Approximate Message Passing Reconstruction of Quantitative Acoustic Microscopy Images

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Abstract—A novel framework for compressive sensing (CS) data acquisition and reconstruction in quantitative acoustic microscopy (QAM) is presented. Three different compressive sensing patterns, adapted to the specifics of QAM systems, were investigated as an alternative to the current raster-scanning approach. They consist of diagonal sampling, a row random and a spiral scanning pattern and they can all significantly reduce both the acquisition time and the amount of sampled data. For subsequent image reconstruction, we design and implement an innovative technique, whereby a recently proposed approximate message passing (AMP) method is adapted to account for the underlying data statistics. A Cauchy maximum a posteriori (MAP) image denoising algorithm is thus employed to account for the non-Gaussianity of QAM wavelet coefficients.

The proposed methods were tested retrospectively on experimental data acquired with a 250-MHz or 500-MHz QAM system. The experimental data were obtained from a human lymph node sample (250 MHz) and human cornea (500 MHz). Reconstruction results showed that the best performance is obtained using a spiral sensing pattern combined with the Cauchy denoiser in the wavelet domain. The spiral sensing matrix reduced the number of spatial samples by a factor of 2 and led to an excellent PSNR of 43.21 dB when reconstructing QAM speed-of-sound images of a human lymph node. These results demonstrate that the CS approach could significantly improve scanning time, while reducing costs of future QAM systems.

Index Terms — Quantitative Acoustic Microscopy, Compressive Sensing, Approximate Message Passing, Cauchy distribution

I. INTRODUCTION

Although introduced more than 30 years ago, quantitative acoustic microscopy (QAM) is still a “new” imaging technology employed to investigate soft biological tissue at microscopic resolution by eliciting its mechanical property when irradiated with very high frequency ultrasound [1]. Specifically, by processing RF echo data, QAM yields two-dimensional (2D) quantitative maps of the acoustical and mechanical properties of soft tissues (e.g., speed of sound, acoustic impedance, and acoustic attenuation). Therefore, QAM provides a novel contrast mechanism compared to histology photomicrographs and optical and electron microscopy images [4]. To date, our group and others have successfully used QAM to investigate a wide range of soft biological tissues such as liver samples, lymph nodes, retina, and even living cells [1]–[5]. Several of these recent studies were performed using QAM systems equipped with spherically focused single-element transducers having center frequencies of 250 MHz or 500 MHz which yielded 2D maps of acoustic properties with spatial resolutions of 7 and 4 \( \mu \)m, respectively [2], [3]. Currently, QAM requires a complete 2D raster scan of the sample to form images, thus yielding a large amount of RF data when using a conventional spatial sampling scheme (e.g., 2 and 1 \( \mu \)m steps at 250 and 500 MHz, respectively). Likewise, sonography techniques exploiting ultrasound need to acquire considerable amount of data (thus significantly exceeding the Nyquist rate) in order to perform high resolution digital beamforming. Therefore, compressive sensing (CS) has been intensively studied as a breakthrough to overcome the limitation of contemporary technology [13].

From this perspective, this paper presents a novel approach to improve the efficiency of QAM RF data acquisition and reconstruction by developing a dedicated CS scheme. Traditionally, statistical signal processing has been centered in its formulation on the hypotheses of Gaussianity and stationarity. This is justified by the central limit theorem and leads to classical least square approaches for solving various estimation problems. The introduction of various sparsifying transforms starting with the penultimate decade of the last century, together with the adoption of various statistical models that are able to model various degrees of non-Gaussianity and heavy-tails, have led to a progressive paradigm shift [7]. At the core of modern signal processing methodology sits the concept of sparsity. The key idea is that many naturally occurring signals and images can be faithfully reconstructed from a lower number of transform coefficients than the original number of samples (i.e., acquired according to Nyquist theorem) [8]. In this sense, CS could prove to be a powerful solution to decrease the amount of data in QAM and to accelerate the acquisition process at potentially no cost to image quality. In terms of reconstruction, most CS methods rely on \( l_1 \)-norm
minimization using a linear programming algorithm. All these approaches do not exploit the true statistical distribution of the data and are motivated by the inability of the classical least-squares approach to estimate the reconstructed signal. In this work, an approximate message passing (AMP)-based algorithm was designed to reconstruct QAM images from spatially undersampled measurements. AMP is a simplified version of MP (message passing) derived from belief propagation in graphical models [9], and is characterised not only by dramatically reduced convergence times but also by a reconstruction performance equivalent to $l_p$-based methods. AMP uses an iterative process consisting of a sparse representation-based image denoising algorithm performed at each iteration. Hence, selection of a robust denoiser and of the most efficient sparsifying basis are essential issues to address in order to achieve fast convergence and high recovery quality [10]. Our proposed AMP-based QAM imaging framework consists of two major modules: (i) In the data acquisition component of our system, we propose novel techniques for QAM data sampling, by choosing sensing matrices that simultaneously meet CS requirements and take into account the peculiarities of practical QAM acquisition devices, instead of. (ii) In the image reconstruction component, we design and test a wavelet domain AMP-based approach, which exploits underlying data statistics through the use of a Cauchy-based MAP algorithm\(^1\).

The paper is structured as follows: Section II covers the essential, necessary background on QAM, CS and AMP. Section III introduces the main theoretical contributions of this work. Specifically, in Section III-A, we present the derivation of the wavelet-based Cauchy denoiser and in Section III-B we describe three different sensing patterns for QAM. Section IV compares the performance of our proposed method with that of existing CS reconstruction techniques, including previously proposed AMP algorithms. Finally, conclusions and future work directions are detailed in Section VI.

II. THEORETICAL BACKGROUND

A. Quantitative Acoustic Microscopy

In QAM, a high-frequency (e.g., > 50 MHz), single-element, spherically-focused (e.g., F-number < 1.3) transducer transmits a short ultrasound pulse and receives the RF echo signals reflected from the sample which consists of a thin section of soft tissue affixed to a microscopy slide (Fig. 1). The echo RF data is composed of two main reflections ($S_1$ and $S_2$ in Fig. 1): $S_1$ originates from the interface between the coupling medium (degassed saline) and the specimen and $S_2$ from the interface between the specimen and the glass substrate interface as illustrated in Fig. 1. At each scan location, the RF data is digitized, saved, and processed offline to yield values of speed of sound ($c$), acoustic impedance ($z$) and attenuation ($\alpha$) [14]. Signal processing also requires the use of a reference signal obtained from a region devoid of sample ($S_0$ in Fig. 1). Briefly, the ratio of the Fourier transform of a sample signal, $S$ and $S_0$, is computed and fit to a forward model to estimate the time of flight differences between $S_1$ and $S_2$ and $S_0$. These time differences are used to estimate $c$ in the sample as well as the tissue thickness (i.e., $d$ in Fig. 1) at that location. The forward model fit also provides the amplitude of $S_1$, which is used to estimate $z$ of the sample. Finally, the amplitude of $S_2$, its frequency dependence, and the previously-estimated tissue thickness are used to estimate $\alpha$ [14]. The transducer is raster scanned and the values obtained at each scan location are then combined to form quantitative 2D parameter maps. Fig. 2 shows the working principle of QAM as well as the 500-MHz QAM system used in this

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\(^1\)An initial version of this algorithm was presented in [11], but the work therein was focused on natural images.
study (Fig. 1), namely the transducer and the thin sample affixed to a microscopy slide. Samples are obtained from fixed or frozen samples sectioned using a microtome or a cryotome. In the case of fixed samples, the paraffin is removed and the sample is rehydrated before imaging. In the case of frozen samples, the sample is thawed and rehydrated before imaging. These protocols are common and used in histopathology. Following QAM data acquisition, the samples can be stained and imaged using optical microscopy approaches and histology photomicrographs can easily be coregistered with QAM images. Therefore, successfully applying a CS approach to QAM acquisition could significantly reduce the amount of QAM images. Therefore, successfully applying a CS approach to QAM acquisition could significantly reduce the amount of QAM images.

C. Approximate Message Passing Reconstruction

In the context of CS, AMP reconstructs an original image from a reduced number of linear measurements by performing elementwise denoising at each iteration. Indeed, the AMP algorithm can be interpreted as recursively solving an image denoising problem. Specifically, at each AMP iteration, one observes a noise perturbed original image. Reconstructing the image amounts to successive noise cancellations until the noise variance decreases to a satisfactory level. The algorithm can be succinctly summarised mathematically through the following two steps:

\[ x^{t+1} = \eta_t(\Phi^T z^t + x^t), \]  
\[ z^t = y - \Phi x^t + \frac{1}{\delta} z^{t-1}(\eta_{t-1}(\Phi^T z^{t-1} + x^{t-1})), \]

where \( x, y, z \) and \( \delta \) denote a sparse signal, observation, residual and undersampling ratio \( M/N \) respectively. \( \eta (\cdot) \) is a function that represents the denoiser, \( \eta (\cdot) \) is its first derivative and \( \eta (\cdot)^T \) is the classical conjugate transpose notation. Given \( x = \theta \) and \( z = y \) as an initial condition, the algorithm iterates sequentially (2) and (3) until satisfying a stopping criterion or reaching a pre-set iteration number. The last term of the right hand side in (3) is referred to as the Onsager reaction term and is also acknowledged to contribute to balancing the sparsity-undersampling tradeoff [9], [21].

An extended wavelet-based AMP system can be generated by integrating a wavelet transform (denoted by \( W \)) into (2) and (3) using the following transformation.

\[ y = \Phi W^{-1} \theta_x + n, \]

where \( W^{-1} \) denotes the inverse wavelet transform, \( W \) and \( \theta_x \) becomes the sparse representation of \( x \) within wavelet domain. Introducing \( \Theta \) as the new notation for \( \Phi W^{-1} \), we get the following expressions:

\[ \theta_x^{t+1} = \eta_t(\Theta^T z^t + \theta_x^t), \]  
\[ z^t = y - \Theta \theta_x^t + \frac{1}{\delta} z^{t-1}(\eta_{t-1}(\Theta^T z^{t-1} + \theta_x^{t-1})). \]

The subsequently defined denoising algorithms seek to denoise the elements of \( \theta_q^t = \Theta^T z^t + \theta_x^t \) corresponding to the contaminated wavelet coefficients. To simplify the following notation, the \( i \)th element of \( \theta_q^t \) is defined as \( \theta_{q,i}^t = v \) and the \( i \)th element

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of the denoised output $\theta_k^{t+1}$ is defined as $\theta_k^{t+1} = \hat{w}$ (a denoised estimate of the true coefficient $w$).

The most important design consideration is arguably the choice of the shrinkage (denoising) function, $\eta$, in (5) above. In the following, we introduce two previously defined functions [6], [21], which will be later used for comparison purpose within our experimental results in Section IV-A.

**Soft Thresholding (ST) denoiser:**

$$\hat{w} = \eta(v) = \text{sign}(v) (|v| - \tau) \cdot \mathbb{1}_{(|v| > \tau)},$$

$$\eta'(v) = \mathbb{1}_{(|v| > \tau)},$$  \hspace{1cm} (7)

where $\mathbb{1}_{(\cdot)}$ is the indicator function. The threshold $\tau$ is defined as the $M^{th}$ largest magnitude value of $\theta_q^t$ [9].

**Amplitude-scale-invariant Bayes Estimator (ABE) denoiser:**

$$\hat{w} = \eta(v) = \frac{(v^2 - 3\sigma^2)_+}{v},$$

$$\eta'(v) = \mathbb{1}_{(v^2 > 3\sigma^2)} \cdot \left(1 + 3 \left(\frac{\sigma}{v}\right)^2\right),$$  \hspace{1cm} (8)

where $\sigma^2$ is the noise variance at iteration $t$ and $(\cdot)_+$ is the right handed function where $(u)_+ = 0$ if $u \leq 0$ and $(u)_+ = u$ if $u > 0$.

As far as the CS reconstruction of conventional ultrasound images is concerned, the denoiser in (8) was shown to achieve better performance than IRLS and $l_p$ programming [29]. Therefore, we hypothesize that ABE should also be a successful criterion for QAM images and consequently, we shall use it for benchmarking our method. Fig. 3 illustrates the behavior of the denoising function for four different denoisers, of which ABE and the Cauchy-based denoisers that is introduced in the subsequent section can be regarded as a compromise between Soft-thresholding and Hard-thresholding [24]. The labels on the horizontal and vertical axes correspond to corrupted wavelet coefficient and their denoised version respectively. Subscript $i$ represents the index of each element which implies element-wise denoising, as stated before.

### III. CAUCHY-AMP FOR COMPRESSED QAM IMAGING

This section describes the key features of wavelet-based Cauchy-AMP together with the practical sensing patterns as a novel approach for QAM CS reconstruction.

**A. Wavelet based Cauchy-AMP**

Wavelet coefficients provide a sparse representation for natural images. In addition, they can be accurately modelled using heavy tailed distributions such as the $\alpha$–stable distribution [22], [23]. The Cauchy distribution is a special case of the $\alpha$–stable family which not only has a heavy tailed form but has a compact analytical probability density function given by [11]:

$$P(w) = \frac{\gamma}{w^2 + \gamma^2};$$  \hspace{1cm} (9)

where $\gamma$ and $\gamma$ are the wavelet coefficient value and the dispersion parameter (controlling the spread of the distribution) respectively. Given (9), a maximum a posteriori (MAP) estimator method (10) can lead to the derivation of explicit formulae (14) estimating a clean wavelet coefficient $w$ from an observed coefficient observation $\nu$ contaminated with additive Gaussian noise (i.e. $n = \nu - w$ with noise variance $\sigma^2$) [12].

$$\hat{w} = \arg \max_w P_{\nu|w}(w|\nu).$$  \hspace{1cm} (10)

The posterior probability $P_{\nu|w}(w|\nu)$ can be expressed as (11) by Bayes’ theorem

$$P_{\nu|w}(w|\nu) = P_{\nu|w}(\nu|w)P_w(w) \big/ P_\nu(\nu).$$  \hspace{1cm} (11)

Assuming $P_{\nu|w}(\nu|w) \sim N(0, \sigma^2)$, the logarithmic form of (10) is given in (12) which is mathematically more intuitive. The evidence $P_\nu(\nu)$ is constant for all inputs and therefore can be ignored.

$$\hat{w}(\nu) = \arg \max_w \left[ -\frac{(\nu - w)^2}{2\sigma^2} + \log \left(\frac{\gamma}{w^2 + \gamma^2}\right) \right].$$  \hspace{1cm} (12)

To find the solution to (12), take the first derivative of the terms in the bracket relative to $w$ and set to zero:

$$\hat{w}^3 - \nu \hat{w}^2 + (\gamma^2 + 2\sigma^2) - \gamma^2 v = 0.$$  \hspace{1cm} (13)

Using Cardano’s method, the estimate of $w$ can be found in (14) of which first derivative is (15).

$$\hat{w} = \eta(v) = \frac{v}{3} + s + t;$$  \hspace{1cm} (14)

$$\hat{w}' = \eta'(v) = 1/3 + s' + t',$$  \hspace{1cm} (15)

where $s$ and $t$ are values determined by $v$ and $\sigma^2$ iteratively updated at each iteration together with a constant value $\gamma$. $\sigma^2$
is estimated as the variance of the \( z \) vector defined in (6). \( s \) and \( t \) are defined as:

\[
\begin{align*}
    s &= \frac{3}{2} + dd, & t &= \frac{3}{2} - dd, \\
    dd &= \sqrt{p^2/27 + q^2/4}, \\
    p &= \gamma^2 + 2\sigma^2 - \nu^2/3, \\
    q &= \nu\gamma^2 + 2\nu^3/27 - (\gamma^2 + 2\sigma^2)\nu/3.
\end{align*}
\]

\( s' \) and \( t' \) are found as follows:

\[
\begin{align*}
    s' &= \frac{q}{2dd} + dd', & t' &= \frac{q}{2dd} - dd', \\
    dd' &= -p'^{3/2} + q'^{3/2}, \\
    p' &= -2\nu/3, \\
    q' &= -2\sigma^2/3 + 2\gamma^3/3 + 2\nu^2/9.
\end{align*}
\]

### B. Practical sensing patterns for QAM

Theoretically, optimal sensing matrices based on randomness are impractical for QAM data acquisition because RF data are typically acquired continuously as the motor stages are moved. Therefore, this paper investigates three practical sensing schemes, which can be easily implemented using servo motors. The diagonal sensing schemes raster scans oblique lines using a constant predefined angle which is used to vary the measurement rate, i.e., a smaller angle leads to denser sampling. The row random sensing pattern is a naive but practical attempt to preserve randomness. Data are also collected using a practical raster scanning approach, but only on randomly selected rows using the random number generator (rng) of Matlab. An input value for rng is used to dictate the measurement rate. Finally, the spiral sensing scheme is also a practical sensing scheme which originates in the center of area to be sampled and spreads out following a spiral pattern. The pace of the spreading is parametrized and used to prescribe the measurement rate. For comparison purposes, the spiral pattern was truncated to cover the same square area as the other two patterns. In actual experiments, the scanned area by the spiral pattern would consist approximately of a circle passing through the four corners of the square.

Fig. 4 illustrates all sensing schemes used to sample data from a target composed of \( 256 \times 256 \) pixel. A measurement rate of 20\% is shown for all three schemes and the white pixels corresponds to the area to be spatially sampled.

### IV. SIMULATION RESULTS

Two different sets of experiments have been conducted and results are reported in Sections IV-A and IV-B. The objective of the first set of experiments was to evaluate the performance of the proposed Cauchy-AMP algorithm. The second set of experiments shows the interest of the proposed sampling schemes in QAM and the ability of Cauchy-AMP algorithm to recover high quality images from the resulting under-sampled data. In addition to visual inspection, the peak signal to noise ratio (PSNR) and the structural similarity (SSIM) index [32] were used to assess the quality of the reconstructed images by comparing them to the corresponding fully-sampled quantitative maps.

#### A. Simulation A: reconstruction results with random sensing schemes

The objective of this subsection is to validate the efficiency of the proposed Cauchy-AMP algorithm in comparison to al-

### Table I

**Numerical Results of Recovery Quality (Gaussian Random Sensing)**

<table>
<thead>
<tr>
<th>Method</th>
<th>SSIM</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cauchy</td>
<td>0.841</td>
<td>39.08</td>
</tr>
<tr>
<td>ABE</td>
<td>0.811</td>
<td>39.01</td>
</tr>
<tr>
<td>ST</td>
<td>0.724</td>
<td>38.41</td>
</tr>
<tr>
<td>IRLS</td>
<td>0.698</td>
<td>34.97</td>
</tr>
<tr>
<td>LILS</td>
<td>0.459</td>
<td>32.14</td>
</tr>
</tbody>
</table>

### Table II

**The Comparison of Execution Time: The Averaged Values Over 30 Trials**

<table>
<thead>
<tr>
<th>Method</th>
<th>Runtime (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cauchy</td>
<td>3.52</td>
</tr>
<tr>
<td>ABE</td>
<td>2.86</td>
</tr>
<tr>
<td>ST</td>
<td>2.82</td>
</tr>
<tr>
<td>IRLS</td>
<td>62.88</td>
</tr>
<tr>
<td>LILS</td>
<td>4.10</td>
</tr>
</tbody>
</table>

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Fig. 4. The proposed three different types of sampling masks, (a) Diagonal, (b) Row random and (c) Spiral.
ternative methods, previously proposed for CS reconstruction. Two of these were described in Section II-C and, while also AMP-based, they use ST and ABE as shrinkage functions in eq. (7) and (8). In addition, we also compare to conventional CS reconstruction algorithms, including the L1LS method (iteratively reweighed least squares):\[ \hat{x} = \min_{x} \| x \|_p \text{ subject to } y = \Phi x. \] (19)

When it comes to estimating a sparse vector characterised by an $\alpha$-stable distribution, (19) leads to solving an $l_p$ minimisation problem. Thus, in order to choose the optimum value of $p$, we employed the approach described in [30]. This approach was found to be superior to existing $l_p$ solvers when applied to CS reconstruction of conventional B-mode images.

The incoherence between the sensing matrix and the sparsifying transform is important in CS applications. Hence, in order to fairly evaluate the performance of the reconstruction algorithms, the results in this section are obtained with two random sensing matrices widely used in the CS literature:
image projections on random Gaussian vectors and point-wise multiplication with Bernoulli vectors formed by uniformly random distributed zeros and ones.

Two experimental data volumes were used, from which impedance maps were estimated point-wise using the method in [14]. The first was acquired from a spatial-resolution target consisting of small bars of known width and spacing. Because the chrome used to form those bars is deposited using photolithography, the metal thickness (i.e., ~ 0.12 µm) is much smaller than the wavelength at 250 MHz (i.e., ~ 6 µm) and therefore only an effective acoustic impedance ($z_{eff}$) can be estimated. The amplitudes of the reflected signal ($A$) and the reference signal ($B$) were used to calculate $z_{eff}$ using the following equation from the pressure reflection law [15]:

$$z_{eff} = \frac{z_w (1 + R_{ref} \frac{z_w}{z_g})}{(1 - R_{ref} \frac{z_w}{z_g})},$$

(20)

where $z_w$ is the known acoustic impedance of water and $R_{ref}$ is the pressure reflection coefficient between water and glass slide:

$$R_{ref} = \frac{z_g - z_w}{z_g + z_w},$$

(21)

where $z_g$ is the known acoustic impedance of the glass slide. The second data volume was acquired from a 12-µm thick section obtained from a lymph node excised from a colorectal cancer patient using the 250-MHz QAM system. For both Gaussian and Bernoulli measurement matrices, the reconstruction results correspond to a measurement rate of 25%, i.e., the ratio between the number of CS measurements and the number of pixels in the fully sampled QAM image.

1) Gaussian random measurements: Figs. 5 and 6 illustrate the impedance images obtained with the five reconstruction algorithms from Gaussian random measurements. It highlights that IRLS and L1LS methods severely distorted the fully-sampled image compared to the AMP-based algorithms. By closely comparing the AMP-based methods, one may remark that Cauchy-AMP shows a tendency of noise removal with a slightly excessive smoothing effect, whereas ST-AMP and ABE-AMP suffer from several reconstruction artefacts. Table I regroups the PSNR and SSIM values corresponding to the results in Figs. 5 and 6. Additionally, Table II provides the runtime of the five methods, averaged over 30 trials. All the algorithms were implemented in Matlab R2014a environment and executed on a desktop computer equipped with a 2.6GHz Intel(R) Core(TM) i7 – 6500C processor with 8GB RAM.

AMP algorithms outperform the two conventional recovery approaches IRLS and L1LS. Particularly, Cauchy-AMP yields the most accurate results compared to its AMP counterparts, at the cost of an execution time marginally higher than ABE- and ST-AMP. The execution time increase per iteration is explained by the number of parameters to be estimated during the de-noising process. Indeed, Cauchy-AMP requires the estimation of an extra parameter compared to ST- and ABE-AMP, i.e. the dispersion parameter that determines the spread of the Cauchy distribution. However, the extra computational cost per iteration is significantly mitigated by the faster convergence of Cauchy-AMP as revealed in Fig. 7.

2) Bernoulli random measurements: The above overall evaluation confirms that AMP-based algorithms are the most promising QAM recovery methods from under-sampled data. Nevertheless, measurements obtained by linear projections on Gaussian vectors are not of practical use in QAM. As explained previously, QAM data is acquired point-wise by raster scanning the sample. Thus, Bernoulli random measurements corresponding to random spatial positions are further adapted to QAM acquisition system. Therefore, the three AMP-based methods are tested in this section on the same image used previously but on Bernoulli randomly sampled data. Similar to the previous results, the proposed Cauchy-AMP outperforms ABE- and ST-AMP algorithms. The three reconstructed images are shown in Figs. 8 and 9. The corresponding quantitative results are regrouped in Table III.

### Table III

<table>
<thead>
<tr>
<th>Method</th>
<th>SSIM</th>
<th>PSNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cauchy</td>
<td>0.851</td>
<td>38.72</td>
</tr>
<tr>
<td>ABE</td>
<td>0.837</td>
<td>38.56</td>
</tr>
<tr>
<td>ST</td>
<td>0.815</td>
<td>37.88</td>
</tr>
</tbody>
</table>

#### B. Simulation B: reconstruction results with sensing schemes dedicated to QAM

The results shown in the previous section proved the superiority in reconstructing QAM images of the proposed Cauchy-AMP algorithm against four well-established methods. However, Gaussian random measurements are impractical for QAM data acquisition. Similarly and although technically possible, it would be inefficient to move the transducer to transmit and receive ultrasound signals at spatial locations following a Bernoulli random measurements. Therefore, in this second set of simulations, the three AMP-based algorithms...
are employed to assess the relevance of the practical sensing patterns proposed in this study (see Section III-B) for QAM imaging.

The simulations used experimental results obtained from three real QAM maps. The first map corresponds to the speed of sound (SoS) map obtained from a human cornea sample using the 500-MHz QAM system. The two other maps are SoS and impedance maps obtained using the 250-MHz QAM system on a human lymph node thin section obtained from a colorectal cancer patient. The fully sampled images correspond to standard raster scanning at conventional spatial scanning frequencies, resulting into a pixel size of 1 \( \mu \)m per 1 \( \mu \)m and respectively of 2 \( \mu \)m per 2 \( \mu \)m for the 500 MHz and 250 MHz data.

All the AMP-algorithm investigations were performed using measurement ratios ranging from 20% to 60% of the data obtained using the conventional raster scanning approach. Fig. 10 displays the resulting PSNR values and permits comparing the QAM image reconstruction quality between the three proposed sensing patterns and the three AMP-based reconstruction algorithms. Blue regions in these images were not included in quantitative analyses because they were devoid of tissues. Independent of the image or algorithm under consideration, results indicate that the spiral pattern always provided the highest PSNR, followed by the diagonal pattern. Figs. 11, 12 and 13 show the fully-sampled images and the ones recovered by the three algorithms from data generated with the three considered patterns for a measurement rate of 40%. Also for the sake of the quantitative evaluation corresponding to the figures, the numerical results are in the Table IV offered. Overall, visual inspections of these images are consistent with the results shown in Fig. 10, the spiral pattern provide the best result and the quality of the reconstructions improves with measurement ratio. The images obtained with a measurement ratio of 40%, i.e. Figs. (11, 12, 13) illustrate nicely the relative performance of each tested sensing pattern. Fig. 10 reveals that Cauchy denoising provides better PSNR values. The corresponding 2D maps are shown in (b)-(d) of Figs. (11, 12, 13). The row random pattern results shows many artifacts appearing as “transverse” lines. In contrast, the spiral and diagonal sensing patterns do not contain any visually-apparent artifacts. In order to determine which of these two sensing patterns performs better for QAM, one can arguably see in Figs. 11-13 that the dense yellow area are better reconstructed using the spiral than the diagonal sensing pattern. This visual assessment is consistent with the quantitative results shown in Fig. 10 and Table IV. Another potential benefit of spiral pattern resides in significantly reduced scanning time. QAM estimation time is typically less important than QAM data acquisition time because tissue properties may change during scanning. Nevertheless, while the proposed AMP approach significantly decreases scanning time, it turns out that it also significantly decreases image formation time, because QAM parameter estimation is done independently on each RF line and is much more time consuming than AMP (cf. Table II). For example, in the case of the 40% spiral, scanning time is reduced by more than 80% because in conventional raster scanning most of the time is spent accelerating and decelerating in each scan line, whereas the spiral is a smooth continuous curve which can be scanned at almost constant speed with servo motors. In addition, initial parameter estimation time is also reduced by 60% prior to the application of the AMP algorithm. The raster scanning and parameter estimation times for the lymph node example (Fig. 12 and Fig. 13) were approximately 20 and 15 minutes. The 40% spiral AMP approach would reduce these times to approximately 4 and 8 minutes. In conclusion, these sets of simulations reveal that combining a spiral sensing pattern with a measurement ratio of 40%, and a Cauchy-AMP...
Fig. 10. PSNR results as a function of the measurement rate, the sensing pattern (diagonal, random rows or spiral) and recovery algorithm (proposed Cauchy-AMP, ABE-AMP and ST-AMP): (a) Cauchy, (b) ABE and (c) ST of Human cornea at 500 MHz of SOS mode. (d) Cauchy, (e) ABE and (f) ST of Human lymph node at 250 MHz of SOS mode. (g) Cauchy, (h) ABE and (i) ST of Human lymph node at 250 MHz of impedance mode.

recovery is the best compromise between a practical spatial sampling pattern easily implementable with servo motors and image reconstruction quality for QAM imaging.

However in three graphs (right column) corresponding to ST-AMP of Fig. 10, intractable problems are found. To date researches associated with CS recovery normally have been reporting that the relationship between recovery quality and measurement ratio is a monotonically increase or decrease within usually simulated range. By contrast, ST-AMP shows unfamiliar results. Indeed CS has been constructed on the premise of sensing matrices satisfying mathematical completeness such as RIP and incoherence to ensure perfect recovery of sparse signals. Nevertheless, since this work prioritized the aspect of practical implementation of sensing strategy, the proposed sensing patterns unavoidably followed deterministic sensing trajectories rather than randomness dominating CS sensing arena owing to meeting essential conditions stated above [34]. From this perspective, further study will focus on the development of practical sensing schemes retaining the pertinent properties of random matrices [35].

V. CONCLUSIONS

In this paper, we introduced a new framework for compressive sampling reconstruction of QAM images together with associated sampling patterns. We proposed and tested three compressive sampling measurement matrices, with a view of reducing both acquisition time and the amount of samples required, while taking into account the constraints imposed by the design of current experimental QAM systems. Specifically, we assessed the relative merits of diagonal sampling, row random sampling and spiral scanning as underlying patterns in designing a CS measurement matrix. We adopted an approximate message passing strategy for the image reconstruction component of our framework, owing to its similarity to $l_p$ minimisation. In particular, in the multiscale wavelet domain, we employed a Cauchy-based MAP estimation algorithm.
to perform the image denoising step required by an AMP algorithm.

We tested our methods in comparison with various compressive image reconstruction algorithms, when applied to QAM data. Our results showed improved performance both with respect to alternative AMP techniques that use different denoising strategies as well as to other, more standard, approaches to CS reconstruction, which employ $l_1$-norm, or $l_p$ minimisation.

The results of this study could prove invaluable in QAM imaging. CS has the potential to yield significantly improved scan times, smaller datasets, faster image formation without degrading image quality. Moreover, CS approaches would reduce experimental challenges currently encountered in QAM imaging. For example, the spiral sampling approach could be implemented on cheap, potentially less precise, servo motors. In addition, reducing scan time would reduce changes that the tissue properties may suffer during scan and would limit temperature variations, particularly in the coupling medium, which can greatly effect speed of sound estimates. Therefore, CS approaches could potentially bring about a new generation of QAM systems which would be lower costs and simpler to use. Finally, the study of dominant factors having an effect on the convergence rate of the proposed method is definitely an interesting study that will be part of our future work.

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Fig. 12. Reconstruction results of SoS (m/s) map estimated from human lymph node data acquired at 250 MHz: (a) original fully sample data at pixel resolution of 2 µm per 2 µm, (b-d) reconstructed images with the proposed Cauchy-AMP algorithm for spiral, diagonal and row random sampling patterns, (e-g) reconstructed images with ABE-AMP algorithm for spiral, diagonal and row random sampling patterns, (h-j) reconstructed images with ST-AMP algorithm for spiral, diagonal and row random sampling patterns. All the results correspond to a measurement ratio of 40%.


