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Rotational Stiffness of Precast Beam-Column Connection using Finite Element Method

N Hashim[1] **and J Agarwal**[2]
[1]UTM Razak School Engineering and Advanced Technology, Universiti Teknologi Malaysia, Malaysia,
[2]Department of Civil Engineering, University of Bristol, United Kingdom

E-mail: [1]fazilah.kl@utm.my, [2]j.agarwal@bristol.ac.uk

Abstract. Current design practice in structural analysis is to assume the connection as pinned or rigid, however this cannot be relied upon for safety against collapse because during services the actual connection reacts differently where the connection has rotated in relevance. This situation may lead to different reactions and consequently affect design results and other frame responses. In precast concrete structures, connections play an important part in ensuring the safety of the whole structure. Thus, investigates on the actual connection behavior by construct the moment-rotation relationship is significant. Finite element (FE) method is chosen for modeling a 3-dimensional beam-column connection. The model is built in symmetry to reduce analysis time. Results demonstrate that precast billet connection is categorized as semi-rigid connection with S_{ini} of 23,138kNm/rad. This is definitely different from the assumption of pinned or rigid connection used in design practice. Validation were made by comparing with mathematical equation and small differences were achieved that led to the conclusion where precast billet connection using FE method is acceptable.

1. Introduction

Connections play an important role in providing structural integrity. Lack of understanding, poor design of connection and ignoring the actual response of connection in structural analysis may lead to inadequate structural design and consequently lead to disaster. If the structure is being designed to resist unpredictable actions like earthquakes or accidental loads, incorrect assumption may easily lead to underestimate (or overestimate) of the structural strength and significantly susceptible to collapse. Some guideline explicitly highlights the need of continuity and full capacity in the connections to increase strength of whole structures [1]. This requirement becomes more critical for prefabricated construction systems where the strength is largely governed by the capacities of connections. Previous studies also stated that realistic continuity of load path between members through connection can increase the structural robustness and integrity and limits the extent of collapse [2, 3] and yet these are still lack of reliable information in the connection data especially related to the strength, durability, characteristic, sustainability and performance. However, in practice of frame design, a connection is typically assumed either as simply pinned or fully rigid, where pinned connection is considered as simple because it does not transfer any moment between beam and column and usually adopted for quick construction in low risk area but this type of connection may easily trigger instability leading to collapse. However, the fact shows that actual response of most connections is neither pinned nor rigid because under load services, connection is usually rotated relatively and it should be categorised as semi-rigid connections [4-8]. This paper investigates the rotational stiffness of precast billet connection in order to apply actual behavior for frame design analysis.

1.1 Moment-Rotation through Beam- line method

Application of semi-rigid connection in frame analysis is widely represented by rotational spring which is expressed through a moment-rotation (*M-θ)* relationship of the connection. From the *M-θ,* 3 main connection characteristics can be expressed: the ultimate moment capacity (*MRd, kNm*), rotational capacity (*θ, radian)* of the connection and the rotational stiffness (*S, kNm/radian*). In the elastic stage this relationship is assumed as linear however, nature behaviour of beam-column under gravity load is always nonlinear due to the effect of geometry and material nonlinearities.

The *S* used for semi-rigid connections is represented by a secant stiffness (S_{sec}) obtained from the gradient of the *M-θ* relationship. Figure 1 shows a nonlinear *M-θ* curve with 2 types of gradient - initial stiffness (S_{ini}) and S_{sec}. S_{ini} can be used to represent the connection by linear spring element but the value is often too high [9]. Secant stiffness is calculated by dividing the S_{ini} with the stiffness modification coefficient (η) which is based on connection type and generally obtained through a beam-line method. A gradient of *M-θ* relationship represent the stiffness of the connection, it is usually comply in the spring element. The beam line method, developed by [10], is used to obtain a stiffness value from *M-θ* curve. A line is plotted across *M-θ* curve as shown in Figure 1. Point A represents bending moment ($wL^2/12$) with zero rotation from a rigid connection and point B is the theoretical maximum rotation ($wL^2/24EI$) obtained from pinned connection. Intersection point between beam-line A-B and *M-θ* curve (i.e. point C) represents moment (M_E) and rotation (θ_E) at the beam end. The ratio of M_E and θ_E gives a secant stiffness value (S_E) which is used for rotational spring of semi-rigid connection.

The difficult part in developing *M-θ* is to determine the rotation of the connection (i.e. the relative deformation between beam and column). A study assumed that the relative rotation of beam-column is combination of 2 deformations: first, due to the elongation of beam tensile reinforcement which is anchored into the column and second, from the flexural deformation of beam end at the region of discontinuity [11,12]. Other study suggested that rotation of the connection should be the summation of 3 deformations [5]: deformation at the beam-column interface due to joint opening, deformation within the connection zone due to beam-end curvature along a plastic hinge length (*lp*) and rotational deformation within the connection zone due to column curvature over a distance equal to the depth of the column (*hcol*) beyond the top and bottom of the beam (i.e. twice *hcol*). The relative rotation is expressed by total beam rotation under gravity loading at the beam end rotation minus the column rotation. Moment and rotation were considered at predicted plastic hinge location which was taken as half of beam height plus 100mm from column face.

1.2 Mathematical Equations

Analytical equations by [5] were derived based on the results of 28 experiments conducted on welded and billet type precast connections built with and without floor. Equation (1) can be used to calculate beam-end moment and rotation of precast concrete connection. A combination of 3 deformations were contribute to the total relative rotation: (i) rotation due to joint opening between beam-column interface, (ii) rotation due to beam end curvature within plastic hinge length (*lp*) and (iii) rotation due to column curvature within connection zone. Thus the total relative rotation is calculated as:

$$\emptyset_c = \frac{F}{\lambda d_e} + \frac{M_{Rc} l_p}{E_c I_b} + \frac{M_{Rc} h_{col}}{E_c I_c} \tag{1}$$

where *F* is tensile force of connector, λ is a stiffness of fixing cleat, dowel or plate, d_e is the effective depth (*mm*) of beam, x_c is depth to the neutral axis of connector (*mm*), h_{col} is the height of column, E_c is theYoung's Modulus of concrete, I_b and I_c are second moments of area of beam and column (mm^4), respectively.

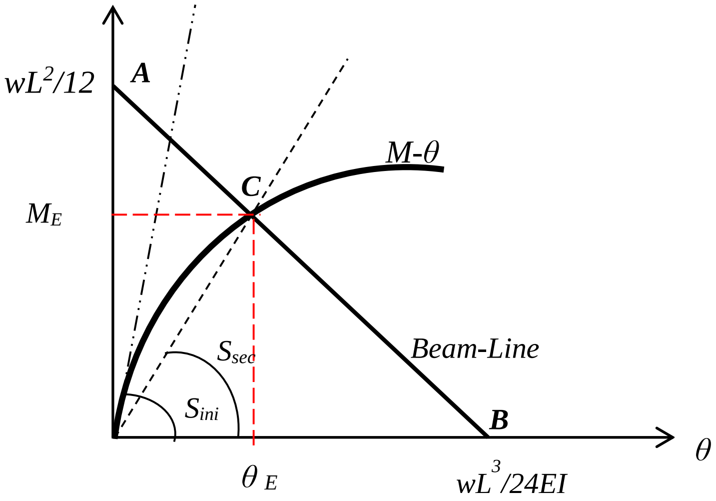

Figure 1. Secant stiffness using beam-line method.

Moment of resistance of the connector (M_{RC}) is given as equation (2), where x_c is depth to the neutral axis ($x_c=F/(0.67f_{cu}0.9b)$ in *mm*, $0.67f_{cu}$ is compressive strength of grout or mortar (taken as 67% of the compressive strength of concrete f_{cu}) and F (in *kN*) is the force of all reinforcing components at the connection (e.g. bolt, dowel). The secant stiffness (S_{RC}) is calculated as equation (3) if the moment-rotation relationship of connection exceeds the requirement of beam-line, while beam-end secant stiffness (S_E) without floor slab is calculated as equation (4).

$$M_{RC} = F(d - 0.45x_c) \tag{2}$$

$$S_{RC} = \frac{M_{RC}}{(M_R - M_{RC})} 2E_c I/L \tag{3}$$

$$S_E = 0.9S_{RC} \tag{4}$$

Finally, moment of resistance (M_E, *kNm*) at beam-end is:

$$M_E = M_R \left[\frac{0.29M_{RC}}{M_R - M_{RC}}\right] - 0.09 \tag{5}$$

where M_R is the moment resistance of the beam.

Analytical equations developed by [13] are used to calculate beam end characteristics of precast connections. Parametric study of 28 FE models of the connections were conducted using statistical measures (R^2) and Standard Error of Estimates (*SEE*). As a result, the ultimate moment resistance (M_u, *kNm*), initial rotational stiffness (S_{ini}, *kNm/rad*), secant rotational stiffness (S_{sec}, *kNm/rad*) and rotation capacity (θ_c, *radian*) were derived as equations (6-9) where f_{cu} is the concrete compression strength in N/mm^2, b_b is the breadth of beam (*mm*), d_e is the effective depth of beam (*mm*), E_c is the Young's Modulus of concrete (N/mm^2), I_c is the second moment area of column (mm^4) and h_{col} is the column height (*mm*). However, only a rotational stiffness is needed for spring element characteristic, thus equation (7) can be directly used for validation of FE result. For the purpose of comparison, mathematical equations from [5] and [13] were taken into consideration.

$$M_u = 9.428 \, x \, 10^{-8} f_{cu} b_b {d_e}^2 + 2.746 \, x \, 10^{-9} E_c I_c / h_{col} \tag{6}$$

$$S_{ini} = 2.79\ x\ 10^{-5} f_{cu} b_d {d_e}^2 + 1.8\ x\ 10^{-7} E_c I_c / h_{col} \tag{7}$$

$$S_{sec} = 2.13x10^{-5} f_{cu} b {d_e}^2 + \frac{1.9x10^{-7} E_c I_c}{L_c} \tag{8}$$

$$\theta_c = 1.80x10^{-12} f_{cu} b_d {d_e}^2 + \frac{1.39x10^{-3} E_c I_c}{L_c} - 4.022x10^{-5} d_e + 0.0152 \tag{9}$$

2. FE Modelling of Beam-Column

2.1 Dimension and Material Properties

Dimensions of beam-column model were design based on typical load of building of medium-storey residential building. Column height was taken from the mid-height of the ground floor to the mid-height of the first floor column. The length of the beam was taken as half of its typical span. The beam was supported on the rectangular steel billet which was built-in with the column. A steel angle located on the top of beam, a bolt and a dowel were used to connect the column and beam. Grout was placed between the beam and column and the beam and steel billet with 10mm distance between them. Grout between dowel and beam was not applied because the *couple model* was adopted for simplicity. The top of the column was restrained in y and z directions while the bottom column was restrained in x, y and z directions (i.e. translations only). The end of the beam was restrained in x-direction for a symmetry boundary condition of the beam. Details on dimensional and material properties used in the model are presented in Figure 2 and Table 1.

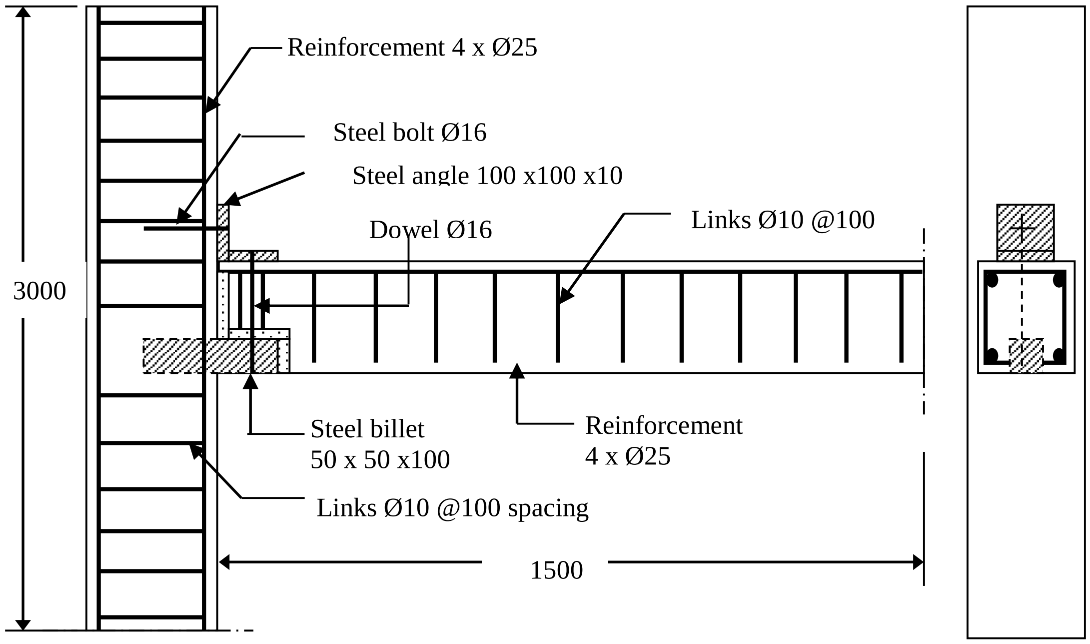

Figure 2. A layout of precast billet connection model (*dimension in mm*)

Table 1. Material properties of precast billet connection model (*Source: Eurocode 2 and Eurocode 3*)

Material	Young's Modulus[a] (N/mm^2)	Grade (N/mm^2)	Poisson's Ratio[b](ν)	Density[c] (kg/m^3)	Strength (N/mm^2)
Concrete: Beam (300 x 350mm)	32000	30	0.2	2400	30

Column (400 x 400mm)					
Grout	20000	20	0.2	2000	20
Reinforcement	200000	-	0.3	7850	500 (main) 275 (stirrup)
Mechanical joint (steel billet and angle)	200000	-	0.3	7850	350
Dowel bar	200000	-	0.3	7850	240
Bolt -Grade 4.6 and 16 mm diameter	200000	M16 (4:6)	0.3	7850	240

[a] *Young's Modulus: (cl.3.2.4.3), (cl.3.3.4.4)*
[b] *Poisson's ratio: (cl.3.1.2.5.3)*
[c] *Density steel bar:(cl.3.2.3)(Eurocode 2, 1992)*

Typical value of concrete strain at maximum stress is taken as 0.002 while ultimate strain is assumed as 0.0032. Typical stress-strain relationship for concrete is calculate based on the European Concrete Committee (*CEB*) where stress is assumed constant to the ultimate strain after reached maximum strength. The pre-tension force ($F_{p,cd}$) for bolts is applied. Effect of torque on the bolt element is applied in terms of initial strain. For a comparison, a model of a typical reinforced concrete in-situ with monolithic connection was built using similar dimension and material properties.

2.2 Modeling using FE Method

FE software named ANSYS version 12 was used [14]. The creation of a model requires the setting of the element types, assigning the real constants, applying material properties and the modelling and meshing of the structure. The first step in FE modelling is to choose an *Element type*. For concrete, SOLID65 was chosen, LINK8 for steel reinforcement and SOLID45 was used for other mechanical joints. The discrete method was chosen in modeling embedded steel bar in the concrete element with an assumption that full bonding between concrete and steel reinforcement were applied where friction and bond-slip response were ignored. The steel angle was modelled as an L-shape without considering the fillet area. A bolt consists of head, stud and nut were used to fix the steel angle to the concrete. Modelling a bolt as a solid model leads to a large number of elements and consequently has high potential for convergence problems. Thus, it has been simplified into 4 models solid bolt model, couple model, spider model and no model [15,16]. Couple model was chosen to reduce number of elements where bolt stud is modelled using a beam element with a capability for a tension force. The nut and head of the bolt are modelled by coupled nodes function which couples the degree of freedom (DOF) between line nodes and angle element. The tension force is transferred through the coupled nodes, LINK10 which is a line element was used to model the bolt and dowel. Torsion effect or pre-tension in bolt is applied as initial strain (ε_o) [17] from EC2 where this is the simplest approach to model clamping effect compared to temperature or pre-tension element.

The interaction between the different materials was modelled using a contact element to transfers loads between 2 elements [18] and in this study, surface-to-surface option was chosen. TARGE170 (3-D target segment) as target element and CONTA173 (3-D 4-noded) as source element, which are compatible with SOLID45 and SOLID65, were used. The surface-to surface method is compatible for discontinuous surfaces between volumes and there is no restriction on the shape of the target surface. The coefficient of friction (CoF), μ_f, between 2 solid surfaces in contact is needed to represent forces between 2 materials. It is expressed as μ_f = F/N where *F* is tangential force and *N* is a normal force [19]. Studies on the coefficient value are very limited, this study used value of 0.8 between grout and concrete (beam and column) [20] and value of 0.4 is common between steel and concrete (Steel billet with beam, steel plate with concrete) [19]. Meshing or discretization is a function that generates elements and nodes ANSYS offers 2 options for meshing; automatic mesh or free mesh and mapped mesh. For automatic, the elements fit automatically into the chosen area or volume model based on the size specified by user. The mapped mesh requires a quadrilateral element for area and brick/hexahedral element for volume. For the best solution speed, a coarse mesh is applied for the larger area or volumes and a finer mesh sizes is used for areas or volumes with stress concentrations. Beam and column were meshed with typical mesh size of 50x50mm. Small size elements were used at joint area around steel plate. A boundary in FE model is applied in terms of constraint (e.g. displacement) or loading (e.g. point load, pressure, moment and temperature). FE analysis is a powerful, yet, complex procedure and to keep the model simple only half of the column height and half of the beam span were modelled with appropriate boundary conditions. Top and the bottom of the column were specified as pinned connection in *y* and *z* direction to restrain any movement in these directions. Pins were applied to represent the location of contra-flexure point which is assumed halfway along the height of column. End of beam was assigned with symmetry boundary condition (i.e. restraint in *x*-direction). Load was applied monotonically along the beam width of the beam-end. The FE model of the precast billet connection, including all details of reinforcement bar, boundary condition and mechanical steel joint are shown in Figure 3.

3. Results and Discussion

The models were loaded gradually until a maximum bending moment is achieved. Intersection of beam line with the *M-θ* curve is used to determine the secant stiffness of the connection. Comparison between both *M-θ* clearly depicted in Figure 4. As results, a rotational stiffness value of 23138kNm/rad was obtained. Looking at the same amount of load, rotation for the monolithic connection was 0.00126radians while for precast billet connection was 0.018radians (i.e. 0.0167radians more than the other). As expected, precast billet will rotate more than monolithic model due to its flexibility between beam-column components. It has lower gradient and hence resulted to lower stiffness than the monolithic connection. The precast billet connection model had S_{ini} value of 23,138kNm/rad as compared with 19,3636kNm/rad for monolithic connection model.

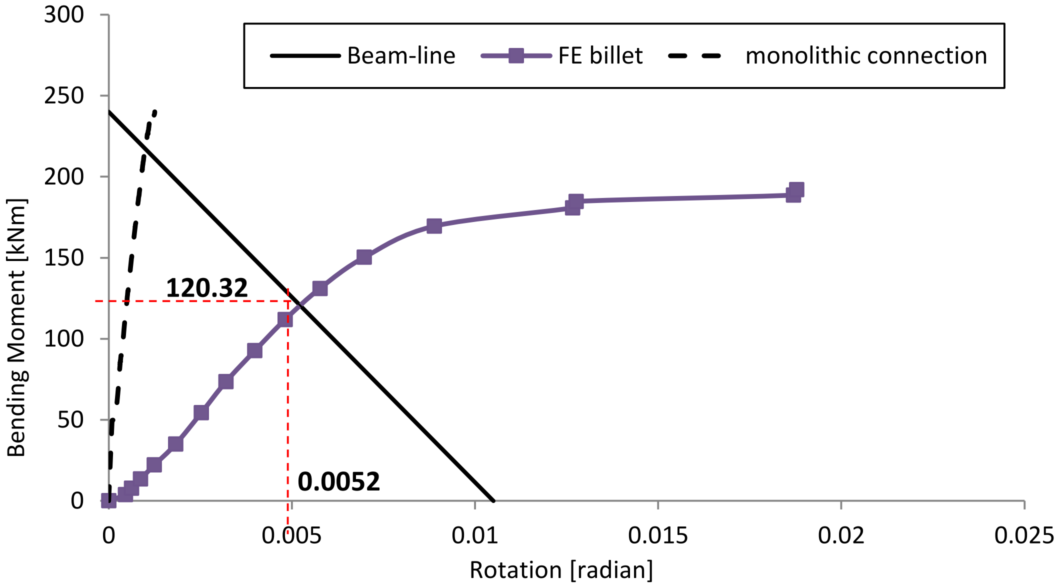

Figure 3. A comparison of *M-θ* relationship using beam-line method between precast billet connection and monolithic connection

3.1 Validation with Mathematical Model

S_{ini} from FE model is 23138kNm/rad which is 13% more from [13] (S_{ini} =20162kNm/rad) and 4% more for [5] (S_{ini} = 22290kNm/rad). These differences are expected due to the several simplifications that applied in the FE model and analysis. For example, the FE model did not consider bar-slip effect (i.e.

fully-bonded relationship was assumed between steel reinforcement and concrete) and not take into account cracking and crushing effect to reduce computational effort. Whereas, both [13] and [5] equations were developed based on actual structural response from experiments where these effects were considered. These small differences and reasons thereof led to the conclusion that rotational stiffness of precast billet connection using FE method is acceptable.

4. Conclusion

A precast billet connection was modelled using a 3-Dimensional FE method in order to rotational stiffness to represent semi-rigid connection in frame analysis. The FE method is very sensitive where small variations could easily lead to convergence problems in analysis but it is economical approach compared to laboratory testing. Based on the derived M-θ relationship, the precast billet connection is classified as a low strength semi-rigid connection with a rotational stiffness of 23138kNm/rad. The value is considered acceptable since only small differences were found when compared with existing analytical equations. Further studies are needed to account for other type of precast connections hence a database of connection characteristic can be developed.

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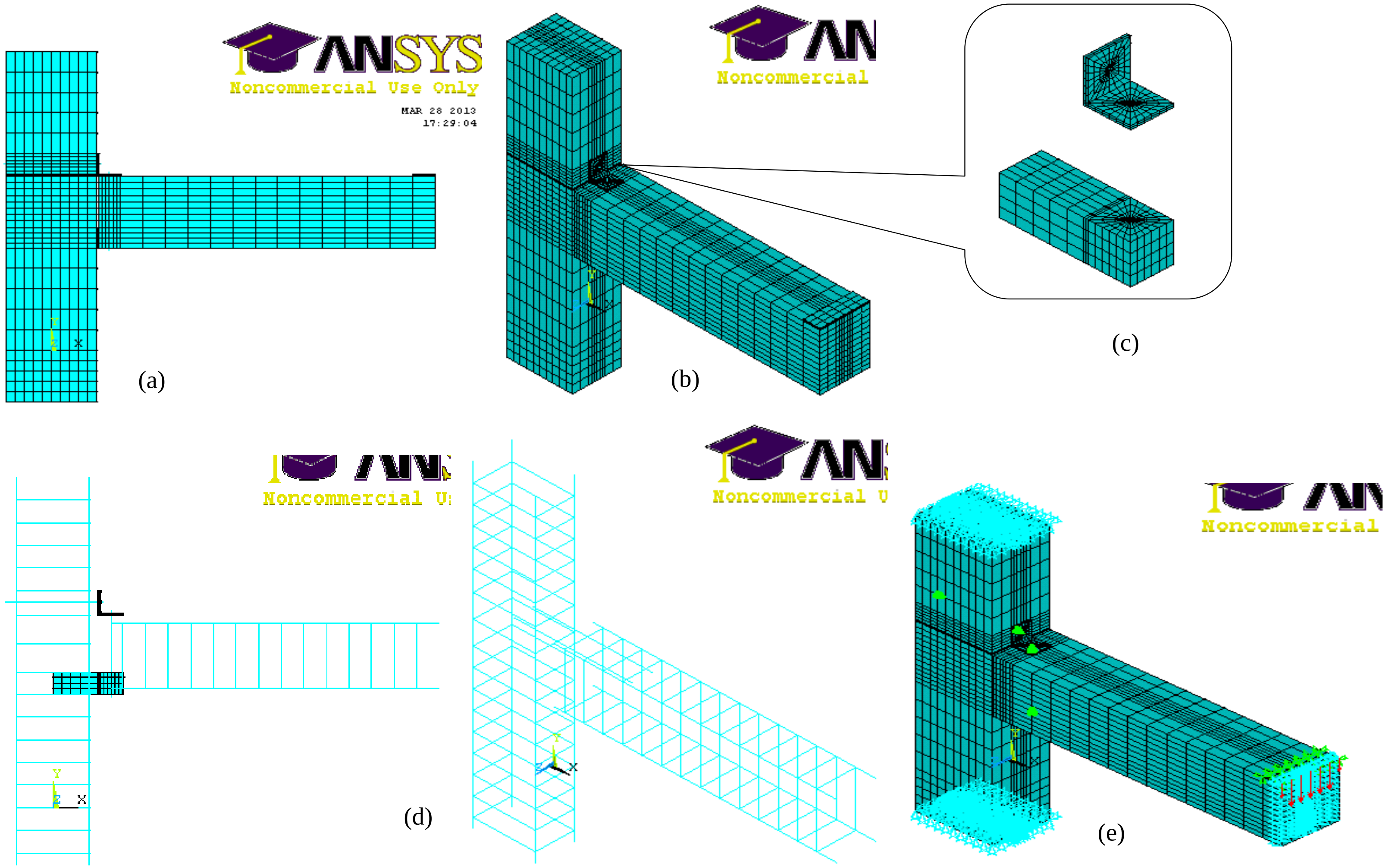

(a) (b) (c) (d) (e)

Figure 4. FE modelling of the precast billet connection, (a) Front view, (b) isotropic view, (c) steel angle and steel billet (d) reinforcement (e) boundary condition and loading applied at the beam-column model.

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