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Nonlinear Phased Array Imaging

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ABSTRACT

A technique is presented for imaging acoustic nonlinearity within a specimen using ultrasonic phased arrays. Acoustic nonlinearity is measured by evaluating the difference in energy of the transmission bandwidth within the diffuse field produced through different focusing modes. The two different modes being classical beam forming, where delays are applied to different element of a phased array to physically focus the energy at a single location (parallel firing) and focusing in post processing, whereby one element at a time is fired and a focused image produced in post processing (sequential firing). Although these two approaches are linearly equivalent the difference in physical displacement within the specimen leads to differences in nonlinear effects. These differences are localized to the areas where the amplitude is different, essentially confining the differences to the focal point. Direct measurement at the focal point are however difficult to make.

In order to measure this the diffuse field is used. It is a statistical property of the diffuse field that it represents the total energy in the system. If the energy in the diffuse field for both the sequential and parallel firing case is measured then the difference between these, within the input signal bandwidth, is largely due to differences at the focal spot. This difference therefore gives a localized measurement of where energy is moving out of the transmission bandwidth due to nonlinear effects.

This technique is used to image fatigue cracks and other damage types undetectable with conventional linear ultrasonic measurements.

Keywords: Phased arrays, nonlinearity, diffuse field, fatigue cracks

1. INTRODUCTION

Nonlinear ultrasonic methods have been proposed for years as a technique to increase sensitivity to damage1-3. These have shown significant increase in sensitivity to fatigue state and the ability to detect previously invisible features such as closed cracks. This however typically comes at the cost of measurement complexity, where harmonics are measured between two transducers at two different frequencies and often use specific high performance amplifiers. In addition most techniques make measurements that are averaged over a region of material. Harmonic generation is a particularly good example of this, whereby an input signal is put into a component and as this propagates, harmonics of the input signal are generated. The resulting harmonic signal is then measured at a remote location. In doing this it is impossible to localize the measurement. That is all that is known is that the nonlinearity has originated somewhere along the ray path, it could however be largely equipment nonlinearity or be uniformly distributed. While this is not a problem if a materials condition is to be tracked over time, it causes significant issues if the feature of interest must be localized within a region or in the absence of a reference measurement. This paper demonstrates an alternative that alleviates these issues. Here a single ultrasonic phased array is used to map nonlinearity in a specimen. Firstly the theoretical background of the approach is described. Its performance for the detection of closed cracks is then demonstrated and a comparative measurement made between linear and nonlinear results as a fatigue crack is grown in a compact test specimen.

2. THEORETICAL BACKGROUND

2.1 Nonlinear modality

Linear imaging using phased arrays has been used for decades to localize damage within the interior of a component. Through the application of relative delays to the parallel transmission of discrete elements, beam forming may be controlled, typically to achieve steering or focusing. More recently, an alternative approach to array imaging commonly referred to as full matrix capture has been employed4. Here, rather than transmitting on elements in parallel, time-domain responses from all transmitter-receiver pair combinations are acquired sequentially to yield the so-called full matrix of data. Delay laws are then applied in postprocessing to emulate the equivalent delayed parallel transmission. The work
reported here identifies how differences in the wave propagation of these transmissions techniques can be used to image acoustic nonlinearity.

From this point on, the focusing of ultrasound through postprocessing of sequential transmissions and physical focusing through parallel element transmission shall be referred to as *sequential* and *parallel* focusing, respectively. Although these two approaches are linearly equivalent, the same is not true of nonlinear propagation. When performing parallel focusing, the absolute acoustic pressure seen by the material is higher at the focal point than in any of the individual transmission cycles during the equivalent sequential focusing. Consequently, there is a larger transfer of energy from the fundamental harmonic to other frequencies due to nonlinear phenomena at the focal point in the parallel focusing case.

If we consider the magnitude of this effect it can be seen that the difference is significant. Consider, for example, the case of longitudinal wave propagation through an elastic solid with an isotropic bulk nonlinearity truncated to the second order. For the parallel and sequentially focused transmission of an N element array, the absolute wave amplitude U experienced at the focal point differs by a factor of N. The amplitude of second-harmonic waves generated is proportional to $U^2$, and hence, the amount of energy lost from the fundamental wave is proportional to $U^4$. The acoustic energy lost from the fundamental wave for the single transmission cycle in parallel focusing is, therefore, $N^3$ greater than that lost through the summation of the N transmission cycles necessary for the equivalent sequential focusing. Given that ultrasonic arrays with order N=100 elements are now commonplace, the difference in nonlinear energy loss from the fundamental between parallel and sequential focusing is significant and measurable. Furthermore, because the physical amplitude difference between parallel and sequential focusing is only large close to the focal point the difference in energy loss from the transmission frequency band is predominantly local to this point. This is the reason why the proposed technique is able to spatially map nonlinearity. It is important to note that this could not be achieved by simply applying parallel focusing at two different amplitudes. In that case, amplitude scaling between the two transmissions is experienced along the whole wave propagation path, rather than just at the focal point, resulting in much poorer nonlinear localization. Furthermore, the obtained measurements would not separate instrument and material nonlinearity.

More generally, even restricting consideration to low-order nonlinear dynamics, sonification of common nonlinear media will generate not only harmonics but also subharmonics and frequencies corresponding to sum and difference combinations of the transmission bandwidth. The property common to all these nonlinear processes is a transfer of energy from the incident frequency band. Consequently, inspection of the fundamental frequency provides a metric sensitive to a wide range of acoustic nonlinearity.

### 2.2 How to measure nonlinear changes

Having shown that the differences in the movement of energy between parallel and sequential firings at the focal spot are significant we will now explore how they may be measured. The movement of energy from the input frequency to the second harmonic is localised to the region of the focal spot with some small point spread function around this point. Elsewhere the acoustic amplitude remains unchanged. Thus in order to measure the nonlinear changes due to the two excitation methods we must measure the amplitude at the focal spot. Without access to the interior of any specimen (which is practically unlikely) this cannot be done directly. The developed solution relies on making measurements of the diffuse field. The diffuse field is illustrated here for two signals captured at different time delays in Figure 1. In these responses it is clear that in the diffuse field the energy is spread uniformly throughout the time window and cannot be attributed to any specific feature.

In providing information as to what is happening at the focal spot two properties of the diffuse field are vital. First, energy sampled at any point within the diffuse field is proportional to the total energy in the system. Second, since transmission, linear propagation, and nonlinear self-interaction are the same for the two focusing methods (due to using the same amplitude throughout), relative differences in energy within the transmission bandwidth are restricted to nonlinear losses occurring local to the focal point. A measurement of relative energy of the transmission bandwidth within the diffuse field, therefore, provides a measure of acoustic nonlinearity at the focal point.
2.3 How to make nonlinear measurements

As the diffuse field can be used to infer the differences in the energy at the focal point this provides the tool to measure local acoustic nonlinearity. The procedure is to make a measurement of the diffuse field using a sequential firing. This can then be focussed at the specific point to be assessed in post processing by applying the appropriate phase delays to the captured signals. The energy present within the transmission bandwidth for the sequential capture can then be calculated as in Equation 1.

\[
E_S(\mathcal{F}) = \sum_{k=1}^{N} \left( \int_{\omega_0}^{\omega_0} \omega^2 \left| \sum_{j=1}^{N} F_{j,k}(\omega)e^{i\omega\delta_1(r)} \right|^2 d\omega \right)
\] (1)

Where \( \omega \) is frequency, \( F \) the fourier transformed full matrix of data and the exponent describes the delay law to focus at a given point. Similarly, the energy in the parallel transmission can be calculated using Equation 2.

\[
E_P(\mathcal{F}) = \sum_{k=1}^{N} \left( \int_{\omega_0}^{\omega_0} \omega^2 |H_k(\mathcal{F},\omega)|^2 d\omega \right)
\] (2)

Where \( H \) is the response to the parallel firing at the focal point, therefore including the delays. As these describe the energy present in both captures, the difference between the two will then characterise the level of nonlinearity present at the focal point. Since the parallel firing will have greater physical amplitude at the focal point there is a greater loss of energy from the transmission bandwidth. Thus the nonlinear metric is formulated as in Equation 3.

\[
\gamma(\mathcal{F}) = \frac{E_S(\mathcal{F}) - E_P(\mathcal{F})}{E_S(\mathcal{F})}
\] (3)

The resulting metric is normalised by the sequential energy to give a percentage change. This implementation requires only a single ultrasonic array and knowledge of acoustic velocity within a material. Since these requisites are the same as
for linear imaging, the proposed nonlinear imaging modality is immediately available to many applications where linear ultrasonic array imaging is currently applied. In addition it does not preclude the production of a standard linear array image.

3. EXPERIMENTAL DEMONSTRATION

Having described the assumptions and methodology behind the nonlinear metric a series of tests were performed.

3.1 Closed fatigue crack

Firstly a fatigue crack was grown in a 30mm x 30mm x 100mm Aluminium specimen. This is a tightly closed crack and is difficult to image with conventional means. The array is coupled to the top surface of the specimen, perpendicular to the crack. The particular array used is a 5 MHz 64 element linear array with a pitch of 0.63mm. This is controlled using a Micropulse FMC array controller to make measurements. This configuration is illustrated here in Figure 2.

![Figure 2. Experimental configuration showing position of array relative to closed fatigue crack.](image_url)

In order to image the nonlinearity using the proposed methodology, a frame of sequentially captured diffuse field data is recorded. Then at every point that the nonlinearity is to be calculated a physical beam is formed at that location (the parallel capture) and the diffuse field response is recorded. Then at every pixel to be measured the energy is the parallel diffuse field is calculated. This is then compared to the equivalent beam formed by post processing using the sequential capture data. These metrics can be used to produce an image of the relative energy in the specimen. An example of the parallel and sequential energy image for the above sample are shown in Figure 3.

These images do not show any obvious differences between them. With the crack and its location not resolvable. It is only when the difference between them is studied as in Figure 3(c) that the crack can be resolved. It is worth noting that the difference in the energy between the two captures is of the order of 2%, thus it is not surprising that the changes are not directly resolvable in the energy images. While the crack is visible in the linear image it is of low amplitude and its true extent is not correctly recreated in Figure 3(d). This can be seen with reference to the faint dotted line in Figure 3(c) and (d). Here the nonlinear image accurately reveals the tip of the crack. Another feature that is important to note is that the nonlinear image suppresses the linear features in the image. This is most obvious in the total absence of a back wall reflection. While not significant here this feature is important to the sample tested in the next section.
3.2 Closed fatigue crack growing from hole

The sample tested in the previous section was modified by drilling two holes through the specimen. One of these is drilled such that the edge of the hole is 5mm behind the crack tip and the other in clean material. This is done to mimic a major class of defect seen in practice, that of damage growing from fasteners.

In the linear image the crack cannot be directly imaged. The echo from the hole itself is of such magnitude that it swamps any direct reflection from the crack. Some breakup in the echo can be noted, indicating that there may be something there. However this would not be sufficient to make a decision in a practical context. In contrast to this the nonlinear image shows a clear response from the location of the tightly closed crack. The location and extent of this indication correlates well with the size of the crack in practise. This feature is resolvable primarily as a result of the suppression of the signals from linear features. This highlights a key strength of this technique. As both inspections are made using exactly the same hardware both can and should be made. The data they provide is complimentary and together these two approaches produce a useful picture of the material state.

3.3 Growing fatigue cracks

Having shown the ability of the approach to localize and detect cracks growing from large linear features, this section will explore the detection limits. An Aluminium compact test specimen was made and cyclically fatigue loaded to produce a fatigue crack from an initiation notch. Initially nonlinear and linear ultrasonic measurements and micrographic
studies were made every 10,000 cycles up to 40,000. By this point the fatigue crack had grown to 630μm in length. An example linear image of the sample is shown here in Figure 5.

Figure 5. Linear TFM image from CT specimen at 0 fatigue samples. The array is at the top firing into the material. At the bottom of the figure are the holes corresponding to the mounting points of the specimen and the external geometry. The spot at around -20dB in the middle of the image is the reflection from the tip of the initiation notch where the crack will grow. This region was selected for further study to track the growth of the fatigue crack. A series of close ups of this region with increasing numbers of fatigue cycles is shown below in Figure 6.

Figure 6. Linear TFM images from CT specimen at the crack initiation notch at, (a) 0 cycles, (b) 10000 cycles, (c) 20000 cycles, (d) 30000 cycles, (e) 35000 cycles and (f) 40000 cycles
Throughout this level of change in fatigue there are no clearly resolvable changes in the position of the initiation notch. This is not necessarily surprising as even at the maximum extent the fatigue crack is around half a wavelength in size and not orientated advantageously. If this is compared to results made using the nonlinear imaging approach the outcome is however significantly different. This is shown here in Figure 7.

![Nonlinear images from CT specimen at the crack initiation notch at, (a) 0 cycles, (b) 10000 cycles, (c) 20000 cycles, (d) 30000 cycles, (e) 35000 cycles and (f) 40000 cycles](image)

Between 0 and 10000 cycles there is no significant change in the nonlinear response from the starter notch, however from 20000 cycles on, with a crack length of 1/6 of a wavelength, there are clearly measurable changes. Both the amplitude of the nonlinear response (shown in Figure 8) and the position of the peak are tracking the extent of the fatigue crack well. In addition the suppression of the linear features as evidenced by the image at 0 cycles means that the resulting indication is unambiguous and represents a useful diagnostic tool.

![Changing nonlinear amplitude with number of fatigue samples](image)
Having demonstrated the ability of the nonlinear approach to detect the fatigue crack at sub wavelength levels, the fatigue cycling was continued to determine at what point the crack became visible with linear imaging and to determine if the position of the tip was tracked in the nonlinear imaging.

Figure 9. Nonlinear images from CT specimen at the crack initiation notch at, (a) 0 cycles, (b) 40000 cycles, (c) 60000 cycles and (d) 80000 cycles

Figure 9 shows the resulting nonlinear images with changing number of fatigue cycles. The position in the maximum of the nonlinear amplitude has moved by 4mm through the figures. This correlates well with the change in crack length measured by micrographs, and suggests the technique is a useful approach for sizing. This is in comparison to the 0 and 80000 cycle linear results shown here in Figure 10.

Figure 10. Linear TFM images from CT specimen at the crack initiation notch at, (a) 0 cycles, (b) 80000 cycles
Although the later image shows some breakup at the notch tip (as in the two hole specimen) there is no positive identification of the tightly closed fatigue crack growth. If measurements are made from a more preferential direction, such as the side, the crack growth can be measured with a high frequency array but sensitivity is still much reduced compared to the nonlinear metric.

4. CONCLUSIONS

The proposed nonlinear measurement approach has demonstrated an ability to not only detect nonlinearity but localise it within a specimen. This is done using standard components and from a single measurement location. The resulting measurements work as a good compliment to existing array imaging approaches and uses all of the same hardware. The suppression of linear features in the images is particular advantageous and allowed growing fatigue cracks to be detected far earlier than would otherwise be possible and sized with a greater degree of accuracy than linear approaches would afford.

Although promising there is significant development work to be carried out on the approach. Most importantly in determining what the lowest level of nonlinearity that can be detected is. Closed cracks are highly nonlinear and to detect fatigue damage in a part, significant increases in the measurement sensitivity will be required. In addition the performance of the approach with changing depth should be characterised to allow accurate quantitative data to be extracted. Nonetheless this is a promising technique that warrants further investigation.

REFERENCES