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Analysis of Fiber-Optic Strain-Monitoring Data from a Prestressed Concrete Bridge

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Supplemental Data

Calculations of the effects of steel relaxation, concrete shrinkage and creep are presented, according to both *Eurocode 2: Design of Concrete Structures* CEN (2004) and the approach given by Collins and Mitchell (1997). Further details are also given in Webb (2010, 2014).

S1 Eurocode approach

S1.1 Prestressing Steel Calculations

For class 2 low relaxation prestressing tendons, as used in this case, Eurocode 2 gives an expression for the loss of stress due to relaxation, $\Delta\sigma_{pr}$, at time t (measured in hours) (CEN, 2004, Eq. 3.29).

$$\frac{\Delta\sigma_{pr}}{\sigma_{pi}} = 0.66\rho_{1000}e^{9.1\mu}\left(\frac{t}{1000}\right)^{0.75(1-\mu)}10^{-5} \qquad \text{(S1)}$$

σ_{pi} is the initial stress in the steel after the short term losses discussed in the main paper. ρ_{1000} is the relaxation loss after 1,000 hrs, which is taken as 2.5% for class 2 prestressing steel (CEN, 2004, cl 3.3.2(6)) and μ is defined by the equation given in CEN (2004) cl 3.3.2(7), reproduced as Eq. S2:

$$\mu = \frac{\sigma_{pi}}{f_{pk}} \qquad \text{(S2)}$$

where f_{pk} is the characteristic value the tensile strength of the steel. Since the area of the steel tendons, A_s, is known, the loss in prestressing force due to relaxation can be calculated using Eq. S3:

$$\Delta P_r = A_s \Delta\sigma_{pr} \quad \text{(S3)}$$

S1.2 Concrete Shrinkage Calculations

Over time concrete loses moisture and hence decreases in volume, an effect known as shrinkage. This is normally assumed to produce a uniform compressive strain, ε_{cs}, throughout the concrete, the rate of which depends on the relative humidity and the surface area of the specimen. There are two components of shrinkage strain, the drying shrinkage strain, ε_{cd}, and the autogenous shrinkage strain, ε_{ca}. given as Eq. S4 below (CEN, 2004, Eq. 3.8):

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca} \quad \text{(S4)}$$

The final value of autogenous shrinkage strain, $\varepsilon_{ca}(\infty)$, depends on concrete strength given as Eq. S5 below (CEN, 2004, Eq. 3.12):

$$\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10)10^{-6} \quad \text{(S5)}$$

The autogenous shrinkage strain develops rapidly during the hardening process (CEN, 2004, Eq. 3.11 and 3.13) (reproduced here as Eq. S6 and S7):

$$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty) \quad \text{(S6)}$$

Where:

$$\beta_{as}(t) = 1 - e^{-0.2t^{0.5}} \quad \text{(S7)}$$

with t, in this case, measured in days.

Drying shrinkage strain develops more slowly as water gradually migrates through the hardened concrete and depends on a number of factors given as Eq. S8 (CEN, 2004, Eq. 3.9):

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) k_h \varepsilon_{cd,0} \quad \text{(S8)}$$

The notional size of the cross section, h_0, is given by Eq. S9 below (see the note below Eq. 3.10 in CEN (2004), p. 33):

$$h_0 = \frac{2A_c}{u} \quad \text{(S9)}$$

where A_c is the cross sectional area of the concrete and u is the length of the perimeter exposed to drying. Beam sections with a large perimeter will have a much greater area over which moisture loss can occur. The coefficient, k_h, in Eq. S8 depends on the notional size of the cross section and according to CEN (2004, Table 3.3) when h_0 = 100, k_h = 1.00; when h_0 = 200, k_h = 0.85; when h_0 = 300, k_h = 0.75 and when h_0 is greater than or equal to 500, k_h = 0.70. $\beta_{ds}(t,t_s)$ is a coefficient which describes the development of drying shrinkage with time given as Eq. S10 (CEN, 2004, Eq. 3.10):

$$\beta_{ds}(t, t_s) = \frac{t - t_s}{(t - t_s) + 0.04\sqrt{(h_0)^3}} \quad \text{(S10)}$$

where t_s is the age of the concrete in days at the end of curing, when drying shrinkage begins. The basic drying shrinkage strain, $\varepsilon_{cd,0}$, is given here as Eq. S11 (based on CEN, 2004, Eq. B.11):

$$\varepsilon_{cd,0} = 0.85\left[(220 + 110\alpha_{ds1})e^{\left(-\alpha_{ds2}\frac{f_{cm}}{10}\right)}\right]10^{-6}\beta_{RH} \quad \text{(S11)}$$

where f_{cm} is the mean cylinder strength of the concrete (see Table 3.1 in CEN, 2004) and Eq. S12:

$$f_{cm} = f_{ck} + 8 \quad \text{(S12)}$$

and β_{RH} is a coefficient which depends on the relative humidity, *RH* (see CEN, 2004, Eq. B.12 and Eq. S13):

$$\beta_{RH} = 1.55\left[1 - \left(\frac{RH}{100}\right)^3\right] \quad \text{(S13)}$$

The constants α_{ds1} and α_{ds2} depend on the class of cement being used, and in this case have the values 6 and 0.11 respectively (see cl B.2(1) in CEN, 2004, p. 204).

The total uniform shrinkage strain, ε_{cs}, can then be used to calculate the loss in prestressing force due to shrinkage, given as Eq. S14:

$$\Delta P_s = \varepsilon_{cs} E_s A_s \qquad \text{(S14)}$$

S1.3 Creep of Concrete

The final effect to consider is creep of concrete, where the strains increase over time for concrete under a constant stress, primarily due to further losses of moisture. This is particularly noticeable in prestressed structures, since the concrete is under a large compressive stress due to the force in the tendons. Creep behavior is normally modelled by calculating an effective Young's Modulus, $E_{c,eff}$ for the concrete which decreases with time (given as Eq. S15):

$$E_{c,eff} = \frac{E_c}{\phi(t,t_0)} \qquad \text{(S15)}$$

The creep coefficient, $\phi(t,t_0)$, is given by Eq. S16 (CEN, 2004, Eq. B.1):

$$\phi(t,t_0) = \phi_0 \beta_c(t,t_0) \qquad \text{(S16)}$$

t_0 is the age of the concrete at the point when the load was first applied, although this must first be modified to take into account the effect of different types of cement, as given by Eq. S17 (CEN, 2004, Eq. B.9):

$$t_0 = t_{0,T}\left(\frac{9}{2+{t_{0,T}}^{1.2}} + 1\right)^{\alpha} \geq 0.5 \qquad \text{(S17)}$$

where $t_{0,T}$ is the actual age of the concrete when the load was applied and α depends on the cement class used, in this case it has a value of 1.

ϕ_0 is the notional creep coefficient which is given by Eq. S18 (CEN, 2004, Eq. B.2):

$$\phi_0 = \phi_{RH} \beta(f_{cm}) \beta(t_0) \qquad \text{(S18)}$$

This comprises a number of factors which are described in Table S1.

Table S1: Factors affecting notional creep coefficient

Factor	Allows for effect of	Reference (CEN, 2004)
$\phi_{RH} = \left[1 + \frac{1 - \frac{RH}{100}}{0.1\sqrt[3]{h_0}}\alpha_1\right]\alpha_2$ for f_{cm} > 35MPa	relative humidity	Eq. B.3b, p. 202
$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$	concrete strength	Eq. B.4, p. 202
$\beta(t_0) = \frac{1}{0.1 + {t_0}^{0.2}}$	concrete age at loading	Eq. B.5, p. 202
$\alpha_1 = \left(\frac{35}{f_{cm}}\right)^{0.7}$	concrete strength	Eq. B.8c, p. 203
$\alpha_2 = \left(\frac{35}{f_{cm}}\right)^{0.2}$	concrete strength	Eq. B.8c, p. 203

$\beta_c(t,t_0)$ is then used to describe the development of creep with time after loading, given as Eq. S19 (CEN, 2004, Eq. B.7):

$$\beta_c(t, t_0) = \left[\frac{t - t_0}{\beta_H + t - t_0}\right]^{0.3} \quad \text{(S19)}$$

Where,

$$\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 \leq 1500\alpha_3 \quad \text{(S20)}$$

(CEN, 2004, Eq. B.8b)

and

$$\alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5} \quad \text{(S21)}$$

(CEN, 2004, Eq. B.8c).

There are now three components which must be added together to calculate the predicted strains in the beam section: strains due to the self-weight of the concrete can be calculated, using $E_{c,eff}$, to account for the effects of creep. Secondly, strains due to the now reduced prestressing force can be calculated as before, using equations 4, 5 and 6 in the main paper. Again the effective concrete modulus, $E_{c,eff}$, should be used to account for the effects of creep. Finally, the uniform shrinkage strain, ε_{cs}, needs to be added.

S2 Collins and Mitchell (1997) approach

In this section calculations are presented following the approach given by Collins and Mitchell (1997), however some alterations to the notation have been made to ensure consistency throughout this paper.

S2.1 Prestressing steel calculations

Collins and Mitchell (1997) give an expression for the stress, σ_p, in prestressing tendons at time t (measured in hours) given as Eq. S22 below (Collins and Mitchell, 1997, Eq. 3-28):

$$\frac{\sigma_p}{\sigma_{pi}} = 1 - \frac{\log t}{10}\left(\frac{\sigma_{pi}}{\sigma_y} - 0.55\right) \tag{S22}$$

where σ_{pi} is the initial stress in the steel, t is the time in hours, and σ_y is the yield strength of the steel, taken as 90% of the ultimate strength of the steel for low relaxation strands. The loss in prestressing force due to relaxation can then be calculated as in Eq. S3.

S2.2 Concrete shrinkage calculations

The rate at which moisture is lost is assumed to depend on the surface area of concrete exposed to the air and the humidity of the air. This loss of moisture produces a uniform compressive strain, ε_{sh}, in the concrete given as Eq. S23 below (Collins and Mitchell, 1997, Eq. 3-19):

$$\varepsilon_{sh} = k_s k_h \left(\frac{t}{35+t}\right) \times 0.00051 \times 1.2 \qquad \text{(S23)}$$

Where t is the age of the concrete in days, k_h is a correction factor for relative humidity, and k_s is a time dependent correction factor for size. k_h and k_s are obtained from Collins and Mitchell (1997) Fig. 3-18. The extra factor of 1.2 is to account for the increased shrinkage due to early exposure of the beam when the concrete was still moist (cf. Collins and Mitchell, 1997, p. 75).

S2.3 Creep of concrete

Creep behavior is modelled by calculating an effective Young's Modulus for the concrete which decreases with time (Eq. S24) (Collins and Mitchell, 1997, Eq. 3-12):

$$E_{c,eff} = \frac{E_{ci}}{1+\phi(t,t_i)} \qquad \text{(S24)}$$

The creep coefficient, ϕ, is given by Eq. S25 (Collins and Mitchell, 1997, Eq. 3-10):

$$\phi(t,t_i) = 3.5 k_c k_f \left(1.58 - \frac{H}{120}\right) t_i^{-0.118} \frac{(t-t_i)^{0.6}}{10+(t-t_i)^{0.6}} \qquad \text{(S25)}$$

where t is the time in days, t_i is the age of the concrete at release, and H is the average relative humidity. k_c is a time dependent correction factor for size from Collins and Mitchell (1997) Fig 3-12, p. 69 and k_f is a correction factor to account for concrete strength, given by Eq. S26 (Collins and Mitchell, 1997, Eq. 3-11):

$$k_f = \frac{1}{0.67+\left(\frac{f'_c}{62}\right)} \qquad \text{(S26)}$$

where f_c' is the cylinder strength of the concrete.

S3 Composite Section Analysis Calculations

For the in-situ concrete, the changes in strain, $\Delta\varepsilon$ (Eq. S28), and curvature, $\Delta\kappa$ (Eq. S27), since the section became composite are only caused by new self equilibrating moments:

$$\Delta\kappa = -\frac{M_p}{E_i I_i} + \frac{X_p a}{E_i I_i} \quad \text{(S27)}$$

$$\Delta\varepsilon = \frac{M_p a}{E_i I_i} - \frac{X_p a^2}{E_i I_i} - \frac{X_p}{E_i A_i} + \varepsilon_{cs,i} \quad \text{(S28)}$$

where E_i is the effective modulus of the in-situ concrete, I_i, is the second moment of area of the in-situ deck slab, A_i is the cross sectional area of the deck slab, and $\varepsilon_{cs,i}$ is the shrinkage strain in the in-situ concrete slab, calculated as before, but with different values of t and t_0 to take into account the different age of the in-situ concrete. The interaction between the precast and in-situ concrete can be analyzed by superimposing self equilibrating moments and longitudinal shear forces along the join between the two layers of concrete (Figure S1).

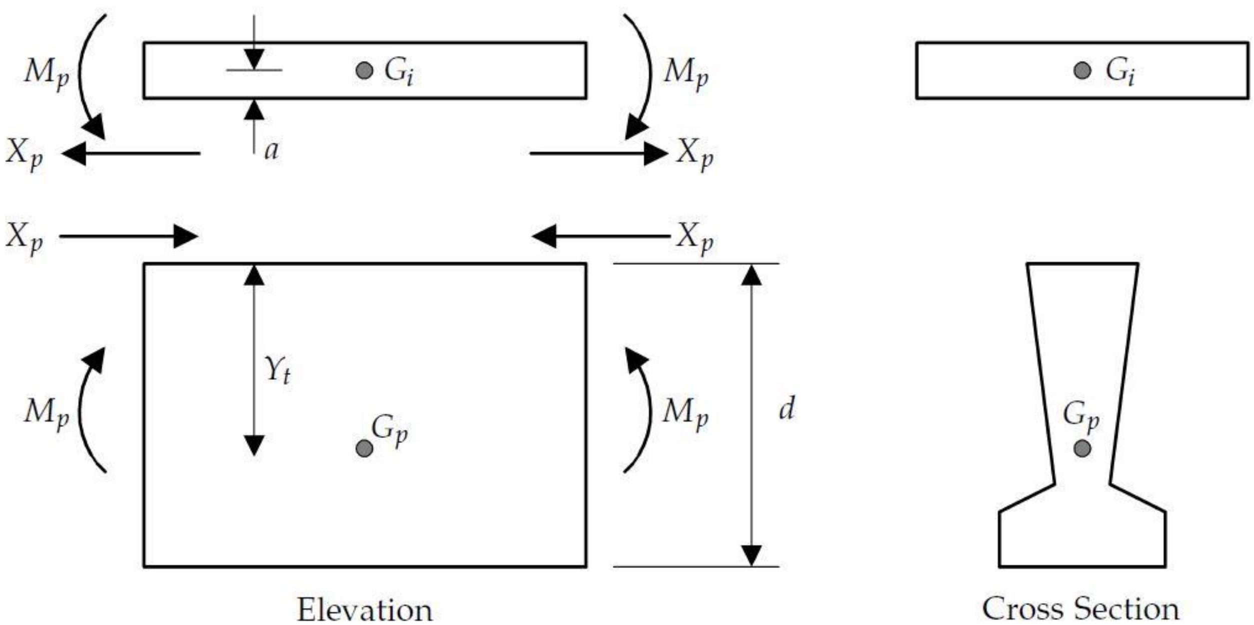

Figure S1: Self equilibrating force system due to differential creep and shrinkage

Once the deck concrete has hardened the two layers are joined together and therefore the changes in strain and curvature along the joint must be the same in both layers. These two conditions of compatibility can be used to derive expressions to determine M_p and X_p.

For the precast concrete, the additional curvature and strain since the section became composite is found as given in Eq. S29 and Eq. S30:

$$\Delta\kappa = \frac{M_p}{E_p I_p} + \frac{X_p Y_t}{E_p I_p} + \frac{\varepsilon_t - \varepsilon_b}{d} - \frac{\varepsilon_{tc} - \varepsilon_{bc}}{d} \quad \text{(S29)}$$

$$\Delta\varepsilon = \frac{M_p Y_t}{E_p I_p} + \frac{X_p {Y_t}^2}{E_p I_p} + \frac{X_p}{E_p A_p} + \varepsilon_t - \varepsilon_{tc} \quad \text{(S30)}$$

where: E_p is the effective modulus of the precast concrete, I_p is the second moment of area of the precast beam and A_p is the cross sectional area of the precast beam. ε_t and ε_b are the strains at the top and bottom fibers of the precast beam which would be caused by the applied loads (including creep and shrinkage) if no composite action occurred. ε_{tc} and ε_{bc} are the top and bottom fiber strains at the time the section became composite, and d is the depth of the precast beam.

To satisfy compatibility of curvature, Eq. S27 and Eq. S29 can be equilibrated:

$$-\frac{M_p}{E_i I_i} + \frac{X_p a}{E_i I_i} = \frac{M_p}{E_p I_p} + \frac{X_p Y_t}{E_p I_p} + \frac{\varepsilon_t - \varepsilon_b}{d} - \frac{\varepsilon_{tc} - \varepsilon_{bc}}{d}$$

$$M_p\left(-\frac{1}{E_i I_i} - \frac{1}{E_p I_p}\right) + X_p\left(\frac{a}{E_i I_i} - \frac{Y_t}{E_p I_p}\right) = \frac{\varepsilon_t - \varepsilon_b - \varepsilon_{tc} + \varepsilon_{bc}}{d} \quad \text{(S31)}$$

Compatibility of strain can similarly be achieved, by equilibrating Eq. S28 and Eq. S30:

$$\frac{M_p a}{E_i I_i} - \frac{X_p a^2}{E_i I_i} - \frac{X_p}{E_i A_i} + \varepsilon_{cs,i} = \frac{M_p Y_t}{E_p I_p} + \frac{X_p {Y_t}^2}{E_p I_p} + \frac{X_p}{E_p A_p} + \varepsilon_t - \varepsilon_{tc}$$

$$M_p\left(\frac{a}{E_i I_i} - \frac{Y_t}{E_p I_p}\right) + X_p\left(-\frac{a^2}{E_i I_i} - \frac{1}{E_i A_i} - \frac{{Y_t}^2}{E_p I_p} - \frac{1}{E_p A_p}\right) = \varepsilon_t - \varepsilon_{tc} - \varepsilon_{cs,i} \quad \text{(S32)}$$

These two equations can be written in matrix form and solved to determine the values of M_p and X_p (Eq. S33):

$$\begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix} \begin{bmatrix} M_p \\ X_p \end{bmatrix} = \begin{bmatrix} \frac{\varepsilon_t - \varepsilon_b - \varepsilon_{tc} + \varepsilon_{bc}}{d} \\ \varepsilon_t - \varepsilon_{tc} - \varepsilon_{cs,i} \end{bmatrix} \qquad \text{(S33)}$$

where

$$\alpha = -\frac{1}{E_i I_i} - \frac{1}{E_p I_p} \; ; \; \beta = \frac{a}{E_i I_i} - \frac{Y_t}{E_p I_p} \; ; \; \gamma = -\frac{a^2}{E_i I_i} - \frac{1}{E_i A_i} - \frac{{Y_t}^2}{E_p I_p} - \frac{1}{E_p A_p} \qquad \text{(S34)}$$

To determine the total strains in the beam, this force and moment are applied to the precast section and the extra strains produced can then be added to the strains found previously, ε_t and ε_b.

S4 Continuous Beam Analysis

The analysis so far has considered each of the bridge's spans as a simply supported beam. However, when the in-situ concrete deck was poured, concrete was also cast in the gaps between the beams at the tops of the internal piers. This means the bridge must now been treated as a continuous structure over all three spans. Extra moments are then induced at the intermediate supports which oppose any further creep and shrinkage strains. These extra moments are required to ensure compatibility of rotations along the length of the beam.

Firstly, the rotations at the ends of each simply supported span are found by superimposing the effects of a uniform load and a constant moment (Figure S2).

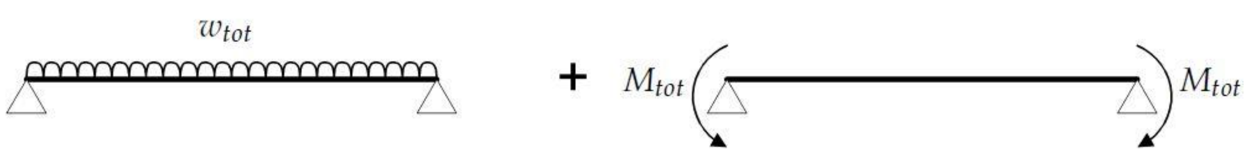

Figure S2: End rotations of beam

The uniform load, w_{tot}, consists of the self-weight of the beam and the in-situ concrete deck. M_{tot} comprises the moments generated by the prestressing force as well as the effects of M_p and X_p due to differential creep and shrinkage. The total end rotation, θ, is then given by Eq. S35:

$$\theta = \frac{M_{tot}L}{2E_pI_p} - \frac{w_{tot}L^3}{24E_pI_p} \quad \text{(S35)}$$

where E_p is the effective modulus of precast concrete.

If it is assumed that the bridge became a continuous structure when the deck was added, then moments will be induced at the supports to ensure that any further rotations, $\Delta\theta$, satisfy compatibility over the supports (Eq. S36):

$$\Delta\theta = \theta - \theta_c \quad \text{(S36)}$$

where θ_c is the rotation at the time the beams became continuous. Figure S3 shows the three spans, the new induced moments, and the rotations they cause.

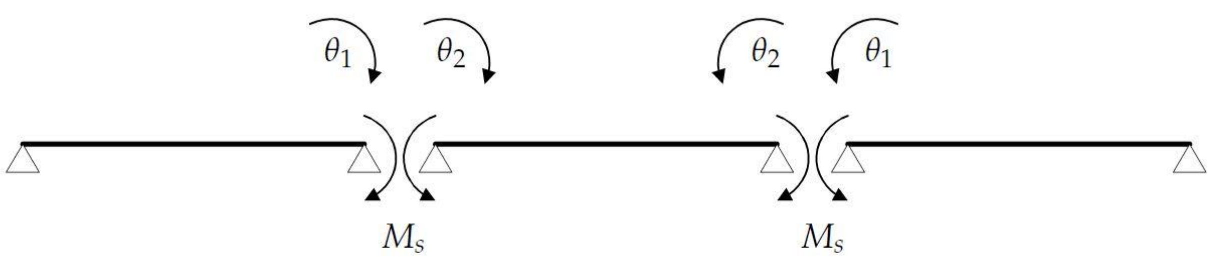

Figure S3: Extra moments induced at supports

Since the bridge is almost symmetrical the two induced moments are assumed to be equal. As these moments are applied to the entire composite cross section not just the precast beam, it is necessary to calculate the second moment of area of the composite cross section. Due to differences in stiffness between the precast and in-situ concrete, the area of the in-situ concrete must be multiplied by the modular ratio of the two concretes, E_i/E_p.

The rotations θ_1 and θ_2 can then be calculated from standard results (Eq. S37 and S38):

$$\theta_1 = \frac{M_sL}{3E_pI_c} \quad \text{(S37)}$$

$$\theta_2 = -\frac{M_S L}{3E_p I_c} - \frac{M_S L}{6E_p I_c} = -\frac{M_S L}{2E_p I_c} \tag{S38}$$

where I_c is the second moment of area of the composite section about its centroid.

M_s can then be determined by enforcing compatibility of rotations (Eq. S39, S40 and S41):

$$\Delta\theta + \theta_1 = -\Delta\theta + \theta_2 \tag{S39}$$

$$\Delta\theta + \frac{M_S L}{3E_p I_c} = -\Delta\theta - \frac{M_S L}{2E_p I_c} \tag{S40}$$

$$M_S = -\frac{12}{5}\frac{E_p I_c}{L}\Delta\theta \tag{S41}$$

This moment is applied to the entire composite cross section, so the extra strains that it causes can be calculated using the composite section's properties. These strains can then be added to the previous results to give the total resulting strain state.

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