



Moylett, D., Linden, N., & Montanaro, A. (2017). *Quantum speedup of the Travelling Salesman Problem for bounded-degree graphs*. Poster session presented at Heilbronn Quantum Algorithms Day 2017, Bristol, United Kingdom.

Publisher's PDF, also known as Version of record

[Link to publication record in Explore Bristol Research](#)
PDF-document

This is the final presented version of the poster.

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

Quantum speedup of the Travelling Salesman Problem for bounded-degree graphs

Dominic J. Moylett^{1,2,3}, Noah Linden⁴ and Ashley Montanaro⁴

¹Quantum Engineering Technology Labs, H. H. Wills Physics Laboratory and Department of Electrical & Electronic Engineering, University of Bristol, BS8 1FD, United Kingdom

²Quantum Engineering Centre for Doctoral Training, H. H. Wills Physics Laboratory and Department of Electrical & Electronic Engineering, University of Bristol, BS8 1FD, United Kingdom

³Heilbronn Institute for Mathematical Research, University of Bristol, BS8 1SN, United Kingdom

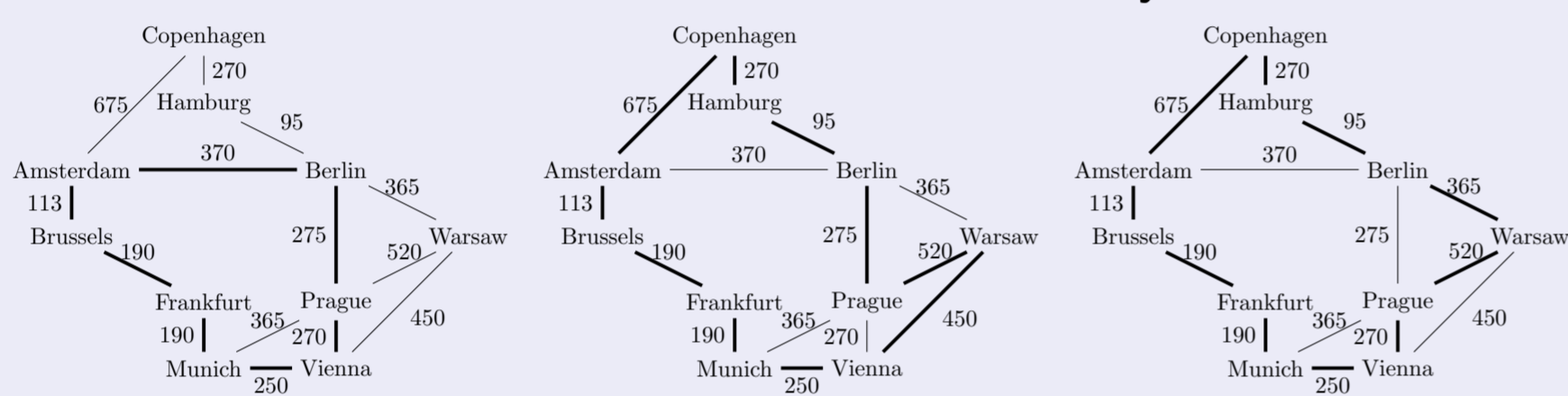
⁴School of Mathematics, University of Bristol, BS8 1TW, United Kingdom

Main Results

- ▶ We show near-quadratic speedups for the Travelling Salesman Problem (TSP) when the degree of any vertex is at most 4.
- ▶ This is through applying a quantum speedup for backtracking [Mon15] to two TSP algorithms [XN16a,XN16b].
- ▶ We then demonstrate polynomial speedups up to degree-6.
- ▶ See Physical Review A 95(3), 032323 (2017) [arXiv:1612.06203] for further details.

1. The Travelling Salesman Problem

- ▶ Let G be a graph with n vertices and m edges.
- ▶ A cycle H on G is *Hamiltonian* if it visits every vertex in G .
- ▶ The TSP is to find the shortest Hamiltonian cycle.



(a) ✗ Not a Hamiltonian cycle. (b) ✗ Not the shortest Hamiltonian cycle. (c) ✓ Solution to the TSP. (Length: 2938 Minutes)

- ▶ The best general classical algorithms take exponential time in n .

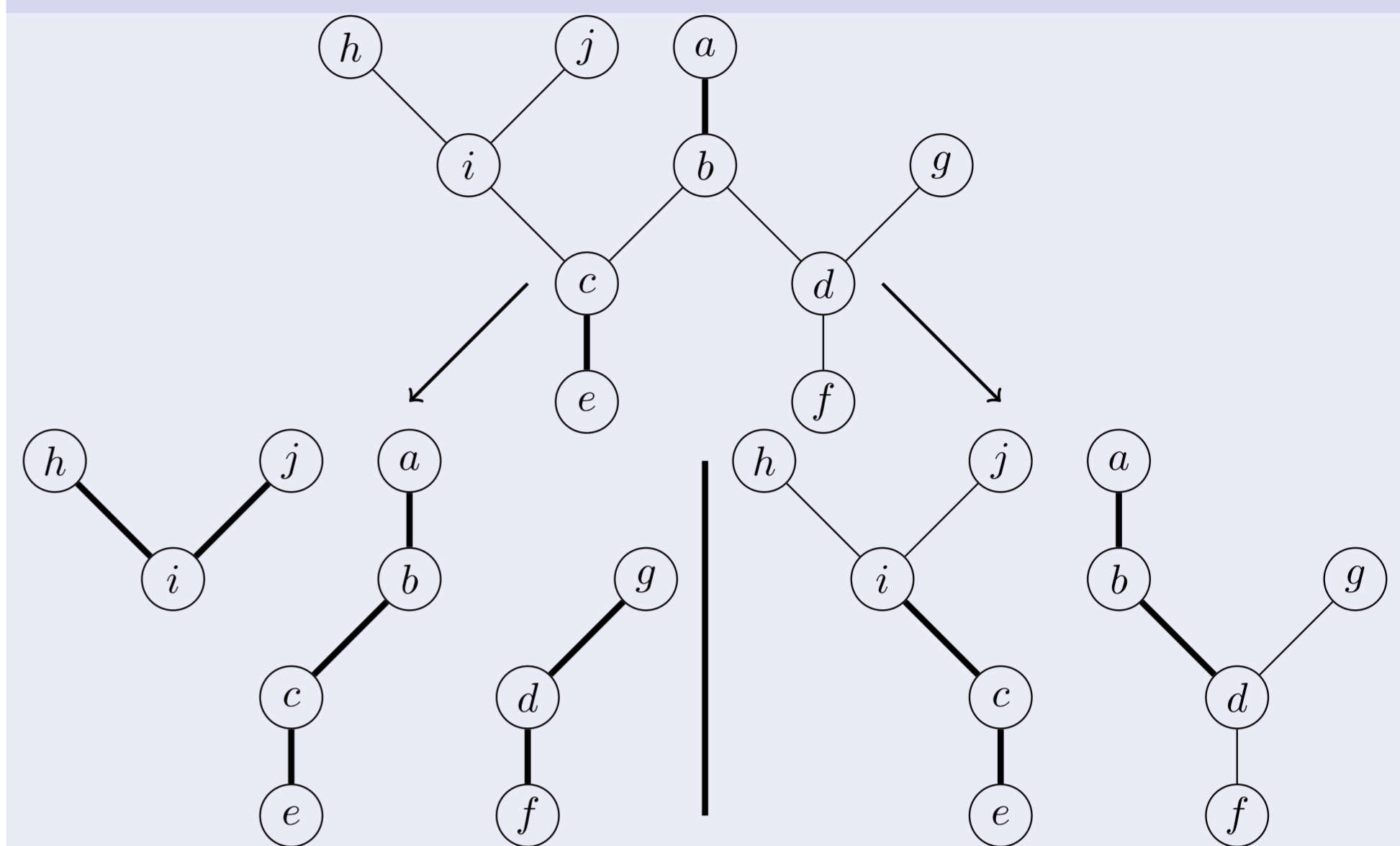
2. Backtracking algorithms

- ▶ Backtracking algorithms are a way of solving constraint satisfaction problems.
- ▶ They have two parts:
 1. A predicate, which checks if the constraints are satisfiable;
 2. and a heuristic, which chooses the next variable to assign.
- ▶ When called with a partial assignment, the predicate checks if the constraints are satisfiable. If not, we return.
- ▶ Otherwise, the heuristic picks a variable which we assign a value to and recursively call ourselves with this new partial assignment.
- ▶ Montanaro [Mon15] developed a quantum backtracking algorithm which has a quadratic speedup for finding a solution.

3. Quantum speedup for degree-3 graphs

- ▶ Backtracking algorithms can solve the TSP with “forced” edges which we must travel down and “removed” edges which we must avoid travelling on.
- ▶ The predicate checks if a Hamiltonian cycle is possible, and the heuristic selects another edge to force or remove.
- ▶ The best backtracking algorithm on degree-3 graphs runs in $O^*(2^{3n/10})$ time and polynomial space [XN16a].
- ▶ We apply [Mon15] to this algorithm to find a Hamiltonian cycle, failing to find one when one exists with probability δ , in $O^*(2^{3n/20} \log(1/\delta))$ time, where O^* hides polynomial factors.
- ▶ We find the shortest Hamiltonian cycle with bounded error (finding a sub-optimal cycle or no cycle) via binary search with $O(\log L \log \log L)$ overhead, where L is the longest edge length.

4. Example backtracking step



- ▶ Forcing bc , as shown on the left, means that b and c are incident to two forced edges, so ci and bd are removed. Now d and i are of degree 2, so edges df , dg , hi and ij are forced.
- ▶ Removing bc , as shown on the right, means that b and c are of degree 2, so edges bd and ci are now forced.

5. Expanding to higher-degree graphs

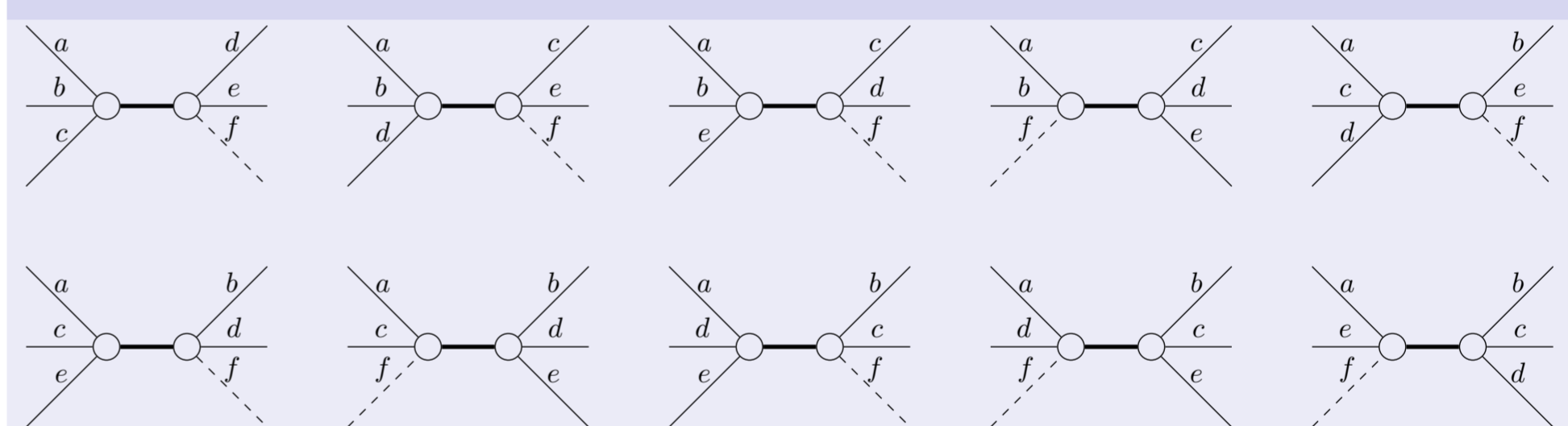


Figure: Ways of splitting a vertex of degree 5 or 6 into two lower-degree vertices.

- ▶ For degree-4 graphs, we apply the same technique to the algorithm of [XN16b] in $O^*(1.301^n \log L \log \log L)$ time.
- ▶ Other speedups can be found by breaking higher-degree vertices into degree 4 vertices connected by forced edges.
- ▶ We find the shortest way of splitting each vertex via [DH99].
- ▶ For degree-5/6 graphs, there are 10 ways of splitting each vertex, of which 6 will preserve the shortest Hamiltonian cycle. Thus we get an additional $O((10/6)^{n/2})$ overhead.
- ▶ For degree-7 graphs, this method is slower than classical algorithms for the general TSP [HK62, Bjö14].

References

- [Bjö14] A. Björklund, SIAM Journal on Computing, 43(1):280–299 (2014)
- [DH99] C. Dürr and P. Høyer, arXiv:quant-ph/9607014 (1999)
- [HK62] M. Held and R. Karp, Journal of the Society for Industrial and Applied Mathematics, 10(1):196–210, (1962)
- [Mon15] A. Montanaro, arXiv:1509.02374 (2015)
- [XN16a] M. Xiao and H. Nagamochi, Algorithmica 74(2):713–741, (2016)
- [XN16b] M. Xiao and H. Nagamochi, Theory of Computing Systems, 58(2):241–272, (2016)