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Patents as Negotiating Assets: Patenting versus Secrecy for Startups*

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We analyse a sequential innovation model and show that relatively narrow patent rights can facilitate a market in which startups' patents are traded as negotiating assets. In this market, the trade of patents, on top of monopoly profits, conveys an extra surplus from the patents' capacity to affect future tech-transfer negotiations. This surplus, which stems from a patent's potential ability to exclude infringers and the corresponding enforcement spillovers that patents confer, may incentivize innovations that would not have been possible under trade secrecy, improving social welfare. (JEL Codes: O31, O32, O34)

Keywords: Patents, trade secrets, startups, takeovers.

1 Introduction

A boost in patenting has been apparent in high-tech industries despite declining average value of patents and the perception that patents are ineffective in appropriating the returns from R&D (Hall *et al.*, 2014; Cohen *et al.*, 2000). This phenomenon is known as the “patent paradox” (Parchomovsky and Wagner, 2005). For example, IBM was granted a record 8,088 patents in 2016¹ to add to its 50,000 strong patent portfolio, as well as having accumulated patents by acquiring more than 150 firms since 2000. Google has also expanded its 30,000 strong portfolio through active acquisition of 200 mainly startup firms since it was founded in 1998, at a rate of one per week of late. Cisco has made 132 acquisitions since 2000, and Facebook and Twitter more than 50 each since 2010.

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¹See <<https://www-03.ibm.com/press/us/en/presskit/42874.wss>>.

Such recent phenomena surrounding patents can be attributed to the multifaceted role patents may play owing to the inherent difficulty of demarcating technological boundaries. In particular, patents can often be protected more efficiently in an intellectual property dispute as part of a bundle, especially when the technology in question is cumulative (Hall and Ziedonis, 2001). Lanjouw and Schankerman (2004) refer to such economies of scale as “enforcement spillovers”. The enhanced bargaining power of dominant incumbent firms owning large patent portfolios poses a problem for startups because a startup commonly lacks production capacity, and so often needs to rely on technology transfer for revenue. But taking out a patent makes a startup vulnerable to threats of infringement litigation when bargaining with an incumbent firm (Shapiro, 2003). As a result, it is unlikely to realize the full value of its patented technology. The startup may therefore prefer trade secrecy to a patent for fear of provoking such costly legal disputes (Graham *et al.*, 2009). Since patents are better than secrecy at promoting knowledge diffusion and technological spillover, there may be significant welfare implications, especially as startups play an outsized role in US net job creation (Hathaway, 2013).

However, the startup’s own innovation also carries the potential to wield enforcement spillover in the future and such prospects may induce strategic interactions not captured in the discussion above. This paper provides a dynamic analysis of such effects on the equilibrium outcome and the overall welfare. The main finding is that patenting allows the startup to claim a share of such potential future benefits and thus, may outperform trade secrecy in motivating startup innovation in industries where incumbent firms hold large patent portfolios. In particular, we show that overt trading of technology ownership via patents facilitates transfer of future benefits of innovation to the current startup, thereby motivating startup innovations that would not be possible otherwise.

We analyze a dynamic model of sequential innovation *à la* Scotchmer (1991), in which an incumbent with a large patent portfolio faces a sequence of startups who innovate in a cumulative fashion. It negotiates agreements on sharing technology with the startups as they arise. Its share of the available surplus increases as it accumulates more patents because then it becomes a more powerful potential litigant due to enforcement spillover. However, the available surplus consists of more than just the monopoly profit from exploiting the patent currently being bargained over—it also includes the additional revenue the incumbent expects in future technology negotiations as a consequence of its enhanced bargaining power from taking over the ownership of the current startup patent at issue. We show that the effect that the enlarged available surplus has on the startup’s bargaining share may overshadow that of its weakened legal position due to potential infringement when it takes out a patent. Consequently, trade of patents as negotiating

assets may incentivize startup innovation activities that would not have taken place under trade secrets, improving the social welfare.

Our analysis suggests that the incumbent's bargaining power should not accumulate too quickly, lest the incumbent should become too powerful a litigant too quickly, killing off the innovation incentives for startups prematurely. Nor should it accumulate this power too slowly, because then the dynamic effect feeding into the startup's bargaining share would be too weak to incentivize innovation. Overall, the need for the balanced bargaining power our model prescribes reflects the following observation: For the transfer of benefits from the future to the present to materialize, there must be a foreseeable sequence of impending innovations to be adequately incentivized and exploited. In fact, in the absence of unfolding innovations, trade secrets prevail. In terms of policy, patent protection must therefore be sufficiently narrow in its scope to allow for derivative products and applications.

The idea that relatively weak intellectual property protection promotes innovation is not new (Bessen and Maskin, 2009; Fershtman and Markovich, 2010). We revive it as a way to foster transfer of future benefits of innovation to the initial innovator through patent sales. By doing so, we also revisit a problem that was first investigated by Scotchmer in a cumulative environment, namely, the difficulty of incentivising both early and follow-on innovations at the same time owing to a holdup problem. Scotchmer proposed *ex ante* profit-sharing agreements as a possible solution (Scotchmer, 1991; Green and Scotchmer, 1995). As *ex ante* agreements are often collusive and impractical, we suggest that takeovers may work better than other *ex post* agreements when a series of small innovators operate under uncertain IP rights *à la* Lemley and Shapiro (2005). Thus, patents not only facilitate a market for ideas (Gans and Stern, 2003), but they may also function as a channel for intertemporal transfer of their value.

The choice between secrecy and patenting has been explored relatively recently in economics, e.g., by Horstmann *et al.* (1985), Anton and Yao (2004), Denicolo and Franzoni (2004), Kultti *et al.* (2006, 2007) and Kwon (2012). These papers study how patents fare relative to trade secrets for firms facing competition from similar rivals. The current paper adds to this literature by studying startups facing potential threats from dominant incumbents.

The paper is organized as follows. Section 2 provides a brief overview of the environment relevant to the model. Section 3 describes and analyses a simple static model as a benchmark. Section 4 conducts the main analysis of a dynamic model and characterizes the unique equilibrium. Section 5 presents simulations and comparative statics, followed by policy implications in Section 6. Section 7 concludes the paper.

2 The relevant facts and assumptions

Our argument is buttressed by two asymmetries. The first one is between patents and trade secrets (TS). An inventor may try to protect her innovative idea/technology from unauthorized uses by disclosing its particulars and registering ownership in the form of a patent. Due to the inherent difficulty of confining the boundaries of technologies, however, patenting risks inadvertently inviting infringement allegations from patent holders of neighbouring technologies. Alternatively, an inventor may try to keep the details of her technology hidden from outsiders under TS, which, if successful, would have the advantage of precluding potential infringement allegations.

However, sometimes TS are more of a wishful thinking. For example, pharmaceutical technologies are often unmasked via re-engineering, and this unveiling can potentially invite infringement allegations.² Lacking a registered ownership, the inventor is ill-positioned to defend her right to the disputed technology in such circumstances.³ We label a technology that reveals enough to invite infringement allegations as a “revealing” technology. A technology is “non-revealing” otherwise, i.e., if it remains concealed under TS. We assume in the formal model that a technology will be non-revealing with a commonly known probability $\theta \in (0, 1)$, and revealing with probability $1 - \theta$.

The second asymmetry is between startups and incumbents. Here, incumbents refer to established firms with an added advantage in protecting their intellectual property (IP) owing to their accumulated patent portfolios (Cf. Lanjouw and Schankerman, 2004). In contrast, startups are fledgling entities that hold only one asset, their IP, and they lack the expertise and funds to engage in expensive legal battles.⁴ Thus, incumbents are favourably positioned in legal IP disputes, which we capture in the model as an increased probability of winning an infringement lawsuit.⁵ In addition, startups lack the capability to impose their might upon other startups via infringement allegations, with a view to gradually

²The case of Adrenalin (Patent No. 730,176 of 1903) and its rival product Adrin is an early example: *Parke-Davis & Co. v. H. K. Mulford Co.*, United States Circuit Court for the Southern District of New York, 189R 95 (C.C.S.D.N.Y. 1911).

³Even though most national patent laws provide for prior user rights (allowing TS holders who were secretly using a process that is subsequently patented to continue to use it), they tend to be narrow in scope in that they do not allow for use in another jurisdiction and often do not cover improvements in the process. Consequently, the TS holder is at a disadvantage even if she was the first to invent.

⁴The startups we have in mind are not like the young Intel, or Google, who (as startups) invented game changing technologies that allowed them to compete against behemoths.

⁵Alternatively, it can be viewed as the enhanced bargaining power of the incumbent in a generalized Nash bargaining model, or as a lower litigation cost. Our results are robust to these changes as all of these have analogous effects of increasing the incumbent’s bargaining share in takeover deals.

building their own patent portfolios and competing one-on-one against incumbents.⁶

In order not to exacerbate the asymmetry between the two parties more than is necessary we assume that justice is swift, there are no preliminary injunctions, and that the incumbent lacks “blocking” patents. Assuming otherwise would empower the incumbent more at the bargaining table, as prolonged court battles and legal restraints regarding the use of the disputed technology are more damaging to the startup.⁷ Also, we do not consider the possibility of a counter-accusation of invalidity, the prospect of which reverses the power bias in the bargaining table toward the startup.⁸ These effects can be captured in our model through the parameter depicting the probability of either party prevailing in court.

Among various forms of technology transfer pertinent to our context, the aforementioned future benefit as negotiating assets can materialize only when the transfer confers sole ownership to the incumbent. This is the case with startup takeovers (or patent sales), but ownership doesn’t change hands in other frequently observed forms of technology sharing such as licensing (or cross licensing) and patent pools. To facilitate exposition, therefore, we only consider takeovers and licensing agreements from now on.

3 A static model as the benchmark

This section illustrates that patents are handicapped relative to TS in a static model. The result also acts as a benchmark for the main dynamic analysis of the next section.

Specifically, we consider two firms in an industry operating under a single cumulative technology. Firm 1 is an established incumbent, holding a patent portfolio. Firm 2 is a

⁶In such occasions the startup must compete in its role as a litigant with the incumbent. Thus, its infringement suit against a future startup will be met by simultaneous allegations from the incumbent. As the startup will have to defend its acquisition against the incumbent and its large patent portfolio, standard due diligence procedure would require the termination of takeover attempts due to outstanding legal obligations.

⁷Swift trials allow the model to abstain from elaborating on the damages. The yardstick used by courts in deriving the damages is either the accumulated royalties resulting from a hypothetical licensing agreement, or the foregone profits from the sale of the infringing good. Both of these are minimal if justice is swift. Preliminary injunctions, by halting the use of the technology, enhance the plaintiff’s negotiating power, see Lanjouw and Lerner (2001) and Lemley and Shapiro (2007).

⁸The usual counter-accusation of invalidity, even if successful, would impact only a small number of individual patents within a large portfolio, the effect of which in our model is merely the reduction of the value of filing a lawsuit for the incumbent. As such, allowing it would not change the substance of our analysis unless the possibility of invalidation is comprehensive enough for the incumbent’s expected payoff from filing a lawsuit to fall drastically.

startup who has to decide whether to invest in an R&D project that costs c , or not. We assume that c is a random variable whose value is 0 or $C > 0$ with probabilities $\eta \in (0, 1)$ and $1 - \eta$, respectively. The realized value of c is firm 2's private information but η is common knowledge.⁹ The assumption that only the startup may invest in R&D is made for expositional ease and our main insights extend to the case in which the two firms engage in an R&D race.¹⁰

If firm 2 chooses not to invest in R&D, the market stays unchanged and the game ends with payoffs normalized as 0 for both firms. If firm 2 invests c , it develops a new technology that has a full commercial value of $V > 0$.¹¹ Upon development, firm 2 observes whether the invented technology is revealing or not (with probabilities $1 - \theta$ and θ , respectively) and decides whether to obtain one single-claim patent as a testimony to its innovativeness or keep as TS. For expositional ease we assume that patenting is free.¹²

If firm 2 keeps a non-revealing technology as TS, the technology remains hidden and no action is available for the incumbent.¹³ Hence, the startup garners a payoff of V by safely commercializing the technology and the incumbent's payoff is zero.

If firm 2 obtains a patent or keeps a revealing technology as TS, the technology gets unveiled and will be perceived as potentially infringing on one or more of the incumbent's patents. Three options are available to the incumbent in this case, namely, filing a suit, seeking a technology-sharing agreement, and doing nothing. We discuss the outcomes from these options below, which vary depending on whether the technology is patented or not. We then provide a full characterization of the subgame-perfect equilibrium of the static game in Proposition 1.

⁹This is in line with Bessen and Maskin (2009). The lower value of c is set at 0 for purely expositional ease: our results are intact so long as it is less than $\bar{\alpha}_2$ to be defined shortly in equation (1).

¹⁰In this case, if the incumbent and the startup can win the race with identical probabilities, it is relatively straightforward to see that the dynamic effects of a takeover deal support our main message, namely, that the future benefit of a takeover deal can motivate the startup's innovation activity that would not take place otherwise. If the incumbent is less likely to win, then it may find it optimal to save its own R&D cost and pursue a takeover deal of the startup's innovation instead. This is reminiscent of the open innovation paradigm (Chesbrough, 2003).

¹¹Our results extend straightforwardly to the cases that investing c leads to an innovation with a known probability less than (rather than equal to) 1.

¹²Our main results continue to hold as long as the cost of patenting is not too large relative to V .

¹³The same qualitative result will prevail if we assume that a technology-sharing agreement is possible in this case, too, but at worse terms for the startup than when it is protected by a patent.

3.1 Patents

Consider the case that firm 2 patents its technology, leaving three options for firm 1 as above. If firm 1 files a suit alleging that firm 2's technology is infringing on its patents, the outcome is uncertain. In the spirit of Lemley and Shapiro (2005), we assume that in this case the court finds firm 2's technology infringing (firm 1 wins) with a commonly known probability $p \in (0, 1)$, invalidating firm 2's single-claim patent; but finds it non-infringing (firm 2 wins) with the probability $1 - p$, confirming firm 2's right to commercialize the technology. We interpret a higher p as reflecting a stronger stance of the court toward IP protection. Following the court's decision, the winner solely commercializes the new technology and reaps a profit of V , and the payoff to the losing party is zero.¹⁴ Regardless of the outcome, going through the legal battle is costly for both parties involved, which we reflect by assuming that litigation incurs a monetary cost of $\ell > 0$ to both.

The second option is for firm 1 to seek a takeover or licensing agreement with the startup. In line with the literature,¹⁵ we model such an agreement as a Nash bargaining where the disagreement/threat points are the expected surpluses when firm 1 files a lawsuit. In fact, such out-of-court settlements are widespread and account for 95% of all infringement cases (Lanjouw and Schankerman, 2004). We present our analysis presuming equal bargaining powers between the two firms, but our main qualitative results remain intact for a wide range of unequal bargaining powers. Note that a takeover and a licensing agreement are indistinguishable in the static model, because the ownership of the patent *per se* does not affect the maximum value of the technology, V , to be shared.

The Nash bargaining outcome is derived as follows. Conditional on firm 2 having developed and patented a new technology, the disagreement/threat points are the expected surpluses from an infringement lawsuit, i.e., $d_1 = pV - \ell$ and $d_2 = (1 - p)V - \ell$ for firms 1 and 2, respectively. Since V is the maximum possible industry profit from the technology, the Nash bargaining set is defined as $B = \{(\bar{s}_1, \bar{s}_2) \in \mathfrak{R}_+^2 \mid \bar{s}_1 + \bar{s}_2 \leq V\}$ where \bar{s}_i denotes the bargaining share of firm $i = 1, 2$ (the bar above s_i is designatory of the static model). Since B is compact and convex, there is a unique Nash bargaining outcome (\bar{s}_1, \bar{s}_2) that

¹⁴In practice firm 1 may be unable to capture the whole value of V for various reasons. For example, there could be residual tacit knowledge not apparent in the document yet needed for the realisation of the full value of the ideas. Or, the revoked patent may contain ideas which are of merit to users, yet they have been judged as having failed the fulfilment of the substantial requirements of patentability; such ideas are effectively in the public domain and any third party can free ride on them. To reflect this, one may assume that an incumbent reaps a profit of $bV \leq V$, where $b \in (0, 1)$. The paper's main results hold regardless of the value of b .

¹⁵E.g., Green and Scotchmer (1995), Chang (1995), Crampes and Langinier (2002), and Shapiro (2003).

solves $\max_{(\bar{s}_1, \bar{s}_2) \in B} (\bar{s}_1 - d_1)(\bar{s}_2 - d_2)$, expressed as the following functions of p (Nash, 1950):

$$\bar{s}_1 = pV \quad \text{and} \quad \bar{s}_2 = (1 - p)V. \quad (1)$$

The third option is for firm 1 to do nothing, in which case firm 2's payoff is V from commercializing the technology and firm 1's payoff is 0. Note that this option is strictly dominated by that of pursuing Nash bargaining for firm 1 because $\bar{s}_1 > 0$. Moreover, as $\bar{s}_1 > d_1$ and $\bar{s}_2 > d_2$, both firms will find it optimal to pursue a technology-sharing agreement *à la* Nash bargaining, instead of litigation, if firm 2 patents its technology.

3.2 Trade secrets

Consider the case that firm 2 decides to keep its innovation as TS. If the technology is non-revealing, firm 1 reaps V and firm 2 earns nothing as explained earlier.

If firm 2 keeps a revealing technology as TS, on the other hand, the details of technology get exposed and firm 1 has the same three options as above. However, firm 1 stands a better chance of winning from filing a suit in this case because firm 2, without a patent, would find it harder to establish how its technology diversifies from the incumbent's patents. Hence, the expected court outcome is more favourable for firm 1 than when firm 2 patented its technology. As this provides a stronger threat point for firm 1, the Nash bargaining outcome is also better (worse) for firm 1 (firm 2). That is, both litigation and a technology-sharing agreement lead to a worse outcome for firm 2 when it keeps a revealing technology as TS rather than under a patent. Since doing nothing is the worst option for firm 1 either way, it follows that patenting is optimal for firm 2 when its technology turns out to be revealing.

3.3 Static equilibrium

As is shown above, if the technology is revealing firm 2 optimally patents its technology, after which a Nash bargaining will ensue generating payoffs \bar{s}_1 and \bar{s}_2 for firms 1 and 2, respectively. If the technology is non-revealing, on the other hand, firm 2 opts for TS over patenting because V is larger than \bar{s}_2 . Consequently, it is optimal for the startup to invest in R&D provided that the expected surplus, $\bar{s}_2 + \theta(V - \bar{s}_2)$, exceeds c . Thus, the subgame-perfect equilibrium of the static game is summarized as below.¹⁶

Proposition 1: *In the static model, the startup innovates as long as $\bar{s}_2 + \theta(V - \bar{s}_2) \geq c$ and protects its technology by keeping it as TS if non-revealing and by patenting it if*

¹⁶The obtained equilibrium is effectively subgame-perfect (although this is technically incorrect as c is private information) because c is a sunk cost that does not have any bearings on future decision making.

revealing. In the latter case, Nash bargaining ensues over a patented innovation, leading to a takeover or licensing, with payoffs \bar{s}_1 and \bar{s}_2 for firms 1 and 2, respectively.

To recapitulate, in the static model TS dominate patenting whenever the technology is non-revealing. This corroborates the finding that patents are used more in industries where TS are ineffective in concealing the technology, such as the pharmaceutical and chemical industry (Hall *et al.*, 2014). In subsequent sections we demonstrate that patents can fare better than TS in a dynamic model even when the technology is non-revealing.

4 A dynamic analysis

We explained that patents may be disadvantaged relative to TS in the static model due to litigation risks from the incumbent. The core insight of this section is that the very same reason may render patenting more attractive than TS in a dynamic environment (at least in early stages of the innovation process), and as a result motivate startup innovations that would not have been otherwise possible. To illuminate this, we consider environments in which high cost startups would never innovate in the static model. As the startup's bargaining share, \bar{s}_2 , obtains a maximum value of V when $p = 0$, this is the case if

$$C > V, \tag{2}$$

which we henceforth assume. Since V captures only the producers surplus, (2) does not mean that high-cost startups should not innovate. It is socially efficient for them to innovate if the total social surplus, which also includes the consumers surplus, exceeds C .

4.1 Model

We extend the static model to a dynamic model of infinite periods. In each period, the static game of Section 3 is played as the stage game between a long-lived incumbent (firm 1) and a new startup (firm 2) that arrives at the market. The key difference from the static model is that the incumbent's patent portfolio may grow. In particular, if the incumbent acquires new patents via takeovers the technological territory covered by its portfolio expands, increasing the likelihood that it will prevail in future patent-infringement suits. In this regard, we assume that legal power increases as the portfolio size gets bigger, but at a decreasing rate. That it increases at a decreasing rate is a logical consequence of the fact that the chance of prevailing in court is bounded above by 1. To capture this we re-define p , the probability of firm 1 winning an infringement suit, as a function of the degree of IP protection, denoted by $z \in (0, 1)$, and the size of firm 1's patent portfolio, measured

by the number of patents in its portfolio. An increase in z (which can be considered as patent breadth) implies that the courts take a tougher stance on infringement, increasing p .

To facilitate presentation, we make two indexing conventions. First, since the continuation game from any period is fully described by the size of firm 1's portfolio at the beginning of that period, we index the period by the size of firm 1's portfolio. Second, since what matters in the analysis is the accumulation of patents on top of the incumbent's initial portfolio, we index the size of the initial portfolio as the base size of 1, and each patent added to it increases the portfolio size by one. Hence, period 1 designates the initial period (of the base portfolio size of 1) and period $t \geq 1$ designates any period that starts with firm 1's portfolio of size t . So long as firm 1 has added one patent every period from the initial period, our indexing coincides with the natural indexing of periods by natural numbers. Two consecutive periods are indexed the same, however, if the incumbent's portfolio did not grow in the first of the two periods either because firm 2 did not innovate or because it did but the incumbent did not buy the patented innovation. For expositional clarity we do not model expiration of patents, the effect of which is discussed later in Section 6.

On account of the above, p is a function of z and t that satisfies

$$\partial p / \partial z > 0, \quad \partial p / \partial t > 0, \quad \text{and} \quad \partial^2 p / \partial t^2 < 0.$$

To be clear about its dependence on z and t , we denote p by $p_z(t)$ in the sequel.

The order of moves in each period t is as follows. First, a startup (firm 2) arrives and decides whether to innovate or not contingent on its R&D cost which is 0 and C with probabilities η and $1 - \eta$, respectively. If firm 2 does not innovate, nothing happens until the next period starts. If firm 2 innovates, it observes whether the developed technology is revealing or not, and decides whether to patent it (with zero cost) or to keep it as TS.

If firm 2 decides to patent, firm 1 then decides whether to file a suit or pursue an agreement *à la* Nash bargaining. If a suit is filed, both parties incur a legal cost of ℓ , and firm 1 wins with probability $p_z(t)$; firm 2 wins with probability $1 - p_z(t)$. The winner gets a surplus of V , leaving a surplus of 0 to the losing party. If an agreement is pursued, the Nash bargaining outcome results over the total producers surplus of V , plus, in case of a takeover, the additional benefits that would accrue to firm 1 in future deals due to its enlarged portfolio. Note that such additional benefits are absent when firm 1 wins an infringement suit because the patent is revoked due to failing to fulfill the substantial requirements of patentability. To avoid the replacement effect, as in Bessen and Maskin (2009), it is assumed that V , the profits that can be generated from commercializing the

new technology, are incremental values.

If firm 2 keeps its technology as TS, what happens next depends on whether the technology is revealing or not. If it is revealing, then firm 1 has the same options as above and, as explained in Section 3.2, litigation results in firm 1 winning with a higher probability than $p_z(t)$. If it is non-revealing, firm 2 obtains a payoff of V from commercializing the technology and firm 1 gets 0. The startup in each period maximizes its expected surplus of that period, net of the innovation costs when relevant. The incumbent maximizes the expected present value of its profit stream with a discount factor $\delta \in (0, 1)$.

4.2 Equilibrium

We start with two observations that facilitate the exposition of equilibrium analysis. First, when the incumbent and the startup negotiate a transfer of a patented technology, the size of pie to bargain over is larger for a takeover than a licensing deal because the former increases the future bargaining power of the incumbent. This means that both parties prefer to bargain over a takeover. Therefore, we take it for granted that they negotiate a takeover deal whenever possible via a Nash bargaining. Second, this implies that the bargaining outcome is better in the dynamic model than in the static model, while the outcomes from litigation and doing nothing are the same as in the static model (because there is no expansion in the incumbent's portfolio). Hence, in the dynamic model the incumbent continues to prefer pursuing a technology-sharing agreement to litigation or doing nothing whenever technology transfer is feasible (i.e., apart from the case of non-revealing TS), which we also take for granted in the equilibrium analysis that follows.

We now proceed with a formal analysis of the dynamic model and characterize the unique subgame-perfect equilibrium. Consider an arbitrary equilibrium of the dynamic game. Let $X(t)$ denote firm 1's value at the beginning of period t , which is the discounted sum of its expected payoff stream from period t onward. Let $Q(t)$ denote the probability that a takeover takes place in period t . Then, the Bellman equation is

$$X(t) = (1 - Q(t))\delta X(t) + Q(t)(s_1(t) + \delta X(t + 1)) \quad (3)$$

because firm 1's value does not change in the next period if there is no takeover deal in the current period; however, if there is a takeover, firm 1 captures the bargaining surplus over the current innovation, $s_1(t)$, plus the next period's value which is $X(t + 1)$.

If a startup innovates and patents its technology in period t , Nash bargaining will ensue leading to a takeover deal as explained above. As the bargaining outcome is always positive for the startup, a low-cost startup always innovates and then patents when the technology turns out to be revealing. Hence, $Q(t) \geq \eta(1 - \theta) > 0$.

In the Nash bargaining of period t , the total surplus from a takeover deal is the sum of the commercial value of the technology and the extra surplus of the incumbent in future technology negotiations, i.e., $V + \delta(X(t+1) - X(t))$. This is the size of the pie on the bargaining table. As both parties must accept the court's decision as explained above if the case was litigated, the threat points are the expected payoffs from the court outcomes minus the legal costs, i.e., $d_1 = p_z(t)V - \ell$ and $d_2 = (1 - p_z(t))V - \ell$. Since the Nash bargaining set in this case is $B(t) = \{(s_1, s_2) \in \mathfrak{R}_+^2 \mid s_1 + s_2 \leq V + \delta(X(t+1) - X(t))\}$, the Nash bargaining outcome (s_1, s_2) that solves $\max_{(s_1, s_2) \in B(t)} (s_1 - d_1)(s_2 - d_2)$ is,

$$s_1(t) = p_z(t)V + \frac{\delta(X(t+1) - X(t))}{2} \quad (4)$$

$$s_2(t) = (1 - p_z(t))V + \frac{\delta(X(t+1) - X(t))}{2}. \quad (5)$$

Plugging $s_1(t)$ back into equation (3) and rearranging, we get

$$X(t+1) - X(t) = \frac{2(1 - \delta)}{3Q(t)\delta}X(t) - \frac{2p_z(t)}{3\delta}V, \quad (6)$$

a difference equation that characterizes the sequence $X(t)$. In addition, plugging (6) into (5), we get

$$s_2(t) = \frac{1 - \delta}{3Q(t)}X(t) + \frac{3 - 4p_z(t)}{3}V. \quad (7)$$

At this point, observe that given the values of innovation cost $c \in \{0, C\}$ and the payoff V of keeping a non-revealing technology as a TS, in light of (2), in equilibrium one of the following three cases must hold in each period t :

- [I] $s_2(t) \geq C$ and both high-cost and low-cost startups innovate and always patent their technology.
- [II] $V \leq s_2(t) \leq C$ and a low-cost startup always innovates and patents its technology when it is non-revealing (for certain if $V < s_2(t)$) as well as when it is revealing; and a high-cost startup may innovate if $s_2(t) = C$ but with a probability less than 1 (and always patents its technology).¹⁷
- [III] $s_2(t) \leq V$ and only low-cost startups innovate and keep their technology as TS if it is non-revealing (and patent it if it is revealing).¹⁸

¹⁷We require "with a probability less than 1" here just to distinguish [II] from [I].

¹⁸It is not possible that $s_2(t) = V$ and a low-cost startup patents its technology with a positive probability when it is non-revealing. This is because there would be a takeover of the patented technology in such a case, enhancing the incumbent's current value $X(t)$ via the strengthened future bargaining power; this in turn would mean a reduced stake to bargain over because $X(t+1) - X(t)$ shrinks and consequently, a reduced $s_2(t) < V$. See the proof of Proposition 3 in Appendix A.

To explain the equilibrium heuristically, note that as the portfolio size gets arbitrarily large, the extra value of an additional patent becomes negligible, and therefore, $X(t+1) - X(t) \rightarrow 0$ as $t \rightarrow \infty$. Because

$$\lim_{t \rightarrow \infty} s_2(t) = (1 - p_z(\infty))V < V \quad (8)$$

from (5), case [III] above holds for all sufficiently large t , say $t \geq T^*$ for some $T^* < \infty$ to be pinned down below. Hence, the equilibrium sequence $X(t)$ for $t \geq T^*$ solves (6) when $Q(t) = \eta(1 - \theta)$, which is an increasing and convergent sequence, as formalized in the next result. Although the solution to (6) when $Q(t) = \eta(1 - \theta)$ is pertinent only for $t \geq T^*$ as a part of the equilibrium, it proves useful to present the solution sequence, denoted by $X^*(t)$, for all natural numbers $t \geq 1$.

Proposition 2: *The sequence $X^*(t)$ that solves (6) when $Q(t) = \eta(1 - \theta)$ is unique, monotonically increases at a decreasing rate, i.e., $X^*(t) - X^*(t-1) > X^*(t+1) - X^*(t) > 0$ for all $t > 1$, and converges to*

$$X^*(\infty) = \frac{\eta(1 - \theta)p_z(\infty)}{1 - \delta}V \quad \text{as } t \rightarrow \infty. \quad (9)$$

Proof: See Appendix A.

Let $s_2^*(t)$ denote the startup's bargaining share when $X(\cdot) = X^*(\cdot)$, i.e.,

$$s_2^*(t) := (1 - p_z(t))V + \frac{\delta(X^*(t+1) - X^*(t))}{2}.$$

Note that $s_2^*(t)$ strictly decreases in t because $p_z(t)$ increases while $X^*(t+1) - X^*(t)$ decreases in t . Letting T^* be the smallest t such that $s_2^*(t) \leq V$, the equilibrium of the dynamic game is characterized below.

Proposition 3: *There is a unique equilibrium of the dynamic game.*

- (a) [I] or [II] holds for $t < T^*$ while [III] holds for $t \geq T^*$, so $X(t) = X^*(t)$ for $t \geq T^*$.
- (b) For any $\tilde{t} > 1$ there is $\underline{\delta}(\tilde{t}) < 1$ such that if $\delta > \underline{\delta}(\tilde{t})$ and [I] holds in some period $t \leq \tilde{t}$, then [I] holds in all periods 1 through t .

Proof: See Appendix A.

Roughly speaking, the proof starts from the observations that $X^*(t)$ is the lower bound of the incumbent's values because $Q(t)$ is bounded below by $\eta(1 - \theta)$, and that $X^*(t)$ is increasing at a decreasing rate from Proposition 2. So, if $X^*(t+1) - X^*(t)$ is not enough to induce high-cost innovation, i.e., $t \geq T^*$, then high-cost innovation cannot be induced in any future period, say $t' > t$, because that would require a higher benefit from obtaining

an additional patent in period t' , i.e., $X(t'+1) > X^*(t'+1)$, which in turn would require $X(t'+2) > X^*(t'+2)$ and so on, contradicting the observation that $X(\cdot)$ must converge to (9) in the limit. This establishes that $X(t) = X^*(t)$ for all $t \geq T^*$. On the other hand, for any $t < T^*$, as $X^*(t+1) - X^*(t)$ is large enough so that $s_2^*(t) > V$ and $X(t+1)$ is no lower than $X^*(t+1)$, it follows that $s_2(t)$ must exceed V and that even non-revealing innovations will be patented, establishing that [I] or [II] prevails for $t < T^*$. Furthermore, if $s_2(t)$ exceeds C so that even a high-cost startup innovates in some $t < T^*$, then it can be shown recursively that the same holds for all preceding periods for δ close to 1.

Thus, the dynamic equilibrium typically goes through a few phases. Initially, both types of startups innovate, always patenting their innovation. In the next phase, only low-cost startups innovate and patent their innovation even when it is non-revealing. In the final phase, only low-cost startups innovate and keep as TS when non-revealing. This transition of phases stems from the insight that when the incumbent's patent portfolio is small, a large impact of a takeover on future bargaining outcomes boosts the size of the pie to be bargained over, increasing the startup's bargaining share enough to motivate even a high-cost startup to innovate (who would not otherwise innovate, due to (2)). As this process continues, the future benefit from increasing the portfolio size diminishes, reducing the startup's share so that high-cost startups stop innovating at some stage and then low-cost startups stop patenting when their technology turns out to be non-revealing.

5 Simulation and comparative statics

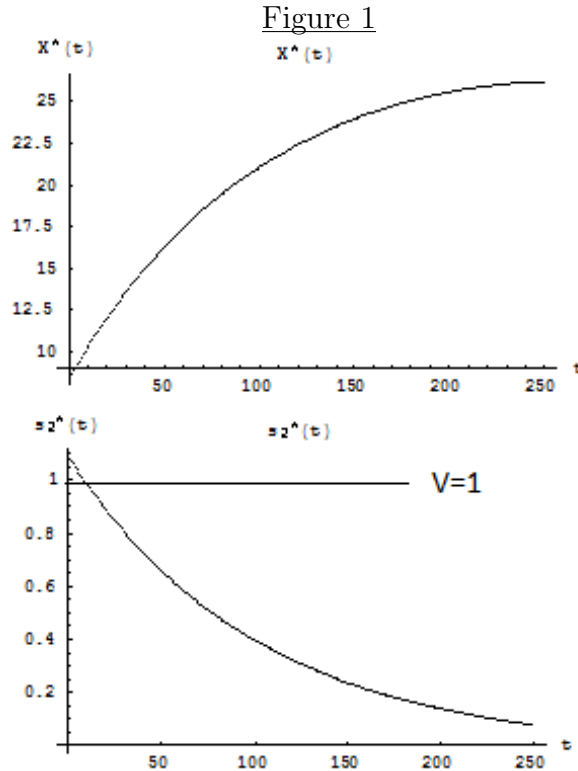
Even though our argument seems intuitive, its recursive nature and its discontinuity at T^* makes it difficult to visualize its fine details. For this reason, the recursive model is now simulated. What we aim to demonstrate is: a) that the sequence $X(\cdot)$ converges and behaves in the fashion described above, and in doing so uncover the model's comparative statics, and b) that the dynamic effects in fact motivate high-cost startups to innovate who wouldn't do so otherwise. For this purpose, we run an algorithm outlined in Appendix B for values that lead to a relatively high T^* . By changing the parameter values, we find different values of T^* without altering the qualitative properties of the equilibrium.

For the simulation, we need to fix the function $p_z(t)$ representing the incumbent's probability of winning an infringement lawsuit. We argued earlier that $p_z(t)$ increases at a decreasing rate in t , and increases in z . Although empirical estimates are rather scarce on this measure, such properties are in line with the findings of Lanjouw and Schankerman (2004) that the marginal protective power of portfolio size is positive but slowing down. We capture this by setting $p_z(t) = 1 - (1 - z)^t$. To provide an example, when $z = .01$

a firm with a portfolio made up of 100 patents stands a 63% chance of winning its case, and an increase by 1 patent raises this by .36%. Note that by changing z we can affect the speed of convergence of $p_z(t)$ to 1; we elaborate on this point in the next section.

For the other parameter values, V is normalized to one ($V = 1$) and $C = 1.0001 > V$ to satisfy (2), ensuring that innovations by high-cost startups are only possible in a dynamic model. Furthermore, by noting that $\bar{s}_2(p) \leq \bar{s}_2(0) = 1$, and that $p_z(t)$ is constant at 0 for all t if $z = 0$, a $C > 1$ ensures that IP protection (i.e. $z > 0$) is necessary for the innovations that are envisioned here. The model is simulated for $\delta = .97$ (the incumbent is patient), $\theta = .1$ (frequently the innovation is easy to re-engineer) and $\eta = .95$ (a very entrepreneurial environment), and the comparative statics on δ , θ , and η are reported.

Using the above values of V , C , δ , θ , η and for the aforementioned $p_z(t)$ with a $z = .01$, the simulation is commenced by calculating the unique sequence $X^*(t)$ of Proposition 2, that solves (6) when $Q(t) = \eta(1 - \theta)$, for $t = 1, 2, \dots$ and the corresponding $s_2^*(t)$ for $t = 1, 2, \dots$. Having the $s_2^*(t)$ at hand we find the T^* which is the smallest t such that $s_2^*(t) \leq V$. In this case $T^* = 10$. This routine corresponds to the first three steps of the algorithm outlined in Appendix B. Figure 1 plots $X^*(t)$ and $s_2^*(t)$. By changing δ , θ , and η throughout their parameter range we find that: i) a decrease in η leads to a lower T^* , ii) an increase in θ provides a smaller T^* , and iii) lowering δ decreases T^* .



Specifications: Iterations = 250, $z = .01$, $\eta = .95$, $\theta = .1$, $\delta = .97$, $V = 1$, $C = 1.00001$

Having derived a $T^* = 10$, the attention is recursively shifted to $t = 1, 2, \dots, T^* - 1$. Specifically, steps 4 and 5 of the algorithm (see Appendix B) are run to find the values of $X(t)$, $s_2(t)$, $\alpha(t)$, $\beta(t)$ and $Q(t)$, and in the process it is verified that $s_2(t) > V$ and $X(t) > X^*(t)$. The results are included in Table 1.

Table 1

t	$X(t)$	$s_2(t)$	$\alpha(t)$	$\beta(t)$	$Q(t)$
1	8.78772	1.07454	1	1	1
2	8.96204	1.06309	1	1	1
3	9.13314	1.05173	1	1	1
4	9.30104	1.04047	1	1	1
5	9.46573	1.02931	1	1	1
6	9.62722	1.01825	1	1	1
7	9.7855	1.00728	1	1	1
8	9.94057	1.00001	1	0.298882	0.964944
9	10.0999	1	0.219863	0	0.875887

Specifications: Iterations=250, $z=.01$, $\eta=.95$, $\theta=.1$, $\delta=.97$, $V=1$, $C=1.00001$;

$\alpha(t)$ is the probability that a low-cost startup patents its innovation when it is non-revealing, and $\beta(t)$ is the probability that a high-cost startup innovates.

6 Policy implications

An interesting policy-relevant question naturally arises from our analysis: what is the optimal level of IP protection, z , and the optimal patent length that provide innovation incentives for startups for the longest possible duration? In terms of z , the core logic of our analysis points to the following intuition: If z is excessive, the marginal protective power that an extra patent brings to the incumbent is large initially but quickly dwindles as a result of accumulating its power too rapidly, killing off the positive effect on startup innovation prematurely. If z is too small, on the other hand, the marginal protective power of an extra patent is too limited to have any real impact on the startup's innovation incentives. We confirm this intuition by running the simulation of the previous section (for the same values of C , V , δ , θ , and η) for various values of z , with a view to establishing the levels of z under which high-cost innovations are induced for the longest duration. The results are in Table 2 below.

Table 2

z	$p_z(t = 100)$	T^*
0.0000001	0.00001	1
0.000001	0.0001	4
0.00001	0.001	13
0.0001	0.01	14
0.001	0.095	14
0.01	0.63	10
0.1	0.99	3
1	1	1

Specifications: Iterations=250, $z = .01$, $\eta = .95$, $\theta = .1$, $\delta = .97$, $V = 1$, $C = 1.00001$

Table 2 lists the T^* for the values of z in $\{1/10^7, 1/10^6, 1/10^5, 1/10^4, 1/10^3, 1/10^2, 1/10, 1\}$. To put these levels of IP protection into perspective, the probabilities of winning the case for a portfolio of 100 patents are also listed. For $z = 1/10^7$, this probability is $1/10^5$, while for $z = 1$ this probability is 1. Table 2 posts the non-monotone relationship between z and T^* . Specifically, the lowest and highest z provide the same $T^* = 1$, with the intermediate values providing higher levels of T^* . For example, $z = 0.001$, which corresponds to $p_z(t = 100) = 9.5\%$, leads to the highest $T^* = 14$.

The term of a patent counterbalances the growth of a portfolio size due to takeovers, as patents exit the portfolio after a maximum period of 20 years. In view of this, consider the following example: if one sets the patent length as T^* then in every period (as long as the portfolio's patents are evenly spaced) one patent expires as a new patent is added. This observation suggests an alternative interpretation of T^* , namely, the maximal patent length that allows for innovation by high-cost startups *ad infinitum* by constraining the incumbent's portfolio size below a threshold.

7 Concluding Remarks

We have shown that patents may outperform trade secrets in a dynamic environment for the very reason that the opposite is true in static settings. This is because the potential future value of patents as negotiating assets can be materialized and shared through an appropriate channel of patent trading. In this sense, we are advocating a potential role of patents as a negotiating asset that can be traded for intertemporal transfer of value, a

novel function of patents that has been largely overlooked hitherto.¹⁹

More generally, such a role of patents may provide a way to address a fundamental conflict of incentive provision in cumulative innovation environments, first observed by Scotchmer (1991): A first innovation would not be incentivized adequately if it is to be deemed narrow in its scope, while it would stifle subsequent innovations if to be deemed overly broad. For mutual benefits, therefore, a compromise needs to be found between current and future innovators to share the surpluses from future innovations facilitated by earlier innovations. Our analysis suggests that appropriate trading of patents may help achieve this goal insofar as patents carry, in addition to the ownership of the technology, a partial claim on follow-on innovations via their role as a negotiating asset in the shadow of infringement litigation.

Appendix A

Proof of Proposition 2: First, note that $X^*(t)$ is bounded below (by 0) and above because the maximum surplus in each period is bounded and $\delta < 1$. If $X^*(t+1) \leq X^*(t)$, then the right hand side of equation (6), with $Q(t) = \eta(1 - \theta)$ and X replaced by X^* , would be non-positive and, furthermore, its value would strictly decrease when evaluated for $t + 1$ because $X^*(t+1) \leq X^*(t)$ and $p_z(t+1) > p_z(t)$. This would mean that $X^*(t+2) - X^*(t+1) < X^*(t+1) - X^*(t) \leq 0$. Applying the same argument repeatedly, we deduce that if $X^*(t+1) \leq X^*(t)$ then the sequence should decrease forever at an increasing rate after t , which is a contradiction because the sequence is bounded below; therefore, we conclude that $X^*(t+1) - X^*(t) > 0$ for all t . Since the sequence is bounded above, it further follows that it must converge. The limit value, $X^*(\infty)$ in (9), is obtained by setting $X^*(t+1) = X^*(t)$ and $p_z(t) = p_z(\infty)$ in equation (6) and solving for $X^*(t)$.

To show uniqueness, suppose to the contrary that there are two sequences, $\{X^*(t)\}$ and $\{X^{**}(t)\}$, that satisfy (6) with $Q(t) = \eta(1 - \theta)$, such that $X^{**}(t') = X^*(t') + \gamma$ for some $\gamma > 0$ and t' . By (6), we have $X^{**}(t'+1) = X^*(t'+1) + (1 + \frac{2(1-\delta)}{3\eta(1-\theta)\delta})\gamma > X^*(t'+1) + \gamma$ and by repeating the same calculation, $X^{**}(t) > X^*(t) + \gamma$ for all $t \geq t'$. This is impossible because both sequences should converge to the same limit as proved above, proving the uniqueness.

¹⁹Patents were not originally envisioned as strategic assets. Article 1.8.8 of the US Constitution describes patents as limited time monopolies bestowed to innovators as to promote science. Their ability to be used as bargaining chips (irrespective of the uses the embodied technology finds) is a relatively new one. It rests on a 1908 Supreme Court decision, *Continental Paper Bag Co. v. Eastern Paper Bag Co.*, which established the principle that patent holders have no obligation to use their patents in production.

Lastly, to show that $X^*(t) - X^*(t-1) > X^*(t+1) - X^*(t)$, note from equation (6) that

$$X^*(t+1) - X^*(t) - X^*(t) + X^*(t-1) = \frac{2(1-\delta)}{3\eta(1-\theta)\delta}(X^*(t) - X^*(t-1)) - \frac{(p_z(t) - p_z(t-1))(b+1)}{3\delta}V. \quad (10)$$

If $X^*(t+1) - X^*(t) \geq X^*(t) - X^*(t-1)$ for some t , it would follow from equation (10) that $X^*(t+2) - X^*(t+1) \geq X^*(t+1) - X^*(t)$ because $0 < p_z(t+1) - p_z(t) < p_z(t) - p_z(t-1)$ due to the assumption that $\partial^2 p / \partial t^2 < 0$. Furthermore, $X^*(t+1) - X^*(t)$ would increase in t by repeated application of the same argument. This is impossible because the sequence $X^*(t)$ converges as shown above, hence we conclude that $X^*(t) - X^*(t-1) > X^*(t+1) - X^*(t)$. *Q.E.D.*

Proof of Proposition 3: We start by showing that in any equilibrium $X(t)$ increases for all t sufficiently large. To prove by contradiction, suppose otherwise, i.e., $X(t+1) - X(t) \leq 0$ for arbitrarily large t , say t' . Then, $s_2(t') < V$ by (5) and $Q(t') = \eta(1-\theta)$ by [III]. Note from possible cases [I]-[III] that $Q(t) \geq \eta(1-\theta)$ for all t . Therefore, the RHS of (6) is negative for $t = t' + 1$, i.e., $X(t'+1) - X(t') \leq 0$. Applying the same argument repeatedly, we deduce that $X(t+1) - X(t) \leq 0$ for all $t \geq t'$ and thus, $Q(t) = \eta(1-\theta)$ for all $t \geq t'$ by [III]. This would mean that $X(t)$ for $t \geq t'$ solves (6) when $Q(t) = \eta(1-\theta)$, but this would contradict Proposition 2. This establishes that

$$X(t) \text{ strictly increases for all sufficiently large } t, \text{ say } t > T^o. \quad (11)$$

Moreover, from the fact that $X(t)$ is bounded above by $X(t) \leq V/(1-\delta)$ because the maximum possible surplus for the economy (hence, for firm 1) is V in each period, it further follows that

$$\limsup_{t \rightarrow \infty} X(t+1) - X(t) = 0. \quad (12)$$

Below, we first show that any equilibrium must satisfy the properties of part (a) of the Proposition, in the process of which we construct the unique equilibrium. Then, we prove part (b).

Supposing that an equilibrium exists, we verify that it satisfies the properties of part (a) of the Proposition. This verification consists of a few steps.

The first step is to show that [III] holds for $t \geq T^*$, i.e., $X(t) = X^*(t)$, $s_2(t) = s_2^*(t)$ and $Q(t) = \eta(1-\theta)$ for all $t \geq T^*$ where T^* is the smallest t such that $s_2^*(t) \leq V$ as defined in the main text. It is straightforward from (12) and (5) that $s_2(t) < V$ for all sufficiently large t and thus, from [III] that $Q(t) = \eta(1-\theta)$. It further follows, therefore, that the $X(t)$ solves (6) when $Q(t) = \eta(1-\theta)$ for all sufficiently large t and as a result, by Proposition 2,

[III] holds for all sufficiently large t . Now, with a view to reaching a contradiction, suppose that there exists $t \geq T^*$ that $s_2(t) \neq s_2^*(t)$. Without loss of generality, suppose that t is the largest such t . Note that $s_2(t) < s_2^*(t)$ would mean $Q(t) = \eta(1 - \theta)$ by [III] because $s_2^*(t) \leq V$ for all $t \geq T^*$ and thus, $s_2(t) = s_2^*(t)$ by (5) and (6), a contradiction. Hence, $s_2(t) \neq s_2^*(t) \leq V$ would imply that $s_2(t) > s_2^*(t)$ which in turn would imply $X(t) < X^*(t)$ by (5) and $Q(t) < \eta(1 - \theta)$ by (6), but this is impossible because $Q(t) \geq \eta(1 - \theta)$ always holds by [I]-[III]. This proves that [III] holds for all $t \geq T^*$.

Next, consider $t = T^* - 1$. Note, in particular, that $s_2^*(T^* - 1) > V$. Given $X(T^*) = X^*(T^*)$, from (6) we deduce that $X(T^* - 1)$ is equal to $X^*(T^* - 1)$ when $Q(t) = \eta(1 - \theta)$ and strictly increases as $Q(t)$ increases. Consequently, from (5) we deduce that $s_2(T^* - 1)$ is equal to $s_2^*(T^* - 1)$ when $Q(t) = \eta(1 - \theta)$ and strictly decreases as $Q(t)$ increases. We represent $X(T^* - 1)$ and $s_2(T^* - 1)$ as $X(T^* - 1, q)$ and $s_2(T^* - 1, q)$ to indicate their dependence on the value $q \in [\eta(1 - \theta), 1]$ of $Q(t)$. Note that $Q(t) = \eta(1 - \theta + \alpha\theta) \in [\eta(1 - \theta), \eta]$ if a low-cost startup innovates and patents with probability α when non-revealing and a high-cost startup never innovates; $Q(t) = \eta + (1 - \eta)\beta \in [\eta, 1]$ if a low-cost startup always innovates and patents and a high-cost startup innovates with probability β and always patent if innovates. The following two cases are possible.

(i) Suppose $V < s_2^*(T^* - 1) \leq C$. In this case, a high-cost startup would never innovate in period $T^* - 1$ because $s_2(T^* - 1, q) < C$ for all $q \in (\eta(1 - \theta), 1]$. If $s_2(T^* - 1, \eta) \geq V$, then $s_2(T^* - 1, q) > V$ for any $q \in [\eta(1 - \theta), \eta)$ and thus, a low-cost startup must innovate in period $T^* - 1$ and always patent (i.e., $\alpha = 1$) in equilibrium. If $s_2(T^* - 1, \eta) < V$, then $s_2(T^* - 1, q) = V$ for a unique $q \in (\eta(1 - \theta), \eta)$, say \hat{q} , and thus, a low-cost startup must innovate in period $T^* - 1$ and patent with probability $\hat{\alpha} \in (0, 1)$ when non-revealing in equilibrium, where $\hat{\alpha}$ is the unique value of α that solves $\eta(1 - \theta + \alpha\theta) = \hat{q}$.

(ii) Suppose $C < s_2^*(T^* - 1)$. If $s_2(T^* - 1, 1) \geq C$, then $s_2(T^* - 1, q) > C$ for any $q \in [\eta(1 - \theta), 1)$ and thus, both types of startup must innovate in period $T^* - 1$ and always patent (i.e., $\alpha = \beta = 1$) in equilibrium. Consider the alternative case that $s_2(T^* - 1, 1) < C$. If $s_2(T^* - 1, \eta) > C$, then a low-cost startup must always innovate and patent while a high-cost startup must innovate and patent with probability $\hat{\beta}$ in period $T^* - 1$ where $\hat{\beta}$ is the unique value of β that solves $s_2(T^* - 1, \eta + (1 - \eta)\beta) = C$. If $V < s_2(T^* - 1, \eta) \leq C$, then a low-cost startup must always innovate and patent while a high-cost startup must not innovate. If $s_2(T^* - 1, \eta) \leq V$, then a high-cost startup must not innovate while a low-cost startup must innovate and patent with probability $\hat{\alpha}$ when non-revealing in period $T^* - 1$ where $\hat{\alpha}$ is the unique value of α that solves $s_2(T^* - 1, \eta(1 - \theta + \alpha\theta)) = V$.

We have shown above that [I] or [II] holds for $t = T^* - 1$. In particular, $X(T^* - 1) >$

$X^*(T^* - 1)$ and thus,

$$\tilde{s}_2(T^* - 2) := \frac{2 - p_z(T^* - 2)(b + 1)}{2}V + \frac{\delta(X(T^* - 1) - X^*(T^* - 2))}{2}$$

is larger than $s_2^*(T^* - 2) > V$. Therefore, we can apply an analogous argument as above (with $\tilde{s}_2(T^* - 2)$ playing the role of $s_2^*(T^* - 2)$) to deduce that [I] or [II] holds for $t = T^* - 2$. In addition, as $X(T^* - 1) > X^*(T^* - 1)$ and the probability of takeover is higher than $\eta(1 - \theta)$ in [I] and [II], it further follows that $X(T^* - 2) > X^*(T^* - 2)$. Consequently, we may apply analogous argument recursively to establish that [I] or [II] holds for all $t < T^*$. Note that $s_2(t) > V$ and (5) ensure that $X(t)$ increases in t . Note that we have constructed the unique equilibrium and that this construction proves part (a).

(b) Fix an arbitrary $t > 0$. Evaluate (6) for one period early to get an expression for $X(t) - X(t - 1)$, then add $\frac{2(1-\delta)}{3Q(t-1)\delta}X(t)$ on both sides and rearrange to get

$$X(t) - X(t - 1) = \frac{2(1 - \delta)}{3Q(t - 1)\delta + 2(1 - \delta)}X(t) - \frac{p_z(t - 1)(b + 1)}{3Q(t - 1)\delta + 2(1 - \delta)}Q(t - 1)V. \quad (13)$$

Subtract (6) from (13) side by side to get

$$\begin{aligned} & X(t) - X(t - 1) - X(t + 1) + X(t) \\ &= 2(1 - \delta)X(t) \left(\frac{1}{3Q(t - 1)\delta + 2(1 - \delta)} - \frac{1}{3Q(t)\delta} \right) \\ & \quad + \left(\frac{p_z(t)(b + 1)}{3\delta} - \frac{p_z(t - 1)(b + 1)}{3Q(t - 1)\delta + 2(1 - \delta)}Q(t - 1) \right) V \\ & \rightarrow \lim_{\delta \rightarrow 1} 2(1 - \delta)X(t) \frac{Q(t) - Q(t - 1)}{3Q(t)Q(t - 1)} + \frac{p_z(t) - p_z(t - 1)}{3}(b + 1)V > 0 \end{aligned} \quad (14)$$

as $\delta \rightarrow 1$ where the inequality follows as long as $Q(t) = 1$, which is the case when [I] pertains in period t . Therefore, for each $t > 1$ there is $\delta(t) < 1$ such that if $\delta > \delta(t)$ and [I] pertains in period t , then $X(t) - X(t - 1) > X(t + 1) - X(t)$ so that $s_2(t - 1) > s_2(t)$ by (5) and thus [I] pertains in period $t - 1$ as well. For any $\tilde{t} > 1$, part (b) is proved by taking $\underline{\delta}(\tilde{t}) = \max\{\delta(2), \delta(3), \dots, \delta(\tilde{t})\}$. Q.E.D.

Appendix B

We describe below the simulation steps used to calculate the unique equilibrium of the dynamic model reported in Section 5.

- (1) Fix the parameter values (try various values): $C > V = 1$, η , z , δ , $\theta \in (0, 1)$.

- (2) Calculate the unique sequence $X^*(t)$ of Proposition 2, that solves (6) when $Q(t) = \eta(1 - \theta)$, for $t = 1, 2, \dots$. Also calculate $s_2^*(t)$ for $t = 1, 2, \dots$.
- (3) Find T^* which is the smallest t such that $s_2^*(t) \leq V$.
- (4) For all $t \geq T^*$, define $X(t) = X^*(t)$ and $s_2(t) = s_2^*(t)$.
- (5) Consider $t = T^* - 1$. Define

$$\widehat{X}(T^* - 1, q) := \left(\frac{3q\delta}{3q\delta + 2(1 - \delta)} \right) X(T^*) + \left(\frac{3q\delta}{3q\delta + 2(1 - \delta)} \right) \frac{p_z(T^* - 1)(b + 1)}{3\delta} V$$

and

$$\widehat{s}_2(T^* - 1, q) := \frac{2 - p_z(T^* - 1)(b + 1)}{2} V + \frac{\delta(X(T^*) - \widehat{X}(T^* - 1, q))}{2}.$$

Verify that $\widehat{s}_2(T^* - 1, \eta(1 - \theta)) > V$.

- i) If $V < \widehat{s}_2(T^* - 1, \eta(1 - \theta)) \leq C$, calculate $\widehat{s}_2(T^* - 1, \eta)$. If $\widehat{s}_2(T^* - 1, \eta) \geq V$, then set $\alpha(T^* - 1) = 1$. If $\widehat{s}_2(T^* - 1, \eta) < V$, then set $\alpha(T^* - 1) = \hat{\alpha} \in [0, 1]$ such that $\widehat{s}_2(T^* - 1, \eta(1 - \theta + \hat{\alpha}\theta)) = V$. Set $\beta(T^* - 1) = 0$.
- ii) If $C < \widehat{s}_2(T^* - 1, \eta(1 - \theta))$, calculate $\widehat{s}_2(T^* - 1, 1)$. If $\widehat{s}_2(T^* - 1, 1) \geq C$, then set $\alpha(T^* - 1) = 1$ and $\beta(T^* - 1) = 1$.
- iii) If $C < \widehat{s}_2(T^* - 1, \eta(1 - \theta))$ and $\widehat{s}_2(T^* - 1, 1) < C$, calculate $\widehat{s}_2(T^* - 1, \eta)$. If $\widehat{s}_2(T^* - 1, \eta) > C$, then set $\alpha(T^* - 1) = 1$ and $\beta(T^* - 1) = \hat{\beta} \in [0, 1]$ such that $\widehat{s}_2(T^* - 1, \eta + (1 - \eta)\beta) = C$. If $V < \widehat{s}_2(T^* - 1, \eta) \leq C$, then set $\alpha(T^* - 1) = 1$ and $\beta(T^* - 1) = 0$. If $\widehat{s}_2(T^* - 1, \eta) \leq V$, then set $\alpha(T^* - 1) = \hat{\alpha} \in [0, 1]$ such that $\widehat{s}_2(T^* - 1, \eta(1 - \theta + \hat{\alpha}\theta)) = V$ and set $\beta(T^* - 1) = 0$.

Set $Q(T^* - 1) = \eta(1 - \theta + \theta\alpha(T^* - 1)) + (1 - \eta)\beta(T^* - 1)$. This is the probability with which an innovation is patented by the startup in period $T^* - 1$. Thus, define

$$X(T^* - 1) = \widehat{X}(T^* - 1, Q(T^* - 1)) \quad \text{and} \quad s_2(T^* - 1) = \widehat{s}_2(T^* - 1, Q(T^* - 1))$$

Verify that $X(T^* - 1) > X^*(T^* - 1)$.

- (6) Repeat all the steps of (5) above when T^* is replaced by $T^* - 1$ everywhere, to determine $Q(T^* - 2)$ through $\alpha(T^* - 2)$ and $\beta(T^* - 2)$ and thereby, $X(T^* - 2)$ and $s_2(T^* - 2)$. Backwardly, repeat the steps of (5) when T^* is replaced by $T^* - 2$ everywhere to determine $X(T^* - 3)$ and $s_2(T^* - 3)$, then when T^* is replaced by $T^* - 3$ everywhere to determine $X(T^* - 4)$ and $s_2(T^* - 4)$, and so on until $X(1)$ and $s_2(1)$ are determined.

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