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Theoretical derivation of artificially cemented granular soils strength

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ABSTRACT

This paper provides a theoretical derivation for the unconfined compression strength of artificially cemented granular soils. The proposed developments are based on the concept of superposition of failure strength contributions of the soil and cement phases. The granular matrix obeys the critical state soil mechanics concept, while the strength of the cemented phase can be described using the Drucker-Prager failure criterion. In the process, the analytical relation is suitably adjusted to parallel a recently proposed empirical relationship that links unconfined compression strength of artificially cemented granular soils to an adjusted porosity/cement ratio parameter. While the proposed analytical relation fits well the experimental data for different granular soils and cement curing time, further parametric analysis offers the possibility to explore the effect of some material parameters on the unconfined compression strength of artificially cemented granular soils.

Keywords: Portland cement, porosity, granular soils, porosity/cement ratio.

1 INTRODUCTION

Over the last decades, there has been an increasing interest in exploring techniques to artificially improve the mechanical performances of soils in order to meet progressively stricter operational criteria for geotechnical structures. Mixing soils with small amounts of binding agents, such as cement, aims to reproduce the stable internal structure of naturally cemented or weakly bonded soils. Artificial cementation of granular soils results in an increased stiffness and peak strength (e.g. Saxena and Lastrico, 1978; Dupas and Pecker, 1979; Clough et al. 1981) associated with a more dilative response (e.g. Lade and Overton 1989; Schnaid et al. 2001) and a pronounced post-peak brittleness (e.g. Abdulla and Kiousis 1997a;

Wang and Leung, 2008). It also gives rise to some tensile strength (e.g. Leroueil and Vaughan, 1990; Clough et al. 1981). However, laboratory experiments under high confining pressures have shown that the beneficial strength and stiffness contributions of the cementing bonds can be reduced or even suppressed. The cemented bonds can break under the applied stress leaving the frictional resistance of the sand matrix to become again the controlling strength component (Coop and Atkinson, 1993; Rabbi and Kuwano, 2012).

Artificial cementation of granular soils has been successfully applied to control excessive displacement/settlement of shallow foundations, in slope protection for earth dams, to prevent liquefaction of loose granular soils and in pavement base layers (e.g Saxena et al. 1988; Porbaha et al. 1998; Gallagher and Mitchell, 2002; Thomé et al. 2005; Mitrani and Madabhushi 2010). These are geotechnical applications which have in common a low confining stress level. In these situations, composite's strength characterisation from unconfined compression tests may offer relevant results towards an appropriate soil/cement mixture design (e.g. Gomez and Anderson 2012).

The unconfined compressive strength (q_u) of artificially cemented granular soils is positively influenced by an increase in cement content and decrease in porosity (e.g. Clough et al. 1981 and Maccarini 1987). However, it has been recently found by Consoli *et al.* (2007) that an adjusted porosity/cement ratio (η/C_{iv}^c) can be plotted against unconfined compressive strength to describe a unique hyperbolic relationship for a given soil and cement type (see Fig. 1a) of the following form:

$$q_u = B \left(\frac{\eta}{C_{iv}^c} \right)^b \quad (1)$$

where porosity (η) is expressed as percentage of the volume of voids divided by total volume of the specimen while volumetric cement content (C_{iv}) is expressed as percentage of the volume of cement divided by the total volume of the specimen. Consoli *et al.* (2009a,b, 2011a,b, 2014a,b and 2015) and Consoli (2014) have successfully shown and confirmed the usefulness of such ratio in controlling strength of several soils and binders. The exponent, c , on the top of C_{iv} is found to be about 1.0 for clean granular soils treated with Portland cement (Consoli *et al.* 2010, 2013) and smaller than unity for granular soils contaminated by fines, silts and/or clays, (Consoli *et al.* 2012) as exemplified in Fig.1a. The particle size distributions for the three analysed materials are also reported in Fig.1b.

Beyond the effect of the properties of the constituents (granular soil matrix and binder), the unconfined compressive strength of artificially cemented granular soils is also affected by other testing variables, such as curing time (e.g. Rabbi and Kuwano, 2012). As shown in Fig.2 (Consoli *et al.* 2013), for each curing period, it is still possible to fit the experimental data with relations of type (1) for fixed c and b and different B values.

A number of theoretical models have been proposed for predicting the overall constitutive behaviour of cemented soils. These are generally based on modifications of state boundary surfaces and hardening rules adopted for granular soils (e.g. Hirai *et al.*, 1989; Sun and Matsuoka, 1999) or they consider the cemented soil as a multiphase material imposing the superposition of the stress contributions of cement bonds and sand grains (e.g. Abdulla and Kioussis 1997b; Vatsala *et al.* 2001). However, for unconfined compression test conditions, it has been shown that the simple relationship proposed in Eq. (1) can successfully predict the strength of a broad range of cemented soils using only three model parameters. Nevertheless, the empirical nature of its derivation has not permitted to establish meaningful connections

between its governing coefficients and relevant material properties. In contrast, this paper approaches the problem of artificially cemented soil from a different perspective by proposing an analytical relationship for the unconfined compression strength based on the superposition of failure strength contributions of the individual constituents: soil matrix and cement. Insight into the physical meaning of governing coefficients for the unconfined compression strength unconfined compression strength is gained by conveniently adjusting the analytically derived relationship to parallel the empirical hyperbolic relationship (1). Finally, a parametric analysis exploring the effect of some material parameters on the unconfined compression strength of cemented soils is conducted.

2 THEORETICAL MODEL

The artificially cemented soil composite material is composed by the soil phase (the granular matrix) and the cement phase. It is assumed that:

- 1) The composite cemented material is isotropic.
- 2) The behaviour of the cemented soil at the failure point is determined by superposing the strength contributions of both phases (similarly to stress superposition approach used by Abdulla and Kioussis 1997b; Vatsala et al., 2001).
- 3) The failure of the composite cemented material occurs as a result of a simultaneous failure of both the cemented and the soil matrix phases.
- 4) Strain compatibility between the composite and its two phases, soil and cement, applies (similarly to the parallel spring approach assumed in Vatsala et al., 2001).

By using a volumetric averaging approach (Diambra et al., 2011; Diambra et al., 2013; Diambra and Ibraim 2015), the failure stress state of the composite (σ) can be derived from the failure stresses of its constituents (σ_m and σ_c for the sand matrix and cement phase, respectively) by the following relationship:

$$\sigma = \mu_m \sigma_m + \mu_c \sigma_c \quad (2)$$

where μ_m and μ_c are the volumetric concentrations of soil and cement in the composite material, respectively. It should be noted that the volumetric cement concentration μ_c equals $C_{iv}/100$. The relation (2) can be expanded in the terms of total mean (p) and deviatoric (q) stress components to satisfy the axisymmetric stress conditions of an unconfined compression test:

$$\begin{bmatrix} q \\ p \end{bmatrix} = \mu_m \begin{bmatrix} q_m \\ p_m \end{bmatrix} + \mu_c \begin{bmatrix} q_c \\ p_c \end{bmatrix} \quad (3)$$

It is now necessary to define the stress failure conditions for the two phases.

Failure for the cement phase

It is considered that the strength of the cement phase is described by the Drucker-Prager failure criterion, which can be expressed in terms of the deviatoric and isotropic stresses as follows:

$$q_c = c_c + M_c p_c \quad (4)$$

where the terms c_c and M_c can be linked to both the uniaxial compressive, σ_c^c , and tensile, σ_c^t , strengths of the cement binder by the following expressions:

$$c_c = 2 \frac{\sigma_c^c}{1-\beta} \quad (5)$$

140 and

$$141 \quad M_c = 3 \frac{\beta+1}{\beta-1} \quad (6)$$

142 where β represents the ratio between the uniaxial compression and extension strengths:

$$143 \quad \beta = \frac{\sigma_c^c}{\sigma_c^t} \quad (7)$$

144 ***Failure for the soil phase***

145 The strength of granular soils is generally represented using the density dependent deviatoric
146 stress and mean stress ratio, (q_m/p_m) . In soil constitutive modelling, it is customary to link the
147 strength of the material with a state parameter (ψ), which quantifies the difference between
148 the current density state from the corresponding one at critical state (Been and Jefferies,
149 1985). For the following modelling development, it is simpler to define the state parameter
150 in terms of the material porosity (η for current porosity and η_{cs} for the corresponding porosity
151 at the critical state) using the following definition:

$$152 \quad \psi = \frac{\eta_{cs}}{\eta} \quad (8)$$

153 where $\psi > 1$ represents a state on the loose side of the critical state line (CSL), while $\psi < 1$
154 represents a state on the dense side of the CSL.

155 The granular soil stress ratio at failure can then be expressed by the following expression:

$$156 \quad \frac{q_m}{p_m} = M^* = M \left(\frac{\eta_{cs}}{\eta} \right)^a \quad (9)$$

157 where M^* is the peak strength, M is the critical state strength and a is a model parameter
158 which links the peak strength to the state parameter, ψ . It is also assumed that grain crushing,

which may affect the strength of the soil matrix and the overall failure mode of cemented sands (Leroueil and Vaughan, 1990), does not occur during loading.

Strength relationship for artificially cemented soil

By substituting (4) and (9) into equation (3), the following expression for the unconfined compressive strength (q_u) is derived:

$$q_u = 3\mu_c \left(\frac{c_c + M_c p_c - M^* p_c}{3 - M^*} \right) \quad (10)$$

In order to have a relationship based solely on cement and soil phase strengths, an estimation of the confining stress of the cement phase, p_c , is required. It appears reasonable first to assume that, under unconfined compression test conditions, the cemented soils exhibit a quasi-elastic behaviour up to the peak strength conditions (Consoli *et al.*, 2009a). Thus, the following relationship between radial (ε_r) and axial (ε_a) composite strains can be considered:

$$\varepsilon_r = -\nu \varepsilon_a \quad (11)$$

where ν is the cemented soil composite Poisson's ratio. Based on the strain compatibility assumption, the cement phase is subjected to the same strain field and, assuming similarly an elastic behaviour, it is possible to derive the ratio K_c between the cement deviatoric and isotropic stresses, q_c and p_c respectively:

$$\frac{q_c}{p_c} = K_c = 3 \frac{1+\nu}{1+\nu_c} \frac{1-2\nu_c}{1-2\nu} \quad (12)$$

where ν_c is the cement Poisson's ratio. Combination of (4) and (12) provides the following expression for the isotropic stress at failure of the cement phase:

$$p_c = \frac{c_c}{K_c - M_c} \quad (13)$$

which can also be expressed in terms of the compression strength and the uniaxial stress ratio by substituting (5) and (6). Further substitution of (13) into (10) leads to the following relationship for the unconfined compression strength:

$$q_u = \frac{6\mu_c \sigma_c^c}{K_c(1-\beta)+3(\beta+1)} \left(\frac{K_c - M \left(\frac{\eta_{cs}}{\eta} \right)^a}{3 - M \left(\frac{\eta_{cs}}{\eta} \right)^a} \right) \quad (14)$$

As can be observed, (14) provides a direct expression of the unconfined strength of the cemented soil as function of the porosity (η) and the cement content (μ_c) variables, with $\mu_c = C_{iv}/100$, while the parameters are constants relative to both the soil and cement phases, as summarised in Table 1. It should be noted that in this development, which refers to unconfined compression conditions only, the value of the soil porosity at the critical state is considered as a soil constant independent of the mean effective stress.

3 MODEL PREDICTIONS

The validity of the unconfined compression strength expression proposed by (14) is assessed here by direct comparison with experimental data provided by unconfined compression test results performed on three different artificially cemented sands which have been cured for different time periods. The three selected granular matrices, whose particle size distribution is given in Fig. 1b, are quartz-feldspar rich sands which are not expected to crush under typical foundation loading. The simulation exercise has been performed for the following conditions:

- 1) Uniform Osorio sand + early strength Portland cement cured at 2, 7 and 28 days (Consoli *et al.* 2013);
- 2) Well-graded Porto silty sand + early strength Portland cement (Consoli *et al.* 2012); and

3) Botucatu residual soil + early strength Portland cement (Consoli *et al.* 2007).

3.1.1 Calibration of model parameters

As shown in Table 1, the model requires the calibration of six parameters, three for the soil matrix and three for the cement phase. The values of the constants relative to the sand matrix have been selected based on triaxial experimental results and the assumed values are also indicated in Table 1. The values of critical state strength ratio, M (linked to the critical state friction angle φ), for Osorio sand, Botucatu residual soil and Porto silty sand have been derived from Dos Santos *et al.* (2010), Ferreira and Bica (2006) and Rios *et al.* (2012), respectively. Based on the same published results, the specific values of the porosity at the critical state (η_{cs}) have been derived from the critical states line defined in the specific volume - mean effective stress plane ($v-\ln p'$).

The estimation of the parameter α for both Osorio sand and Botucatu residual soil required some careful inspection of a set of published triaxial tests by Dos Santos *et al.* (2010), and Schnaid *et al.* (2001), respectively and was based on a calibration exercise between data provided by (9) and the peak strength observed in the triaxial tests, as shown in Fig. 3. Values of $\alpha = 1.3$ for Osorio sand and $\alpha = 3.5$ for Botucatu residual soil were found.

Owing to the lack of available triaxial data for the Porto silty sand, a value of $\alpha = 4$, in line with the value selected for the Botucatu soil, has been selected considering that both soils present similar particle size distributions (Fig. 2).

The value of the model's parameter, β , relative to the cement phase has been selected based on typical ranges of the uniaxial compression and extension cement strength values for the Portland cement (Table 1). Following (12), the value of the parameter K_c is dependent on the

Poisson's ratios of the cemented soil material and the cement phase. Extensive experimental characterisation of the elastic properties of cemented soils by Felt and Abram (1957) suggests values of the Poisson's ratio for cemented sand and silts between 0.22 and 0.31 with a median value of about 0.26, while typical values of Poisson's ratio for mortar matrix are around 0.2, as suggested by Narayam Swamy (1971). These lead to a value of $K_c \approx 4$, which is assumed for all three tested soils. Imposing all the values of the parameters above, the strength of the cementing phase, σ_c^c , was determined by matching the unconfined compression strength given by relation (14) with the experimental results. For each material the calibration process used three randomly selected unconfined compression test results. A curve fitting procedure was also employed by Abdulla and Kioussis (1997b) and Vatsala et al. (2001) to calibrate the strength of the cementing bonds. Like in this work, the former authors used unconfined compression strength results but, for the sake of simplicity, a negligible frictional contribution of the sand matrix was assumed. Instead, Vatsala et al. (2001) based their calibration on tests performed at very low confining pressure to maximise the relative bonds' strength contribution. The strengths σ_c^c values for the Botucatu sand and the Porto silty soil (Table 1) are larger than that for the Osorio sand because of the better grading and larger proportion of fines for the former soils.

3.1.2 Simulations

A direct comparison between model predictions of the unconfined compression strength (q_u) using the parameter listed in Table 1 and the results from the laboratory tests versus the η/C_{iv} ratio is shown in Fig. 4 for the three different types of cemented soils (different densities and cement contents) cured for 7 days.

The model predicts reasonably well the unconfined compression strength including the hyperbolic relationship between the two variables. For the Porto silty sand (Fig. 4b) the

experimental data do not converge in a unique curve (Rios et al., 2012) and the model captures this experimental observation too. Moreover, the Porto silty sand cemented material shows a considerably larger strength than the others which equally the model automatically predicts. This is due to both the larger cement strength σ_c^c if compared with the Osorio sand and the values of the parameters assumed for the soil matrix if compared with the Botucatu soil. Ismail et al. (2002a) pointed out that the properties of the soil grains, size and shape, have also a primary influence on the strength of the cemented soil, which is increased by decreasing the particle size and by using round or non-angular soil grains. These soil features are all included in the parameters M , α , η_{cs} of the matrix which are indeed primarily governed by the shape and size distribution of the grains.

The effect of curing time on the unconfined compressive strength of the cemented soil is shown in Fig. 5, where additional experimental results for cemented Osorio sand at 2 and 28 curing periods are compared with the model prediction. As suggested by Rabbi and Kuwano (2012), the curing time dependency on the overall strength can be captured by assuming an increase of the compressive strength of the cement phase. In these simulations, σ_c^c varies from 30 MPa, to 50 MPa and 70 MPa for 2, 7 and 28 curing days, respectively.

4 DISCUSSION

4.1.1 Parallelism with empirical formula

The proposed relationship (14) based on theoretical considerations has a different form compared with the empirically based relation (1) proposed by Consoli *et al.* (2007). However, the trends in Figures 1, 2, 4 and 5, show that both approaches fit well the experimental data. In order to make a parallelism between the two relationships, relationship (14) should be

267 further expressed as a linear function of the peak strength of the soil M^* ($M^*=M(\eta_{cs}/\eta)^a$) and
 268 this can be achieved by introducing the following approximation:

$$269 \quad \frac{K_c - M^*}{3 - M^*} \cong M^* (-0.6 + 0.45 K_c) \quad (15)$$

270 The use of (15) in to (14) yields to the following expression for the unconfined compression
 271 strength:

$$272 \quad q_u = \frac{6 M \sigma_c^c (-0.6 + 0.45 K_c) \eta_{cs}^a}{100 (K_c(1-\beta) + 3(\beta+1))} \left(\frac{C_{iv}}{\eta^a} \right) \quad (16)$$

273 A full parallelism with the empirical formula (1) proposed by Consoli *et al.* (2007, 2010, 2012)
 274 can be completed if (16) is further manipulated to be expressed in the following form:

$$275 \quad q_u = \frac{6 M \sigma_c^c (-0.6 + 0.45 K_c) \eta_{cs}^a}{100 (K_c(1-\beta) + 3(\beta+1))} \left(\frac{\eta}{C_{iv}^{\frac{1}{a}}} \right)^{-a} = B \left(\frac{\eta}{C_{iv}^{\frac{1}{a}}} \right)^{-a} \quad (17)$$

276 where

$$277 \quad B = \frac{6 M \sigma_c^c (-0.6 + 0.45 K_c) \eta_{cs}^a}{100 (K_c(1-\beta) + 3(\beta+1))} \quad (18)$$

278 As shown in Fig. 6, the transformation introduced by (15) appears to have a limited effect on
 279 the model predictions Following the suggestion by Consoli *et al.* (2007), the horizontal axis is
 280 now expressed using the adjusted porosity/cement ratio (η/C_{iv}^c), where the exponent has
 281 been imposed equal to $1/a$ following (17). It can be noticed that the imposed transformation
 282 of the horizontal axis leads both the experimental data and the model prediction for a given
 283 soil to collapse in an almost unique line.

284 A closer observation of Equations (17) and (18) can now offer some insight on the meaning
 285 and the factors governing the parameters introduced by the empirical relation (1):

- 1) The proposed formulation in (17) suggests that the exponent b and c in (1) are dependent on the soil matrix related parameter a , with $c=1/a$, and $b=-a$. This corroborates well the experimental findings by Consoli *et al.* (2007) who suggested a dependency of c coefficient on the grading of the soil matrix.
- 2) It follows that a rather straightforward relationship $b=-1/c$ can be obtained. This is confirmed by the data plotted in Fig. 1 with $b=-1.5\approx-1=-1/c$ and $c=1$ for Osorio sand, $b=-3.4\approx-3.57=-1/c$ and $c=0.28$ for Botucatu residual soil, and $b=-4.2\approx-4.77=-1/c$ with $c=0.21$ for Porto silty sand.
- 3) Analysis of (18) suggests that the multiplying parameter B in (1) is the result of combined properties of the sand matrix and cement phase. The key governing parameters seem to be the frictional strength of the matrix (M), and the strength of the cemented phase, σ_c^c .
- 4) The multiplying factor B is also controlled by the value of the exponent a , which is controlled by the sand matrix properties. Analysis of the values given by Consoli *et al.* (2007, 2012, 2013) (Fig.1) demonstrates that large values of the exponent a lead to higher values of B , as suggested by relationship (18).

4.1.2 Parametric analysis

A parametric analysis to investigate the relative importance of the constituents' parameters is performed. The effect of the variation of the cement phase parameters - compressive strength, σ_c^c , between 10 and 500MPa, cement strength ratio, β , from -4 to -8, and cement stress ratio, K_c , between 3.2 and 6 – on the unconfined compression strength is provided in Fig. 7 a-b-c. The basic values of the soil parameters used in this exercise are as follows: $\sigma_c^c=50$ MPa, $\beta=-6$, $K_c=4$, $M=1.3$, $\eta_{cs}=0.35$ and $a=3$. By comparing these three plots, it is rather clear that the strength of the cement binder is by far the leading parameter, corroborating the

experimental results of Maccarini (1987), Ismail et al. (2002b) and Haeri et al. (2006). The Poisson's ratio, and the uniaxial strength ratio have a limited influence. The partial effect of the cement stress ratio K_c suggests that the necessary modelling assumption on the deformation behaviour of the cemented soil imposed in (12) has little effect on the final model predictions, thus increasing the confidence on the proposed developments.

The further influence on the predictions of the three model parameters related to the sand matrix are shown in Fig. 7d-e-f. It is clear that the matrix strength ratio M (or friction angle φ) has some effect but much more limited if compared to the strength properties of the cement phase. However, much more caution should be paid particularly to the effects of the critical state porosity and strength porosity dependency as both parameters have some connection with the adjusted porosity-cement ratio variable. Fig. 7e suggests that materials with larger critical state porosity η_{cs} have larger unconfined strength for the same η/C_{iv} ratio and this would rather be misleading considering that these materials will be characterised by larger value of porosities (thus a larger value of η/C_{iv} ratios) for the same relative density. However, a faster decrease of the unconfined compressive strength (q_u) with increasing η/C_{iv} ratio can be observed for larger values of η_{cs} . Analysis of the trends in Fig. 7f must be carried out with similar caution. While it appears that increasing values of a would lead to a rightward shift of the hyperbolic curve, it must be remembered that the horizontal axis is also affected by the assumed value of a . While a direct conclusion cannot be drawn, the analysis of the three soils considered in this study suggests that large values of a are related to soils with larger fine content, which also show a larger unconfined compressive strength as experimentally observed by Ismail et al. (2002a).

From the whole parametric analysis, it clearly appears that the information on the soil matrix mechanical behaviour embedded in the parameters M , α , η_{cs} and a number of three unconfined compression test results to calibrate the value of the cement bond strength σ_c^c may be sufficient to provide satisfactory predictions of the cemented soil strength over a large spectrum of cement contents and porosities.

4.1.3 Further applicability of the proposed model

The proposed theoretical model has been primary developed for artificially cemented soils, stabilised with Portland cement and fabricated in controlled laboratory conditions. Application to different cement types can be carried out by assuming different values for the cementation bonding parameters. Nevertheless, it must be considered that different binders such as calcite or gypsum, may also affect some macroscopic behavioural features leading to prior and more brittle yielding (Ismail et al., 2002b). This may contradict some of the model assumptions such as the simultaneous failures of the composite constituents.

The proposed model could also be used to predict the unconfined compressive strength of naturally bonded soils. Nevertheless, such assessment should account for the inhomogeneity and variability generally associated with soils formed in a natural environment. As suggested by Abdulla and Kioussis (1997b), further difficulties may arise from the estimation of the exact percentage of cementation. However, this variable may be combined with the strength of the cemented bonds to be considered as an unique model parameter. Further challenges may arise from the presence of anisotropic cementation, arising from the soil formation process, which may lead to anisotropic failure conditions as experimentally observed by Anagnostopoulos et al. (1991) and Coop and Atkinson (1993) or as considered in the theoretical model by Gao and Zhao (2012). Further modifications of the model describing the

strength of the soil matrix may also be necessary for the perspective application to crushable calcareous soil matrices. CONCLUSIONS

A theoretical derivation for the unconfined compression strength of artificially cemented granular soils based on the concept of superposition of failure strength contributions of the soil and cement phases was proposed. Comparison with experimental data and with empirical relations relating the unconfined compression test to the porosity/cement ratio were conducted. The following conclusions can be drawn:

- The use of the superposition of strength contributions of the soil and cement phase, critical state and dependence of shear strength on the state parameter concepts permitted the elaboration of the theoretical framework;
- The theoretical model developed herein satisfactorily simulates experimental data that related q_u of artificially cemented granular soils and η/C_{iv}^c , capturing the effects of material density, bonding and curing periods.
- The convenient adjustment of the theoretical formulation developed herein to parallel a widely used empirical hyperbolic relationship between the unconfined compressive strength (q_u) of cemented soil with an adjusted porosity/cement ratio (η/C_{iv}^c) has shed some light on both the significance and material properties governing the parameters of the empirical relationship; as expected, the compressive strength of the cement binder is by far the key leading parameter in controlling the unconfined compression strength of the cemented soils. This appears dependent to the particle size distribution of the soil matrix. The ratio of the compression/extension uniaxial cement strengths and the strength of the soil matrix have also a visible influence but indeed much less important than the cement matrix strength.

- The proposed theoretical framework, based on calibration of three parameters relative to the soil matrix and three unconfined compression tests results for the strength of the cemented bonds (the two remaining model parameters of the cemented bonds have a limited influence on the model outcomes), enables the prediction of the unconfined compressive strength of cemented soils over a large spectrum of cement contents and porosities. Development of such relationship can offer significant guidance towards the design of specific soil/cement mixture design to satisfy required strength criteria.
- Extension of the proposed model to different types of cement and natural soils could be pursued but considerations on occurrence of grain crushing (especially for calcareous soils), anisotropy of soil fabric and cementation, inhomogeneity of material properties and precise assessment of the exact percentage of cementation among others, should be introduced.

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References

Abdulla, A. A., and Kioussis, P. D. (1997a). "Behavior of cemented sands—I. Testing." *International Journal for Numerical and Analytical Methods in Geomechanics*, 21(8), 533-547.

398 Abdulla, A. A., and Kioussis, P. D. (1997b). "Behavior of cemented sands—II.
399 Modelling." *International Journal for Numerical and Analytical Methods in Geomechanics*, 21(8),
400 549-568.

401 Anagnostopoulos, A. G., Kalteziotis, N., Tsiambaos, G. K., and Kavvadas, M. (1991). "Geotechnical
402 properties of the Corinth Canal marls." *Geotechnical and Geological Engineering*, 9(1), 1-26

403 Been, K. and Jefferies, M.G. (1985). "A state parameter for sands." *Géotechnique* 35(2), 99-112.

404 Clough, G. W., Sitar, N., Bachus, R. C., and Rad, N. S. (1981). "Cemented sands under static
405 loading." *Journal of Geotechnical Engineering Division*, 107(6), 799-817.

406 Consoli, N.C. (2014). "A method proposed for the assessment of failure envelopes of cemented
407 sandy soils". *Engineering Geology*, 169, 61-68.

408 Consoli, N.C.; Foppa, D.; Festugato, L. and Heineck, K.S. (2007). "Key parameters for strength
409 control of artificially cemented soils". *Journal of Geotechnical and Geoenvironmental*
410 *Engineering*, 133, 197-205.

411 Consoli, N.C.; Viana da Fonseca, A.; Cruz, R.C. and Heineck, K.S. (2009a). "Fundamental parameters
412 for the stiffness and strength control of artificially cemented sand". *Journal of Geotechnical and*
413 *Geoenvironmental Engineering*, 135, 1347-1353.

414 Consoli, N.C.; Lopes Junior, L.S. and Heineck, K.S. (2009b). "Key parameters for the strength
415 control of lime stabilized soils". *Journal of Materials in Civil Engineering*, 21, 210-216.

416 Consoli, N.C.; Cruz, R.C.; Floss, M.F. and Festugato, L. (2010). "Parameters controlling tensile and
417 compressive strength of artificially cemented sand". *Journal of Geotechnical and*
418 *Geoenvironmental Engineering*, 136, 759-763.

419 Consoli, N.C.; Cruz, R.C. and Floss, M.F. (2011a). "Variables controlling strength of artificially
420 cemented sand: Influence of curing time". *Journal of Materials in Civil Engineering*, 23, 692-696.

421 Consoli, N.C.; Viana da Fonseca, A.; Cruz, R.C. and Silva, S.R. (2011b). "Voids/cement ratio
 422 controlling tensile strength of cement treated soils". *Journal of Geotechnical and*
 423 *Geoenvironmental Engineering*, 137, 1125-1131.

424 Consoli, N.C.; Viana da Fonseca, A.; Cruz, R.C.; Silva, S.R. and Fonini, A. (2012). "Parameters
 425 controlling stiffness and strength of artificially cemented soils". *Géotechnique*, 62, 177-183.

426 Consoli, N.C.; Festugato, L.; Rocha, C.G.; Cruz, R.C. (2013). "Key parameters for strength control of
 427 rammed sand-cement mixtures: Influence of types of Portland cement". *Construction and*
 428 *Building Materials*, 49, 591-597.

429 Consoli, N.C.; Lopes Junior, L.S.; Consoli, B.S. and Festugato, L. (2014a). "Mohr-Coulomb failure
 430 envelopes of lime-treated soils". *Géotechnique*, 64, 165-170.

431 Consoli, N.C.; Rocha, C.G. and Saldanha, R.B. (2014b). "Coal fly ash-carbide lime bricks: An
 432 environment friendly building product". *Construction and Building Materials*, 69, 301-309.

433 Consoli, N.C.; Festugato, L.; Consoli, B.S. and Lopes Junior, L.S. (2015). "Assessing failure envelopes
 434 of soil fly ash lime blends". *Journal of Materials in Civil Engineering*, 26, 04014174-1 -
 435 04014174-8.

436 Coop, M. R., and Atkinson, J. H. (1993). "The mechanics of cemented carbonate
 437 sands." *Géotechnique*, 43(1), 53-67.

438 Diambra, A., Ibraim, E. Russell, A.R. and Muir Wood, D. (2011): "Modelling the undrained
 439 response of fibre reinforced sands." *Soils and Foundations* 51(4), 625-636.

440 Diambra, A., Ibraim, E. Russell, A.R. and Muir Wood, D. (2013) "Fibre reinforced sands: from
 441 experiments to modelling and beyond." *International Journal for Numerical and Analytical*
 442 *Methods in Geomechanics*, 37(15), 2427-2455.

443 Diambra, A. and Ibraim, E. (2015) "Fibre-reinforced sand: interaction at the fibre and grain
 444 scale." *Géotechnique*, 65(4), 296-308.

445 Dos Santos, A.P.S., Consoli, N.C. and Baudet, B.A. (2010). "The mechanics of fibre-reinforced
 446 sand." *Géotechnique*, 60(10), 791-799.

447 Dupas, J. M., and Pecker, A. (1979). "Static and dynamic properties of sand-cement." *Journal of the*
 448 *Geotechnical Engineering Division*, 105(3), 419-436.

449 Felt, E. J., and Abrams, M. S. (1957). Strength and elastic properties of compacted soil-cement
 450 mixtures. Am Soc Testing and Matls Spec Tech Publ Portland Cement Assoc R and D Lab Bull.

451 Ferreira, P.M.V. and Bica, A.V.D. (2006). "Problems in identifying the effects of structure and
 452 critical state in a soil with a transitional behaviour." *Géotechnique*, 56(7), 445-454.

453 Gallagher, P. M., and Mitchell, J. K. (2002). "Influence of colloidal silica grout on liquefaction
 454 potential and cyclic undrained behavior of loose sand." *Soil Dynamics and Earthquake*
 455 *Engineering*, 22(9), 1017-1026.

456 Gao, Z., and Zhao, J. (2012). "Constitutive modeling of artificially cemented sand by considering
 457 fabric anisotropy." *Computers and Geotechnics*, 41, 57-69.

458 Gomez, N.S., and Anderson, D.N. (2012). "Soil cement stabilization - mix design, control and
 459 results during construction." ISSMGE - TC 211 International Symposium on Ground
 460 Improvement IS-GI Brussels 31 May - 1 June 2012,

461 Haeri, S. M., Hamidi, A., Hosseini, S. M., Asghari, E., and Toll, D. G. (2006). "Effect of cement type on
 462 the mechanical behavior of a gravely sand." *Geotechnical and Geological Engineering*, 24(2),
 463 335-360.

464 Hirai, H., Takahashi, M., and Yamada. M. (1989). "An elastic-plastic constitutive model for the
 465 behavior of improved sandy soils." *Soils and Foundations* 29(2), 69-84.

466 Ismail, M. A., Joer, H. A., Randolph, M. F., and Meritt, A. (2002a). "Cementation of porous materials
 467 using calcite." *Geotechnique*, 52(5), 313-324.

468 Ismail, M. A., Joer, H. A., Sim, W. H., and Randolph, M. F. (2002b). "Effect of cement type on shear
 469 behavior of cemented calcareous soil." *Journal of Geotechnical and Geoenvironmental*
 470 *Engineering*, 128(6), 520-529.

471 Lade, P. V., and Overton, D. D. (1989). "Cementation effects in frictional materials." *Journal of*
 472 *Geotechnical Engineering*, 115(10), 1373-1387.

473 Leroueil, S., and Vaughan, P. R. (1990). "The general and congruent effects of structure in natural
 474 soils and weak rocks." *Géotechnique*, 40(3), 467-488.

475 Maccarini, M. (1987). "Laboratory studies of weakly bonded artificial soil." PhD thesis, University
 476 of London.

477 Mitrani, H., and Madabhushi, S. G. (2010). "Cementation liquefaction remediation for existing
 478 buildings." *Proceedings of the Institution of Civil Engineers-Ground Improvement*, 163(2), 81-94.

479 Porbaha, A., Tanaka, H., and Kobayashi, M. 1998. "State of the art in deep mixing technology. Part
 480 II: Applications." *Ground Improvement*, J. ISSMGE, 22, 125-139.

481 Rabbi, A. T. M. Z., and Kuwano, J. (2012). "Effect of Curing Time and Confining Pressure on the
 482 Mechanical Properties of Cement-Treated Sand." In *GeoCongress 2012: State of the Art and*
 483 *Practice in Geotechnical Engineering*, 996-1005, ASCE.

484 Rios, S., Viana da Fonseca, A. and Baudet, B.A. (2012). "Effect of the porosity/cement ratio on the
 485 compression of cemented soil." *Journal of Geotechnical and Geoenvironmental*
 486 *Engineering*, 138(11), 1422-1426.

487 Saxena, S. K., and Lastrico, R. M. (1978). "Static properties of lightly cemented sand." *Journal of*
 488 *Geotechnical Engineering Division*, 104(GT12), 1449-1464.

489 Saxena, S. K., Reddy, K. R., and Avramidis, A. S. (1988). "Liquefaction resistance of artificially
 490 cemented sand." *Journal of Geotechnical Engineering*, 114(12), 1395-1413.

491 Schnaid, F., Prietto, P.D.M. and Consoli, N.C. (2001). "Characterization of cemented sand in triaxial
492 compression." *Journal of Geotechnical and Geoenvironmental Engineering*, 127(10), 857-868.

493 Sun, D. A., and Matsuoka, H. (1999). "An elastoplastic model for frictional and cohesive materials
494 and its application to cemented sands." *Mechanics of Cohesive-frictional Materials*, 4(6), 525-
495 543.

496 Swamy, R. N. (1971). Dynamic Poisson's ratio of Portland cement paste, mortar and
497 concrete. *Cement and Concrete Research*, 1(5), 559-583. Thomé, A., Donato, M., Consoli, N. C.,
498 and Graham, J. (2005). Circular footings on a cemented layer above weak foundation
499 soil. *Canadian Geotechnical Journal*, 42(6), 1569-1584.

500 Vatsala, A., Nova, R., and Murthy, B. S. (2001). Elastoplastic model for cemented soils. *ASCE Journal*
501 *of Geotechnical and Geoenvironmental Engineering*, 127(8), 679-687.

502 Wang, Y. H. and Leung, S. C. (2008). Characterization of cemented sand by experimental and
503 numerical investigations. *Journal of Geotechnical and Geoenvironmental Engineering*, 134(7),
504 992-1004

505

506 **Notation list**

507	a	Parameter linking peak strength to state parameter
508	B	Multiplying parameter in Empirical relationship (1)
509	b	Exponent of empirical relationship (1)
510	c	Exponent of empirical relationship (1)
511	c_c	Cohesion of the cement phase
512	C_{iv}	Volumetric cement content (expressed in percentage)
513	M	Critical state strength ratio for the soil

514	M_c	Slope of the failure line for the cement phase in the q_c - p_c plane
515	M^*	Peak strength ratio for the soil
516	K_c	Cement stress ratio
517	p	Mean stress of the cemented soil
518	p_c	Mean stress of the cement phase
519	p_m	Mean stress of the soil matrix
520	q	Deviator stress of the cement soil
521	q_c	Deviator stress of the cement phase
522	q_m	Deviator stress of the soil matrix
523	q_u	Unconfined compressive strength for the cemented soil
524	β	Ratio between the uniaxial compression and extension strengths
525	φ	Friction angle for the soil matrix
526	ε_a	Axial strain for cemented soil
527	ε_r	Radial strain for cemented soil
528	ν	Poisson's ratio for cemented soil
529	ν_c	Poisson's ratio for cement phase
530	μ_c	Volumetric cement concentration
531	μ_m	Volumetric soil matrix concentration
532	ψ	State parameter
533	η	Porosity
534	η_{cs}	Porosity at critical state
535	σ_c^c	Uniaxial compressive strength of the cement
536	σ_c^t	Uniaxial tensile strength of the cement
537		

Table 1. Parameters of the proposed model

Symbol	Variable	Values		
		Osorio sand	Porto silty sand	Botucatu residual soil
M	Critical state soil strength ratio	1.33 ($\varphi \approx 33^\circ$)	1.35 ($\varphi \approx 33.5^\circ$)	1.25 ($\varphi \approx 31.5^\circ$)
η_{cs}	Critical state soil porosity	44	36	30
a	Parameter governing dependence of soil strength on its density	1.3	4	3.5
σ_c^c	Cement phase compressive strength	30 MPa (2 days during) 50 MPa (7 days curing) 70 (28 days curing)	90 MPa (7 days curing)	90 MPa (7 days curing)
β	Uniaxial compression and extension cement strength ratio	-6	-6	-6
K_c	Cement stress ratio	4	4	4

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Fig. 1. (a) Variation of unconfined compressive strength (q_u) with adjusted porosity/cement ratio for three soil types treated with early strength Portland cement (adapted from Consoli *et al.* 2007, 2012, 2013) after 7 days curing time; (b) particle size distribution of the sandy soils

Fig. 2. Variation of unconfined compressive strength (q_u) with adjusted porosity/cement ratio for Osorio sand treated with early strength Portland cement and cured for different lengths of time (adapted from Consoli *et al.* 2013).

Figure 3. Calibration of parameter a for Osorio sand and Botucatu residual soil using peak strength data from triaxial tests by Dos Santos *et al.* (2010) and Schnaid *et al.* (2001), respectively.

Figure 4. Comparison between model prediction and experimental results for three different types of cemented soils cured for 7 days: (a) Osorio sand, (b) Porto silty sand and (c) Botucatu residual soil. (d) reports a direct comparison between model predictions and experimental results for the three soils.

Figure 5. Comparison between model predictions and experimental results for cemented Osorio sand cured for: (a) 2 days and (b) 28 days. (c) reports a direct comparison between model predictions and experimental results for the three curing periods available for cemented Osorio sand.

Figure 6. Comparison between experimental data, theoretical prediction and approximated formula (17).

Figure 7. Influence of parameters related to the cement (a, b and c) and soil matrix phases (d,e, and f) on model predictions. Basic values of the soil parameters used in this exercise are: $\sigma_c^c = 50$ MPa, $\beta = -6$, $K_c = 4$, $M = 1.3$, $\eta_{cs} = 0.35$ and $\alpha = 3$.