NLOS Identification and Mitigation for Geolocation Using Least-squares Support Vector Machines

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Abstract—**This paper examines the problem of non-line-of-sight (NLOS) identification and mitigation for geolocation signals in mobile networks. A ray tracing tool is used to simulate a mobile radio network with fixed base stations and thousands of mobile stations. The channel data between these mobile stations and base stations is used to extract parameters or features that are used for classification. Techniques for NLOS identification using a Least-Squares Support Vector Machine (LSSVM), are devised, producing greater than 98 percent accuracy for the proposed location specific approach, and 87 percent for the location independent approach. Respective LSSVM NLOS mitigation techniques are also proposed and evaluated. A usage context for the location specific approach is suggested, where the approach can help in addressing some of the challenges of next-generation wireless systems like massive MIMO.**

Keywords—localization; support-vector-machines; TDOA; TOA

I. INTRODUCTION

Accurate geolocation in urban areas is a challenge. Systems that normally use Global Navigation Satellite Systems (GNSS) based positioning, like GPS, are impacted by the inability to get a clear line-of-sight connection to the required, at least 3 satellites, because of the urban canyon. There has been a resurgence in interest in the mobile network-based positioning systems for various reasons [1-2]. With massive MIMO being a candidate technology for next-generation wireless systems, there is an opportunity for improved positioning accuracy. The drive for mobile network positioning is further incentivised by the potential system benefits that can be obtained from positioning, in next generation systems using technologies like mmWave and Massive MIMO. Positioning can help in designing new hand-over strategies, new resource allocation techniques, reducing pilot contamination and also reducing device transmit power. As the mobile device moves from a line-of-sight (LOS) position to a highly shadowed non-line-of-sight (NLOS) position, the change in received power may be very sharp and abrupt, such that closed-loop power control algorithms may fail to adapt quickly enough, leading to power control errors. If power control algorithms can take the location information, together with knowledge of the environment for each instantaneous position, power control errors could be reduced. Also pilot allocation can be made such that all mobiles with similar angle-of-arrivals (AOAs) are prevented from sharing the same pilot [13].

Measurements like time delay of signals, received power levels and the angle of arrival or departure of signals from/to a mobile device, are key to mobile positioning. Their usage usually assumes a direct signal path between the base station (BS) and the mobile (MS). In reality, there may be no such direct path, especially in urban areas and other highly shadowed environments. There is a need to take into account the effects of multipath. A common approach is to identify signals that are the result of line-of-sight (LOS) propagation and those that are non-line of sight (NLOS). For non-line-of-sight signals, mitigation techniques can then be applied to compensate for the positive range bias that occurs for NLOS signals. When enough line-of-sight signals are present, a preference of such signals can be applied in the selection of signals to use for positioning. This work demonstrates that even when a mixture of LOS and NLOS signals exist, simply applying a prioritisation of the LOS signals, then ground reflections before incorporating NLOS signals, can greatly improve the location accuracy. Approaches for NLOS identification include considering the geometry of the channel to estimate the distance travelled by multipath rays [3], considering polarisation of the signal where every polarisation change is considered to be a reflection, or also using some statistical characterisation [4]. Most techniques involve statistical approaches that require determination of the joint probability distributions of the underlying features, the outcome of which becomes very heuristic. A comprehensive survey of NLOS identification and mitigation techniques is provided in [8].

Support Vector Machines have been demonstrated to be effective for NLOS classification in an indoor environment [9-10] and are the subject of this study. The Least-squares Support Vector Machine (LS-SVM) is used to perform both NLOS identification and mitigation. This technique does not require any statistical modelling of LOS and NLOS channels, therefore it performs both tasks under a common framework. The two approaches proposed in this study yield a location specific scheme and a location independent scheme. The LSSVM has been previously studied for location independent NLOS identification in [9]. The main differences of their work, to our second scheme, are that, their work is based on measurements from a real-time indoor environment and also they use different localisation strategies.

II. EXPERIMENTAL SETUP

A. *Ray-tracing setup*

In this study, the ray-tracing software developed at the University of Bristol is used to provide ray prediction data for NLOS identification and localisation. The ray tracing tool incorporates a real-world Laser Illuminated Detection & Ranging (LIDAR) database of the City of Bristol in the United Kingdom. Hundreds of base station to mobile station links were simulated to generate 3D ray data. For simplicity 2D positioning algorithms are used, but an extension to 3D is straightforward. 36 Base stations were carefully placed in a 6x6 grid that covered a 1km^2 area of Central Bristol. The BS-BS distance of approximately 300m was considered to give a dense coverage which provided a large enough number of rays per BS-MS link. Ten thousand MS positions are then randomly placed within the grid and ray tracing performed for each BS-MS link (Fig. 1). Because of the random placement of MSs, some were placed in locations with no useful signal such as court yards. This meant that, no ray data was generated for those links and consequently those mobile positions were excluded from the study. The ray-tracing parameters (Table I) were chosen to match the massive MIMO testbed at Bristol University [5]. The key outputs from the ray-tracing results are the BS location, Actual MS location (for comparison with estimated location), Azimuth Angle-of-Departure (AOD) and AOA on BS and MS respectively, Elevation AOD and AOA on BS and MS respectively and received power and time delay for each ray.

B. *Assumptions*

This study assumes a network with base stations that are capable of obtaining reliable AOA information, possibly through the use of antenna arrays like in massive MIMO systems. The BSs are assumed to be capable of resolving individual rays. Next-generation wireless systems are likely to have enough bandwidth for this. Noise in the measured values as presented in the ray-tracer outputs, is neglected. Received power, AOA/AOD information and time delay estimation done in the ray tracing software is considered to be accurate enough for purposes of this study. No measurement noise modelling is built into the algorithms used. Actual NLOS identification (used to compare and evaluate the proposed techniques), is based on the experimental setup knowledge. Ground-reflected rays (GR rays) are taken as those that are single bound ground reflected or rooftop diffracted rays. Other NLOS scenarios excluding GR rays are considered to be “pure NLOS”. The designations GR and pure NLOS are used here to distinguish between these scenarios.

C. *Pre-processing algorithms*

Localisation algorithms in this study make use of the received power, time delay and NLOS/LOS classification, for each ray. Priority was given to LOS rays, for localization to produce the curves in Fig 4. For a given BS-MS link, if multiple rays have the same LOS classification, then the ray with the least delay is selected. This was based on empirical observations that indicated choosing rays with least delay produced better localization performance than selection based on received power level. For 3 BS TDOA, each mobile station will select 3 BSs within its proximity, whose rays have least time delays, and use those rays for localization. A ray prioritization scheme was developed, to pick LOS rays first, and if not present, or not enough for the localization scheme desired, then GR rays are selected, before pure NLOS rays are used. Ground reflected rays are given preference over pure NLOS rays because range errors produced by ground reflected paths (and rooftop diffracted paths) are smaller than other NLOS scenarios in most cases. Also ground reflected multipath may be irresolvable from the LOS path since they generally exceed the temporal and spatial resolution capabilities of measurement systems. The severity of this issue depends on the antenna patterns, the BS/MS heights and also how far the MS is, from the base station [7].

D. *Localisation techniques*

1) *TDOA with 3 Base stations*

TDOA processing is based on trilateration, with at least 3 base stations. A hyperbolic curve with two base stations located at its foci, gives a constant time delay difference. Fig. 2 illustrates the setup. BS1 and BS2 have their hyperbolic curve R2-R1, and BS1 and BS3 have their hyperbolic curve R3-R1. R1, R2 and R3 are the distances between the target (mobile) and each respective base station. The intersection of the two hyperbola gives the position of the mobile.

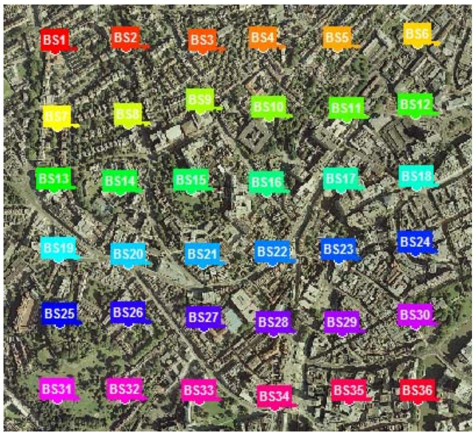

Fig 1. Base station deployment

TABLE I. RAY-TRACING PARAMETERS

Parameter	Value
Environment	1km^2 area of Central Bristol
Frequency	3.5GHz
BS transmit Power	32dBm
BS height	15m above clutter
MS height	1.5m
Receiver sensitivity	-120dBm
Antennas	Isotropic

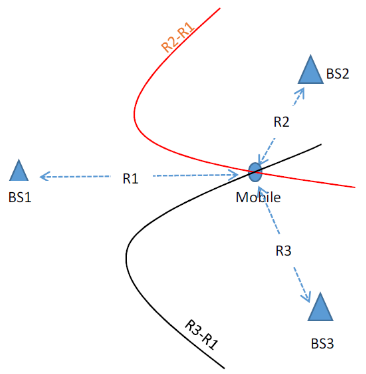

Fig. 2. TDOA geometry

Chan's algorithm [6] is used to find the solution to the mobile position. His proposed solution demonstrated to come closer to the Cramer-Rao Lower Bound (CRLB).

2) Simple Hybrid TOA + AOA

The estimated MS range is used together with the AOA to compute the position of the mobile, in 2D. This assumes a LOS mobile position. Where there are no LOS rays, the range estimate, obtained via 3 BS TDOA can be used. From Fig. 3, knowing the range *R*, and the angle of arrival θ, the position of the mobile device can be calculated from the following set of equations:

$$\begin{aligned} x &= x_{BS} + R.\cos\theta, \\ y &= y_{BS} + R.\sin\theta \end{aligned} \tag{1}$$

Advantages of using the range obtained by TDOA (as compared to TOA) in this technique are that TOA requires two way communication, which may not be convenient or possible in some applications; and also since TDOA utilizes at least 3 BSs, the link to other surrounding BSs may be better and could result in improved range estimates. However, TDOA range estimates may become very poor if all the 3 links are NLOS.

E. *Localisation perfomance*

The location/positioning error estimate e for each MS *i,* is calculated as the distance between the estimated position and the actual position of the mobile station as obtained from the ray-tracer setup.

$$e_i = \sqrt{[(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2} \tag{2}$$

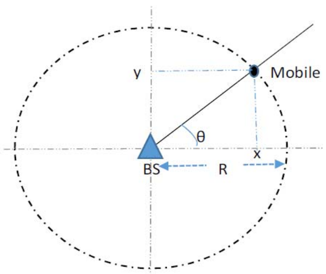

Fig. 3. Using AOA in TOA

Where (x_i, y_i) are actual coordinates for the i^{th} MS taken from the ray-tracing tool, and $(\hat{x}_i, \hat{y}_i)$ are the corresponding estimated coordinates. Location error CDF plots for different scenarios are presented in Fig. 4, which demonstrates the need for NLOS identification.

III. Support Vector Machines

A. *Introduction*

Support Vector Machines (SVMs) are a robust and effective technique for solving non-linear classification and function estimation problems. They were originally introduced in statistical learning theory and structural risk minimisation, where convex optimisation problems are solved. Least-Squares Support Vector Machines (LSSVM) is a reformulation of the standard SVMs in order to solve linear kernel-based systems. The solution is found by solving a set of linear equations instead of quadratic programming (QP) as in standard SVMs. LSSVMs for classification, were first proposed by Suykens and Vendewalle [11] in 1999. They are classified under kernel-based learning methods.

B. *Least-Squares Support Vector Machines*

(1) Classification

The SVM methodology seeks to construct a classifier, which is a function $\Re^n \rightarrow \{+1,-1\}$, of the form;

$$y(x) = sign\left[\sum_{i=1}^{N} \alpha_i y_i \psi(x, x_i) + b\right], \tag{3}$$

given a training set of *N* data points $\{x_i y_i\}_{i=1}^N$ where the i^{th} input is $x_i \in \Re^n$ and $y_i \in \{+1,-1\}$ is the i^{th} label or LOS status corresponding to the x_i input. b is a real constant and α_i are positive real constants, both which form the parameters of the classifier; that are unknown, and can be obtained empirically from equation (4).

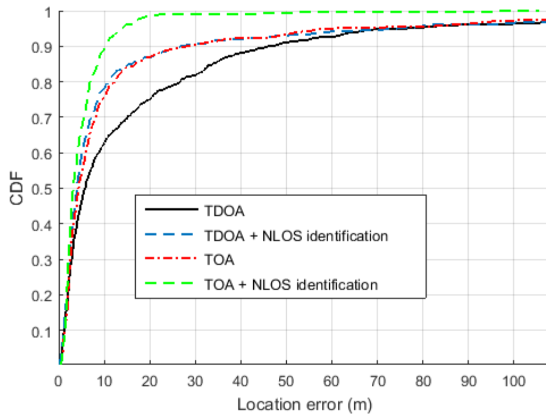

Fig. 4. Location accuracy – For a 10 percent outage, NLOS identification has improved location accuracy by 66% for TOA and by 40% for TDOA.

The function $\psi(x, x_i)$ is called the Kernel and it is typically taken to be $x_i^T x$ for linear SVMs or $\psi(x, x_i) = \exp\left\{\frac{\|x - x_i\|^2}{\sigma^2}\right\}$ for Radial Basis Function (RBF) SVMs, with σ, being a constant. The LS-SVM formulation leads to a linear system that can be written in matrix form as;

$$\begin{bmatrix} 0 & 1_N^T \\ 1_N & \Omega + \gamma^{-1} I_N \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ Y \end{bmatrix}, \qquad (4)$$

where $Y = [y_1 \dots y_N]^T$, $\alpha = [\alpha_1 \dots \alpha_N]^T$, and $1_N = [1 \dots 1]^T$, I_N is an $N \times N$ identity matrix and Ω_{ij} is the kernel matrix which, for linear SVM, can be defined by; $\Omega_{ij} = \phi(x_i)^T \phi(x_j) = K(x_i, x_j)$. The hyper-parameter γ tunes the trade-off between model complexity and level of tolerable training errors.

(2) Regression

In a similar way, one obtains a linear regressor, which is a function $\Re^n \to \Re$, of the form;

$$y(x) = \sum_{i=N}^{N} \alpha_i \psi(x, x_i) + b, \qquad (5)$$

given a training set of *N* data points $\{x_i\, y_i\}_{i=1}^N$ where the i^{th} input is $x_i \in \Re^n$ and $y_i \in \Re$ is the i^{th} "output" to be used for training the regressor.

The LS-SVM formulation and detailed options are available from Vapnik's original formulation [12] and in [11].

IV. Location Specific NLOS Identification

A. Methodology

Data from the ray tracer includes, for each ray, parameters like the BS position, time delay, received power and Angle of Arrival (AOA). This information is what forms the input data points x_i. Geometric information for each ray, together with actual MS coordinates, are used to determine whether each path is either LOS or NLOS. This information formed the output sequence or labels (y_i) for each input point. If a particular i^{th} path/ray is found to be LOS, then $y_i = +1$, otherwise $y_i = +1$ for NLOS path. A total of 22 384 data points were created from ray tracing simulations, with half being LOS and the other half of the paths being NLOS. This forms the training set $\{x_i\, y_i\}_{i=1}^N$ from which various sizes of N can be extracted, making sure that it contains half-half of NLOS and LOS rays. The LSSVM was trained using 10-Fold cross-validation. The 10-Fold cross-validation produces the tuning parameters γ and σ for the LSSVM. These parameters are then used in the training, with different sizes of data points. The RBF kernel was used in all cases because it yields the best validation and test set performance [14]. After training, the new ray tracing data, which represents the data to be used for positioning, is then run against the trained LSSVM. The result for each path is compared against the a posteriori LOS status.

B. Input space and feature selection

The base input space comprises of the BS in a 2D space. The proposition is that, with the knowledge of a particular BS's location and each ray's measurements, the LSSVM should be able to determine if that ray is LOS or NLOS. The input space is extended by including features of interest. In this study, the time delay and received power measurements for each path/ray are considered. The dynamic range of the features is also reduced, by taking their logarithms. After determining the optimum configuration, i.e. training data set size and combination of features, the base input was extended by considering the AOA on the BS. For both classification and mitigation, different sets of training data were constructed with differing content for the NLOS data points potion, where a mixture of ground reflected paths plus other NLOS scenarios produced training data 1 (TD1), all ground reflected paths for training data 2 (TD2) and pure NLOS in training data 3 (TD3). The goal is to evaluate the constitution of training data that gives the best performance. A study to determine a sufficient training set size for the classifier was conducted. The results showed that a data set size of 5000 produced an error probability of around 5%, with only a small reduction in classification error probability with successive doublings of the data set size. On this basis the data set size was chosen to be 10,000 which had error probability approximately 3%.

C. NLOS Mitigation

NLOS propagation leads to positively biased range estimates. Mitigation is achieved by estimating the ranging error. The LSSVM function estimation is used to estimate the error in the measured time delay or alternatively, the error in the corresponding range estimate. The input space (x_i) comprises of the base input (BS position and AOA at BS) and selected features or combinations of them. The output ($y_i = e_i$) is the time delay error. y_i are constructed through calculations of the time delay error, by comparing the expected LOS propagation time delay (given the knowledge of the actual BS and MS locations from the ray tracer setup) and the measured time delay. Where range error is used as the output parameter, it is obtained from the relationship, *range error = c x time delay error*, where *c* is the propagation speed. The LSSVM is then trained and the obtained regressor parameters are used to estimate range errors from a separate data set meant to be used for localisation. Regressor performance evaluation is done by subtracting the regressor output, in its form as range error, from the actual ranges to get the residual range error. Cumulative Distribution Functions (CDF) of the residual range error after mitigation, are plotted together with the original measured range error in Fig. 5. Mitigation is applied to all rays for localisation.

D. Results and Discussion

The results presented in this section demonstrate the performance of the LSSVM classifier and regressor as an NLOS identification and mitigation technique, respectively. Table II shows that best performance is achieved when training data 2 (TD2) is used, i.e. when the training data points consist of half LOS rays and the other half being GR rays. GR rays are good approximations to LOS when ($d >> h_{BS} > h_{ms}$) where d is the distance between BS and MS, h_{BS} and h_{MS} are the BS and MS heights respectively. Training with such data therefore provides a very fine separating hyperplane which reduces classification errors. In the test data considered, no purely NLOS paths (NLOS excluding GR) were misclassified. It is also evident that incorporating AOA and combining the two features, delay and received power, produces the best performance. Fig. 5 shows the CDFs of the ranging error when the LSSVM is trained with different sets of data and features. When training data 2 is used, it can be observed that mitigation performs well for components that originally had small range errors although as the original error grew, mitigation could not offer significant correction. This is mainly because training data 2 contains half LOS and half GR paths, which produce smaller range errors. Training data 1 provides better mitigation for large range errors and TD3 perform better at very large range errors. One can therefore choose the training data to use depending on the network setup, bearing in mind, the expected range errors. Overall, large range errors are hard to mitigate effectively because they also introduce a larger dynamic range for the regressor.

TABLE II. LS-SVM NLOS IDENTIFICATION PERFORMANCE

Features	Probabilities		
	False LOS identification	**Missed LOS identification**	**Identification Error**
Using training data 1 (half LOS , half {GR + pure NLOS})			
Delay (τ)	0.196	0.015	0.296
Received power (α)	0.058	0.008	0.064
τ & α	0.042	0.001	0.048
Using training data 3 (half LOS, half pure NLOS)			
τ	0.237	0.008	0.254
α	0.107	0.007	0.131
τ , α	0.103	0.002	0.114
Using training data 2 (half LOS, half GR rays)			
τ	0.189	0.014	0.221
α	0.048	0.012	0.062
τ , α	0.036	0.001	0.038
logs of (τ,α)	0.034	0.002	0.035
logs of (τ, α) + AOA	0.019	0.0001	0.019

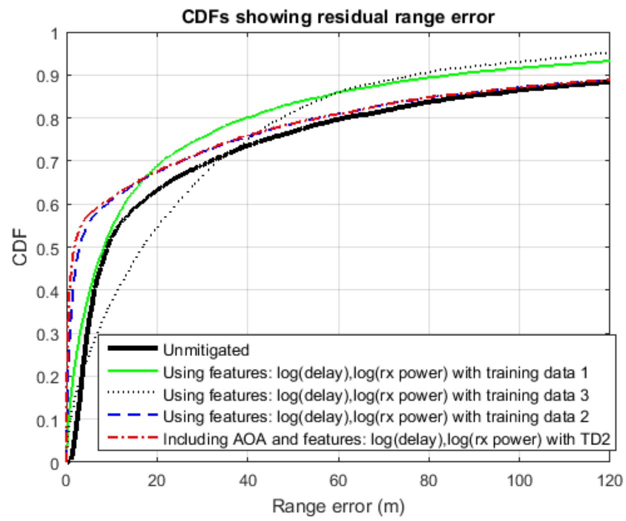

Fig. 5. CDFs of residual ranging error

When range error mitigation is only applied to NLOS rays, it was observed that no improvement results, compared to the case when mitigation was applied to all rays, which suggests that the performance of the LSSVM regressor is mainly affected by NLOS components. The TOA + AOA scheme easily demonstrates the effect of ranging errors on localisation. The algorithm used selects a GR ray whenever there are no LOS rays and reverts to 3 BS TDOA range when all rays are purely NLOS. This allows mitigation to be applied to mainly GR rays which have smaller original range errors. The effect of applying mitigation to this scheme is shown in Fig. 6. It can be noted that for an allowable outage of 20%, NLOS mitigation performs very well with a maximum location error of 10m. However large NLOS range error mitigation may require use of training data 1.

V. LOCATION INDEPENDENT NLOS IDENTIFICATION

A. Methodology

The location specific approach outlined above classifies and mitigates individual rays, between a BS and MS. With the location independent approach, features that are extracted from the channel's impulse response, like delay spread can be used to determine if the link is LOS or NLOS.

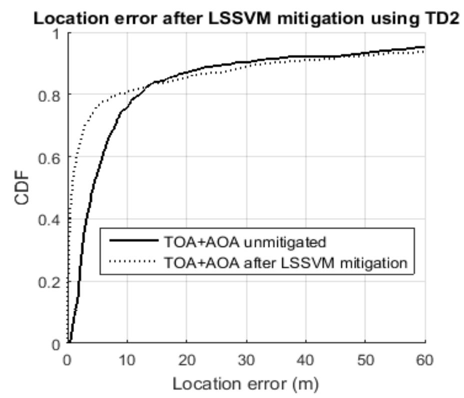

Fig. 6. CDF of location error after mitigation - ~40% improvement for 80% of mobile positions.

Once identification is performed, an assumption is then made that if the link is LOS, it is the first arriving ray that is actually the line of sight one. This ray is then used for localization. For this scheme, the position of the BS is not required. This means that it is possible to re-use parameters from one urban environment, in another similar environment without the need for re-training.

The following features are used;

- Delay of the first arriving path ($\tau_{\min}$)
- Maximum path amplitude $(\alpha_{\max})$
- Mean delay of the channel (τ_a)
- RMS Delay spread (τ_{rms})

Links that produced less than ten paths/rays, from the ray-tracer simulation, were excluded from the analysis. The output sequence used in the training data y_i is taken to be $y_i = +1$ if the BS-MS link contains a LOS ray, and $y_i = -1$ if the link does not contain a LOS path. The RBF kernel was used in all the cases. The fundamental methodology is similar to what is presented under location-specific identification. Training data 2 is used. A training data size of 10,000 was used. Mitigation follows similar methodology as conducted under location-specific mitigation and is applied to the first arriving ray, whose parameters are used for localisation.

B. Results and discussion

It can be noted from Table III that most combinations of 3 or more of these features produce an error probability within 1% of each other. It can be argued that the best performance this approach will give is around 0.13 error probability. The same tuning parameters were used for classification in a different part of the city, using all 4 features, and achieved an error probability of 0.1337 without re-training. This result is comparable to those summarised in [9] using similar features, albeit this being for an outdoor environment. These results suggest that the LS-SVM NLOS classification technique can indeed be extended to an outdoor urban environment.

TABLE III. LOCATION INDEPENDENT LSSVM NLOS IDENTIFICATION PERFORMANCE

Features	Error probability
$\tau_{\min}$	0.3933
τ_a	0.2910
τ_{rms}	0.2143
$\alpha_{\max}$	0.1754
$\tau_a, \tau_{rms}, \alpha_{\max}$	0.1299
$\tau_{\min}, \tau_{rms}, \alpha_{\max}$	0.1320
$\tau_{\min}, \tau_a, \tau_{rms}, \alpha_{\max}$	0.1279

VI. CONCLUSIONS

This study has demonstrated that LSSVM can be used for NLOS identification in outdoor urban environments. Approximately 40% improvement to location accuracy has been demonstrated. This approach can contribute immensely to mobile network-based localisation strategies, which in-turn, can be critical to 5G systems, where geolocation information can be exploited to benefit various subsystems.

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