynamic stall have been described. Both the Beddoes-Leishman and Goman-Khrabrov models have been applied to a NACA 0012 aerofoil and validated against URANS data and are shown to be a suitable candidate for use as a viscous correction. An iterative unsteady coupling method capable of extending dynamic stall models to calculate loads on a finite wing using three-dimensional, unsteady, inviscid potential flow models has been outlined, including a comparison of ULLT and UVLM. ULLT was shown to agree well with UVLM with the benefit of allowing for much faster calculations. The process of updating the three-dimensional potential flow with two-dimensional URANS data has been investigated and early results show that using two-dimensional viscous data can bound the loads to realistic values and reduce the loads with the occurrence of separation, whilst giving better predictions at low angles of attack compared to using ULLT alone. An alternative method for unsteady aerodynamic modelling using Kriging
is presented along with localised results from three training datasets. It is shown that the with sufficient 3D data the Kriging model can produce good agreement. However the main advantage of coupling a 3D unsteady inviscid potential flow model with 2D viscous data is that it avoids the reliance of three-dimensional high-fidelity simulations.

![Reduced Frequency = 0.02](image)

Figure 36 Kriging Results from Model Trained Using Database #3

### VIII. Acknowledgements

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### References

2. Steven R Allmaras and Forrester T Johnson, Modifications and clarifications for the implementation of the Spalart-Allmaras turbulence model. In Seventh International Conference on Computational Fluid Dynamics (ICCFD7), pages 1–11, 2012.


