
Publisher's PDF, also known as Version of record

License (if available):
CC BY-NC-ND

Link to published version (if available):
10.1016/j.compstruct.2018.09.086
10.1016/j.compstruct.2018.09.086

Link to publication record in Explore Bristol Research

PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available: http://www.bristol.ac.uk/pure/user-guides/explore-bristol-research/ebr-terms/
A robust and reliability-based aeroelastic tailoring framework for composite aircraft wings

Muhammad F. Othman,⁎ Gustavo H.C. Silva, Pedro H. Cabral, Alex P. Prado, Alberto Pirrera, Jonathan E. Cooper

A robust and reliability-based aeroelastic tailoring framework for composite aircraft wings

This paper presents a multi-level aeroelastic tailoring framework for the optimisation of composite aircraft wings. The framework is capable of structural sizing and produces detailed composite ply configurations through robust and reliability-based design optimisation, and is demonstrated on a representative regional jet airliner finite element wing box model. The optimisation procedure is divided into two levels. The first level optimises the wing structure for minimum weight subject to multiple constraints including strain, buckling, aeroelastic stability and gust response. These first level solutions are then fed into the second level to be further optimised for robustness or reliability by considering uncertainties in material properties at ply level. Both the principles of robust and reliability-based design optimisation can also be used in combination to ensure a balance between the robustness and reliability of the structural performance. In order to keep computations to an acceptable cost, the second level optimisation employs the Polynomial Chaos Expansion method to approximate the effect of probabilistic uncertainty on structural performance. In comparison to the original benchmark wing, the framework produces an overall weight reduction of 32.1%, despite a 1.5% increase from the first to the second level optimisation that accounts for stochastic design variations.

1. Introduction

The design of aerodynamically tailored aircraft structures—intended for maximum performance and minimum weight—remains a challenging multidisciplinary optimisation problem. Although the possibility, and the consequent benefits, of aerodynamic tailoring through composite materials have been around since the early 1980s, most of the designs proposed by the research community have been based on the so-called 'black metal' approach, which does not exploit anisotropy fully. This somewhat conservative attitude is at odds with the elastic tailoring capabilities offered by composite materials, which, by allowing bending-torsion stiffness coupling terms to be modified, lend themselves to innovative design solutions for improved aerodynamic performance.

A significant amount of work can be found in the literature concerning the design and optimisation of aerodynamically-tailored composite wing structures. Typically, research has focused on optimising wings for minimum structural weight, subject to multiple constraints including stress, strain, buckling, aeroelastic response and gust response. All of these studies report of modifications of the wing's composite panels to design for passively coupled bending-torsional deformations.

Concurrently, considerable efforts have been devoted to develop the aeroelastic tailoring design process to improve both its accuracy and computational efficiency. In particular, in order to circumvent the computational cost associated with detailed, high fidelity wing representations, some authors adopted simplified models, while others resorted to efficient optimisation techniques. Examples of the former case include: Refs. [7,9–12], which approximated composite lifting surfaces as cantilever plates and optimised for aeroelastic stability, neglecting the effects on structural weight; and Refs. [3,7,13–16], which used wing-box models, including skins, spar and ribs, in order to obtain a more accurate structural and aeroelastic performance characterisation. Examples of the latter case, instead, include the work by Kuzmina and Guo [2,3] and Gasbarri et al. [17] that employed multi-objective or hybrid optimisation procedures. In their approach, the...
optimisations are divided into stages to address multiple objectives or
constraints, which leads to enhanced computational efficiency.

Traditionally, aircraft wing structures are designed using determi-
nistic approaches for minimum structural weight, while satisfying
multiple constraints for performance and certification. Designers,
however, are aware that deterministic optimisations, being unable to
account for probabilistic uncertainties, may lead to unreliable or un-
realistic designs. There are two types of uncertainty that can be clas-
sified as ‘epistemic’ and ‘aleatory’. As described by Melchers [18], epis-
temic uncertainty is a type of uncertainty arise from limitation of
knowledge which is often due to lack of understanding about the
physics and human errors. Aleatory uncertainty is an irreducible un-
certainty which is inherent to the system or model. When dealing with
aircraft design, uncertainties may arise from human errors, geometric
and material properties, and from manufacturing processes. If one were
to design for reliability and robustness, these uncertainties should be
quantified accurately. Hence, the growing interest in improving or re-
placing deterministic optimisation procedures for robust and reliability-
based structural design methods.

Including parameter uncertainty in design optimisation implies
solving the problem with sampling or quantification methods such as
Monte Carlo Simulation (MCS) [8,19], Polynomial Chaos Expansion
(PCE) [12,16,20,21] or Stochastic Collocation [22]. MCS is the most
common and straightforward technique to quantify uncertainty in a
model; however, great computational efforts are required to produce
meaningful results. The effectiveness of PCE over MCS has been re-
ported in Refs. [12,20] in relation to plate wing models for aeroelastic
analyses, where it was shown that using a sampling methods such as
PCE reduces the number of runs required to fully characterise the
model’s uncertainty, in comparison to MCS. A finding of obvious po-
tential for uncertainty-based design optimisations.

In the context of aeroelastic design of composite structures, alea-
toric uncertainty arises from a number of sources, including structural
geometry, errors in aerodynamic predictions, variability in material
properties such as material non-homogeneity, fibre misalignment, wa-
viness, wrinkling and defects, as well as manufacturing tolerances and
thickness variations. These uncertainties are to be quantified accurately
in order to produce realistic designs accounting for robustness and re-
liability. The literature reports two main methodologies for uncertainty-
based design optimisation: 1) Reliability-Based Design Optimisation
(RBDO) [12,23–25] and 2) Robust Design Optimisation (RDO) [24,26].
RBDO aims at optimising a design whilst having a particular risk or
target reliability/performance as a constraint. RDO seeks optimal de-
signs about a mean response value thereby maximising robustness via
minimisation of the sensitivity to random parameter variations [24]. A
mixed approach, which employs features of both RDO and RBDO is
thought to be a more effective means to search for robust optima that
also satisfy reliability constraints. Paiva et al. [24] used a mixed RDO-
RBDO approach for the preliminary design of aircraft wings. Their
multidisciplinary approach employs a Krigeing surrogate model to ac-
count for uncertainty in parameters of flight condition.

The application of probabilistic optimisation approaches such as
RBDO and RDO for the aeroelastic tailoring of composite structures has
been reported by several authors [12,23,25]. Scarth et al. [12] and
Manan et al. [23] used simplified analytical models for aeroelastic
stability with uncertainty arising from composite material properties.
These works employ a PCE model for uncertainty evaluation, together
with a singly-constrained RBDO approach, to obtain a reliable design
for maximum instability speed. In contrast, the work presented in this
paper employs a detailed finite element wing box model, together with
a PCE surrogate model for uncertainty quantification, for a multi-con-
strained aeroelastic tailoring optimisation problem. In addition, robust
and reliability-based design methods are used in combination within a
multi-level optimisation framework.

This paper introduces a multi-level aeroelastic tailoring optimisa-
tion approach to determine minimum structural wing weight, subject to
multiple structural and aeroelastic constraints. The optimisation pro-
cedure is divided into two levels: a deterministic optimisation and a
combined implementation of robust and reliability-based design optimi-
sations (RRBDO). Composite material properties and ply thickness
variations are chosen as the parameters carrying uncertainty, with
different levels of variation. A comparison between the RDO, RBDO and
RRBDO approaches for aeroelastic tailoring is presented. It is found that
the proposed multi-level aeroelastic approach enables the designer to
rapidly evaluate minimum weight solutions for composite wings and
also account for considerations of robustness and design reliability.

2. Model definition and analysis methods

A detailed Finite Element (FE) model for the high aspect ratio wing
box of a reference regional jet airliner is used for the analyses presented
in this paper. The model is shown in Fig. 1. Planform and wing box
geometry are depicted in Fig. 2 with dimensions normalised to the wing
semi-span. The structure is modelled in MSC PATRAN 2013 using CQUAD4
shell elements for the spars, ribs and skins and CBAR beam elements for
the stringers. The model comprises three spars, 30 ribs and 14 stringers.
All parts of the primary structure are made of intermediate modulus
carbon/epoxy composite (Hexcel 8552 IM7 [27]), with material prop-
erties listed in Table 1.

A total of 25471 elements and 16453 nodes are used in the struc-
tural mesh to ensure converged results. The aerodynamic panelling,
also shown in Fig. 1, is divided into two sections, with the outer wing
having a higher mesh density compared to the inner wing. The model’s
aerodynamic grid and structural mesh are coupled using a surface
spline to transfer loads from the aerodynamic grid points to the FE
nodes.

For dynamic and aeroelastic analyses, engine and fuel weight are
modelled as concentrated masses, with locations as shown in Fig. 2(g).
The fuel mass is distributed spanwise along the tank centroid line, with
point masses positioned between each spar-rib bay.

Only skin and spar sections are included in the optimisation pro-
cedures, where a total of 41 panels is created, with 11 panels for top
and bottom skins, eight panels for spar 1 and 2, and three panels for
spar 3, as shown in Fig. 2(b). As mentioned in the introduction, the
wing model is optimised for minimum weight with consideration of
robustness and reliability, when subject to multiple constraints in-
cluding strain, buckling, aeroelastic stability and extreme gust loads.
Lamination parameters and laminate thickness are chosen as design
variables and translated into stiffness components to be input into the
FE model.

2.1. Lamination parameters

Lamination parameters provide a compact and computationally
inexpensive mathematical representation for composite lay-ups and
are, therefore, adopted in this paper. In the most general case, by using
The so-called material invariants, 12 lamination parameters and the laminate thickness are sufficient to define the full set of stiffness coefficients describing a composite stack [28]. This compact definition greatly reduces the number of unknown variables required to specify a stacking sequence. In addition, lamination parameters are continuous variables and therefore lend themselves to efficient gradient-based optimisations. The mapping between lamination parameters and lay-ups, however, is not bi-objective. Not all sets of lamination parameters correspond to feasible stacking sequences. Additionally, a point in laminate parameter space may map onto multiple stacking sequences. For these reasons, the aforementioned gradient-based optimisations are usually constrained to operate within feasibility regions defined by inequalities relating the lamination parameters. Stacking sequences are then retrieved from lamination parameters by solution of an additional optimisation problem. An example is given by Kameyama and Fukunaga [29] that utilised lamination parameters as intermediate design variables for the minimum weight optimisation of composite plate wings subject to flutter and divergence constraints.

According to Classical Laminate Theory (CLT) [30], the relation between generalised internal forces and generalised deformations for symmetric laminates is given by

$$\{A\} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \{\varepsilon\}$$

where $A$ and $D$ represent the laminate's stretching and bending stiffnesses, respectively, $N = [N_x, N_y, N_{xy}]^T$ and $M = [M_x, M_y, M_{xy}]^T$ are resultant forces and moments per unit length, and $\varepsilon = [\varepsilon_x, \varepsilon_y, \gamma_{xy}]^T$ and $\kappa = [\kappa_x, \kappa_y, \kappa_{xy}]^T$ are in-plane stretching terms, and bending and twist curvatures. The stiffness components, $A_{ij}$ and $D_{ij}$, can be calculated from the stiffness invariants, $[U]$, and the in-plane and out-of-plane laminate parameters, $\xi^A_k$ and $\xi^D_k$ (where $k = 1, 2, 3, 4$), by means of the following equations:

$$\begin{bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{36} \end{bmatrix} = \begin{bmatrix} 1 & \xi^A_1 & \xi^A_2 \\ 0 & 0 & -\xi^A_4 \\ 1 & -\xi^A_1 & \xi^A_2 \\ 0 & 0 & -\xi^A_4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

and

$$\begin{bmatrix} D_{11} \\ D_{12} \\ D_{22} \\ D_{36} \end{bmatrix} = \frac{t^3}{12} \begin{bmatrix} 1 & \frac{\xi^D_1}{\xi^A_1} & \frac{\xi^D_2}{\xi^A_2} \\ 0 & 0 & -\frac{\xi^D_4}{\xi^A_4} \\ 1 & -\frac{\xi^D_1}{\xi^A_1} & \frac{\xi^D_2}{\xi^A_2} \\ 0 & 0 & -\frac{\xi^D_4}{\xi^A_4} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

where $t$ is the thickness of the laminate.

By defining the non-dimensional through-thickness coordinate, the lamination parameters can be expressed in terms of ply orientation $\theta$ as

$$\xi^A_{1,2,3,4} = \frac{1}{2} \int_{-1}^{1} \left( \cos 2\theta(u), \cos 4\theta(u), \sin 2\theta(u), \sin 4\theta(u) \right) du$$

and

$$\xi^D_{1,2,3,4} = \frac{3}{2} \int_{-1}^{1} \left( \cos 2\theta(u), \cos 4\theta(u), \sin 2\theta(u), \sin 4\theta(u) \right) w^2 du.$$

Bloomfield et al. [28] provide a comprehensive overview on the feasible regions of lamination parameters. In this work, we use the inequalities derived by Fukunaga and Sekine [31], which describe the feasible regions of the four in-plane and out-of-plane lamination parameters. These are

$$-1 \leq (\xi^A_j) \leq 1, \quad (\xi^D_j)^2 + (\xi^A_j)^2 \leq 1,$$

$$2(1 + \xi^A_j)(\xi^D_j)^2 - 4\xi^A_j(\xi^D_j)^4 + (\xi^A_j)^4 - 2(\xi^D_j)^2 + 1)(1-\xi^A_j) \leq 0,$$

where $k = 1, 2, 3, 4$ and $j = A, D$.

Current practice in the design and optimisation of composite laminates is to restrict the design space to balanced and symmetric lay-ups, to enable feasible manufacture. One of the reasons is that non-symmetric laminates tend to warp upon cool down from curing temperature, and unbalanced laminates have shear-extension coupling. For

![Fig. 2. Wing geometry: (a) General dimension and location of point masses; (b) Panel partitions for skin and spar sections.](image-url)
balanced laminates, the lamination parameters $\xi_i$ and $\xi_j$ vanish and hence, the bend-twist coupling stiffness, $D_{16}$ and $D_{26}$, are zero. This effect reduces the influence of anisotropy (bend-twist coupling) on the response of composite structures [32], thereby reducing the design spaces for aeroelastic tailoring. In order to avoid this limitation, non-balanced, symmetric laminates are considered in this work. This decision results in nine design variables for each composite panel in the wing box model (eight lamination parameters plus one laminate thickness), giving a cumulative total of 369 design variables for each level of optimisation.

2.2. Aeroelastic analysis

The aeroelastic stability of the wing box is assessed using MSC Nastran’s SOL 145, which employs the frequency matching ‘p-k’ method to predict the flutter speed, $V_f$. Further details can be found in [33]. Structural frequencies, as well as their modal amplitudes and damping, are obtained from the analysis as functions of air speed. The flutter speed for each mode is found from the value at which the damping becomes negative. A total of 12 modes are considered in the flutter analysis to allow for an adequate representation of the aeroelastic behaviour.

2.3. Gust response

The optimisation of composite wings for gust load alleviation has been explored by various researchers [3,8,34,35]. As specified by aeronautical authorities (CS-25 [36]), the dynamic gust load conditions for an aircraft consists of discrete gust and continuous turbulence (or continuous gust). For discrete gust load, the gust velocity varies in deterministic manner which is represented using ‘1-cosine’ gust profile. For continuous gust load, the gust velocity is assumed to vary in a random manner. The deterministic method employs ‘worst-case’ atmospheric gust approach where there is an idealised ‘discrete’ event that the aircraft encounters during flight time. Herein, the deterministic method is used to analyse the wing’s response in terms of root bending moment (RBM) due to worst-case gust scenario [8,20]. The variation in air velocity, in the direction normal to the aircraft path, is shown in Fig. 3. The expression governing the ‘1-cosine’ gust is given by

$$V_g(t) = \frac{V_{g0}}{2} \left(1 - \cos \frac{2\pi V}{L_g} t\right). \quad (7)$$

where $V_{g0}$ is the peak or design gust velocity, $L_g$ is the gust length and $V$ is the flight speed. In this work, the gust length is chosen to vary from 18 m to 216 m. The design gust velocity and flight speed are set to 20 ms$^{-1}$ and 253 ms$^{-1}$, respectively. MSC Nastran’s SOL 146 is used to evaluate the wing box dynamic aeroelastic response to discrete gusts.

3. Multi-level aeroelastic tailoring

A multi-level optimisation method is proposed for the aeroelastic tailoring of the composite wing box of a reference regional jet airliner and the various optimisation methodologies, algorithms and strategies are detailed in this section.

The objective of the optimisation process is to minimise structural weight, subject to multiple constraints, including strength and aeroelastic stability. Thickness and lamination parameters of the wing box composite panels are used as design variables. The design’s robustness and reliability, when considering stochastic variations of composite ply material properties and thickness, are also assessed.

The optimisation framework comprises two levels, as illustrated in Fig. 4. MATLAB’s implementation of the Particle Swarm Optimisation (PSO) algorithm and MSC Nastran are used to solve the optimisation problem. PSO is a heuristic search method based on simple analogues of collaborative behaviour and swarming in biological populations [37]. Similar to a Genetic Algorithm (GA), PSOs perform population-based searches that depend on exchanges of information between individuals for search progression. PSO is reported to be computationally more efficient than GAs, because the algorithm requires fewer function evaluations [38].

The PCE, as presented by [23,12], is used as a non-sampling-based method to quantify model uncertainty and create surrogate models for robust and reliability-based design optimisation. The PCE method is developed such that the model is assumed to be a black-box with unknown inner structures. The method is straightforward as the model response of random value $y$ is represented by random vector $x$ rather than the distribution density function.

In the first level optimisation, the wing structure is subjected to a static manoeuvre load and optimised for minimum weight with strain, buckling, flutter and gust response constraints. The load distribution due to the aerodynamic loading is obtained from a trim analysis performed with MSC Nastran’s SOL 144. The static manoeuvre load case is run at a Mach 0.82, cruise altitude 10,000 m and acceleration 2.5 g. Fuel load is included to provide a realistic representation of the wing model. A weighted cost function is used to account for the influence of multiple constraints.

Results from the first level are fed to the second level to optimise the design further for robustness and reliability. The effect of uncertainties on flutter response is considered in terms of probability (of failure) density functions (PDF). To keep computational time to acceptable levels, the effect of uncertainties on other first level constraints is not quantified explicitly. However, for consistency, first level responses are imposed as design constraints in the second level.

![Fig. 3. ‘1-cosine’: discrete gust representation ($x_g$ is the flight travel distance).](image-url)
3.1. First level: Deterministic optimisation

The first level optimisation problem is formulated as follows:

\[
\begin{align*}
\text{minimize } & f_{obj}(W(x), f_{strain}(x)), \\
\text{subject to: } & \text{Strain Failure Index, } FI(x) \leq 1 \text{ (Max. Strain)}, \\
& \text{Buckling critical load factor, } \lambda(x) \geq 1, \\
& \text{Flutter speed, } V_f(x) \geq 1.15 V_0 \text{ (Design dive speed)}, \\
& \text{Wing Root Bending Moment, } \max(\text{RBM}(x, L_g)) \leq \max(\text{RBM}_{\text{benchmark}}(L_g)), \\
& x = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6, \varepsilon_{\text{panel,1}}, \varepsilon_{\text{panel,2}}].
\end{align*}
\]

(8)

where

- \(x\) is vector containing the design variables.
- \(\lambda\) is the lowest buckling load factor (ten modes are computed to account for mode switching).
- \(FI\) is the strain Failure Index defined as
  \[
  FI = \max \left( \frac{\varepsilon_1}{\varepsilon_{\text{ allowable}}}, \frac{\varepsilon_2}{\varepsilon_{\text{ allowable}}}, \frac{\varepsilon_3}{\varepsilon_{\text{ allowable}}} \right) \leq 1,
  \]
  (9)
  where \(\{\varepsilon_1, \varepsilon_2, \varepsilon_3\} = \{\varepsilon\}\) are the strain components through the laminate’s thickness, \(\varepsilon_{\text{ allowable}} = 710 \mu \text{ e}\) and \(\varepsilon_{\text{ allowable}} = 4500 \mu \text{ e}\).
- The flutter speed, \(V_f\), is calculated from a conventional \(V_g\) plot as per §2.2, assuming Mach 0.82 and flight dive velocity, \(V_0\), at 10,000 m. Since 12 modes are considered, \(V_f\) is assumed to be the lowest of 12 values at which the damping factor equals zero.
- For the gust constraint, six different values of \(L_g\) as indicated in §2.3 are used in order to compute the maximum \(\text{RBM}\).
The values are selected within the gust gradient distance range (H) of 9 m to 107 m (gust length is twice of gust gradient distance) as specified by European Aviation Safety Agency (EASA). Six values are sufficient to find the critical response for each load quantity.

Finally, the objective function in Eq. (8) is given as

$$f_{obj} = \frac{W(x)}{W_{benchmark}} + f_{1, cost}(x),$$

where $W$ is the wing structural weight (skins and spars only); $f_{1, cost}(x)$ is a cost penalty function defined to account for constraint violations as

$$f_{1, cost} = w_f \times \frac{V_f - V_{Design}}{V_{Design}} + w_g \times \frac{RBM - RBM_{Benchmark}}{RBM_{Benchmark}} + w_{EIG} \times \frac{\lambda_{Design} - \lambda_{min}}{\lambda_{min}}
+ w_{FI} \times \frac{FI - FI_{Design}}{FI_{Design}},$$

and where

$$w_{consti} = w_{consti} \in [0, 1]; \sum_{consti} w_{consti} = 1, \text{ constri} \in \{f, g, EIG, FI\}$$

is the set of weighting coefficients relative to each of the constraints, and the subscript 'Design' denotes desired or allowable values. The design constraints are included in the objective function in the form of cost penalty function. So that, the trade-off between minimum structural weight and optimum constraints values are accounted towards the improved design solution.

By variation of the weighting coefficients, a Pareto front of optimised solutions is obtained. Following the averaging principle defined in [39], the overall best deterministic design is chosen as the Pareto point minimising the expression ($\Sigma$=5), where

$$\Sigma = \frac{W}{W_{min}} + \frac{V_f}{V_{f, max}} + \frac{RBM}{RBM_{min}} + \frac{\lambda}{\lambda_{min}} + \frac{FI}{FI_{max}},$$

and where the subscripts min and max indicate the minimum and maximum values obtained for each parameter from all possible weighting combinations.

### 3.2. Second level: Robust and reliability-based design optimisation

The need for a multi-level optimisation strategy is justified by considerations of computational feasibility. Evaluating full wing box designs, for multiple performance/constraint metrics and by means of finite element models, can be costly and take many minutes per attempted solution. Aiming to quantify the effect of parameter uncertainty on the robustness and reliability of optimised designs, one would have to run a statistically relevant number of stochastic variations for every tentative solution trialled by the optimiser. This requirement makes “all-at-once”, single level approaches computationally impractical. A potential alternative to alleviate the computational burden is to recur to metamodels or surrogate models to approximate system behaviour with functions that are quick to interrogate and evaluate. However, training the surrogates to capture a variety of responses to multiple parameters is similarly computationally expensive and impractical.

To overcome these limitations, the approach adopted in this work is to run a deterministic optimisation first and then pass the output to a second level, to account for uncertainty. In the second level, PCE is used to quantify the effect of uncertainties on some responses only, using the optimised values of the remaining ones as design constraints. This approach guarantees that the second level output, i.e. the final optimised design, is robust and reliable in terms of chosen responses, whilst still meeting all of the constraints imposed on and met by the deterministic optimum.

Reliability-Based Design Optimisation and Robust Design Optimisation are the two main methodologies reported in the literature for probabilistic design optimisation [23–26,40]. In this work, aleatory variations in material stiffness and ply thickness are considered. In particular, and unless stated otherwise, longitudinal and shear modulus, $E_1$ and $G_{12}$, respectively, are assumed to be random variables with coefficient of variation (COV) equal to 0.1. The COV for $t_{p0}$ is assumed to be 0.01. Specific means and standard deviations are given in Table 2.

![PDF](image_url)

**Fig. 5.** Changes in the Probability Density Functions of a generic design response due to (a) Reliability-Based Design Optimisation (RBDO) and (b) Robust Design Optimisation (RDO). RBDO minimises $P_f$, whilst RDO minimises the response variance around a target mean value.

### Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean, $\mu$</th>
<th>Std Dev., $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$ (GPa)</td>
<td>148.0</td>
<td>14.8</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>5.90</td>
<td>0.59</td>
</tr>
<tr>
<td>$t_{p0}$ (m)</td>
<td>$1.83 \times 10^{-4}$</td>
<td>$1.83 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

*Note: Specific means and standard deviations are given in Table 2.*
3.2.1. Reliability-based design optimisation (RBDO)

In RBDO, the goal is for a structure to achieve a target performance whilst attaining a prescribed level of design reliability [24]. Reliability is measured in terms of probability of failure, \( P_f \), i.e. constraint violation, or occurrence of a particular response. \( P_f \) is calculated as the area between the PDF and the target design constraint as shown in Fig. 5(a).

Aircraft designers aim to minimise aircraft weight subject to stress and other constraints, and aeroelastic tailoring helps to meet this objective. The reliability of the designs can be improved by minimising \( P_f \) [23,40], that is by shifting the failure PDF to the right and/or shrinking it. The generalised form of the RBDOs performed in this work is expressed as

\[
\begin{align*}
\text{minimize} \quad & f_{\text{rbdo}}(W(x,p), P_f(x,p)), \\
\text{subject to:} \quad & g_i(x,p) \leq 0, \\
& g_j(x,p) \leq 0, \\
& x_l \leq x \leq x_u,
\end{align*}
\]

(14)

where \( f_{\text{rbdo}} \) is the objective function; \( g_i(x) \) is the reliability constraint; \( g_j(x) \) is the vector set of design constraints for which a reliability target is not established; \( p \) is a vector of constant parameters that do not vary in the optimisation; and \( x \) is bound between lower and upper limits, \( x_l \) and \( x_u \).

The objective function is defined as an aggregate of the structural weight and the probability of failure

\[
\begin{align*}
f_{\text{rbdo}} = w_w \times \frac{W}{W_{\text{det}}} + w_p \times \frac{P_f}{P_{\text{allow}}},
\end{align*}
\]

(15)

where \( W_{\text{det}} \) is the structural weight from the deterministic optimisation, and \( w_w \) and \( w_p \) are weighting coefficients chosen so that \( w_w + w_p = 1 \). Here, the reliability constraint takes the form

\[
\begin{align*}
g_{\text{det}} = P_f - P_{\text{allow}},
\end{align*}
\]

(16)

where \( P_{\text{allow}} \) is the allowable probability of failure. In our case, the probability of exceeding the design flutter speed.

3.2.2. Robust design optimisation (RDO)

RDO aims at optimising a structure placing the targeted performance around a mean value and maximising robustness by minimising sensitivity to random parameter variations [24]. This aim is achieved by minimising the variance and optimising the mean of the response in question, as illustrated in Fig. 5(b). The generalised form of the RDOs performed in this work is

\[
\begin{align*}
\text{minimize} \quad & f_{\text{rdo}}(W(x,p), \mu_f(x,p), \sigma_f(x,p)), \\
\text{subject to:} \quad & g_{\text{upper}}(\mu_f(x,p), \sigma_f(x,p)) \leq \text{USL} \quad \text{or} \quad g_{\text{lower}}(\mu_f(x,p), \sigma_f(x,p)) \geq \text{LSL}, \\
& g_j(x,p) \leq 0, \\
& x_l \leq x \leq x_u,
\end{align*}
\]

(17)

where \( f_{\text{rdo}} \) is the objective function defined in terms of weight, weighting coefficients \((w_w, w_p, w_f): w_w + w_p + w_f = 1\), mean response, \( \mu_f \), and standard deviation, \( \sigma_f \),

\[
\begin{align*}
f_{\text{rdo}} = w_w \times \frac{W}{W_{\text{det}}} + w_p \times \frac{\mu_f - \mu_{\text{det}}}{\mu_{\text{det}}} + w_f \times \frac{\sigma_f}{\sigma_{\text{det}}},
\end{align*}
\]

(18)

and where \( g_{\text{upper}} = \mu_f + n_{\sigma_f} \) and \( g_{\text{lower}} = \mu_f - n_{\sigma_f} \) are design constraints used to define the solution’s robustness. These constraints are bounded by their upper and lower statistical limits, USL and LSL, which are given as functions of the mean and standard deviation of the deterministic optimisation design, \( \mu_{\text{det}} \) and \( \sigma_{\text{det}} \), as

\[
\begin{align*}
\text{USL} = \mu_{\text{det}} + n_{\sigma_{\text{det}}} \quad \text{and} \quad \text{LSL} = \mu_{\text{det}} - n_{\sigma_{\text{det}}}.
\end{align*}
\]

(19)

entailing that feasibility is maintained within \( n \) standard deviations of the optimised mean. In this work, \( n = 6 \) in line with a 6σ design philosophy [41].

3.2.3. Robust and reliability design optimisation (RRBDO)

A combined approach, mixing robust and reliability-based design optimisations (RRBDO), is thought to be more comprehensive than RBDO and RDO individually. Particularly when, as in the case of aeroelastic tailoring, design reliability and robustness are sought together. An RRBDO approach is expected to: (a) improve on RDO solutions by bringing additional reliability; and (b) improve on RBDO with increased robustness. In aeroelastic terms, RRBDO should ensure minimum mean weight with mean constrained responses, such as flutter or stresses, all close to the boundary of failure. Mathematically, this is obtained by combining RDO and RBDO constraints as follows:

\[
\begin{align*}
\text{minimize} \quad & f_{\text{rrbdo}}(W(x,p), \mu_f(x,p), \sigma_f(x,p)), \\
\text{subject to:} \quad & g_{\text{upper}}(\mu_f(x,p), \sigma_f(x,p)) \leq \text{USL}, \\
& g_{\text{lower}}(\mu_f(x,p), \sigma_f(x,p)) \geq \text{LSL}, \\
& g_j(x,p) \leq 0, \\
& x_l \leq x \leq x_u,
\end{align*}
\]

(20)

where the objective function, \( f_{\text{rrbdo}} \), is

\[
\begin{align*}
f_{\text{rrbdo}} = w_w \times \frac{W}{W_{\text{det}}} + f_{\text{cost}} + f_{2,\text{cost}},
\end{align*}
\]

(21)

and the cost penalty functions, \( 5.90 \) and \( f_{2,\text{cost}} \) are defined as

\[
\begin{align*}
f_{\text{cost}} = w_p \times \left( \frac{\mu_f - \mu_{\text{det}}}{\mu_{\text{det}}} \right) + w_f \times \frac{\sigma_f}{\sigma_{\text{det}}}, \quad \text{and} \quad f_{2,\text{cost}} = w_p \times \frac{P_f}{P_{\text{allow}}},
\end{align*}
\]

(22)

where \( w_w, w_p, w_f \) are the weighting factors \((w_w, w_p, w_f): w_w + w_f + w_p = 1\) and all other quantities retain the meaning defined in §3.2.1 and §3.2.2.

4. Stochastic modelling

For reasons of the computational cost arising from the effects of random parameters variations in optimisation algorithms, sampling methods or stochastic modelling techniques are needed to evaluate model performance quickly. Monte Carlo Simulations [8,19], Polynomial Chaos Expansion [12,16,20,21] and Stochastic Collocation [22] are popular tools for uncertainty quantification. MCSs are simple but require a large number of sample analyses for accurate estimations, which is computationally expensive. Other techniques, such as the perturbation method and PCE, have been introduced to overcome this limitation and provide an alternative to MCS. In the context of aeroelastic tailoring, Castrave et al. [42] investigated the influence of uncertain bending and torsional stiffness on the flutter behaviour of a wing using both MCSs and a perturbation method. Their results show a good correlation between the two techniques and a computational advantage for the latter. Similar results were presented by [20,12] using PCE.

The method of choice for our work is PCE, which was derived from the Weiner-Askey Chaos Expansion [21,43]. The PCE model for any second-order random process (i.e. any process with finite variance), \( u(\theta) \), can be written as

\[
\begin{align*}
u(\theta) = a_0 \xi_0 + \sum_{i=1}^{\infty} a_i \Gamma_p[\xi_i(\theta)] + \sum_{i=1}^{\infty} \sum_{i=1}^{\infty} a_{i,j} \Gamma_{ij}[\xi_i(\theta), \xi_j(\theta)] \\
+ \sum_{i=1}^{\infty} \sum_{i=1}^{\infty} \sum_{i=1}^{\infty} a_{i,j,k} \Gamma_{ijk}[\xi_i(\theta), \xi_j(\theta), \xi_k(\theta)] + ... \quad (23)
\end{align*}
\]

where \( \Gamma_p[\xi_i(\theta), ..., \xi_k(\theta)] \) is the polynomial chaos of order \( p \) in the independent random variables \([\xi_i(\theta), ..., \xi_k(\theta)]\), the \( a \) terms are...
deterministic expansion coefficients and $\theta$ is the random character of the quantity involved. The random variables can be modelled using different types of polynomials as described in [12,43]. For example, if $[\xi] = [\xi_1(\theta), \ldots, \xi_n(\theta)]^T$ is a set of standard Gaussian random variables with zero mean and unit variance, $\Gamma_p$ can be expressed with n-dimensional Hermite polynomials [43] as

$$\Gamma_p[\xi_1(\theta), \ldots, \xi_n(\theta)] = (-1)^p \frac{1}{\sqrt{2^p p!}} \frac{d^p}{d\theta^p} \left[ e^{-\frac{\theta^2}{2}} \right]$$

Eq. (23) is often written as

$$u(\theta) = \sum_{p=0}^{\infty} \beta_p \phi_p(\xi(\theta)),$$

where there is a one-to-one correlation between $\Gamma_p[\xi_1(\theta), \ldots, \xi_n(\theta)]$ and $\phi_p(\xi(\theta))$ and between the $\beta$ and the $\theta$ terms. For instance, the 2-dimensional expansion of a 3rd order PCE model based on the Gaussian input $\xi = [\xi_1, \xi_2]^T$ can be written as (see [20])

$$u_{2D} = \beta_0 + \beta_1 \xi_1 + \beta_2 \xi_2 + \beta_{11} (\xi_1^2 - 1) + \beta_{21} (\xi_2^2 - 1) + \beta_{111} (\xi_1^3 - 3\xi_1) + \beta_{211} (\xi_2^3 - 3\xi_2) + \beta_{1111} (\xi_1^4 - 6\xi_1^2 + 3) + \beta_{2111} (\xi_2^4 - 6\xi_2^2 + 3)$$

(26)

where the $\beta_k$ are unknown coefficients to be calculated using a computing test data set. In this work, the flutter speed, $V_f$, is sampled at a series of $N$ points in the design space of the Gaussian material properties. Using Eq. (25), one can write a set of simultaneous equations such that

$$\{V_i\} = \{|\psi| \beta| + |\epsilon|,$$

(27)

where $|\epsilon|$ is the simplification error due to the truncation of expansion. As proposed in [44], a least-squares linear regression model can be used to determine the expansion coefficients. In particular $|\beta|$ is found by minimising of $\sum_i e_i^2$ such that

$$\epsilon = \min \left( \sum_{i=1}^{N} e_i^2 \right).$$

(28)

and hence

$$|\beta| = (|\psi|^{-1}) |\psi|^{-1} |\psi| |\epsilon|.$$

(29)

The resulting coefficients are then fed back to Eqn. (27) to emulate the system response for any combination of random variables and to estimate the statistical properties of the system at reduced computational cost.

A general overview of the PCE method is illustrated in Fig. 6. The Latin Hypercube Sampling (LHS) technique [44] is employed to span the sampling space uniformly, so that a relatively small number of samples is sufficient to construct a surrogate model of acceptable fidelity.

The use of PCE to quantify uncertainty in the optimisation procedure provides a significant reduction in the number of sample runs required in comparison to MCS. A convergence study measuring accuracy of the uncertainty quantification versus the number of samples proves that 30 sample runs are sufficient for the PCE method to converge, which is a 100x less than MCS. The PCE method requires only 6/1000 of the total run time of the MCS. The analysis was conducted on a quad-core Intel Core i7-3770S-CPU @ 3.10 GHz with 8 CPUs and 32 GB RAM. Fig. 7 shows a comparison of the flutter speed distribution obtained from 5000 MCS runs and using PCE models of different order (with random composite material properties as defined in §3.2). An adequate agreement is obtained using low order PCE models, i.e. 3rd order, and a small number of sample runs, which contains overall computational cost.

5. Results and discussion

Results obtained using the optimisation framework detailed in previous sections are presented herein, where the benchmark wing model is tailored deterministically as per §3.1 using different combinations of the weighting factors for each of the responses in the cost function. An ideal deterministic optimum is then selected from the Pareto front generated. Subsequently, by following the methods detailed in §3.2, RBDO, RDO and RRBD are employed to optimise the design for added reliability and/or robustness with minimal structural weight penalty. The effect of uncertainties is quantified for flutter speed and weight. All of the other responses of the deterministic design are kept in the second level optimisation as additional design constraints (q_i) to ensure no deterioration in performance from the first level optimisation.

Henceforth, it is assumed that the random parameters are Gaussian continuous variables. Hermite polynomials are used to construct the polynomial basis in the stochastic model. Thirty LHS sample runs have proven to be sufficient for the analysis.

5.1. Case study 1

5.1.1. First level: Deterministic optimisation

A total of 20 optimisation runs is performed, with the weighting factors for each of the responses (as defined in Eq. (11)) assuming values in $[0, 1]$. These values are chosen using LHS to respect Eq. (12) and are shown in Table 3.

Table 4 presents a summary of the results. In comparison to the benchmark wing, the optimisation reduces structural weight by at least 16.4% (DET9) and up to a maximum of 35.7% (DET8). Interestingly, the lightest solution has a buckling load factor equal to one, suggesting that buckling resistance is critical for minimum weight designs.

Intuitively, cost penalties are incurred when the optimiser is tasked with satisfying multiple constraints. Indeed, the cost function reaches its lowest values for singly constrained optimisations (DET11 to DET14), with the relative reserve factors converging approximatively to the design allowable. A clear example is DET12, for which $w_f = 1$ and RBM/RBMbenchmark is minimum. Similarly, the lowest flutter speed is obtained when $w_f = 1$, i.e. for DET11. Although, it is noted that $V_f$ varies marginally across optimisations, the largest value deviating only 8.1% from $V_f_{Design}$ (DET9).

Further insight into the results can be gained from Fig. 8, where the constraints values of the optimised solutions (for different set of weighting factor) are plotted against the corresponding weighting factor. In theory, the higher the weighting factor, the closer the response should be to its allowable value. This proves to be the case here, which gives confidence into the validity of the underlying calculations.

The overall best design is derived from the Pareto fronts of Fig. 8, by

---

Fig. 6. Overview of the stochastic modelling process using Polynomial Chaos Expansion.
except, of course, for RUN 1, for order PCE using different weighting factors. The aleatoric parameters are shown in Fig. 9, where they are also compared to the benchmark model. Naturally, thickness values are discontinuous and multiples of $t_{\text{guy}}$. For simplicity, blending constraints were not applied at this stage of the study. However, in order to prevent sharp changes of thickness, no more than two plies were allowed to be dropped between adjacent panels.

### 5.1.2. Second level: Reliability-based design optimisation (RBDO)

Following on from the first level, DET1 is further optimised for reliability, assuming stochastic variations of material properties ($E_1$ and $G_{12}$) and composite ply thickness ($t_{\text{guy}}$). The aleatoric parameters are assumed to have the properties reported in Table 2. The PCE method is used for uncertainty quantification, utilising 30 data samples selected using LHS. Reliability is evaluated in terms of the probability of failure, $P_f$, of trialled designs to exceed the minimum flutter speed ($V_f/V_{\text{Design}} > 1$).

The RBDO objective function is formulated in terms of structural weight and probability of failure as indicated by Eq. (15). The allowable probability of failure is set to be equal to the probability of failure of DET1. Hence, $P_{\text{fail}} = 8.5 \times 10^{-7}$. Eleven combinations of the weighting factors, $w_w$ and $w_P$, are used, as indicated in Table 5.

A design is deemed to be more reliable than the baseline when the probability of failure, or the occurrence of flutter at the design speed, is reduced. To ensure overall design feasibility, first level responses, for which the effect of uncertainties is not evaluated (strain, buckling and gust wing root bending moment), are imposed here as optimisation constraints.

RBDO results are summarised in Table 5. For all combinations of weighting coefficients, the wing box is lighter that the benchmark and the values of $P_f$ are lower than $P_{\text{fail}}$. Except, of course, for RUN 1, for which $w_P = 0$. The minimum value is obtained for RUN 10, for which $P_f = 2.448 \times 10^{-7}$.

Fig. 10(a) shows that reductions of $P_f$ are due to the flutter PDFs shifting to the right. However, increases in mean flutter speed are accompanied by greater standard deviations, suggesting that reliability is obtained at the expense of robustness. Fig. 10(b) shows the structural weight PDFs resulting from RBDO. It is interesting to note that the distribution of the structural weight obtained for RUN 2 has a lower mean value compared to the deterministic design. This result indicates that it is possible to minimise structural weight whilst improving design reliability. However, all of the other runs have lower probability of failure and higher mean structural weight, demonstrating that a weight penalty is generally necessary for greater reliability.

The overall best RBDO design is chosen again based on an averaging means of the averaging principle defined in Eq. (13). The best design solution is selected from a Pareto point whose give the lowest value for $\Sigma$. DET1 is found to be the Pareto point to be used as the starting point for the second level optimisation. The corresponding wing box sizing parameters are shown in Fig. 9, where they are also compared to the benchmark model. Naturally, thickness values are discontinuous and multiples of $t_{\text{guy}}$. For simplicity, blending constraints were not applied at this stage of the study. However, in order to prevent sharp changes of thickness, no more than two plies were allowed to be dropped between adjacent panels.

### Table 4: Deterministic optimisation results at different weighting factors.

<table>
<thead>
<tr>
<th>Run</th>
<th>$w_w$</th>
<th>$w_P$</th>
<th>$V_f/\text{Design}$</th>
<th>$\lambda$</th>
<th>$f_{\text{EIG}}$</th>
<th>$f_{\text{FI}}$</th>
<th>$\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DET1</td>
<td>0.669</td>
<td>1.007</td>
<td>0.004</td>
<td>1.006</td>
<td>0.823</td>
<td>0.030</td>
<td>4.974</td>
</tr>
<tr>
<td>DET2</td>
<td>0.646</td>
<td>1.001</td>
<td>0.145</td>
<td>1.042</td>
<td>0.601</td>
<td>0.095</td>
<td>45.284</td>
</tr>
<tr>
<td>DET3</td>
<td>0.681</td>
<td>1.019</td>
<td>0.032</td>
<td>1.057</td>
<td>0.993</td>
<td>0.030</td>
<td>13.426</td>
</tr>
<tr>
<td>DET4</td>
<td>0.676</td>
<td>1.009</td>
<td>0.043</td>
<td>1.022</td>
<td>0.976</td>
<td>0.026</td>
<td>16.486</td>
</tr>
<tr>
<td>DET5</td>
<td>0.647</td>
<td>1.007</td>
<td>0.030</td>
<td>1.062</td>
<td>0.859</td>
<td>0.033</td>
<td>12.472</td>
</tr>
<tr>
<td>DET6</td>
<td>0.698</td>
<td>1.026</td>
<td>0.026</td>
<td>1.020</td>
<td>0.913</td>
<td>0.031</td>
<td>11.395</td>
</tr>
<tr>
<td>DET7</td>
<td>0.683</td>
<td>1.015</td>
<td>0.201</td>
<td>1.029</td>
<td>0.935</td>
<td>0.062</td>
<td>61.986</td>
</tr>
<tr>
<td>DET8</td>
<td>0.643</td>
<td>1.001</td>
<td>0.207</td>
<td>1.000</td>
<td>0.589</td>
<td>0.038</td>
<td>63.235</td>
</tr>
<tr>
<td>DET9</td>
<td>0.836</td>
<td>1.081</td>
<td>0.180</td>
<td>1.008</td>
<td>0.915</td>
<td>0.071</td>
<td>56.143</td>
</tr>
<tr>
<td>DET10</td>
<td>0.654</td>
<td>1.001</td>
<td>0.039</td>
<td>1.029</td>
<td>0.922</td>
<td>0.037</td>
<td>15.195</td>
</tr>
<tr>
<td>DET11</td>
<td>0.681</td>
<td>1.000</td>
<td>0.088</td>
<td>1.052</td>
<td>0.725</td>
<td>0.000</td>
<td>29.121</td>
</tr>
<tr>
<td>DET12</td>
<td>0.661</td>
<td>1.008</td>
<td>0.003</td>
<td>1.024</td>
<td>0.758</td>
<td>0.003</td>
<td>4.741</td>
</tr>
<tr>
<td>DET13</td>
<td>0.709</td>
<td>1.030</td>
<td>0.450</td>
<td>1.001</td>
<td>0.740</td>
<td>0.001</td>
<td>133.535</td>
</tr>
<tr>
<td>DET14</td>
<td>0.736</td>
<td>1.048</td>
<td>0.030</td>
<td>1.212</td>
<td>1.000</td>
<td>0.000</td>
<td>12.897</td>
</tr>
<tr>
<td>DET15</td>
<td>0.663</td>
<td>1.006</td>
<td>0.321</td>
<td>1.000</td>
<td>0.763</td>
<td>0.060</td>
<td>96.381</td>
</tr>
<tr>
<td>DET16</td>
<td>0.772</td>
<td>1.059</td>
<td>0.012</td>
<td>1.095</td>
<td>0.783</td>
<td>0.036</td>
<td>7.646</td>
</tr>
<tr>
<td>DET17</td>
<td>0.712</td>
<td>1.036</td>
<td>0.086</td>
<td>1.002</td>
<td>0.860</td>
<td>0.016</td>
<td>28.682</td>
</tr>
<tr>
<td>DET18</td>
<td>0.645</td>
<td>1.007</td>
<td>0.037</td>
<td>1.010</td>
<td>1.000</td>
<td>0.012</td>
<td>14.737</td>
</tr>
<tr>
<td>DET19</td>
<td>0.646</td>
<td>1.001</td>
<td>0.005</td>
<td>1.408</td>
<td>0.878</td>
<td>0.028</td>
<td>5.778</td>
</tr>
<tr>
<td>DET20</td>
<td>0.663</td>
<td>1.006</td>
<td>0.012</td>
<td>1.028</td>
<td>0.952</td>
<td>0.015</td>
<td>7.546</td>
</tr>
</tbody>
</table>

Fig. 7. Flutter speed responses obtained using MCS and PCE: (a) MCS and PCE using polynomials of different order; (b) MCS and 3rd order PCE using different number of sample runs.
principle, in this case, by accounting for the contributions of structural weight and probability of failure at different weighting factor. In particular, the ideal design is picked to be the one having $\Sigma = W/W_{\min} + P_f/P_{f_{\min}}$ closest to two. This condition is met by RUN 10 in Table 5, which is 31.8% lighter than the benchmark model and only 1.3% heavier than the best deterministic design.

5.1.3. Second level: Robust design optimisation (RDO)

DET1 is now optimised for robustness following the procedure described in §3.2.2. In particular, we seek a wingbox configuration of minimal weight and whose flutter speed distribution, arising from uncertainties in material properties, has mean as close as possible to the deterministic value and minimum standard deviation.

Results are presented in Table 6 and Fig. 11. All design solutions are characterised by weight reductions in comparison to the benchmark model, mean flutter speeds above the target design value. An increase in robustness is demonstrated by smaller standard deviations in comparison to both DET1 and the RBDO solutions. The minimum reduction from $w_{\text{DET1}} = 2.766$ occurs for RUN 1 ($w_f = 0$) and is 2.6%; the maximum one being 24.9% and occurring for RUN 3 ($w_f = 1$). Having used $V_{f,\text{DET1}}$ as an optimisation target, mean flutter speeds cluster uniformly around it. Conversely, all but one RDO solutions have similar or greater weight in comparison to the best deterministic optimum, thus suggesting that an increase in design robustness is likely to be achieved at the expense of weight. Interestingly, some RDO solutions are also sufficiently reliable but these are substantially heavier than their RBDO counterparts.

The overall best RDO design corresponds to RUN 8 and is chosen as the minimiser of $\Sigma_{-3}$, with $\Sigma = W/W_{\min} + P_f/P_{f_{\min}} + \sigma_f/\sigma_{f_{\min}}$. In

![Fig. 8. Pareto plots for (a) Flutter constraint against weighting, $w_f$, (b) RBM constraint against weighting, $w_g$, (c) Buckling constraint against weighting, $w_{\text{EIG}}$ and (d) Strain constraint against weighting, $w_{\text{FI}}$.](image_url)

![Fig. 9. Thickness variation for skin and spar sections for benchmark and deterministic optimum design (DET1).](image_url)
comparison to the overall best RBDO solution, RUN 8 features lower structural weight and mean flutter speed, and smaller standard deviation.

5.1.4. Second level: Robust and reliability-based design optimisation (RRBDO)

RBDO and RDO results show the following trends: 1) As expected, RBDO solutions tend to be more reliable and less robust than RDO ones, and vice versa; 2) Mean flutter speeds are close to but consistently above the design allowable. With RBDO, these values are also consistently above the mean flutter speed of the overall best deterministic design (DET1). While, with RDO, they are uniformly distributed around it; 3) Reliability or robustness are generally achieved at the expenses of weight, the latter imposing greater penalties. An RRBDO approach is thought to be able to provide a better compromise between weight and design robustness and reliability. Results are presented in Table 7 and Fig. 12.

Notably, most flutter speed PDFs cluster closely, with mean values approximately 1% above the allowable. Similarly, all runs result in probabilities of failure below \( P_{f,\text{allow}} \). The lowest value is \( 2.34 \times 10^{-4} \), which is a 97.2% improvement in comparison to the deterministic design. In terms of robustness, RBDO results, although generally worse, are comparable with RDO solutions (\( \sigma_{r,\text{rbdo}} \in [2.077, 2.694] \) vs \( \sigma_{r,\text{rdo}} \in [2.555, 2.788] \)). A slight increase in minimum structural weight is observed for RRBDO designs in comparison to both RDO and RBDO ones \( \left( \frac{W_{\text{min}}}{W_{\text{benchmark}}} = 0.658, \frac{W_{\text{min}}}{W_{\text{benchmark}}} = 0.639, \frac{W_{\text{min}}}{W_{\text{benchmark}}} = 0.669 \right) \).

The increase in structural weight is thought to be due to the increase in mean flutter speed and the decrease in its standard deviation. These variations are necessary to shift flutter PDFs to the right and to shrink them, which enhances design reliability and robustness. In conclusion, RBDO results further support the finding that a weight penalty is necessary to impart some level of robustness and reliability to the design. The overall best RRBDO solutions is RUN 7 with a normalised structural weight of 0.679.

### Table 5

<table>
<thead>
<tr>
<th>Run</th>
<th>Weightings</th>
<th>Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>( w_W )</td>
<td>( w_P )</td>
</tr>
<tr>
<td>1</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.80</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>0.70</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>0.60</td>
<td>0.40</td>
</tr>
<tr>
<td>6</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>7</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>8</td>
<td>0.30</td>
<td>0.70</td>
</tr>
<tr>
<td>9</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>10</td>
<td>0.10</td>
<td>0.90</td>
</tr>
<tr>
<td>11</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### Table 6

<table>
<thead>
<tr>
<th>Run</th>
<th>Weightings</th>
<th>Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>( w_W )</td>
<td>( w_P )</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.750</td>
<td>0.125</td>
</tr>
<tr>
<td>5</td>
<td>0.125</td>
<td>0.750</td>
</tr>
<tr>
<td>6</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>7</td>
<td>0.500</td>
<td>0.250</td>
</tr>
<tr>
<td>8</td>
<td>0.250</td>
<td>0.500</td>
</tr>
<tr>
<td>9</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td>10</td>
<td>0.340</td>
<td>0.330</td>
</tr>
</tbody>
</table>

M.F. Othman et al. Composite Structures 208 (2019) 101–113
A multi-level optimisation approach for the robust and/or reliability-based aeroelastic tailoring of a wing box structure is presented. The optimisation objective is to minimise weight subject to multiple constraints, including strength, buckling and flutter margin. The procedure accounts for stochastic variations in input material design parameters. Based on grounds of computational cost, surrogate modelling with Polynomial Chaos Expansion is preferred to Monte Carlo Simulation for the quantification of the effect of uncertainties on structural weight and flutter speed. The results presented in this paper support the following conclusions:

![Fig. 11. PDF plots of RDO solutions: (a) Flutter speed and (b) Structural weight.](image)

![Fig. 12. PDF plots of RRBDO solutions.](image)

### Table 7

RRBDO solutions for different weighting values for weight, flutter speed mean and standard deviation, and probability of failure.

<table>
<thead>
<tr>
<th>Run</th>
<th>Weighting coefficients Responses</th>
<th>ID</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_{w}$</th>
<th>$w_{\mu}$</th>
<th>$w_{\delta}$</th>
<th>$w_{P_f}$</th>
<th>$\frac{\mu}{\mu_{Design}}$</th>
<th>$\sigma_{\mu}$</th>
<th>$\rho_{\mu}$</th>
<th>$\Xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.669</td>
<td>1.011</td>
<td>2.788</td>
<td>$8.359 \times 10^{-3}$</td>
<td>38.812</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.669</td>
<td>1.010</td>
<td>2.683</td>
<td>$7.989 \times 10^{-3}$</td>
<td>37.190</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.679</td>
<td>1.015</td>
<td>2.561</td>
<td>$8.126 \times 10^{-3}$</td>
<td>37.728</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.669</td>
<td>1.010</td>
<td>2.663</td>
<td>$8.214 \times 10^{-3}$</td>
<td>38.144</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.250</td>
<td>0.669</td>
<td>1.010</td>
<td>2.688</td>
<td>$8.264 \times 10^{-3}$</td>
<td>38.364</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.500</td>
<td>0.250</td>
<td>0.125</td>
<td>0.125</td>
<td>0.125</td>
<td>0.669</td>
<td>1.014</td>
<td>2.555</td>
<td>$2.446 \times 10^{-3}$</td>
<td>4.063</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.250</td>
<td>0.500</td>
<td>0.125</td>
<td>0.125</td>
<td>0.125</td>
<td>0.679</td>
<td>1.010</td>
<td>2.741</td>
<td>$8.011 \times 10^{-3}$</td>
<td>37.306</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.250</td>
<td>0.125</td>
<td>0.500</td>
<td>0.125</td>
<td>0.125</td>
<td>0.669</td>
<td>1.009</td>
<td>2.633</td>
<td>$7.923 \times 10^{-3}$</td>
<td>36.886</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.125</td>
<td>0.250</td>
<td>0.300</td>
<td>0.300</td>
<td>0.300</td>
<td>0.669</td>
<td>1.010</td>
<td>2.584</td>
<td>$7.720 \times 10^{-3}$</td>
<td>36.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.100</td>
<td>0.300</td>
<td>0.300</td>
<td>0.300</td>
<td>0.300</td>
<td>0.669</td>
<td>1.010</td>
<td>2.584</td>
<td>$7.720 \times 10^{-3}$</td>
<td>36.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusions

A multi-level optimisation approach for the robust and/or reliability-based aeroelastic tailoring of a wing box structure is presented. The optimisation objective is to minimise weight subject to multiple constraints, including strength, buckling and flutter margin. The procedure accounts for stochastic variations in input material design parameters. Based on grounds of computational cost, surrogate modelling with Polynomial Chaos Expansion is preferred to Monte Carlo Simulation for the quantification of the effect of uncertainties on structural weight and flutter speed. The results presented in this paper support the following conclusions:
Polynomial Chaos Expansion is capable of quantifying the effects of uncertainties with sufficient accuracy and fewer model runs in comparison to Monte Carlo Simulations, thus enabling probabilistic design optimisation of a full Finite Element wire box model.

Reliability-based optimisation shows that reducing the model’s probability of failure entails a weight penalty and a loss of design robustness.

Optimising for robustness successfully reduces the design sensitivity to stochastic variations at the cost of additional weight. Robust designs can also be sufficiently reliable, but generally at a greater weight penalty in comparison to designs optimised for reliability only.

In general, the model can be optimised for minimal weight and a desired level of reliability or robustness or both. However, enhanced reliability and robustness result in a weight penalty in comparison to the deterministic optimum design.

Simultaneous robust and reliability-based design optimisation successfully provides the best compromise between weight, reliability and robustness.

In comparison to the benchmark wing, the framework produces an overall weight reduction of 32.1% for the test case considered, with a 1.5% increase from the first to the second level optimisation to account for stochastic design variations.

Results follow the same pattern when the coefficient of variations of the aleatoric parameters is changed.

Acknowledgements

The authors would like to acknowledge the support of the EPSRC through funding of the Centre for Doctoral Training in Advanced Composites for Innovation and Science [EP/G036772/1] at the University of Bristol; Emberu S.A.; the Royal Academy of Engineering; and the Malaysian Ministry of Higher Education and Universiti Sains Malaysia for the scholarship to the first author.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.compstruct.2018.09.086.

References