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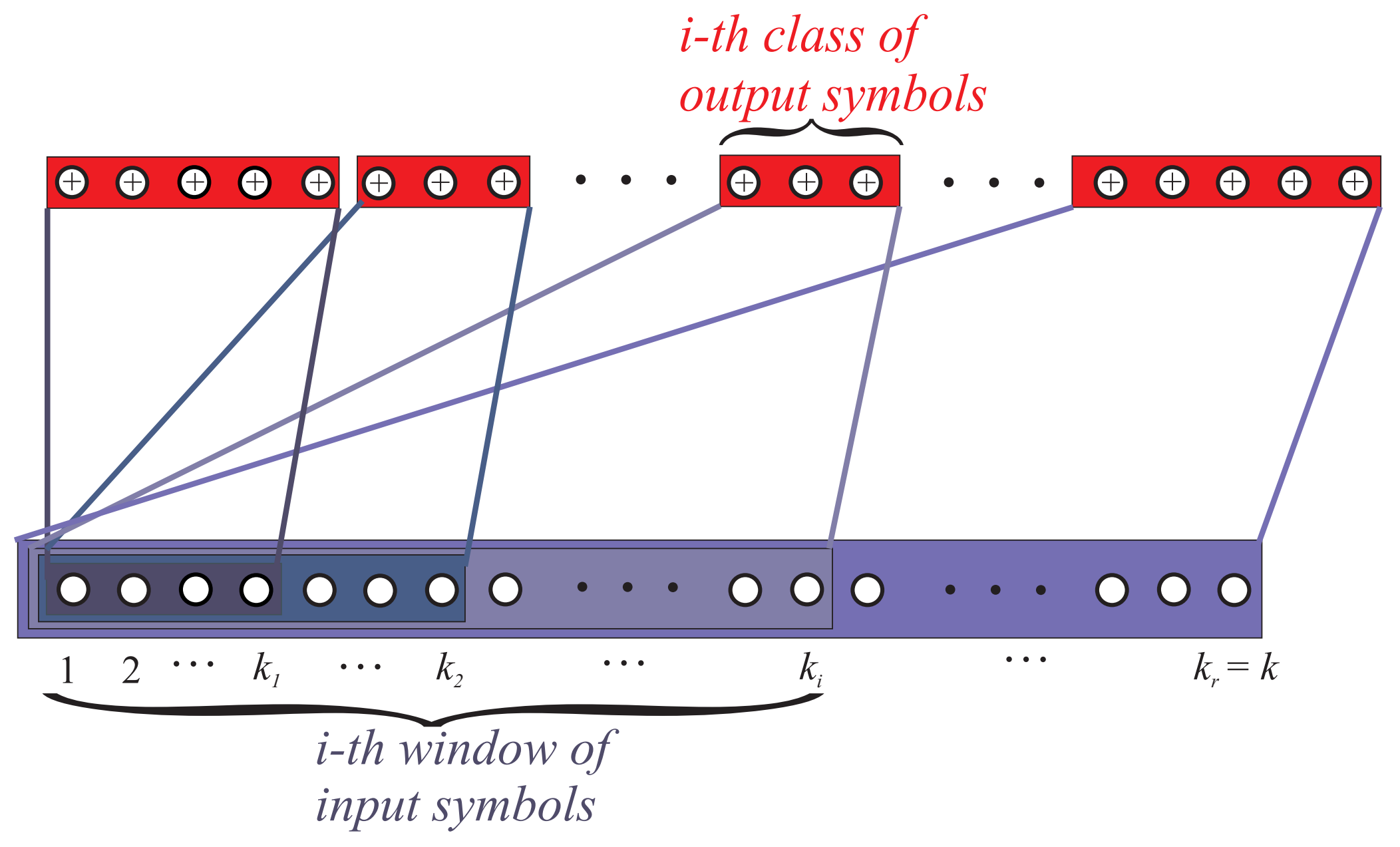
EXPANDING WINDOW FOUNTAIN CODES FOR UNEQUAL ERROR PROTECTION

Dino Sejdinović[1], Dejan Vukobratović[2], Angela Doufexi[1], Vojin Šenk[2] and Robert J. Piechocki[1]
[1]Centre for Communications Research, Dept. of Electrical & Electronic Engineering, University of Bristol, Bristol, UK, Email: {d.sejdinovic, r.j.piechocki, a.doufexi}@bristol.ac.uk
[2]Dept. of Communications and Signal Processing, University of Novi Sad, Novi Sad, Serbia, Email: {dejanv, ram_senk}@uns.ns.ac.yu



EWF codes - generalization of LT codes:

Let us assume that the numbers $k_1, k_2, \ldots, k_r$ such that $k_1 < k_2 < \cdots < k_r = k$ determine the partition of the message block of length k into the groups of input symbols named windows, such that the first k_i input symbols belong to the i-th window. Using windows, we divide the block into the groups of input symbols of unequal importance. The most important class consists of the first $s_1 = k_1$ input symbols in the block, the class of secondary importance consists of the next $s_2 = k_2 - k_1$ input symbols and, in general, the class of the i-th order of importance, $2 \le i \le r$, is made of s_i input symbols with indices $k_{i-1}+1, \ldots, k_i$. We describe the division into importance classes using generating polynomial $\Pi(x) = \sum_{i=1}^{r} \Pi_i x^i$, where $\Pi_i = \frac{s_i}{k}$. We define EWF code $\mathcal{F}_{EW}(\Pi, \Gamma, \Omega^{(1)}, \ldots, \Omega^{(r)})$ as a fountain code which assigns each output symbol to the j-th window with probability Γ_j and encodes chosen window using the LT code with distribution $\Omega^{(j)}(x) = \sum_{i=1}^{k_j} \Omega_i^{(j)} x^i$.

By generalizing the and-or lemma [5], we obtain the erasure probability evolution for input nodes of EWF codes decoded iteratively, as stated in the following lemma.

Lemma 1 . *For EWF code $\mathcal{F}_{EW}(\Pi, \Gamma, \Omega^{(1)}, \ldots, \Omega^{(r)})$, the probability $y_{l,j}$ that the input node of class j is not recovered after l iterations of belief propagation algorithm applied at the overhead ε is*

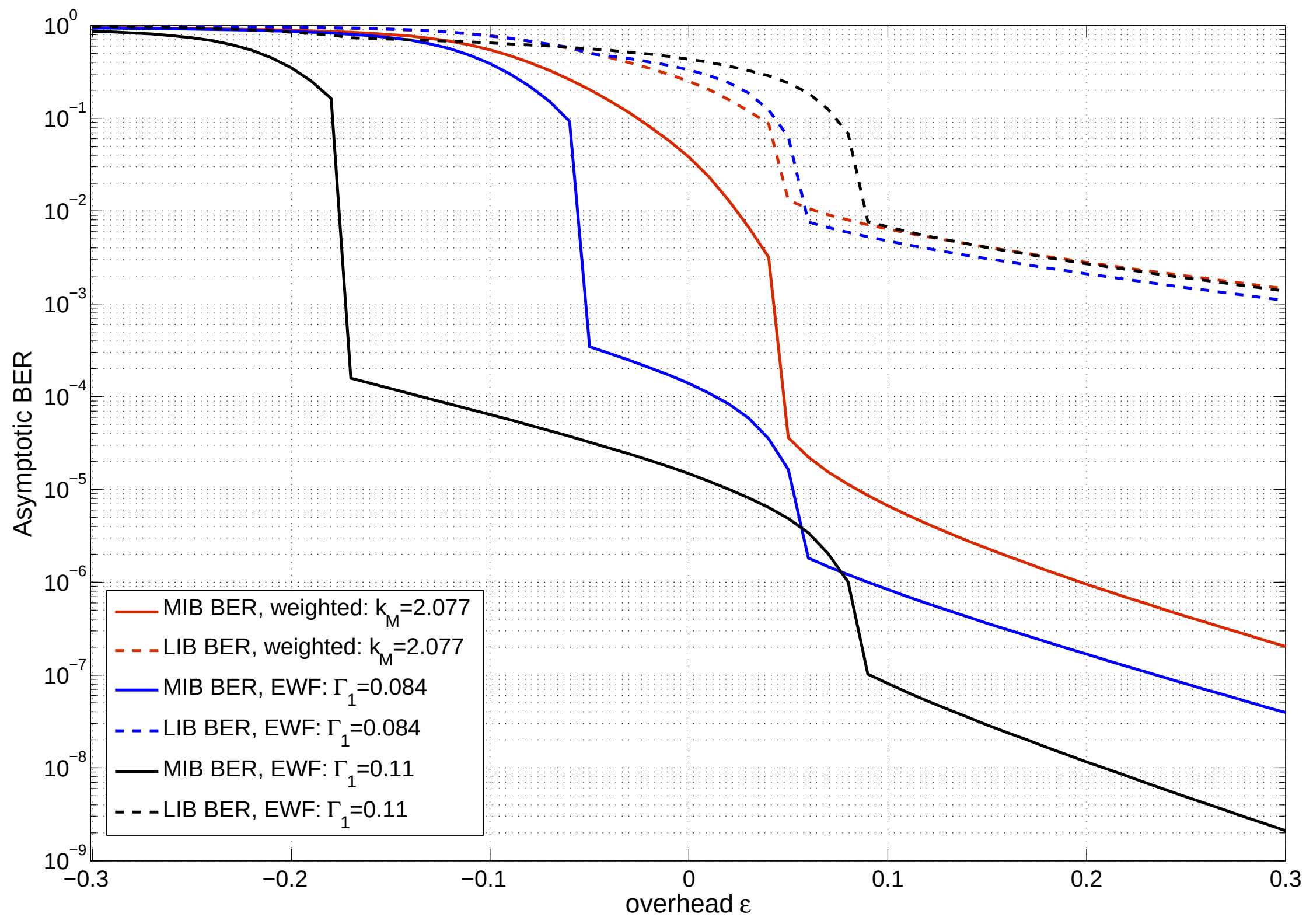
$$\begin{aligned} y_{0,j} &= 1 \\ y_{l,j} &= \exp\left(-(1+\varepsilon)\sum_{i=j}^{r} \frac{\Gamma_i}{\sum_{t=1}^{i}\Pi_t}\Omega^{(i)\prime}\left(1 - \frac{\sum_{m=1}^{i}\Pi_m y_{l-1,m}}{\sum_{t=1}^{i}\Pi_t}\right)\right). \end{aligned}$$

This result allows us to compare asymptotic BER of EWF codes with that of weighted LT codes for the case of two importance classes.

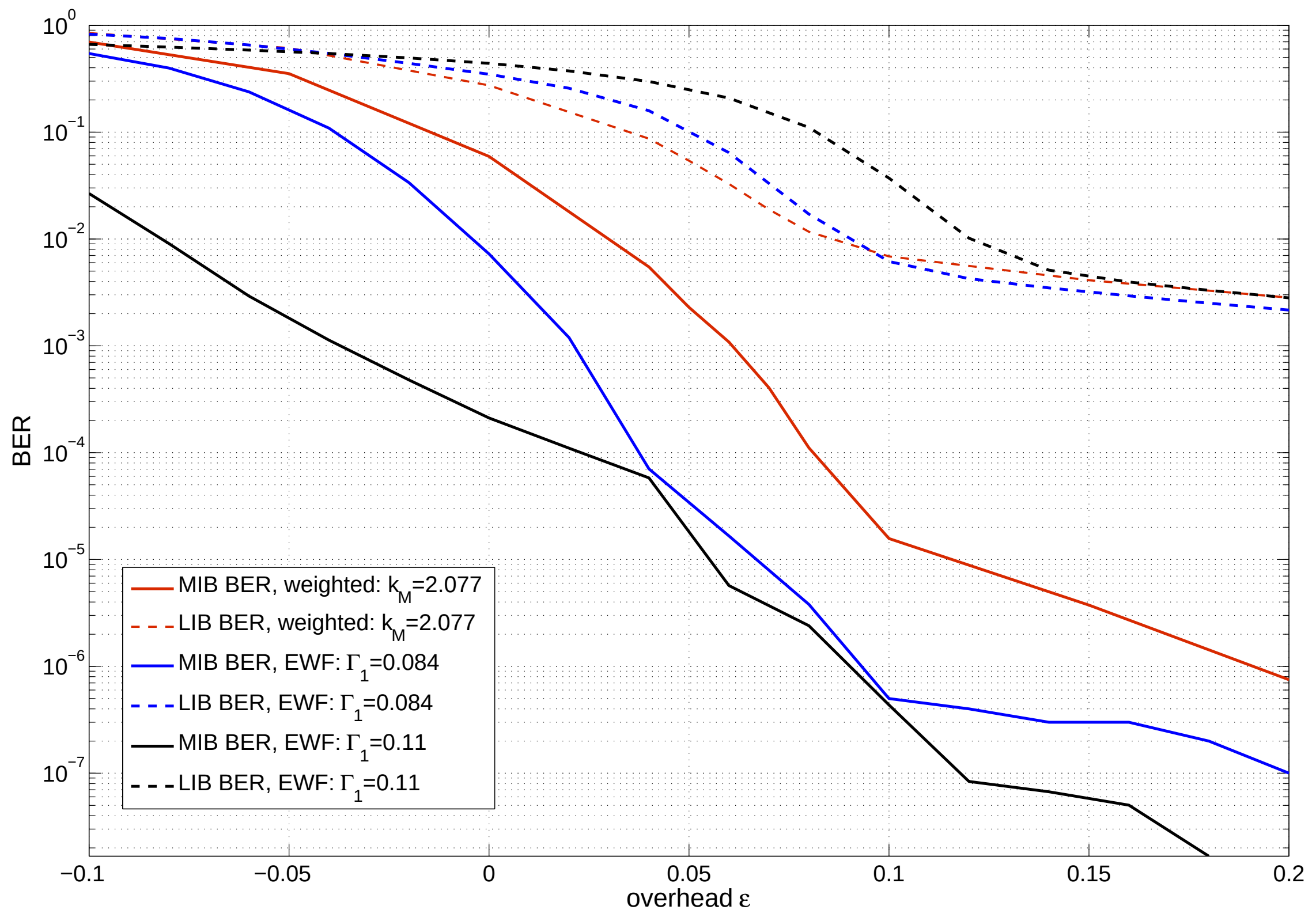
Aim: We introduce and study novel class of fountain codes for erasure channels with unequal error protection (UEP) and unequal recovery time (URT) properties, using a windowing strategy.

Introduction:

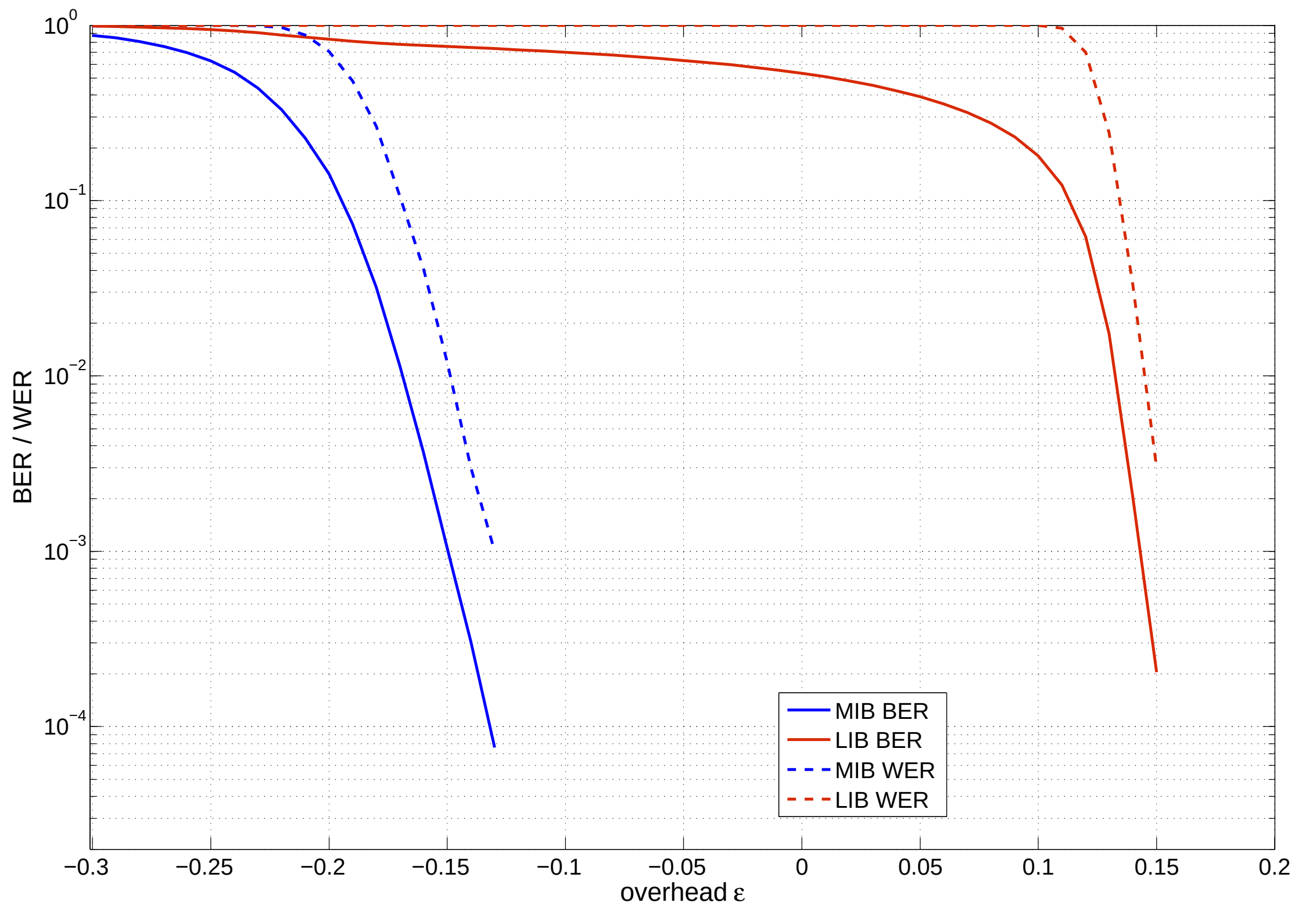
- Fountain codes [1],[2] are an attractive forward error correction solution for multicasting data over erasure channels due to their property of ratelessness; they provide Equal Error Protection (EEP) for all input symbols.
- Many applications benefit from stronger error protection for a certain portion of input symbol, e.g. multicasting image or video files compressed with layered or scalable coders, where input symbols are divided into base layer - the set of the most important symbols, and a number of enhancement layers which progressively improve image or video quality.
- In order to construct fountain codes which provide UEP and URT properties, we use the windowing approach similar to that of [3], which introduces additional parameters in the UEP rateless code design, making it more general and flexible than the weighted approach of [4]; Windowing also provides better performance of UEP/URT scheme, which is confirmed both theoretically and experimentally.
- EWF codes are thus attractive for applications such as delay-constrained multicast of image/video under layered compression.



Asymptotic BER of EWF codes versus the overhead ε and comparison with weighted UEP rateless codes from [4]



BER simulation results for EWF codes on message block with two importance classes



BER/WER simulation results for EWF codes concatenated to a hybrid LDPC+Half high rate precode adopted for 3GPP [6]

Simulation results:

- A message block of length $k = 5000$ with two importance classes, denoted as More Important Bits (MIB) and Less Important Bits (LIB), with $\Pi_1 = 0.1$.
- Using Lemma 1, choose the optimal distribution Γ such that at a fixed overhead MIB BER is minimized, and LIB BER is of the order of magnitude of BER of EEP fountain codes.
- The choice of distribution $\Omega^{(1)}$ used on the MIB window provides an additional design parameter and enables EWF codes to outperform the weighted UEP fountain codes in terms of MIB BER.
- Precoding of EWF codes can be done separately for each importance class and thus, via choice of Γ, we can target the overheads at which full successful decoding of MIB symbols and LIB symbols is expected.
- This way, one can achieve strong URT property necessary for delay-constrained applications - even users with severe channel conditions may be guaranteed the successful reception of the base layer (MIB) [7].

Conclusions:

- We have constructed EWF codes - rateless codes which provide UEP and URT erasure correction properties which use windowing of the data set instead of weighted approach as in previously studied UEP rateless codes.
- The asymptotic analysis of these codes was undertaken - we noted the advantage of EWF codes, additional design parameters which allow lower asymptotic BER values under BP decoding in comparison with previously studied UEP rateless codes.
- We derived lower and upper bounds on ML decoding of EWF codes which confirm their advantages hold under ML decoding.
- Simple precoding scenario allows for targeting the average overheads necessary for successful decoding of different importance classes - strong URT property can be achieved.

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