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## **KINEMATICALLY ENHANCED CONSTITUTIVE MODELLING: A VIABLE OPTION FOR THE SIMULATION OF THE MANUFACTURING OF FULL-SCALE COMPOSITE PARTS**

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### **Abstract**

Earlier work carried out the University of Bristol has shown the possibility to accurately predict consolidation-driven defect formation in composite parts using a ply-by-ply approach. However, using this method, a typical model for a lab-scale specimen can easily reach the hundreds of thousands of degrees of freedom, which makes it unsuitable for the modelling of full-scale components and limits its applicability for the modelling of real industrial cases. The present contribution, proposes to overcome this difficulty using kinematically enhanced constitutive modelling. The paper first discusses the basis of kinematically enhanced modelling for layered structures made of soft anisotropic material. The ability of the proposed framework to significantly reduce the computational cost of the simulation while correctly capturing consolidation and defect formation processes is then established. Finally, conclusions are drawn on the prospect for modelling larger components.

### **1. Introduction**

Predicting defect formation occurring during the consolidation and the curing of composite parts made from the latest generation of highly toughened pre-preg materials remains an ongoing subject of research [1, 2]. In a new approach the present authors suggested that the traditional flow modelling of thermoset-based prepreg that uses the Darcy's law and considers the behaviour of a material point situated in the middle of a fully constrained piece of pre-preg needed to be re-visited. A thorough experimental program [3] was carried out on cruciform shaped pre-preg stacks of various in-plane dimensions and stacking sequences subjected to a range of temperature and pressure cycles. This highlighted the necessity for a new model that takes into account the coupling between squeezing and bleeding flow that has been observed experimentally in the past [4] but largely disregarded. Starting from micro-structural considerations, a phenomenological analytical model that assumes a transition from squeezing to bleeding flow was subsequently proposed [5]. This model was implemented as a hyper-viscoelastic material model for the finite element package Abaqus/Standard and validated against experimental data. The model was then coupled with more traditional thermo-chemical, residual stress and cure shrinkage models for a full description of the chemical and physical changes occurring upon the components consolidation and cure [6]. The ability of this new model to capture wrinkles formed during manufacturing was recently proven for lab-scale samples, representative of industrial geometries [7].

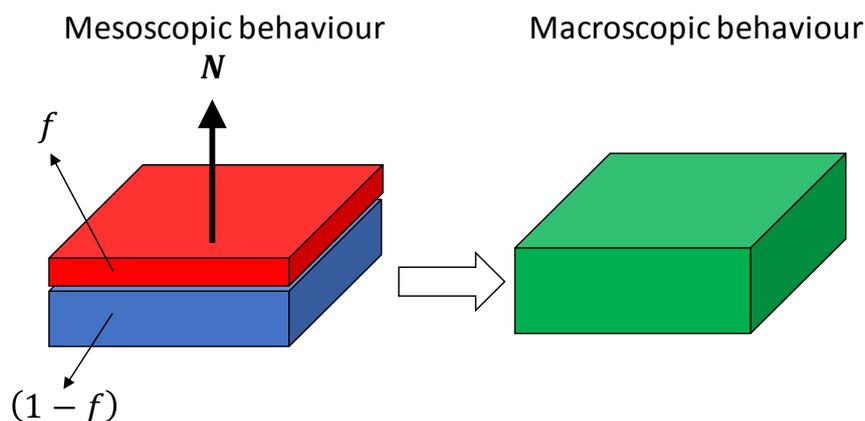
The method used a ply-by-ply approach and a relatively simple contact between the plies is assumed. This allows prediction of very detailed information on wrinkle shape, porosity between the plies and ply interaction. However, a typical model for a lab-scale specimen (e.g. 20 mm-thick sample with 300 mm \* 80 mm in-plane dimensions) can easily reach the hundreds of thousands of elements, with extra degrees of freedom being added for the contacts between the plies, which are also responsible for long analysis times due to their poor convergence in the implicit FE analysis. This clearly prevents the use of the approach for the modelling of full-scale components (i.e. an aircraft wing is typically >20 m long) and therefore limits the applicability of the approach for real industrial cases.

Recently, Dodwell [8] proposed an efficient modelling framework for stacks of uncured prepreg under processing conditions based on Cosserat theory. Good agreement between model predictions and experimental observations of out-of-plane wrinkle instabilities in the corner region of a C-section demonstrator was obtained. The framework was proven to be computationally efficient but somewhat difficult to implement within a commercial FE package. In the present contribution, new effort that have been deployed at the university of Bristol in order to overcome the computational cost of our approach are presented. Dodwell's idea of describing ply interaction using modelling techniques traditionally used to describe crack initiation and propagation in materials is reutilised. It is, however, chosen to use kinematically enhanced constitutive models [9, 10] in the place of Cosserat theory. A model where several plies and interfaces are taken account of within a single element is formulated. The paper is organised as follows: first, the kinematically enhanced modelling framework used is set-up; the model is then validated through simple compaction tests as well as on a more complex (lab-scale) L-shaped laminate; finally, the potential for applying the model to predict wrinkle formation in full-size components is discussed.

## 2. Model Formulation

Modelling strain localisation phenomena at a reduced computational cost is still an ongoing field of research. Over the years, researchers have proposed different techniques ranging from simple and practical methods such as the smeared crack approach, to mathematically more sophisticated ones such as Cosserat, nonlocal/gradient theories or extended finite elements (XFEM). More recently, in the context of quasi-brittle failure of materials, Nguyen and co-workers [9, 10] have proposed a new constitutive modelling framework which, as opposed to the smeared-crack approach, is independent of the type of Finite Element (FE) used, yet is mathematically simpler and easier to implement in a commercial FE package than the XFEM or the Cosserat methods. In this paper, Nguyen's modelling approach is adapted to the context of composite manufacturing and more generally to the mechanical response of layered structures made from soft anisotropic materials.

In the suggested model, the constitutive behaviour of each ply constitutive of the stack follows the hyper-viscoelastic model for prepreg under processing conditions proposed by Belnoue et al. [5]. The interaction between the plies has a strong influence on the apparent bending behaviour of the stack [8] as well as on its compressive response [11] and thus needs to be modelled. To account for this, ply interfaces are represented as thin extra layers of viscous fluid connecting the plies. The ply-scale model already considered ply interaction and its implications on the compressive response of the stack. Hence, in the laminate-scale model the role of interfaces was to include the relative movement of the plies. For consistency of the formulation, the interface modelled in the same way as the plies but without including the stiff elastic response in the fibres direction.



**Figure 1:** Homogenisation procedure for a stack made from 2 layers. A layer can be an uncured piece of prepreg or an interface made of pure resin.

Mandel–Voigt tensor notation is used throughout the paper. We note  $\mathbf{E}$  and  $\mathbf{S}$  respectively as the logarithmic strain tensor and the 2<sup>nd</sup> Piola-Kirchhoff stress tensor of the stack, considered at the macro-scale. At the meso-scale, the logarithmic strain and 2<sup>nd</sup> Piola-Kirchhoff stress of the  $n$ -th layer of the stack are respectively called  $\mathbf{E}^{(n)}$  and  $\mathbf{S}^{(n)}$ . Time derivative quantities are defined using Newton's notation. Finally, the transpose operator of a tensor  $\mathbf{A}$  is denoted  $\mathbf{A}^T$ . Merging of model of individual plies and interfaces in a unified laminate model is implemented through energy balance using Nguyen's framework. In this framework, the homogenisation of two layers made from different materials (see Figure 1) relies on 3 assumptions:

1. The macro strain rate tensor of the stack (considered as a homogeneous block) can be computed from the strain rates of each of the constitutive layers at the mesoscopic level using a simple rule of mixture.

$$\dot{\mathbf{E}} = (1 - f)\dot{\mathbf{E}}^{(1)} + f\dot{\mathbf{E}}^{(2)} \quad (1)$$

2. The strain field is continuous across the plane where the two materials link (i.e. compatibility condition).

$$\begin{aligned} \dot{\mathbf{E}}^{(1)} &= \dot{\mathbf{E}} - f\mathbf{N}\dot{\tilde{\mathbf{E}}}_i \\ \dot{\mathbf{E}}^{(2)} &= \dot{\mathbf{E}}^{(1)} + \mathbf{N}\dot{\tilde{\mathbf{E}}}_i = \dot{\mathbf{E}} + (1 - f)\mathbf{N}\dot{\tilde{\mathbf{E}}}_i \end{aligned} \quad (2)$$

3. The macro stress is obtained from the Hill-Mandel condition, which ensures that the work produced by the macroscopic stress/strain rate should equilibrate the volume-averaged work due to the stresses and strain rates in each constitutive layer.

$$\mathbf{S}^T \dot{\mathbf{E}} = (1 - f)\mathbf{S}^{(1)T} \dot{\mathbf{E}}^{(1)} + f\mathbf{S}^{(2)T} \dot{\mathbf{E}}^{(2)} \quad (3)$$

In eqs. (1-3),  $f$  is the volume fraction of the 2<sup>nd</sup> layer and  $\mathbf{N}$  is the normal vector of the interface where the 1<sup>st</sup> and 2<sup>nd</sup> layers are joined (see Figure 1).  $\dot{\tilde{\mathbf{E}}}_i$  is a kinematic strain enhancement rate. Feeding eqs. 1 and 2 into eq. 3 then leads to the continuity of the traction vector across the interface and the expression of the macroscale 2<sup>nd</sup> Piola-Kirchhoff stress as a function the mesoscale stresses of the layer 1 and 2 following a rule of mixture:

$$\mathbf{S}^{(1)T} \mathbf{N} = \mathbf{S}^{(2)T} \mathbf{N} \quad (4)$$

$$\mathbf{S} = (1 - f)\mathbf{S}^{(1)} + f\mathbf{S}^{(2)} \quad (5)$$

The relationships expressed above allow to link the macroscale apparent behaviour of the stack to the responses of materials 1 and 2 respectively. The relationship between the stress rate and the strain rate in each layer is derived at length in Belnoue et al. [2]. For conciseness, this is expressed here as:

$$\dot{\mathbf{S}}^{(n)} = \mathbf{D}^{(n)} \dot{\mathbf{E}}^{(n)} \quad (6)$$

Introducing eq. (6) into eq. (4) and then replacing  $\dot{\mathbf{E}}^{(1)}$  and  $\dot{\mathbf{E}}^{(2)}$  in the obtained expression using eq. (2) gives rise to:

$$\dot{\tilde{\mathbf{E}}}_i = [(1 - f)\mathbf{N}^T \mathbf{D}^{(2)} \mathbf{N} + f\mathbf{N}^T \mathbf{D}^{(1)} \mathbf{N}]^{-1} \mathbf{N}^T (\mathbf{D}^{(1)} - \mathbf{D}^{(2)}) \dot{\mathbf{E}} = \mathbf{C}^{-1} \mathbf{N}^T (\mathbf{D}^{(1)} - \mathbf{D}^{(2)}) \dot{\mathbf{E}} \quad (7)$$

Deriving eq. (5) with respect to time and reintroducing eq. (7) into the obtained expression then allows to express the macroscopic stress rate as:

$$\dot{\mathbf{S}} = [f\mathbf{D}^{(2)} + (1 - f)\mathbf{D}^{(1)} - f(1 - f)(\mathbf{D}^{(1)} - \mathbf{D}^{(2)})\mathbf{N}\mathbf{C}^{-1}\mathbf{N}^T(\mathbf{D}^{(1)} - \mathbf{D}^{(2)})] \dot{\mathbf{E}} \quad (8)$$

As eq. (8) is of the same mathematical form as eq. (6), a laminate can be constructed by a series of

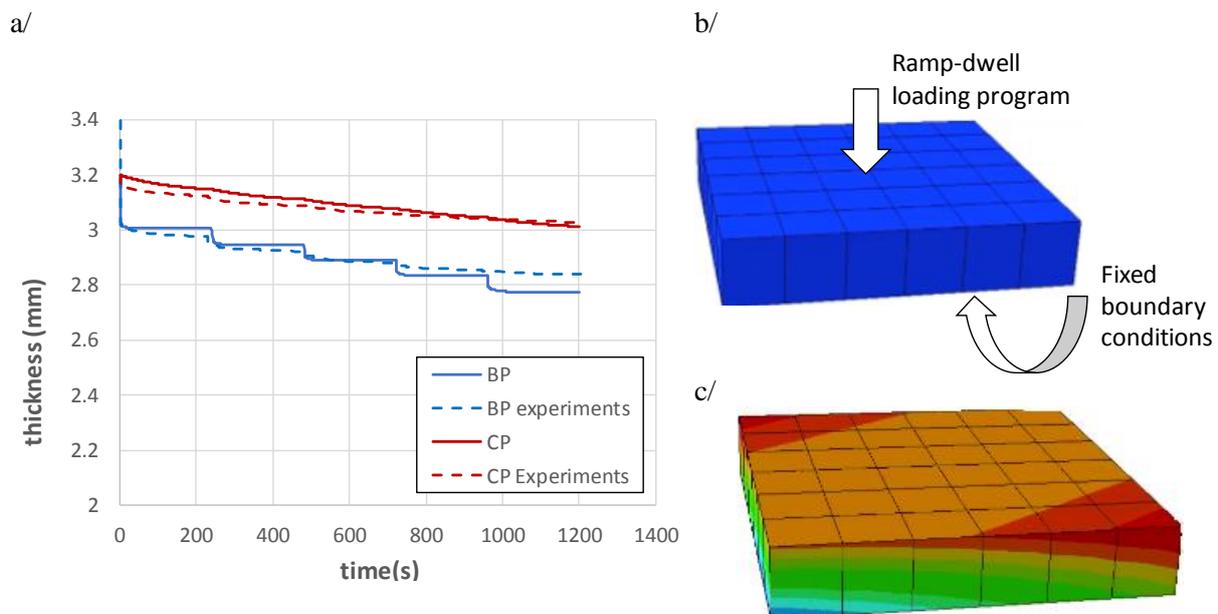
homogenisation steps for each of two layers, which follows the procedure described above. Indeed, a macroscopic response obtained from the homogenisation of layer 1 and 2 can be homogenised with layer 3. The results of this procedure can then be homogenised with layer 4 and so on so forth up to n-th layer of the laminate.

This procedure was implemented in the form of a UMAT subroutine for the implicit FE package Abaqus/Standard. This was facilitated by the fact that everything is formulated in terms of strains rather than degrees of freedom as in the case of similar theories such as Cosserat or XFEM. The scheme relies on the computation, at all time, of the strains in each of the layers of the stack and each of the resin rich regions in-between the plies. This way, a resolution similar to the ply-by-ply approach can be obtained. Yet the FE software is only fed with the stress/strain response of the homogenised block, thus decreasing the number of degrees of freedom seen by the solver and drastically reducing the computational cost of the simulation. Another factor in the reduction of the runtime is the fact that highly strained regions are smeared-out within the laminate homogeneous response thus easing the convergence of the FE scheme. One of the main drawback of the method is that it is very costly in terms of memory usage.

### 3. Model assessment

#### 3.1. Compaction testing

Initial model assessment was performed by simulating some of the compaction experiments on uncured laminates performed by Nixon-Pearson et al. [3] (see Figure 2). Cross-ply  $[90/0]_8$  (CP) and blocked-ply  $[90_4/0_4/90_4/0_4]$  (BP) laminates made from IM7-8552 and with  $15 \times 15 \text{ mm}$  in-plane dimensions were considered. Only the experiments performed at  $T=90^\circ\text{C}$  were simulated. The results are presented in Figure 2. As shown in Figure 2 a/, similar evolution with time of the specimen thicknesses compared to the experiments were observed. The layup effect reported in the experimental paper [1] were well captured, with BP specimens behaving fundamentally differently from CP samples.



**Figure 2:** Simulation of Nixon-Pearson's compaction experiments [3]. a/ Evolution of the specimens' thickness with time. b/ Boundary conditions applied on the FE model. c/ Additional case of a  $[20/0]_8$  layup showing apparent macroscopic through-thickness shearing of the laminate, as indicated by this gradient plot of the magnitude of the displacement field.

Inspired by Sorba et al. [11], who reported that 0/20 laminates under compaction exhibit some relative rotation of the plies, the additional case of a [20/0]<sub>8</sub> laminate of the same geometry and subjected to the same thermo-mechanical loading was also simulated. No experimental data are available for this case. However, Figure 2 c/ shows that the laminate is macroscopically under shear. This corresponds to the relative rotation of the plies observed in [11]. As this deformation mode would not be possible without the interaction between the plies being modelled [8], this additional case allows to gain further confidence that the newly proposed scheme is working properly.

### 3.2. Qualitative assessment under bending

Further qualitative assessment of the role of the interfaces was performed by simulating a cantilever bending test for 3 different layups:

1. A quasi-isotropic layup
2. A UD laminate with the fibres aligned in the direction of the beam.
3. A UD laminate similar to case 2. but in which the plies interfaces have been stiffened so that the plies can not slip relatively to each others as easily.

Dynamic implicit analysis was performed. In all cases, the beams were subjected to self-weight and the temperature was set to 90°C. Results are displayed in Figure 3. As expected the quasi-isotropic layup with fewer fibres aligned with the direction of loading was the most compliant. Cases 2 and 3 also behave differently. The stiff interfaces make case 3 the specimen with the stiffest apparent behaviour.

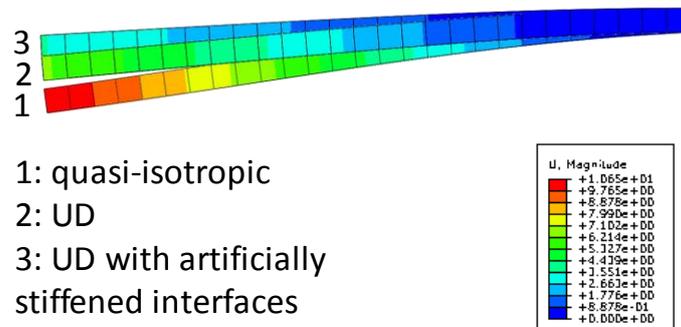
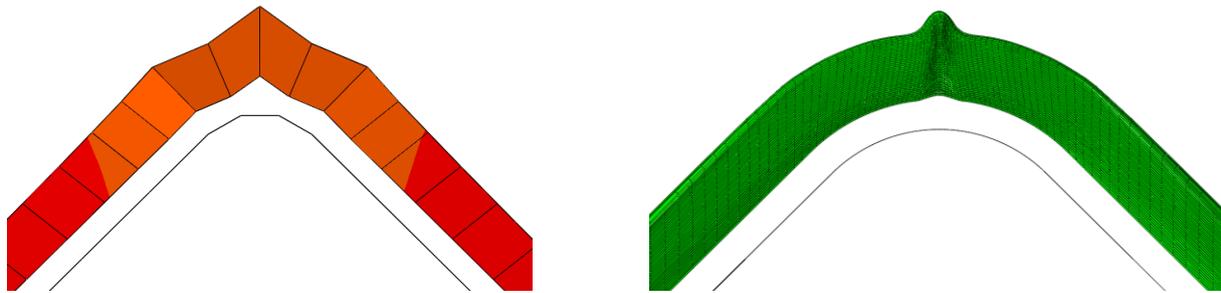


Figure 3: Simulation of a cantilever bending test in the case of 3 different layups.

### 3.3. Consolidation over an external radius

Finally, more validation of the model was performed by simulating the autoclave consolidation over an L-shaped tool of a 6 mm-thick specimen with 300 mm \* 80 mm in-plane dimensions. The tool radius was 15 mm and the material used IM7-8552. Validation of the ply-by-ply model for the exact same case is presented in [7]. As illustrated in Figure 4, the wrinkle severity predicted by the new kinematically enhanced constitutive model is strikingly similar to that made using the high resolution model. Very good agreement is also obtained for the specimen final thickness. More importantly, the new approach allows to reduce the time necessary to run an analysis from 3 days down to 30 mins on the same machine and using the same number of CPUs.



**Figure 4:** The new kinematically enhanced constitutive modelling approach (left) compares very well with the ply-by-ply approach (right) for the prediction of wrinkle formation in thick composite parts.

#### 4. Conclusions and future developments

In this paper, a new modelling approach for the prediction of defects arising in the course of composites manufacturing is proposed. The kinematically enhanced constitutive modelling approach used allows to take account of discontinuities at the material level, making its implementation into commercial FE codes much easier than other methods providing similar degrees of enhancement of the displacement field (i.e. Cosserat models, XFEM etc). It was shown that in the case of a lab-scale specimen a significant reduction of the number of elements required could be achieved, thus suppressing the need for the ply-by-ply modelling approach previously used by the authors of this contribution. The newly proposed model makes the prospect of modelling real-size components much more feasible.

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