**Impact of efficiency, investment, and competition on low carbon manufacturing**

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**Abstract:** Low carbon economy has become a top agenda for many countries following the agreement in the Paris meeting on climate change. In this article, we take price and emission sensitive demand into account and incorporate competition between the two rival manufacturers in the demand function. This research takes more proactive actions incorporating carbon emissions in the strategic and operational decisions, which complements the existing literature on low carbon manufacturing, in which the carbon emissions attribute is often used as a constraint, or only the single manufacturer’s demand is considered. Based on game theory, the pricing and carbon emissions reduction decisions are investigated. Our study contributes to the existing literature on low carbon manufacturing by specifically examining the impact of production efficiency, carbon emissions reduction efficiency, and market power structure on achieving low carbon manufacturing. Through the systematic analysis of optimal pricing and green technology investment decisions to improve the economic and environmental performance under different market power structures, our findings provide valuable managerial implications, which will help many manufacturing firms to make important strategic and operational decisions regarding low carbon manufacturing.

**Keywords:** Low carbon manufacturing; green technology investment; power structure; price competition; emission competition.

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1 Introduction

With the rapid economic development, the global warming has brought serious challenges to human’s survival and development (Hogue et al., 2013). Research has shown that global warming is likely to be at least 90% caused by human beings (IPCC, 2007). Transforming of the mode of human production and life, and realizing the sustainable development of low carbon economy are becoming the focus of global attention (Chen and Hao, 2015). Across different industry sectors, the manufacturing sector is often the single largest contributor to carbon emissions in most developing and developed countries (Fysikopoulos et al., 2014). In order to achieve a more sustainable consumption and production, low carbon manufacturing, often referred as the manufacturing process that generates low carbon emissions intensity through the effective and efficient use of energy and resources (Tridech and Cheng, 2011), has therefore become an important area of research enquiry. Many companies have already started to work on developing low carbon emissions manufacturing practices in order to gain competitive advantage. Nevertheless, a transition to a low carbon manufacturing will require innovation and investment in a range of low carbon technologies and radical changes to operation practices in the industry sector.

To make low carbon manufacturing sustainable, it is essential for companies to develop appropriate pricing policies for low carbon products. As the increased awareness of environmental protection and the change of the consumption habits, customers are more sensitive to low carbon products and willing to pay extra price (Arora, 1995; Bansal and Gangopadhyay, 2003; Björklund, 2011; Paksoy and Özceylan, 2013; Chander and Muthukrishnan, 2015). In addition, customer environmental consciousness will influence carbon emissions reduction strategy and, in return, manufacturer will seek for optimal green technology investments to reduce carbon emissions for winning more customers (Geffen and Rothenberg, 2000; Laroche et al., 2001; Innes, 2006; Zhu and Sarkis, 2007; Sengupta, 2015; Jiang and Chen, 2016; Luo et al., 2016). Therefore, the attribute of low carbon has become an influential aspect for customers. In response, manufacturing firms should at least consider the demand sensitivity in carbon emissions in the decision making of the pricing and emission reduction. These decisions will have a significant impact on manufacturers’ profits.
From manufacturers’ perspective, with the increasing consumer environmental awareness, the competition between manufacturers is no longer only based on economic performance. It also extends to their environmental performance. Therefore, both carbon emissions reduction strategy and pricing policies on low carbon products have become crucial for manufacturing firms. Adding to the complexity of these decisions, most manufacturers are facing intense competition in a competitive market environment. The benefits of competition have been extensively covered in the literature, for instance, Moorthy (1988), Banker et al. (1998), Hall and Porteus (2000), So (2000), Tsay and Agrawal (2000), etc.. These studies examined firms’ optimal operational strategies with competition on quality, pricing, service, and delivery time respectively. They also analysed the influence of competitive intensity and found that the equilibrium levels of quality and service increase while price and delivery time decrease as competition intensifies. More recent work such as Choi and Fredj (2013) and Sang (2014), only focused on price competition. However, it is not clear how the power dynamics in the marketplace will influence manufacturing firms’ decisions on green technology investment and pricing, and the consequential effect on their economic and environmental performance. Power issues including market power (Montgomery, 1985; Ailawadi et al. 1995; Wei and Zhao, 2016) and supply chain power (Huang et al., 2002; Benton and Maloni, 2005; Chen and Wang, 2015) were widely explored in the marketing and operations management literature. The existing studies on market power structure between rival firms or supply chain power structure between supply chain members mainly focused on the profit-related operations decisions like pricing. Few studies have taken environmental performance like carbon emissions into consideration. Furthermore, to our best knowledge, no research has so far examined the impact of market power structure on the decisions of low carbon manufacturing. Therefore, the following key questions are addressed in this research:

1. What are the optimal pricing policy and carbon emissions reduction strategy for the manufacturers under different market power structures?
2. To what extent, do the manufacturers’ production and carbon emissions reduction efficiencies affect their optimal prices, investment on green technology, and maximum profits?
3. What impact do the power structure, price competition and emission competition
have on low carbon manufacturing?

To answer these questions, we consider two rival manufacturers under price and emission sensitive demand. The same product is produced by them and sold to customers with a demand which is determined by their own and competing manufacturer’s price and unit carbon emissions. Using the game theoretical approach, we attempt to obtain the optimal solutions for pricing and green technology investment decisions for both manufacturers in a balanced power structure and an imbalanced power structure respectively. The impact of the production efficiency and carbon emissions reduction efficiency on the manufacturers’ optimal pricing and maximum profits is examined in the balanced power structure. We also explore the impact of power structure in the imbalanced power structure. Through comparing the optimal solutions and performances, this research intends to understand the impact that production efficiency, green technology investment, and market power structure have for low carbon manufacturing.

After a review of the literature in Section 2, the model formulation and assumptions are presented in Section 3. In Section 4 and 5, the pricing and unit carbon emissions polices in Nash and Stackelberg model are discussed respectively, to address Question (1). Section 6 is divided into two parts: Section 6.1 examines the impact of production and carbon emissions reduction efficiencies, which answers Question (2); and Section 6.2 explores the impact of power structure, which answers part of Question (3). Additionally, a numerical example is provided in Section 7 to demonstrate the validity of the proposed models. The numerical analysis also looks at the impact of price competition and emission competition on the optimal decisions, environmental and economic performance, which addresses the remaining part of Question (3). Finally, managerial insights are discussed and directions for future work are considered in Section 8.

2 Literature review

The literature reviewed here primarily relates to three research streams: models with carbon emissions, applications of price and emission sensitive demand, and the impact of power structure on the environmental and organisational performance.
The first relevant literature stream focuses on operation management taking carbon emissions into consideration and much of those are based on the regulation of carbon emissions policies. Penkunh et al. (1997), Dobos (2005), Letmathe and Balakrishnan (2005) and Rong and Lahdelma (2006) did their investigation under different government regulation policies such as carbon tax, carbon cap, and cap-and-trade. They obtained the optimal operations decisions and analysed the impact of regulation policies on these decisions. More recently, Bouchery et al. (2012) investigated optimal order quantity under carbon emissions constraint through an expanded Economic Ordering Quantity (EOQ) model with multi-objective decision. Rosic and Jammernegg (2013) studied the optimal order source and quantity of a single retailer with dual sourcing model under cap-and-trade and carbon tax. Zhang and Xu (2013) discussed the impact of carbon cap and trade price on optimal solution and performance in a context of a multi-item production firm with a stochastic demand. García-Alvarado et al. (2016) extended the work of Ahiska and King (2010) by introducing a cap-and-trade mechanism in an infinite-horizon inventory system. They found that inventory policy could play an important role in compliance with environmental legislation. Wang et al. (2016) presented three mathematical models to study manufacturing/remanufacturing planning issues (e.g. optimal production quantities of a new and remanufactured product) considering capital and/or carbon emissions constraints. They discussed the impact of carbon emissions constraint and found that the carbon emissions constraint will have more distinct influences on the manufacturing/remanufacturing decisions, and under that the manufacturer need more capital to achieve the maximum profit. Although the relevant literature is rich as illustrated above, most of them only consider carbon emissions as a constraint and few of them take more proactive actions incorporating carbon emissions in their strategic and operational decisions.

Among the few studies that incorporate emission sensitive demand or consumer environmental awareness in their own initiative, Yalabik and Fairchild (2011) studied a manufacturer’s optimal price and emissions level under the regulatory penalties and demand decrease caused by emissions. They found that when environmentally sensitive customers are available, the manufacturer has an incentive to reduce carbon emissions through green technology investment. Sengupta (2012) analysed a firm’s decision behaviour on pricing with
environmentally conscious consumers. The research finding indicated that firms would be willing to disclose their environmental performance to gain better market response when they realize their consumers become more aware of environmentally sound products. Hoen et al. (2014) studied transport mode selection decision of a carbon-aware company and found that company can reduce carbon emissions by switching to a different transport mode. Nouira et al. (2014) analysed the impact of emission sensitive customers on manufactures’ profits in the scenarios of price sensitive demand and both price and emission sensitive demand. Their research found that manufacturer must focus on the impact of carbon emissions of production process and material input on the environment. Toptal et al. (2014) investigated one manufacturer’s joint decisions on ordering and investment on emission reduction with condition of three carbon emissions policies, and analytically compared the impact of different policies. Liu et al. (2012), Du et al. (2015) and Zheng et al. (2015) also considered consumer environmental awareness and studied its impact on firms’ operations decisions such as the optimal ordering policies and coordination contracts and their economic and environmental performance. Chen et al. (2016) studied the warehouse management decisions under the cap-and-trade emission policy. They obtained the optimal carbon emissions reduction policy and analysed the role of green technology investment in managing the trade-offs between the economic and environmental performances of warehousing operations. Although a few studies have incorporated carbon emissions in their strategic and operational decisions, often only one manufacturer is studied without considering the market competition. In this research, two competing manufacturers’ optimal pricing and carbon emissions reduction policies are studied with price and emission sensitive demand.

Another relevant research stream looks into the impact of market competition and power structure. Most of these researches, such as Choi (1991), Ertek and Griffin (2002), Cai et al. (2009), Edirisinghe et al. (2011), Zhang et al. (2012), Shi et al. (2013) and Xiao et al. (2014) mainly focus on the supply chain vertical competition between manufacturers and their suppliers or between manufacturers and their customers, and very few studies have attempted to investigate the effect of the market competition between the rival manufacturers on the environmental and organisational performance. Moorthy (1988) examined two identical firms competing on product quality and price. They obtained the quality-price equilibrium strategies
and found that the firm should be differentiating its product from its competitor. Banker et al. (1998) studied the impact of competitive intensity on the equilibrium levels of quality and found that the relationship between quality and competitive intensity depends on the increased competition and other parameters. Hall and Porteus (2000) constructed an explicitly dynamic model of firm behaviour in which firms compete based upon customer service and studied firm’s capacity decisions in respond to customer service and competition pressure. So (2000) assumed that demand is sensitive to both the price and delivery time guarantees. He analysed the optimization problem for the individual firms and then studied the equilibrium solution in a multiple-firm competition. Tsay and Agrawal (2000) studied two independent retailers who use service and retail price to directly compete for end customers. Wu et al. (2012) investigated competitive pricing decisions between two retailers in a two-stage supply chain with horizontal and vertical competition and obtained the optimal policies. The above researches study the impact of the market competition between the rival manufacturers on economic performance. The carbon emissions factor is often neglected despite that more attention has been paid by customers. Chen and Hao (2014) focused on two competing firms’ optimal pricing and production policies with a balanced power structure under emissions tax. Their study found that to achieve a certain emissions reduction percentage, the tax charged from the firm with high production efficiency should be higher than that from the firm with low production efficiency. Luo et al. (2016) examined the role of co-opetitive relationship between the rival manufacturers in achieving low carbon manufacturing. The above literature that considers carbon emissions factor in the horizontal market competition has only examined the balanced power structure and got Nash equilibrium or Bertrand equilibrium. However, not much attention has been paid to Stackelberg equilibrium (firms’ pricing, production and carbon emissions reduction policies) under the imbalanced power structure. Although the effect of imbalanced power structure on firms’ economic performance has been investigated by many studies such as Netessine and Shumsky (2005), Wu et al. (2012), Grauberger and Kimms (2016), to the best of our knowledge, no one has so far examined the impact of market power structures in the context of low carbon manufacturing. This research aims to address the gap in the literature by examining the impact of market power structure on low carbon manufacturing with the consideration of price and emission sensitive demand and
competition

3 Model development and assumption

We consider a situation that two manufacturing firms producing substitute items compete in a same market. We define \( a \) as the primary market size, which can be interpreted as the total market demand for these two products manufactured by manufacturers 1 and 2. Two rival manufacturing firms with different production efficiency are considered. To achieve a more sustainable consumption and production, the manufacturers seek for green technology to “green” product and reduce emissions. We assume the green technology investment as a one-off disposable investment to improve the production process. Through that, initial unit carbon emission \( e_0 \) is decreased to \( e_i \) per product. The investment is \( I_i = t_i (e_0 - e_i)^2 \) (Yalabik and Fairchild, 2011; Choudhary et al., 2015). The quadratic investment function is convexity on \( e_i \), which is attributed to diminishing returns from expenditures (Tsay and Agrawal, 2000; Bhaskan and Krishnan, 2009; Ghosh and Shah, 2011).

Banker et al. (1998) and Tsay and Agrawal (2000) modelled the demand with price and quality level sensitive, and price and service level sensitive, respectively. However, in our setting, environmental performance is an important factor, thus the demand is influenced by both the product price and environmental property. Furthermore, we incorporate competition between the two rival manufacturers in the demand function. This is different to many existing studies, in which the carbon emissions attribute is often used as a constraint or only the single manufacturer’s demand is considered. The manufacturer \( i \) must choose a price level \( p_i \) and a carbon emissions level \( e_i \), and both of those are absolute values. Therefore, the demand faced by manufacturer \( i \) is

\[
q_i = a - b_1 p_i + b_2 p_j - k_1 e_i + k_2 e_j
\]

where \( i, j \in \{1, 2\} \) and \( i \neq j \). Note that \( b_1 > b_2 > 0 \) and \( k_1 > k_2 > 0 \), in which \( b_1 > b_2 > 0 \) means that the influence of the self-price sensitivity is stronger than cross-price sensitivity, and similarly \( k_1 > k_2 > 0 \) means that self-carbon emissions sensitivity is higher than cross-carbon emissions sensitivity. \( b_2 \) and \( k_2 \) can be also defined as price competition and emission competition. Note that the demand function can be rewritten as
\[ q_i = a - (b_1 - b_2)p_i + b_2(p_j - p_i) - (k_1 - k_2)e_i + k_2(e_j - e_i) \]

It is straightforward that every unit by which the manufacturer \( i \) raises (cuts) price \( p_i \) will lose (attract) \( b_1 \) customers because of \( dq_i/dp_i = -b_1 \), that is, \( (b_1 - b_2) \) of these customers will not purchase (would not have purchased) any products from both manufacturers at all, and \( b_2 \) of these customers are diverted to (from) manufacturer \( j \) thanks to \( dq_j/dp_i = b_2 \). And every unit by which the manufacturer \( i \) raises (cuts) carbon emissions level \( e_i \) will lose (attract) \( k_1 \) customers because of \( dq_i/de_i = -k_1 \), that is, \( (k_1 - k_2) \) of these customers will not purchase (would not have purchased) any products from both manufacturers at all, and \( k_2 \) of these customers are diverted to (from) manufacturer \( j \) thanks to \( dq_j/de_i = k_2 \).

The unit production cost of manufacturer 1 and 2 are \( c_1 \) and \( c_2 \) respectively. Note that unit production cost decreases as the production efficiency improves. Without loss of generality, \( q_1 = a - b_1c_1 + b_2c_2 - k_1e_0 + k_2e_0 > 0 \) and \( q_2 = a - b_1c_2 + b_2c_1 - k_1e_0 + k_2e_0 > 0 \). That is, without green technology investment, the demand is always positive when price equals cost. The parameters and variables for model development are donated as the following notations shown in Table 1. The \( A_1, A_2, B, C, D, U_1, U_2 \) and \( U^3 - 2V^2U - k_2^2V^2 \) can be easily proved to be positive due to the non-negativity of the decision variables and boundedness and concavity of the profit function.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Descriptions</th>
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<tbody>
<tr>
<td>( q_1, q_2 )</td>
<td>Production quantities (or customer demands) of manufacturer 1 and 2 respectively</td>
</tr>
<tr>
<td>( p_1, p_2 )</td>
<td>Unit retailing price of manufacturer 1 and 2 respectively</td>
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<tr>
<td>( e_0 )</td>
<td>Initial unit carbon emissions of manufacturer 1 and 2</td>
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<tr>
<td>( e_1, e_2 )</td>
<td>Unit carbon emissions after green technology investments of manufacturer 1 and 2 respectively, ( e_1 &lt; e_0 ) and ( e_2 &lt; e_0 ) (Yalabik and Fairchild (2011), Zhang and Xu (2013), Choudhary et al. (2015), Du et al. (2015) and Luo et al. (2016))</td>
</tr>
<tr>
<td>( c_1, c_2 )</td>
<td>Unit production cost of manufacturer 1 and 2 respectively, ( p_1 &gt; c_1 ) and ( p_2 &gt; c_2 )</td>
</tr>
<tr>
<td>( l_1, l_2 )</td>
<td>The green technology investment of manufacturer 1 and 2 respectively</td>
</tr>
<tr>
<td>( t_1, t_2 )</td>
<td>Green technology investment cost coefficient of manufacturer 1 and 2 respectively</td>
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</table>
\[ U_i = 4b_1 t_i - k_1^2 \]
\[ V_i = 2b_2 t_i - k_1 k_2 \]
\[ A_1 = \frac{(a - b_1 c_1 + b_2 c_2 - k_1 e_0 + k_2 e_0) U_2 + (a - b_1 c_2 + b_2 c_1 - k_1 e_0 + k_2 e_0) V_2}{U_i U_2 - V_i V_2} \]
\[ A_2 = \frac{(a - b_1 c_1 + b_2 c_2 - k_1 e_0 + k_2 e_0) V_1 + (a - b_1 c_2 + b_2 c_1 - k_1 e_0 + k_2 e_0) U_1}{U_i U_2 - V_i V_2} \]
\[ B = \frac{(a - b_1 c + b_2 c - k_1 e_0 + k_2 e_0)(U + V)}{U^3 - 2V^2 U - k_2 V^2}, U = U_{i|t=t} \text{ and } V = V_{i|t=t} \]
\[ C = \frac{(a - b_1 c + b_2 c - k_1 e_0 + k_2 e_0)((U + k_2^2) V + (U^2 - V^2))}{U^3 - 2V^2 U - k_2 V^2}, U = U_{i|t=t} \text{ and } V = V_{i|t=t} \]
\[ D = \frac{a - b_1 c + b_2 c - k_1 e_0 + k_2 e_0}{U - V}, U = U_{i|t=t} \text{ and } V = V_{i|t=t} \]

According to the above assumption, the manufacturer 1’s profit, denoted \( \pi_1(p_1, e_1) \), is
\[ \pi_1(p_1, e_1) = (p_1 - c_1)(a - b_1 p_1 + b_2 p_2 - k_1 e_1 + k_2 e_2) - t_1(e_0 - e_1)^2 \quad (1) \]
The first term means the profit from product sales, and the second term is the green technology investment. Similarly, the manufacturer 2’s profit, denoted \( \pi_2(p_2, e_2) \), is
\[ \pi_2(p_2, e_2) = (p_2 - c_2)(a - b_1 p_2 + b_2 p_1 - k_1 e_2 + k_2 e_1) - t_2(e_0 - e_2)^2 \quad (2) \]

4 Nash model

In the case of Nash game, each manufacturer has equal market power, and both manufacturers make their decisions simultaneously. The order of events is as follows. The manufacturer 1 decides the product retail price and unit carbon emissions to maximize profit given manufacturer 2’s product retail price and unit carbon emissions. At the same time, manufacturer 2 decides the product retail price and unit carbon emissions to maximize profit given manufacturer 1 product retail price and unit carbon emissions. Finally, when the customer demand is realized, the manufacturers gain their revenues.

For the manufacturers’ optimal prices (\( p^n_1 \)) and unit carbon emissions (\( e^n_1 \)) in a Nash game, the following proposition is obtained.

**Proposition 1** In a Nash game, \( p^n_1 = c_1 + 2t_1 A_1 \), \( p^n_2 = c_2 + 2t_2 A_2 \), \( e^n_1 = e_0 - k_1 A_1 \) and \( e^n_2 = e_0 - k_1 A_2 \).

This proposition means that in a Nash game, manufacturer 1’s and 2’s optimal prices and unit carbon emissions are existent and unique.

From proposition 1, we obtain manufacturer 1’s optimal production quantities (\( q^n_1 \)) and
manufacturer 2’s optimal production quantities \( q^*_1 \) and
\( q^*_2 \) respectively as following:

\[
q^*_1 = 2b_1 t_1 A_1 \quad (3)
\]
\[
q^*_2 = 2b_2 t_2 A_2 \quad (4)
\]

From proposition 1, we get the technology investments of manufacturer 1 \( I^n_1 \) and
manufacturer 2 \( I^n_2 \) respectively as following:

\[
I^n_1 = t_1 k^*_1 A^2_1 \quad (5)
\]
\[
I^n_2 = t_2 k^*_2 A^2_2 \quad (6)
\]

Then we get the maximum profits of manufacturer 1 \( \pi^n_1(p^n_1,e^n_1) \) and
manufacturer 2 \( \pi^n_2(p^n_2,e^n_2) \) respectively as following:

\[
\pi^n_1(p^n_1,e^n_1) = t_1 U_1 A^2_1 \quad (7)
\]
\[
\pi^n_2(p^n_2,e^n_2) = t_2 U_2 A^2_2 \quad (8)
\]

Then, the following corollary is obtained.

**Corollary 1** 1) \( p^n_i, q^n_i, I^n_i \) and \( \pi^n_i(p^n_i,e^n_i) \) all increase in \( a \), \( e^n_i \) decrease in \( a \). 2) \( e^n_i \) increases in \( c_i \), \( q^n_i, I^n_i \) and \( \pi^n_i(p^n_i,e^n_i) \) all decrease in \( c_i \). 3) Both \( p^n_i \) and \( q^n_i \) decrease in \( t_i \), \( e^n_i \) increase in \( t_i \).

This corollary examines both manufacturers’ optimal policies and corresponding
technology investment and economic performances when facing the changing external
environment, e.g. primary market size (\( a \)) in a Nash game. From the perspective of economics,
it is easy to understand that as the increasing primary market size will lead to higher price and
production quantity, which is an effective mechanism for manufacturers to balance the
demand and supply. From the perspective of manufacturers, both manufacturers may take a
low price strategy to gain market share in the short term. However, in the long run, a
manufacturer must develop green technology to reduce carbon emissions and gain
competitive advantage. Due to the high green technology investment, the manufacturers will
pass on the additional costs to the end consumers and then set higher prices. Therefore, when
the primary market size expands, the manufacturers can gain more profit by raising price and
increasing green technology investment. Similarly, when the unit production cost increases
(production efficiency decreases), more money will be used to produce goods even though the
product quantity is declining. Therefore, the capital supposed to invest to reduce carbon
emissions level will become less and the unit carbon emissions will go up. As a result, high
production cost (low production efficiency) has weighed on profit growth. An increase of green technology investment cost coefficient (decrease of carbon emissions reduction efficiency) will escalate the unit carbon emissions. Due to the high unit carbon emissions, some customers are not willing to buy that kind of goods. As a result, the manufacturers set a much lower price.

5 Stackelberg model

Generally speaking, there exist two Stackelberg game models, namely, Manufacturer 1-Stackelberg model (Manufacturer 1 dominates Manufacturer 2), and Manufacturer 2-Stackelberg model (Manufacturer 2 dominates Manufacturer 1). However, we focus on the impact of market power structure and let \( c_1 = c_2 = c \) and \( t_1 = t_2 = t \), thus these two manufacturers have no difference and Manufacturer 1-Stackelberg model and Manufacturer 2-Stackelberg model are symmetric. Therefore, just two power structures are analysed in our model. We assume the manufacturer 1 is the Stackelberg leader and the manufacturer 2 is the Stackelberg follower. Manufacturer 1 and 2 make their decisions in sequence. The order of events is as follows. First, manufacturer 2 decides the product retail price and unit carbon emissions given manufacturer 1’s product retail price and unit carbon emissions. Then, manufacturer 1 chooses optimal product retail price and unit carbon emissions using the response function of manufacturer 2 to maximize profit.

For the manufacturers’ optimal prices \( (p_i^s) \) and unit carbon emissions \( (e_i^s) \) in a Stackelberg game, the following proposition is obtained.

**Proposition 2** *In a Stackelberg game, \( p_1^s = c + 2tUB \), \( p_2^s = c + 2tC \), \( e_1^s = e_0 - (k_1U - k_2V)B \) and \( e_2^s = e_0 - k_1C \).*

This proposition means that in a Stackelberg game, manufacturers’ optimal prices and unit carbon emissions are existent and unique.

From proposition 2, we obtain manufacturer 1’s optimal production quantities \( (q_1^s) \) and manufacturer 2’s optimal production quantities \( (q_2^s) \) respectively as following:

\[
q_1^s = 2t(b_1U - b_2V)B \quad (9)
\]
\[
q_2^s = 2tb_1C \quad (10)
\]
From proposition 2, we get the technology investments of manufacturer 1 \((I_1^s)\) and manufacturer 2 \((I_2^s)\) as following:

\[
I_1^s = t(k_1U - k_2V)^2B^2 \tag{11}
\]

\[
I_2^s = tk_1^2C^2 \tag{12}
\]

Then we get the maximum profits of manufacturer 1 \((\pi_1(p_1^s, e_1^s))\) and manufacturer 2 \((\pi_2(p_2^s, e_2^s))\) respectively as following:

\[
\pi_1^s(p_1^s, e_1^s) = t(U^3 - 2V^2U - k_2^2V^2)B^2 \tag{13}
\]

\[
\pi_2^s(p_2^s, e_2^s) = tUC^2 \tag{14}
\]

Then, the following corollary is obtained.

**Corollary 2**  
1) \(p_i^s, q_i^s, I_i^s\) and \(\pi_i^s(p_i^s, e_i^s)\) all increase in a, \(e_i^s\) decrease in a.  
2) \(e_i^s\) increases in c, \(q_i^s, I_i^s\) and \(\pi_i^s(p_i^s, e_i^s)\) all decrease in c.

Similar to Corollary 1, this corollary also explores the effect of primary market size and unit production cost on the two manufacturers’ optimal policies as well as corresponding technology investment and economic performances in a Stackelberg game. From this corollary, we know that when the primary market size is big, higher prices, more products with low unit carbon emissions and more green technology investments will be the optimal. For unit production cost (production efficiency), when the production cost increases (production efficiency decreases), it is optimal to reduce production quantities and green technology investments. However, such an action will result in relatively high unit carbon emissions and worse economic performance.

### 6 Discussions

Here, we attempt to analyse the impact of production efficiency and carbon emissions reduction efficiency on the two rival manufacturers’ optimal pricing policies, production quantities, investments in green technology and maximum profits in a Nash game, and then discuss the impact of the imbalanced market power on the two competing manufacturers’ optimal pricing policies and maximum profits in a Stackelberg game.

#### 6.1 The impact of production efficiency and carbon emissions reduction efficiency

Firstly, to discuss the impact of production efficiency on the two rival manufacturers’ optimal
pricing policies, production quantities, investments in green technology and maximum profit, we define \( c_1 < c_2 \) and \( t_1 = t_2 = t \), that is, the production efficiency of manufacturer 1 is higher than that of manufacturer 2 and both of them have the same carbon emissions reduction efficiency. For the manufacturers’ optimal pricing policies \((p^n_{c_1}, p^n_{c_2}, e^n_{c_1}, e^n_{c_2})\) and optimal production quantity \((q^n_{c_1}, q^n_{c_2})\), optimal investment in green technology \((I^n_{c_1}, I^n_{c_2})\) and maximum profit \((\pi_1^n(p^n_{c_1}, e^n_{c_1}), \pi_2^n(p^n_{c_2}, e^n_{c_2}))\), the following proposition is obtained.

**Proposition 3** In a Nash game, when \( c_1 < c_2 \) and \( t_1 = t_2 = t \), then 1) If \( t \leq \frac{k_2^2 + k_1 k_2}{2b_1} \), then \( p^n_{c_1} \geq p^n_{c_2} \); if \( t > \frac{k_2^2 + k_1 k_2}{2b_1} \), then \( p^n_{c_1} < p^n_{c_2} \). 2) \( e^n_{c_1} < e^n_{c_2} \). 3) \( q^n_{c_1} > q^n_{c_2} \). 4) \( I^n_{c_1} > I^n_{c_2} \). 5) \( \pi_1^n(p^n_{c_1}, e^n_{c_1}) > \pi_2^n(p^n_{c_2}, e^n_{c_2}) \).

From this proposition, we know that low production efficiency or high unit product cost does not necessarily mean high unit retail price, which is also influenced by the green technology investment cost coefficient \((t)\). If the investment cost coefficient \((t)\) is lower than a certain ratio, then the optimal price of manufacturer 1 (higher production efficiency) is higher than that of manufacturer 2 (lower production efficiency). Otherwise, as the investment cost coefficient \((t)\) is higher than the ratio, the relationship between the two optimal prices will be in opposite. This ratio in Nash game is decided by carbon emissions sensitivities of the two manufacturers and the self-price sensitivity of manufacturer 1. Due to the high production efficiency or low unit product cost of manufacturer 1, he can invest more money to obtain greener products than manufacturer 2 with low production efficiency or high unit product cost. Therefore, the unit carbon emissions of manufacturer 1 is lower than that of manufacturer 2. We can also find that no matter what relationship between manufacturers’ prices under two different production efficiencies or unit product costs, the demand for the product with low unit carbon emissions is always higher than that with high unit carbon emissions. That is, the emission level is a more dominant factor as compared to the price level and greener products can gain a large market share. Although the technology investment of manufacturer 1 is more than manufacturer 2, a larger demand contributed by the environmental friendly product can help manufacturer 1 gain more profits. From this proposition we can conclude that under the carbon emissions sensitive demand, manufacturers can still increase their profits by improving production efficiency to reduce production cost, which is similar to the general
model without low carbon manufacturing.

Secondly, to examine the impact of carbon emissions reduction efficiency on the two rival manufacturers’ optimal pricing policies, production quantities, investments in green technology and maximum profits, we define $c_1 = c_2 = c$ and $t_1 < t_2$, that is both of them have the same production efficiency and the carbon emissions reduction efficiency of manufacturer 1 is higher than that of manufacturer 2. As to the manufacturers’ optimal policies ($p_{t_1}^n$, $p_{t_2}^n$, $e_{t_1}^n$, $e_{t_2}^n$) and optimal production quantity ($q_{t_1}^n$, $q_{t_2}^n$), optimal investment in green technology ($I_{t_1}^n$, $I_{t_2}^n$) and maximum profit ($\pi_1^m(p_{t_1}^n, e_{t_1}^n)$, $\pi_2^m(p_{t_2}^n, e_{t_2}^n)$), the following proposition is obtained.

**Proposition 4** In a Nash game, when $c_1 = c_2 = c$ and $t_1 < t_2$, then 1) $p_{t_1}^n > p_{t_2}^n, 2) e_{t_1}^n < e_{t_2}^n, 3) q_{t_1}^n > q_{t_2}^n, 4) I_{t_1}^n > I_{t_2}^n, 5) \frac{t_1}{t_2} \geq \frac{(U_1+V_1)^2u_2}{(U_2+V_2)^2u_1}$, then $\pi_1^m(p_{t_1}^n, e_{t_1}^n) \geq \pi_2^m(p_{t_2}^n, e_{t_2}^n)$; if $\frac{t_1}{t_2} < \frac{(U_1+V_1)^2u_2}{(U_2+V_2)^2u_1}$, then $\pi_1^m(p_{t_1}^n, e_{t_1}^n) < \pi_2^m(p_{t_2}^n, e_{t_2}^n)$.

This proposition means that the optimal price, production quantity and investment in green technology of manufacturer 1 (high emission reduction efficiency) are all higher than that of manufacturer 2 (low emission reduction efficiency). And the optimal unit carbon emissions of manufacturer 1 are lower than that of manufacturer 2. When the ratio between high and low emission reduction efficiency (investment cost coefficient of manufacturer 1 and 2) is higher than $\frac{(U_1+V_1)^2u_2}{(U_2+V_2)^2u_1}$, the maximum profits of manufacturer 1 are higher than that of manufacturer 2. Especially, when the ratio between high and low emission reduction efficiency (investment cost coefficient of manufacturer 1 and 2) is just the same as or lower than $\frac{(U_1+V_1)^2u_2}{(U_2+V_2)^2u_1}$, the maximum profits of manufacturer 1 are equal or less than that of manufacturer 2, that is the manufacturer cannot gain more profits by investing higher emission reduction efficiency green technology. This proposition indicates that although the manufacturer with high emission reduction efficiency has a large investment in green technology and gain more environmentally friendly products, the profits he gets are not the most.

6.2 The impact of power structure
The operations research on power structure often focuses on supply chains including power relationships between suppliers and manufacturers or manufacturers and retailers. Many of them have found that the member of the supply chain who has more power (Stackelberg leader) will gain more benefits or profits (Chen and Wang, 2015). But in our study, the imbalanced power structure of two horizontally competing manufacturers has totally different conclusions compared to the vertical competition between supply chain members. As to the manufacturers’ optimal policies \((p^*_1, p^*_2, e^*_1, e^*_2)\) and maximum profit \((\pi^*_1(p^*_1, e^*_1), \pi^*_2(p^*_2, e^*_2))\), the following proposition is obtained (manufacturer 1 Stackelberg).

**Proposition 5** In a Stackelberg game, 1) if \(t \in \left(0, \frac{k_1 k_2}{2b_2}\right) \cup \left(\frac{k_1 k_2 + k_2^2}{2b_2}, +\infty\right)\), then \(p^*_1 > p^*_2\); \(t \in \left[\frac{k_1 k_2}{2b_2}, \frac{k_1 k_2 + k_2^2}{2b_2}\right]\), then \(p^*_1 \leq p^*_2\). 2) If \((2b_1 k_2 + b_2 k_2 - b_2 k_1)(2b_2 t - k_1 k_2) \geq 0\), then \(e^*_1 \geq e^*_2\); if \((2b_1 k_2 + b_2 k_2 - b_2 k_1)(2b_2 t - k_1 k_2) < 0\), then \(e^*_1 < e^*_2\).

This proposition means that in a Stackelberg game the unit retail price is influenced by the green technology investment cost coefficient \((t)\). When the cost coefficient is relatively low or high, the Stackelberg leader (manufacturer 1) will set higher unit retail price than the follower (manufacturer 2). When the cost coefficient is moderate, the leader’s unit retail price will be lower than that of the follower’s. However, in the vertical competition between supply chain members, the Stackelberg leader will always set high price. When the cost coefficient is equal to the boundary values, then the competing manufacturers’ optimal unit retail price are the same with different market power. The relationship of unit carbon emissions between the two competing manufacturers is contingent on values of parameters describing the green technology investment cost coefficient, sensitive intensity and competitive intensity.

**Proposition 6** In a Stackelberg game, if \((b_2 k_1 k_2 - b_1 k_2^2 - b_2^2 t)(3UV + 2U^2 + k_2^2 V) \geq 0\), then \(\pi^*_1(p^*_1, e^*_1) \geq \pi^*_2(p^*_2, e^*_2)\); if \((b_2 k_1 k_2 - b_1 k_2^2 - b_2^2 t)(3UV + 2U^2 + k_2^2 V) < 0\), then \(\pi^*_1(p^*_1, e^*_1) < \pi^*_2(p^*_2, e^*_2)\).

From proposition 6, we know that in the imbalanced market power structure, the relationship of maximum profits between manufacturers with different market power is uncertain, which is determined by the green technology investment cost coefficient, sensitive intensity and competitive intensity. So it is not always the case that the manufacturer with
more market power gains more profit than that of the manufacturer with less market power.

In a Nash model, when \( c_1 = c_2 = c \) and \( t_1 = t_2 = t \), the optimal prices can be rewritten as \( p_1^n = p_2^n = p^n = c + 2tD \), and so as to the optimal unit carbon emissions, \( e_1^n = e_2^n = e^n = e_0 - k_1D \). The maximum profits is \( \pi_1^n(p_1^n,e_1^n) = \pi_2^n(p_2^n,e_1^n) = \pi^n(p^n,e^n) = TUD^2 \). By cross comparison of Nash and Stackelberg settings, the following proposition can be obtained.

**Proposition 7 (1)** For the Stackelberg leader with more market power: 1) \( p_1^s \geq p^n \). If \( t \in \left(0, \frac{k_1k_2}{2b_2}\right) \), then \( e_1^s < e^n \); if \( t \in \left[\frac{k_1k_2}{2b_2}, \frac{k_1k_2+\gamma}{2b_2}\right] \), then \( e_1^s \geq e^n \); if \( t \in \left(\frac{k_1k_2+\gamma}{2b_2}, +\infty\right) \), then \( e_1^s < e^n \), where \( \gamma = \frac{k_2(U^2-V^2)}{k_1(U+k_2^2)} > 0 \). 2) \( \pi_1^s(p_1^s,e_1^s) \geq \pi^n(p^n,e^n) \).

(2) For the Stackelberg follower with less market power: 1) If \( t \in \left(0, \frac{k_1k_2}{2b_2}\right) \), then \( p_2^s \leq p^n \) and \( e_2^s \geq e^n \); if \( t \in \left(\frac{k_1k_2}{2b_2}, +\infty\right) \), then \( p_2^s > p^n \) and \( e_2^s < e^n \). 2) When \( k_1k_2 - \delta > 0 \), if \( t \in \left(0, \frac{k_1k_2-\delta}{2b_2}\right) \), \( \pi_2^s(p_2^s,e_2^s) > \pi^n(p^n,e^n) \); if \( t \in \left[\frac{k_1k_2-\delta}{2b_2}, \frac{k_1k_2}{2b_2}\right] \), then \( \pi_2^s(p_2^s,e_2^s) \leq \pi^n(p^n,e^n) \); if \( t \in \left(\frac{k_1k_2}{2b_2}, +\infty\right) \), then \( \pi_2^s(p_2^s,e_2^s) > \pi^n(p^n,e^n) \). When \( k_1k_2 - \delta < 0 \), if \( t \in \left(0, \frac{k_1k_2}{2b_2}\right) \), then \( \pi_2^s(p_2^s,e_2^s) \leq \pi^n(p^n,e^n) \); if \( t \in \left(\frac{k_1k_2}{2b_2}, +\infty\right) \), then \( \pi_2^s(p_2^s,e_2^s) > \pi^n(p^n,e^n) \), where \( \delta = \frac{2(U^3-2V^2U-k_2^4V^2)}{V^2+uk_2^4} > 0 \).

Proposition 7 explores a cross comparison of Nash and Stackelberg settings based on manufacturers with different market power. From (1) of Proposition 7, compared to the optimal solutions in a balanced market power structure, in an imbalanced market power structure when one acts as a Stackelberg leader by gaining more competitive advantage and higher market status, he will tend to set a higher price. In addition, the leader will obtain more profits than that in a balanced market power structure. However, for the emission level, the relationship of optimal unit carbon emissions between the two power structures is determined by the investment cost coefficient and the competition of price and emission level. That is, a firm with higher market status frequently carries out the power of pricing but not good environmental indexes to manifest his strong competitive advantages. Although we cannot deny environmental performance’s significance to firms and whole society, it does not play a crucial role in gaining great economic benefits. Meanwhile, the unequal competitive force
contributes to an imbalanced market power, as to the market follower, his optimal operation policies including pricing and emission level decisions have to be changed on the basis of the investment cost coefficient and the competition of price and emission level. In addition, the profit is more or less than before which is not sure. That is, the product with high unit carbon emissions can be priced lower or high-price product can be produced greener to attract more buyers and gain more profit than that in a balanced market power. Therefore, in some sense, the imperfect competition caused by unequal competitive advantages is not a bad situation.

7 Numerical analysis

In this section, a numerical example is provided to demonstrate the feasibility of the mathematical models and analyse the impact of price competition ($b_2$) and emission competition ($k_2$) on the decisions of prices and unit carbon emissions. We also show their impact on total carbon emissions and maximum profits in different power structures. We specify $a = 500$, $b_1 = 25$, $k_1 = 25$, $e_0 = 20$, $c_1 = c_2 = c = 10$, $t_1 = t_2 = t = 15$. We define $k_2 = 15$ and $b_2 = 20$ respectively to investigate the impact of price and emission competition. The results are given in Figure 1-5.

![Figure 1 Impact of price competition and emission competition on optimal prices](image-url)
Figure 2 Impact of price competition and emission competition on optimal unit carbon emissions

Figure 3 Impact of price competition and emission competition on optimal production quantity

Figure 1 to 3 depict that as the price and emission competition intensify, the two competing manufacturers will set higher prices, production quantities, and lower unit carbon emissions. And also we can see as the intension of price competition, the changing extent of optimal prices, production quantities and unit carbon emissions are getting more significant and as the intension of emission competition, the changing extent is relevantly insignificant. Intuitively, as the consumer environmental awareness increases and the emission competition intensifies, the manufacturers will increase investments in green technology to reduce unit carbon emissions and increase production quantities.
Figure 4 Impact of price competition and emission competition on total carbon emissions

Figure 5 Impact of price competition and emission competition on maximum profits

In figure 4, it is interesting to see that as price competition intensifies, the total carbon emissions will decrease after increasing to the peak and as emission competition intensifies, the total carbon emissions will increase all the time. This is partly because the unit carbon emission is lower as the increasing of emission competition, but the low-carbon product demand will largely increase which results in higher total carbon emissions.

Figure 5 shows that as the price and emission competition intensify, the two competing manufacturers will gain more profits and the changing extent of profits caused by price competition is more significant than that of profits caused by emission competition. Intuitively, it seems that both price and emission competition will push up higher production quantity and lower unit carbon emissions and bring benefits for manufacturers to capture more profits, but these improvements are based on higher unit product retail prices and total carbon emissions (always for emission competition and sometimes for price competition). So we can infer that from manufacturers’ perspective, the environment performance of price competition is better...
than that of emission competition.

8 Conclusions and suggestions for further research

In this paper, we study two rival manufacturers’ optimal pricing and emissions reduction decisions with different market power structures. Using non-cooperative game theory, we build pricing and emissions level decision models with price and emissions sensitive demand. Then we derive the manufacturers’ optimal pricing and unit carbon emissions decisions in both the balanced and imbalanced power structures respectively. What’s more, the impact of production and carbon emissions reduction efficiencies are discussed in the balanced power structure and the effect of the imbalanced market power structure on the two competing manufacturers’ optimal pricing decisions and maximum profits are also examined. The main results are as following:

(1) Our findings show that for the two rival manufacturers with different production efficiency under the balanced power structure, the relationship between the two manufacturers’ optimal prices is affected by green technology investment cost coefficient. When the investment cost coefficient is relatively small, the optimal price of manufacturer with high production efficiency is higher than that of manufacturer with low production efficiency, and vice versa. High production efficiency will result in lower unit carbon emissions and higher production quantity, green technology investment and profit.

(2) We also find that for the two rival manufacturers with different carbon emissions reduction efficiency under the balanced power structure, higher carbon emissions reduction efficiency will make the manufacturer produce more, invest more in green technology and get low-carbon goods. This finding is similar to the previous research Luo et al. (2016), who also examined the effect of carbon emissions reduction efficiency under the cap-and-trade policy. In this case, how green a product is depends on the carbon emissions reduction efficiency. Therefore, carbon emissions reduction efficiency is a critical factor that should be carefully considered when adopting green technology. The relationship of the two manufacturers’ profits is determined by their green technology investment cost coefficient. We find that although the manufacturer with high emission reduction efficiency has a larger investment in
green technology and produce more environmentally friendly products. Nevertheless, it does not necessarily lead to more profits.

(3) In the imbalanced power structure, our study shows that power structure have a complex effect on the two manufacturers’ optimal pricing decisions and maximum profits which contingent on the price and emission sensitivity, price and emission competition and the green technology investment cost coefficient. Therefore, the manufacturers who have strong market power do not always obtain more profits than others with less market power, which is different to the findings of other studies on channel and supply chain power structure (Zhang et al. 2012; Chen and Wang 2015; Chen et al. 2016). By exploring a cross comparison of the policies and profits of a manufacturer in Nash and Stackelberg settings, the one when gaining more market power tends to set a higher price and obtain more profits than ever before, but the products he produces may be not as greener as he used to do in a balanced power structure.

(4) Strong price and emission competition make manufacturers set higher unit retail prices and lower unit carbon emissions. Manufacturers can obtain more profits through improving production quantities with an increasing marginal utility caused by price competition and a diminishing marginal utility caused by emission competition, but that may result in larger total carbon emissions and lead to poor environment performance.

This research has three main contributions. First, our paper extends the existing literature on low carbon manufacturing by specifically examining the impact of production efficiency, green technology investment, and market power structure in achieving low carbon manufacturing. Second, our study not only takes price and emission sensitive demand into account, but also incorporates competition between the two rival manufacturers in the demand function. This is different to many existing studies in which the carbon emissions attribute is often used as a constraint and few of them take more proactive actions incorporating carbon emissions in their strategic and operational decisions, and only the single manufacturer’s demand is considered (Rong and Lahdelma, 2006; Bouchery et al., 2012; Toptal et al., 2014). Third, through the systematic analysis of optimal pricing decision and green technology investments to improve the economic and environmental performance of the two competing manufacturers under different market power structures, our findings provide valuable
managerial implications, which will help many manufacturing firms to make important strategic and operational decisions regarding low carbon manufacturing.

The work in our paper can be further extended in several avenues. First, we only consider two rival manufacturers in this research. In fact, one future research avenue is to take multiple manufacturers into consideration. In addition, achieving a low carbon economy requires the coordination of the whole supply chain. It is therefore important to extend the current research from the manufacturing level to the whole supply chain level. Another interesting extension is to look into carbon emissions policies such as mandatory carbon emissions capacity, carbon tax and cap-and-trade, and discuss the impact of various carbon policies on low carbon manufacturing and investment in low carbon technologies on competitiveness. In addition, some business cases could be included to investigate the nature of the investment impact in the long run.

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Appendix

Proof of Proposition 1

From (1), we get \[ \frac{\partial \pi_1(p_1, e_1)}{\partial p_1} = (a - b_1 p_1 + b_2 p_2 - k_1 e_1 + k_2 e_2) - b_1 (p_1 - c_1) \] and \[ \frac{\partial \pi_1(p_1, e_1)}{\partial e_1} = -k_1 (p_1 - c_1) + 2 t_1 (e_0 - e_1) . \] Then, we obtain \[ \frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1^2} = -2 b_1 < 0 , \] \[ \frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1^2} = -2 t_1 \] and \[ \frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1 \partial e_1} = \frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1 \partial p_1} = -k_1 \] , then we get \[ \begin{vmatrix} \frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1^2} & \frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1 \partial e_1} \\ \frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1^2} & \frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1 \partial p_1} \end{vmatrix} = 4 b_1 t_1 - k_1^2 = U_1 > 0 . \] Therefore, \( \pi_1(p_1, e_1) \) is a joint concave function of \( p_1 \) and \( e_1 \). From (2), we can also get that \[ \begin{vmatrix} \frac{\partial^2 \pi_2(p_2, e_2)}{\partial p_2^2} & \frac{\partial^2 \pi_2(p_2, e_2)}{\partial p_2 \partial e_2} \\ \frac{\partial^2 \pi_2(p_2, e_2)}{\partial e_2 \partial p_2} & \frac{\partial^2 \pi_2(p_2, e_2)}{\partial e_2^2} \end{vmatrix} = 4 b_2 t_2 - \]
\[ k_1^2 = U_2 > 0 \]. Therefore, \( \pi_2(p_2, e_2) \) is a joint concave function of \( p_2 \) and \( e_2 \). Let

\[
\frac{\partial \pi_1(p_1, e_1)}{\partial p_1} = \frac{\partial \pi_1(p_1, e_1)}{\partial e_1} = \frac{\partial \pi_2(p_2, e_2)}{\partial p_2} = \frac{\partial \pi_2(p_2, e_2)}{\partial e_2} = 0, \]

we get

\[
(a - b_1 p_1 + b_2 p_2 - k_1 e_1 + k_2 e_2) - b_1(p_1 - c_1) = 0 \quad (1-1)
\]
\[
-k_1(p_1 - c_1) + 2t_1(e_0 - e_1) = 0 \quad (1-2)
\]
\[
(a - b_1 p_2 + b_2 p_1 - k_1 e_2 + k_2 e_2) - b_1(p_2 - c_2) = 0 \quad (1-3)
\]
\[
-k_1(p_2 - c_2) + 2t_2(e_0 - e_2) = 0 \quad (1-4)
\]

From (1-1), (1-2), (1-3) and (1-4), we get \( p_1^n = c_1 + 2t_1 A_1 \), \( p_2^n = c_2 + 2t_2 A_2 \), \( e_1^n = e_0 - k_1 A_1 \) and \( e_2^n = e_0 - k_1 A_2 \). As \( p_1^n > c_1 \) and \( p_2^n > c_2 \), then \( A_1 > 0 \) and \( A_2 > 0 \). This completes the proof.

**Proof of Corollary 1**

Since

\[
0 < A_1 = \frac{(a-b_1 c_1+b_2 c_2-k_1 e_0+k_2 e_0)U_2+(a-b_1 c_2+b_2 c_1-k_1 e_0+k_2 e_0)U_1}{U_1 U_2 - V_1 V_2}
\]

and

\[
0 < A_2 = \frac{(a-b_1 c_1+b_2 c_2-k_1 e_0+k_2 e_0)V_1+(a-b_1 c_2+b_2 c_1-k_1 e_0+k_2 e_0)V_2}{U_1 U_2 - V_1 V_2}
\]

and it is easy to understand that when 
\( p_1 \rightarrow c_1 \), \( p_2 \rightarrow c_2 \), \( e_1 \rightarrow e_0 \) and \( e_2 \rightarrow e_0 \), then \( a - b_1 c_1 + b_2 c_2 - k_1 e_0 + k_2 e_0 > 0 \).

Therefore, we get \( \frac{U_2+V_2}{U_1 U_2 - V_1 V_2} > 0 \) and \( \frac{U_1+V_1}{U_1 U_2 - V_1 V_2} > 0 \). And note that the above analysis is true for any \( t_1 \) and \( t_2 \).

Proposition 1 shows \( \frac{d p_1^n}{d a} = \frac{2t_1(U_2+V_2)}{U_1 U_2 - V_1 V_2} > 0 \), \( \frac{d p_2^n}{d a} = \frac{2t_2(U_1+V_1)}{U_1 U_2 - V_1 V_2} > 0 \), \( \frac{d e_1^n}{d a} = -\frac{k_1(U_2+V_2)}{U_1 U_2 - V_1 V_2} < 0 \) and \( \frac{d e_2^n}{d a} = -\frac{k_1(U_1+V_1)}{U_1 U_2 - V_1 V_2} < 0 \). That is, both \( p_1^n \) and \( p_2^n \) increase in \( a \); both \( e_1^n \) and \( e_2^n \) decrease in \( a \).

From (3), (4), (5), (6), (7) and (8) we get \( \frac{d q_1^n}{d a} = \frac{2b_1 t_1(U_2+V_2)}{U_1 U_2 - V_1 V_2} > 0 \), \( \frac{d q_2^n}{d a} = \frac{2b_2 t_2(U_2+V_2)}{U_1 U_2 - V_1 V_2} > 0 \), \( \frac{d q_1^n(p_1^n, e_1^n)}{d a} = \frac{2t_1 U_1 A_1(U_2+V_2)}{U_1 U_2 - V_1 V_2} > 0 \) and \( \frac{d q_2^n(p_2^n, e_2^n)}{d a} = \frac{2t_2 U_2 A_2(U_1+V_1)}{U_1 U_2 - V_1 V_2} > 0 \). That is, both \( q_1^n \) and \( q_2^n \) increase in \( a \); both \( l_1^n \) and \( l_2^n \) increase in \( a \); both \( \pi_1^n(p_1^n, e_1^n) \) and \( \pi_2^n(p_2^n, e_2^n) \) increase in \( a \).

Define \( f(t_1, t_2) = \frac{U_2+V_2}{U_1 U_2 - V_1 V_2} \) while \( \frac{U_2+V_2}{U_1 U_2 - V_1 V_2} > 0 \). When \( t_1 = t_2 = t \), then \( U_1 = U_2 = \)
U (from Hessian Matrix of Nash model we know that $U > 0$) and $V_1 = V_2 = V$, and we have $f(t, t) = \frac{U + V}{U^2 - V^2} = \frac{1}{U - V} > 0$. That is, for any $t$, we have $U > V$. Replace $t$ with $t_1$ and $t_2$ respectively, we have $U_1 > V_1$ and $U_2 > V_2$. Because of $b_1 > b_2 > 0$ and $k_1 > k_2 > 0$, we can get $b_1U_1 - b_2V_1 > 0$, $b_1U_2 - b_2V_2 > 0$, $4b_1U_2 - 2b_2V_2 > 0$, $4b_1U_1 - 2b_2V_1 > 0$, $k_1^2U_2 - k_1k_2V_2 > 0$ and $k_1^2U_1 - k_1k_2V_1 > 0$. In Stackelberg model, $p^e_1 = c + 2tUB > c$, we have $B > 0$. Because $B = \frac{(a - b_1c + b_2c - k_1k_2V_1)(U + V)}{U^3 - 2V^2U - k_2^2V^2} > 0$, and $a - b_1c + b_2c - k_1e_0 + k_2e_0 > 0$ and $U^3 - 2V^2U - k_2^2V^2 > 0$ (this can be confirmed from Hessian Matrix of Stackelberg model), we have $U + V > 0$ for any $t$. Therefore, replace $t$ with $t_1$ and $t_2$ respectively, we have $U_1 + V_1 > 0$ and $U_2 + V_2 > 0$. Combine $U_1 > V_1$ and $U_2 > V_2$, we can get $U_1 > |V_1| > 0$ and $U_2 > |V_2| > 0$, then we have $U_1U_2 - V_1V_2 > 0$.

Proposition 1 shows $\frac{d e_1^n}{d c_1} = \frac{k_1(b_1U_1 - b_2V_1)}{U_1V_2 - V_1U_2} > 0$ and $\frac{d e_2^n}{d c_2} = \frac{k_1(b_1U_1 - b_2V_1)}{U_2V_1 - V_2U_1} > 0$. That is, $e_1^n$ and $e_2^n$ respectively increases in $c_1$ and $c_2$.

From (3), (4), (5), (6), (7) and (8), we get $\frac{d q_1^n}{d c_1} = -2b_1t_2(b_1U_1 - b_2V_1) < 0$, $\frac{d q_2^n}{d c_2} = -2b_2t_1(b_1U_1 - b_2V_1) < 0$, $\frac{d l_1^n}{d c_1} = -2t_1k_1^2A_1(b_1U_2 - b_2V_2) < 0$, $\frac{d l_2^n}{d c_2} = -2t_2k_1^2A_2(b_1U_1 - b_2V_1) < 0$, $\frac{d \pi_1^n(p_1^n, e_1^n)}{d c_1} = -2t_1U_1A_1(b_1U_1 - b_2V_2) < 0$ and $\frac{d \pi_1^n(p_1^n, e_1^n)}{d c_2} = -2t_2U_2A_2(b_1U_1 - b_2V_1) < 0$. That is, $q_1^n$, $l_1^n$ and $\pi_1^n(p_1^n, e_1^n)$ all decreases in $c_1$ and $q_2^n$, $l_2^n$ and $\pi_2^n(p_2^n, e_2^n)$ all decreases in $c_2$.

Proposition 1 shows $\frac{d p_1^n}{d t_1} = -2k_1^2U_2 - k_1k_2V_2 < 0$, $\frac{d p_2^n}{d t_2} = -2k_2^2U_1 - k_1k_2V_1 < 0$, $\frac{d e_1^n}{d t_1} = \frac{4b_1U_2 - 2b_2V_2}{U_1V_2 - V_1U_2}k_1A_1 > 0$ and $\frac{d e_2^n}{d t_2} = \frac{4b_1U_1 - 2b_2V_1}{U_1V_2 - V_1U_2}k_1A_2 > 0$. That is, $p_1^n$ and $p_2^n$ respectively decrease in $t_1$ and $t_2$; $e_1^n$ and $e_2^n$ respectively increase in $t_1$ and $t_2$.

From (3) and (4), we get $\frac{d q_1^n}{d t_1} = -2k_1U_1 - k_1k_2V_2 < 0$, $\frac{d q_2^n}{d t_2} = -2k_2U_1 - k_1k_2V_1 < 0$. That is, $q_1^n$ and $q_2^n$ respectively decrease in $t_1$ and $t_2$.

This completes the proof.

**Proof of Proposition 2**

In a Stackelberg game, we let $c_1 = c_2 = c$ and $t_1 = t_2 = t$, and the manufacturer 1 is the Stackelberg leader while the manufacturer 2 is the Stackelberg follower. From the Proof of
Proposition 1, we know that
\[
\frac{\partial^2 \pi_2(p_2, e_2)}{\partial p^2_2} \frac{\partial^2 \pi_2(p_2, e_2)}{\partial p \partial e_2} \frac{\partial^2 \pi_2(p_2, e_2)}{\partial e_2 \partial p_2} = 4b_1 t - k_1^2 = U > 0.
\]
Therefore, \(\pi_2(p_2, e_2)\) is a joint concave function of \(p_2\) and \(e_2\). When \(c_1 = c_2 = c\), (1-3) and (1-4) can be rewritten as
\[
(a - b_1 p_2 + b_2 p_1 - k_1 e_2 + k_2 e_1) - b_1 (p_2 - c) = 0 \quad (2-1)
\]
\[-k_1 (p_2 - c) + 2t(e_0 - e_2) = 0 \quad (2-2)
\]
From (2-1) and (2-2), we get
\[
p_2 = -\frac{-2(a + b_1 c + k_2 e_1 + b_2 p_1) t + k_1 (k_1 c + 2 e_0 t)}{u} \quad (2-3)
\]
\[e_2 = -\frac{a k_1 - b_1 c k_1 + e_1 k_1 k_2 + b_2 k_1 p_1 - 4 b_1 e_0 t}{u} \quad (2-4)
\]
Substitute (2-3) and (2-4) into (1), we have
\[
\pi_1(p_1, e_1) = (p_1 - c)[a - b_1 p_1 + b_2 (-\frac{-2(a + b_1 c + k_2 e_1 + b_2 p_1) t + k_1 (k_1 c + 2 e_0 t)}{u}) - k_1 e_1 +
\]
k_2 \left(-\frac{a k_1 - b_1 c k_1 + e_1 k_1 k_2 + b_2 k_1 p_1 - 4 b_1 e_0 t}{u}\right) - t(e_0 - e_1)^2 \quad (2-5)
\]
From (2-5), we get
\[
\frac{\partial \pi_1(p_1, e_1)}{\partial p_1} = \left[a - b_1 p_1 + b_2 \left(-\frac{-2(a + b_1 c + k_2 e_1 + b_2 p_1) t + k_1 (k_1 c + 2 e_0 t)}{u}\right) - k_1 e_1 +
\]
k_2 \left(-\frac{a k_1 - b_1 c k_1 + e_1 k_1 k_2 + b_2 k_1 p_1 - 4 b_1 e_0 t}{u}\right) - t(e_0 - e_1)^2 \quad (2-5)
\]
and
\[
\frac{\partial \pi_1(p_1, e_1)}{\partial e_1} = \left(-\frac{2 b_2 k_2 t}{u} - k_1 - \frac{k_1 k_2^2}{u}\right) (p_1 - c) + 2t(e_0 - e_1)\right) . Then, we obtain \(\frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1 \partial e_1} = -2t\) and \(\frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1 \partial p_1} = -k_1 - \frac{k_1 k_2^2}{u}\) +
\]
\[
\frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1^2} = \left(-\frac{2 b_2 k_2 t}{u} - k_1 - \frac{k_2^2}{u}\right) (p_1 - c) + 2t(e_0 - e_1)\right) . Then, we obtain \(\frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1 \partial e_1} = -2t\) and \(\frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1 \partial p_1} = -k_1 - \frac{k_1 k_2^2}{u}\) +
\]
\[
\frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1^2} = \left(-\frac{2 b_2 k_2 t}{u} - k_1 - \frac{k_1 k_2^2}{u}\right)(p_1 - c) + 2t(e_0 - e_1) = 0 \quad (2-6)
\]
\[
\left(-\frac{2 b_2 k_2 t}{u} - k_1 - \frac{k_1 k_2^2}{u}\right)(p_1 - c) + 2t(e_0 - e_1) = 0 \quad (2-7)
\]
From (2-6) and (2-7), we get \(p_1^* = c + 2t U B\) and \(e_1^* = e_0 - (k_1 U - k_2 V) B\). Substitute
(p_1^e, e_1^e) into (2-3) and (2-4), we can obtain p_2^e = c + 2tC and e_2^e = e_0 - k_1C. As 
\frac{U^3 - 2V^2U - k_2^2V^2}{u^2} > 0, we have \( U^3 - 2V^2U - k_2^2V^2 > 0 \). Due to \( p_1^e, p_2^e > c \), we have \( B, C > 0 \). This completes the proof.

**Proof of Corollary 2**

As we proved in the Proof of Corollary 1, we can easily get \((b_1U - b_2V)(U + V) > 0\) and \((k_1U - k_2V)(U + V) > 0\).

In Stackelberg model, the leader manufacturer 1’s profit function \( \pi_1(p_1, e_1) \) is a joint concave function of \( p_1 \) and \( e_1 \), it must satisfy Hessian

\[
\begin{vmatrix}
\frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1^2} & \frac{\partial^2 \pi_1(p_1, e_1)}{\partial p_1 \partial e_1} \\
\frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1 \partial p_1} & \frac{\partial^2 \pi_1(p_1, e_1)}{\partial e_1^2}
\end{vmatrix} = 0
\]

\[
\frac{U^3 - 2V^2U - k_2^2V^2}{u^2} > 0, \text{ then we have } U^3 - 2V^2U - k_2^2V^2 > 0. \text{ Because optimal price } p_2^e = c + 2tC > c, \text{ we get } C = \frac{(a - b_1c + b_2c - k_1e_0 + k_2e_0)[(U + k_2^2)V + (U^2 - V^2)]}{U^3 - 2V^2U - k_2^2V^2} > 0. \text{ Therefore, combine } a - b_1c + b_2c - k_1e_0 + k_2e_0 > 0 \text{ and } U^3 - 2V^2U - k_2^2V^2 > 0, \text{ we have } (U + k_2^2)V + (U^2 - V^2) > 0.

The above results will be used in the following proof.

Proposition 2 shows

\[
\frac{dp_1^e}{da} = \frac{2tU(U + V)}{U^3 - 2V^2U - k_2^2V^2} > 0, \quad \frac{dp_2^e}{da} = \frac{2t[(U + k_2^2)V + (U^2 - V^2)]}{U^3 - 2V^2U - k_2^2V^2} > 0, \quad \frac{de_1^e}{da} = \frac{k_1(U + k_2^2)V + (U^2 - V^2)}{U^3 - 2V^2U - k_2^2V^2} < 0.
\]

That is, both \( p_1^e \) and \( p_2^e \) increase in \( a \); both \( e_1^e \) and \( e_2^e \) decrease in \( a \).

Form (9), (10), (11), (12), (13) and (14), we get

\[
\frac{dq_1^e}{da} = \frac{2tb_1[(U + k_2^2)V + (U^2 - V^2)]}{U^3 - 2V^2U - k_2^2V^2} > 0, \quad \frac{dq_2^e}{da} = \frac{2tb_1(U^2 - V^2)}{U^3 - 2V^2U - k_2^2V^2} > 0, \quad \frac{dl_1^e}{da} = \frac{2tk_1c(U + k_2^2)V + (U^2 - V^2)}{U^3 - 2V^2U - k_2^2V^2} > 0, \quad \frac{dl_2^e}{da} = \frac{2tk_1c(U + k_2^2)V + (U^2 - V^2)}{U^3 - 2V^2U - k_2^2V^2} > 0.
\]

\[
\frac{dq_1^e(p_1^e, e_1^e)}{da} = 2tB(U + V) > 0 \text{ and } \frac{dq_2^e(p_2^e, e_2^e)}{da} = \frac{2tUC(U + k_2^2)V + (U^2 - V^2)}{U^3 - 2V^2U - k_2^2V^2} > 0.
\]

That is, \( q_1^e, q_2^e, l_1^e, l_2^e, \pi_1^e(p_1^e, e_1^e) \) and \( \pi_2^e(p_2^e, e_2^e) \) all increase in \( a \).

Proposition 2 shows

\[
\frac{de_1^e}{dc} = \frac{(k_1U - k_2V)(b_1 - b_2)(U - V)}{U^3 - 2V^2U - k_2^2V^2} > 0, \quad \frac{de_2^e}{dc} = \frac{k_1(1 - b_2)(U + V)(b_1 - b_2)}{U^3 - 2V^2U - k_2^2V^2} > 0.
\]

That is, both \( e_1^e \) and \( e_2^e \) increase in \( c \).

From (9), (10), (11), (12), (13) and (14), we get

\[
\frac{dq_1^e}{dc} = \frac{2tb_1(U + V)(b_2 - b_2)}{U^3 - 2V^2U - k_2^2V^2} < 0.
\]
\[
\frac{d q_2^n}{d c} = -\frac{2b_1 t(b_1-b_2)[(U+k_2^2)V+(U^2-V^2)]}{u^3-2v^2u-k_2^2v^2} < 0 , \quad \frac{d t_1^n}{d c} = -\frac{2t(k_1U-k_2V)^2B(b_1-b_2)(U+V)}{u^3-2v^2u-k_2^2v^2} < 0 , \quad \frac{d t_2^n}{d c} = -\frac{2tk_2^2(b_1-b_2)[(U+k_2^2)V+(U^2-V^2)]}{u^3-2v^2u-k_2^2v^2} < 0 , \quad \frac{d \pi_1^n(p_1^n, e_1^n)}{d c} = -2tB(b_1-b_2)(U+V) < 0 \]

and

\[
\frac{d \pi_2^n(p_2^n, e_2^n)}{d c} = -2tUC(b_1-b_2)[(U+k_2^2)V+(U^2-V^2)] < 0. \]

That is, \( q_1^n, I_1^n, \pi_1^n(p_1^n, e_1^n), q_2^n, I_2^n \) and \( \pi_2^n(p_2^n, e_2^n) \) all decreases in \( c \). This completes the proof.

**Proof of Proposition 3**

1) From proposition 1, we get \( p_{c_1}^n - p_{c_2}^n = \frac{(c_1-c_2)(-k_1^2+k_2^2+2b_1t)}{U+V} \). So, if \( t < \frac{k_1^2+k_2^2}{2b_1} \), then \( p_{c_1}^n > p_{c_2}^n \); if \( t = \frac{k_1^2+k_2^2}{2b_1} \), then \( p_{c_1}^n = p_{c_2}^n \); if \( t > \frac{k_1^2+k_2^2}{2b_1} \), then \( p_{c_1}^n < p_{c_2}^n \).

2) From proposition 1, we get \( e_{c_1}^n - e_{c_2}^n = \frac{k_1(c_1-c_2)(b_1+b_2)}{U+V} < 0 \). That is, \( e_{c_1}^n < e_{c_2}^n \).

3) From (3) and (4), we get \( q_{c_1}^n - q_{c_2}^n = 2b_1 t(A_1-A_2) \). Because \( e_{c_1}^n < e_{c_2}^n \), we get \( A_1 > A_2 \). Therefore, \( q_{c_1}^n - q_{c_2}^n > 0 \). That is, \( q_{c_1}^n > q_{c_2}^n \).

4) From (5) and (6), we get \( I_{c_1}^n - I_{c_2}^n = t k_2^2(A_1^2-A_2^2) > 0 \). That is, \( I_{c_1}^n > I_{c_2}^n \).

5) From (7) and (8), we get \( \pi_1^n(p_{c_1}^n, e_{c_1}^n) - \pi_2^n(p_{c_2}^n, e_{c_2}^n) = tU(A_1^2-A_2^2) > 0 \). That is, \( \pi_1^n(p_{c_1}^n, e_{c_1}^n) > \pi_2^n(p_{c_2}^n, e_{c_2}^n) \). This completes the proof.

**Proof of Proposition 4**

1) From proposition 1, we get \( p_{t_1}^n - p_{t_2}^n = \frac{2k_1(k_1+k_2)(t_2-t_1)(a-b_1c+b_2c-k_1e_0+k_2e_0)}{u_1u_2-v_1v_2} > 0 \). That is, \( p_{t_1}^n > p_{t_2}^n \).

2) From proposition 1, we get \( e_{t_1}^n - e_{t_2}^n = -\frac{2k_1(2b_1+b_2)(t_2-t_1)(a-b_1c+b_2c-k_1e_0+k_2e_0)}{u_1u_2-v_1v_2} < 0 \). That is, \( e_{t_1}^n < e_{t_2}^n \).

3) From (3) and (4), we get \( q_{t_1}^n - q_{t_2}^n = 2b_1 t(A_1-A_2) \). Because \( e_{t_1}^n < e_{t_2}^n \), we get \( A_1 > A_2 \). Therefore, \( q_{t_1}^n - q_{t_2}^n > 0 \). That is, \( q_{t_1}^n > q_{t_2}^n \).

4) From (5) and (6), we get \( I_{t_1}^n - I_{t_2}^n = tk_2^2(A_1^2-A_2^2) > 0 \). That is, \( I_{t_1}^n > I_{t_2}^n \).

5) From (7) and (8), we get \( \pi_1^n(p_{t_1}^n, e_{t_1}^n) - \pi_2^n(p_{t_2}^n, e_{t_2}^n) = \frac{t_1u_1}{(u_1+v_1)^2} \frac{(u_1+v_1)^2}{t_2u_2} > 1 \), that
is if \( \frac{t_1}{t_2} > \frac{(U_1+V_1)^2-u_2}{(u_2+v_2)^2-u_1} \), then \( \pi^n_1(p_{t_1}, e_{t_1}^n) > \pi^n_2(p_{t_2}, e_{t_2}^n) \); if \( \frac{t_1 u_1}{(U_1+V_1)^2} \cdot \frac{(u_2+v_2)^2}{t_2 u_2} = 1 \), that is if \( \frac{t_1}{t_2} = \frac{(U_1+V_1)^2 u_2}{(u_2+v_2)^2 u_1} \), then \( \pi^n_1(p_{t_1}, e_{t_1}^n) = \pi^n_2(p_{t_2}, e_{t_2}^n) \); if \( \frac{t_1 u_1}{(U_1+V_1)^2} \cdot \frac{(u_2+v_2)^2}{t_2 u_2} < 1 \), that is if \( \frac{t_1}{t_2} < \frac{(U_1+V_1)^2 u_2}{(u_2+v_2)^2 u_1} \), then \( \pi^n_1(p_{t_1}, e_{t_1}^n) < \pi^n_2(p_{t_2}, e_{t_2}^n) \). This completes the proof.

**Proof of Proposition 5**

1) From proposition 2, we get \( p_1^s - p_2^s = \frac{2(a-b_1c+b_2c-k_1e_0+k_2e_0)(-k_1k_2+2b_2 t)(-k_1k_2-k_2^2+2b_2 t)}{u_3-2u_2^2-u_2^2 v^2} \), if \( t \in (0, \frac{k_1 k_2^2}{2 b_2}) \) or \( \frac{k_1 k_2^2+k_2^2}{2 b_2} \), then \( p_1^s > p_2^s \); if \( \frac{k_1 k_2^2}{2 b_2} \), then \( p_1^s = p_2^s \) \( t \in (\frac{k_1 k_2^2}{2 b_2}, \frac{k_1 k_2^2+k_2^2}{2 b_2}) \), then \( p_1^s < p_2^s \).

2) From proposition 2, we get \( e_1^s - e_2^s = \frac{2(a-b_1c+b_2c-k_1e_0+k_2e_0)(b_2 t-k_1 k_2)(b_2 t-k_1 k_2)}{u_3-2u_2^2-u_2^2 v^2} \), if \( (2b_1 k_2+b_2 k_2-b_2 k_1)(2b_2 t-k_1 k_2) > 0 \), then \( e_1^s > e_2^s \); if \( 2b_1 k_2+b_2 k_2-b_2 k_1 = 0 \) or \( 2b_2 t-k_1 k_2 = 0 \), then \( e_1^s = e_2^s \); if \( (2b_1 k_2+b_2 k_2-b_2 k_1)(2b_2 t-k_1 k_2) < 0 \), then \( e_1^s < e_2^s \). This completes the proof.

**Proof of Proposition 6**

From (13) and (14), we get \( \pi^n_1(p_{t_1}^o, e_{t_1}^o) - \pi^n_2(p_{t_2}^o, e_{t_2}^o) = \frac{4(a-b_1c+b_2c-k_1 e_0+k_2 e_0)^2 v^2 v (2b_2 t-k_1 k_2-b_2 k_2 t)(3 U V+2 U^2+k_2^2 V)^2}{u_3-2v^2 u-k_2^2 v^2} \), if \( (b_2 k_1 k_2-b_1 k_2^2-b_2 k_2^2 t)(3 U V+2 U^2+k_2^2 V) > 0 \), then \( \pi^n_1(p_{t_1}^o, e_{t_1}^o) > \pi^n_2(p_{t_2}^o, e_{t_2}^o) \); if \( t = \frac{b_2 k_1 k_2-b_1 k_2^2}{b_2^2} \) or \( \frac{k_1 k_2^2}{2 b_2} \) or \( 3 U V+2 U^2+k_2^2 V = 0 \), then \( \pi^n_1(p_{t_1}^o, e_{t_1}^o) = \pi^n_2(p_{t_2}^o, e_{t_2}^o) \); if \( (b_2 k_1 k_2-b_1 k_2^2-b_2 k_2^2 t)(3 U V+2 U^2+k_2^2 V) < 0 \), then \( \pi^n_1(p_{t_1}^o, e_{t_1}^o) < \pi^n_2(p_{t_2}^o, e_{t_2}^o) \). This completes the proof.

**Proof of Proposition 7**

(1) For the Stackelberg leader with more market power

1) \( p_1^s - p^n = 2 t \frac{v^2(u+k_2^2)}{(u-v)(u-2v^2-u^2 v^2 k_2^2)} \). Due to \( U > 0 \), \( U-V > 0 \), \( U^3-2UV^2-V^2 k_2^2 > 0 \) and \( V^2 \geq 0 \), \( \frac{v^2(u+k_2^2)}{(u-v)(u-2v^2-u^2 v^2 k_2^2)} \geq 0 \). Therefore, \( p_1^s \geq p^n \). \( e_1^s - e^n = \frac{k_1(u+k_2^2)}{(u-v)(u-2v^2-u^2 v^2 k_2^2)} (2b_2 t-k_1 k_2)(y-2b_2 t+k_1 k_2) \), where \( y = \frac{k_1(u^2-v^2)}{k_1(u+k_2^2)} > 0 \). Due to \( U > 0 \) and \( U^2-V^2 > 0 \), \( \gamma > 0 \). Because of \( U-V > 0 \) and \( U^3-2UV^2-V^2 k_2^2 > 0 \),
\[
\frac{k_1(U+k^2)}{(U-V)(U^3-2UV^2-V^2k^2)} > 0. \text{ Therefore, if } t \in \left(0, \frac{k_1k_2}{2b_2}\right), \text{ then } e^*_1 < e^n; \text{ if } t \in \left[\frac{k_1k_2}{2b_2}, \frac{k_1k_2+y}{2b_2}\right], \text{ then } e^*_1 \geq e^n; \text{ if } t \in \left(\frac{k_1k_2+y}{2b_2}, +\infty \right), \text{ then } e^*_1 < e^n.
\]

2) \[\pi^*_1(p^*_1, e^*_1) - \pi^n(p^n, e^n) = \frac{v^2(U^2+Uk^2)}{(U-V)(U^3-2UV^2-V^2k^2)} \]
Due to \(U > 0, U - V > 0, U^3 - 2UV^2 - V^2k^2 > 0\) and \(V^2 \geq 0, \frac{v^2(U^2+Uk^2)}{(U-V)(U^3-2UV^2-V^2k^2)} \geq 0\). Therefore, \[\pi^*_1(p^*_1, e^*_1) \geq \pi^n(p^n, e^n).\]

(2) For the Stackelberg follower with less market power

1) \[p^*_2 - p^n = \frac{(2b_2t-k_1k_2)(V^2+Uk^2)}{(U-V)(U^3-2UV^2-V^2k^2)} \text{ and } e^*_2 - e^n = -\frac{(2b_2t-k_1k_2)(V^2+Uk^2)}{(U-V)(U^3-2UV^2-V^2k^2)} \]
Due to \(U > 0, U - V > 0, U^3 - 2UV^2 - V^2k^2 > 0\) and \(V^2 \geq 0, \frac{v^2(U^2+Uk^2)}{(U-V)(U^3-2UV^2-V^2k^2)} \geq 0\). Therefore, if \(t \in \left(0, \frac{k_1k_2}{2b_2}\right), \text{ then } p^*_2 \leq p^n \text{ and } e^*_2 \geq e^n; \text{ if } t \in \left(\frac{k_1k_2}{2b_2}, +\infty \right), \text{ then } p^*_2 > p^n \text{ and } e^*_2 < e^n.\]

2) \[\pi^*_2(p^*_2, e^*_2) - \pi^n(p^n, e^n) = \frac{(V^2+Uk^2)^2}{(U-V)(U^3-2UV^2-V^2k^2)} \frac{(2b_2t-k_1k_2)(2b_2t-k_1k_2+\delta)}{V^2+Uk^2} \]
where \(\delta = \frac{2(U^3-2UV^2-V^2k^2)}{V^2+Uk^2} \). Due to \(U > 0 \text{ and } U^2 - V^2 > 0, \delta > 0\). When \(k_1k_2 - \delta > 0, \text{ if } t \in \left(0, \frac{k_1k_2-\delta}{2b_2}\right), \text{ then } \pi^*_2(p^*_2, e^*_2) \geq \pi^n(p^n, e^n); \text{ if } t \in \left[\frac{k_1k_2-\delta}{2b_2}, \frac{k_1k_2}{2b_2}\right], \text{ then } \pi^*_2(p^*_2, e^*_2) \leq \pi^n(p^n, e^n); \text{ if } t \in \left(\frac{k_1k_2}{2b_2}, +\infty \right), \text{ then } \pi^*_2(p^*_2, e^*_2) > \pi^n(p^n, e^n)\).

This completes the proof.

References


