Is There a Baseflow Budyko Curve?

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Abstract  There is no general theory to explain differences in baseflow between catchments, despite evidence that it is mainly controlled by climate and landscape. One hypothesis is that baseflow fraction (the ratio between baseflow and precipitation) can be primarily attributed to the aridity index (the ratio between potential evapotranspiration and precipitation), that is, that there is a “baseflow Budyko curve.” Comparing catchment data from the United States and the United Kingdom shows, however, that aridity is not always a good predictor of baseflow fraction. We use the revised Ponce-Shetty annual water balance model to show that there is no single “baseflow Budyko curve” but rather a continuum of curves emerging from a more universal model that incorporates both climate and landscape factors. In humid catchments, baseflow fraction is highly variable due to variations in a catchment’s wetting potential, a parameter that describes catchment storage capacity. In arid catchments, vaporization limits baseflow generation, which leads to lower variability in baseflow fraction. Generally, when the magnitude of precipitation is important, the aridity index only partly explains baseflow response. Adapting the model to explain variability of the baseflow index (the ratio between baseflow and total streamflow) shows that the aridity index is generally a poor predictor of baseflow index. While the wetting potentials and other parameters are obtained by fitting the Ponce-Shetty model to annual catchment data, their links to physical properties remain to be explored. This currently limits the model’s applicability to gauged catchments with sufficiently long records.

Plain Language Summary  Baseflow originates from stored water (e.g., groundwater) and sustains river flow in dry periods, which makes it an important water resource. Baseflow is known to vary with climate and landscape properties such as geology or vegetation, but there is no universal theory to explain this variability. To explore baseflow variability, we use data from several hundred catchments in the United States and the United Kingdom. We investigate whether a catchment’s baseflow fraction, that is, the fraction of rainfall that becomes baseflow, can be attributed primarily to the aridity index, a commonly used climate index. The aridity index is defined as the ratio between potential evapotranspiration (available energy) and precipitation (available water). We find that in humid catchments (low aridity index), baseflow cannot be attributed primarily to the aridity index. Rather, a catchment’s capacity to store water determines how much precipitation becomes baseflow. In arid catchments (high aridity index), the aridity index can be seen as the primary determinant of baseflow fraction. It strongly influences how much of the precipitation can be evaporated back to the atmosphere and thus cannot become baseflow. These results might help to assess how water availability (in the form of baseflow) changes with changing climate and land use.

1. Introduction

Baseflow is defined as flow derived from groundwater and other delayed sources and thus sustains streamflow also during dry periods (Hall, 1968; Smakhtin, 2001). Understanding how baseflow varies with changing climate and landscape properties is crucial for various issues related to water quantity and quality (e.g., Beck et al., 2013; Buttle, 2018; Price, 2011; Smakhtin, 2001). Population growth is linked to an increase in freshwater demand for agriculture, industry, and human consumption, and water shortages pose a threat even in humid regions (Price, 2011). Baseflow is essential for ecosystem functioning and provides habitat for stream biota (Poff et al., 1997; Price, 2011). Furthermore, baseflow is important with respect to water quality issues (chemistry and temperature) such as effluent load from wastewater treatment plants (Ficklin et al., 2016; Smakhtin, 2001). If we want to understand how humans impact baseflow, we need to understand what determines baseflow under (near-)natural conditions.

Many studies found that baseflow is correlated with climate and landscape properties such as soils, geology, topography, and vegetation, but a universal relationship or general theory is yet to be found (Price, 2011).
Geology was found to be the key variable in various regional studies (e.g., Bloomfield et al., 2009; Longobardi & Villani, 2008; Neff et al., 2005). Similarly, soil classes (which are correlated with geology) were used to explain baseflow variability in the United Kingdom (Boorman et al., 1995) and Europe (Schneider et al., 2007). Schneider et al. (2007) found that soils were less influential toward southern Europe, which might be attributed to differences in topography and climate. Van Dijk (2010) explored catchments in Australia and concluded that climate was the most important control on baseflow, while Lacey and Grayson (1998) found that for southeastern Australia vegetation-geology groups were the main influence. In summary, the studies that found landscape properties to be most influential were usually of regional nature and thus investigated catchments with relatively similar climates. Continental studies and the first global study by Beck et al. (2013) led to somewhat inconclusive results. While some key landscape and climate characteristics could be identified, the underlying processes remain to be explained. The influence of lakes (Neff et al., 2005) and snow (Beck et al., 2013), that is, baseflow generating mechanisms different than groundwater discharge, further complicates the analysis.

Baseflow is usually quantified by the baseflow index (BFI), the long-term ratio between baseflow and total streamflow. Alternatively, we can use the baseflow fraction $K_B$ (Sivapalan et al., 2011), defined as the ratio between mean annual baseflow $\bar{Q}_b$ and precipitation $\bar{P}$ (cf. to the runoff ratio, the ratio between total streamflow $\bar{Q}$ and precipitation $\bar{P}$). $K_B$ has the advantage that it relates baseflow to precipitation, a climate input that is (mostly) independent of catchment form. The similarity to the runoff ratio allows us to investigate $K_B$ in the context of the Budyko hypothesis. A disadvantage of $K_B$ is that we need both streamflow and rainfall data.

The Budyko hypothesis (Budyko, 1974) is a widely applied empirical top-down approach in catchment hydrology (Wang et al., 2016). It hypothesizes that the ratio between mean annual actual evapotranspiration $\bar{E}_a$ and precipitation $\bar{P}$ is primarily a function of the ratio between mean annual potential evapotranspiration $\bar{E}_p$ and precipitation $\bar{P}$; that is, the aridity index $\varphi = \bar{E}_p / \bar{P}$. As $\bar{E}_a$ is usually not available, $\bar{Q}$ might be used instead (Andréassian & Perrin, 2012). Figure 1a shows a Budyko-type plot for catchments in the United States and the United Kingdom (data sources will be explained in section 2.2). The catchments fall relatively close to a single curve, the so-called Budyko curve, for which various model equations exist (see, e.g., review by Wang et al., 2015). Is there a similar behavior for baseflow, that is, a baseflow Budyko curve? That is, is the aridity index the primary control on baseflow fraction? Wang and Wu (2013) modeled the relationship between baseflow fraction and aridity by means of a Budyko-type curve that approaches unity for increasing humidity. Similarly, Sivapalan et al. (2011) reported “that the fraction of precipitation partitioned to slow flow is highest in wet catchments (as high as 0.7) and decreases with increasing aridity.” Both stud-
ies analyzed Model Parameter Estimation Experiment (MOPEX) data (Duan et al., 2006), that is, data from the contiguous United States. Redoing this analysis with data from the United States and the United Kingdom reveals a different behavior. We can see from Figure 1b that the fraction of precipitation that becomes baseflow does not always increase with decreasing aridity index but decreases for many humid catchments. The data presented in Figure 1 suggest that the influence of climate aridity on baseflow fraction is not straightforward or universal. This reinforces the variability in the literature on the relative importance of climate and landscape characteristics. Is there a way to quantify and/or parametrize these relative important? How can we model baseflow variability in a process-based way? As a framework for addressing these questions, we will use the revised Ponce-Shetty model (Ponce & Shetty, 1995a, 1995b; Sivapalan et al., 2011) to model catchment water balance at the annual scale. The Ponce-Shetty model has been described as a functional model (Sivapalan et al., 2011) as it focuses on how water is partitioned, stored and released, that is, catchment's functions (Black, 1997; Wagener et al., 2007). This approach is promising as it goes beyond mere empiricism by representing processes such as the partitioning of water at the annual scale. The processes and the respective parameters are arguably highly abstracted and connecting emergent parameters to catchment characteristics remains challenging (Sivapalan et al., 2011). This approach, however, allows us to investigate large samples of catchments and thus enables us to explore catchment (dis-)similarity and patterns, which eventually might be synthesized to new catchment-scale theory (Harman & Troch, 2014; McDonnell et al., 2007; Sivapalan, 2005; Wagener et al., 2007). In the face of environmental change (Milly et al., 2008), process-based models that allow for extrapolation are more needed than ever (Wagener et al., 2010).

We will use the revised Ponce-Shetty annual water balance model to obtain and investigate a theoretical model of baseflow fraction (and BFI) as a function of mean annual climate variables (Sivapalan et al., 2011). We will fit the Ponce-Shetty model to catchments in the United States and the United Kingdom to obtain catchment-scale parameter values defining how water is partitioned at the annual scale (Ponce-Shetty parameters; described in section 2). We will then assess whether this approach has the potential to explain the variability in baseflow fraction (and BFI) shown in Figure 1b and the apparently differing behavior exhibited by the catchments in the United Kingdom.

2. Theory and Data

2.1. Theory

2.1.1. Annual Water Balance Model

The revised Ponce-Shetty model (Sivapalan et al., 2011) is a functional approach to water balance modeling following Horton (1933), L'vovich (1979), and Ponce and Shetty (1995a, 1995b). A catchment's annual water balance is conceptualized as a two-stage partitioning process. First, precipitation \( P \) is partitioned into fast flow \( Q_f \) (direct runoff and fast subsurface flow) and wetting \( W \) (water that is being stored). The stored water is then further partitioned into vaporization \( V \) (water returned to the atmosphere) and baseflow (slow flow) \( Q_b \). Fast flow and baseflow combined yield total streamflow \( Q \). Interannual water storage change and other water gains or losses such as intercatchment groundwater flows are assumed to be negligible. Figure 2 shows a schematic of the model.

The balance equations for the two partitioning stages are as follows:

\[
P = Q_f + W \tag{1}
\]

\[
W = Q_b + V. \tag{2}
\]

The balance equations for the whole catchment are as follows:

\[
P = V + Q \tag{3}
\]

\[
Q = Q_f + Q_b. \tag{4}
\]

These balance equations are used to determine \( V \) (from equation (4)) and \( W \) (from equation (1)). Data sources for \( Q \) and \( P \) and the estimation of \( Q_f \) and \( Q_b \) are described in the following subsections.
2.1.2. Baseflow Estimation

To obtain an estimate of fast flow and baseflow, we perform a hydrograph separation using digital filtering techniques. Following Troch et al. (2009) who reported that the choice of the filter has no significant influence on annual water balance metrics (they analyzed the Horton index), many subsequent studies used only one hydrograph separation technique (e.g., Harman et al., 2011; Sivapalan et al., 2011). Since in the original Troch et al. (2009) paper only 33 catchments were analyzed, we perform a comparative analysis of baseflow separation methods for all the catchments investigated here. We use the one-parameter Lyne-Hollick digital filter (Lyne & Hollick, 1979), which is applied forward, backward, and forward again using a filter parameter of 0.925. As an alternative, we test the U.K. Institute of Hydrology smoothed minima method (Institute of Hydrology, 1980). Both filters have the advantage of being only minimally parameterized (one parameter) and thus being easily applied to a large sample of catchments. Knowing $P$, $Q$ (both measured), $Q_f$, $Q_b$ (both estimated), we can then calculate $V$ and $W$.

2.1.3. Ponce-Shetty Equations

Based on empirical observations, Ponce and Shetty (1995a) presented a mathematical model of the two-stage partitioning, which was reintroduced by Sivapalan et al. (2011). The form of the equations follows the curve number runoff equation (NRCS, 2004), which is an empirical equation that satisfies conservation of mass. The idea of two competing processes (here: fast flow vs. wetting and baseflow vs. vaporization) was later generalized by means of the so-called proportionality hypothesis, and the maximum entropy production principle was identified as a possible thermodynamic basis for this mathematical form (Wang et al., 2015; Wang & Tang, 2014; Zhao et al., 2016).

The first partitioning stage is modeled as follows:

\[ Q_f = \begin{cases} 
0, & \text{if } P \leq \lambda P WP \\
\frac{(P-\lambda P WP)^2}{P+1-2\lambda P WP}, & \text{if } P > \lambda P WP
\end{cases} \tag{5} \]

\[ W = \begin{cases} 
P, & \text{if } P \leq \lambda P WP \\
\frac{(P-\lambda P WP)^2}{P+1-2\lambda P WP}, & \text{if } P > \lambda P WP
\end{cases} \tag{6} \]

As $P \to \infty$, $Q_f \to P - WP$, $W \to WP$. \tag{7}
where \( \lambda_p \) is the fast flow initial abstraction coefficient and \( W_p \) is the wetting potential. Their product \( \lambda_p W_p \) is the fast flow generation threshold. This form is convenient as \( \lambda_p \) ranges between 0 and unity (Ponce & Shetty, 1995a). The second partitioning stage is modeled as follows:

\[
Q_b = \begin{cases} 
0, & \text{if } W \leq \lambda_w V_p \\
\frac{(W-\lambda_w V_p)^2}{W(1-2\lambda_w W_p)}, & \text{if } W > \lambda_w V_p 
\end{cases}
\]

(8)

\[
V = \begin{cases} 
W, & \text{if } W \leq \lambda_w V_p \\
W - \frac{(W-\lambda_u V_p)^2}{W(1-2\lambda_u W_p)}, & \text{if } W > \lambda_u V_p 
\end{cases}
\]

(9)

where \( \lambda_w \) is the baseflow initial abstraction coefficient and \( V_p \) is the vaporization potential. Their product \( \lambda_w V_p \) is the baseflow generation threshold.

Figure 3 shows curves derived from the Ponce-Shetty model equations. Both the \( P-W \) plot (Figure 3a) and the \( W-V \) plot (Figure 3c) start at the origin and approach a limit (their potentials). The wetting potential \( W_p \) can be seen as some sort of storage capacity of a catchment. The vaporization potential \( V_p \) can be seen as some sort of energy limit (somewhat analogous to potential evapotranspiration). The \( P-Q_f \) plot (Figure 3b) and the \( W-Q_b \) plot (Figure 3d) start to rise after a certain threshold and then rise without a (theoretical) limit. The precipitation threshold is a minimum amount of rainfall required to generate fast slow. The baseflow threshold is a minimum amount of wetting required to generate baseflow. This reflects the idea that if there is only little rain (or wetting), the water will not reach the stream and evaporate (e.g., interception). The physical meaning of these parameters is somewhat ambiguous as they are emergent parameters representing processes over a large area (catchment) and over a long time (years). Links to physical (observable) catchment characteristics remain to be explored but will be discussed qualitatively in section 4.
2.1.4. Rescaled Form of the Ponce-Shetty Equations

In order to compare between catchments, the (mean annual) Ponce-Shetty variables can be normalized using the Ponce-Shetty parameters (Sivapalan et al., 2011). We define two rescaled driving variables: rescaled (mean annual) precipitation $\tilde{P}$ and a rescaled vaporization potential $\tilde{V}_p$.

$$\tilde{P} = \frac{P - \lambda_p W_p}{(1 - \lambda_p)W_p}$$  \hspace{1cm} (11)

$$\tilde{V}_p = \frac{V_p - \lambda_w V_p}{(1 - \lambda_p)W_p}.$$  \hspace{1cm} (12)

2.1.5. Catchment Indices

We define two catchment indices: the baseflow fraction $K_B$ (note that this definition is slightly different from the usual definition as it includes the parameter $\lambda_w V_p$) and the BFI.

$$K_B = \frac{Q_b}{\tilde{P} - \lambda_w V_p}$$  \hspace{1cm} (13)

$$\text{BFI} = \frac{Q_b}{Q},$$  \hspace{1cm} (14)

We can approximate these indices using the rescaled driving variables (equations (11) and (12); for the full derivation of $K_B$ see Sivapalan et al., 2011, and for the derivation of BFI see Appendix A):

$$K_B = \frac{P}{(1 + \tilde{P})(1 + \tilde{V}_p, P)}$$  \hspace{1cm} (15)

$$\text{BFI} = \frac{1}{(1 + \tilde{P})(1 + \tilde{V}_p)}.$$  \hspace{1cm} (16)

These expressions can be used to model the observed catchment indices (equations (13) and (14)). These equations are functions of two variables ($\tilde{V}_p$ and $\tilde{P}$) and not just a single variable such as aridity (which might be defined here as rescaled aridity index $\tilde{\phi} = \frac{\tilde{V}_p}{\tilde{P}}$). Note that in the derivation of these equations we assume a parameter $K = \frac{\lambda_p W_p - \lambda_w V_p}{(1 - \lambda_p)W_p}$ (not presented here for brevity) to be 0. This assumption led to insignificant differences, which is consistent with Sivapalan et al. (2011).

2.2. Data

We use data from the contiguous United States and Great Britain. CAMELS (Addor et al., 2017a; Newman et al., 2015) includes daily precipitation, potential evapotranspiration, and streamflow data as well as a wide range of catchment attributes for 671 catchments in the contiguous United States. The U.K. Benchmark Network (UKBN2; Harrigan et al., 2017) describes catchments in the United Kingdom that are near natural. It consists of 146 catchments whereof 8 catchments in Northern Ireland are not considered. The data are obtained from different sources. Daily streamflow data, catchment characteristics, and catchment boundaries are obtained from the NRFA (National River Flow Archive, 2018), precipitation data from CEH-GEAR (Tanguy et al., 2016), and potential evapotranspiration data from CHESS-PE (Robinson et al., 2016). We trim the daily data to contain only full water years (starting 1 October). We then aggregate daily data to obtain annual data, which are used to calibrate the Ponce-Shetty model for each catchment. For all other calculations we use mean annual data, that is, data averaged over all full water years. To obtain a suitable data set, we remove some of the catchments according to the following criteria:

- Catchments with areas smaller than 10 km² as measurement errors and catchment delineation errors tend to be significant for very small catchments.
- Catchments with records shorter than 15 years as calibrating the Ponce-Shetty model requires many annual values. This threshold is chosen to remove some rather short and thus potentially unreliable records while trying to keep enough catchments for the ongoing analysis.
Table 1
Comparison of Mean Annual Baseflow $Q_b$, Ponce-Shetty Parameters, $K_b$, and BFI Using Different Baseflow Separation Techniques (Lyne-Hollick Filter and U.K. Institute of Hydrology Method)

<table>
<thead>
<tr>
<th>Metric</th>
<th>$Q_b$ (mm)</th>
<th>$W_p$ (mm)</th>
<th>$\lambda_p$ (-)</th>
<th>$V_p$ (mm)</th>
<th>$\lambda_W$ (-)</th>
<th>$K_b$ (-)</th>
<th>BFI (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson correlation</td>
<td>1.00</td>
<td>0.84</td>
<td>0.98</td>
<td>0.95</td>
<td>0.97</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>Spearman correlation</td>
<td>1.00</td>
<td>0.99</td>
<td>0.96</td>
<td>0.99</td>
<td>0.95</td>
<td>0.99</td>
<td>0.96</td>
</tr>
<tr>
<td>Median RE</td>
<td>0.07</td>
<td>0.05</td>
<td>0.17</td>
<td>0.05</td>
<td>0.31</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Median AE</td>
<td>11</td>
<td>159</td>
<td>0.01</td>
<td>147</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Note. The relative error (RE) is defined as $RE = \left| 1 - \frac{x_a}{x_b} \right|$. The absolute error (AE) is defined as $AE = \left| x_a - x_b \right|$.

-Catchments where snow and lakes are influential, as these processes are not considered in the Ponce-Shetty model. We remove catchments with fractions of precipitation falling as snow >0.2 and catchments with significant surface water bodies. The latter is done by removing UKBN2 catchments with FARL < 0.8 (a parameter quantifying the influence of lakes and reservoirs) and CAMELS catchments with frac_water > 0.05.
-Catchments with runoff ratios larger than unity in any year of record ($Q/P > 1$), resulting in negative vaporization values ($V < 0$), as this indicates significant water balance issues and thus violates the assumptions of the Ponce-Shetty model.

The final data set consists of 571 out of 817 catchments.

3. Results
3.1. Baseflow Estimation
Table 1 shows several metrics comparing results obtained using the Lyne and Hollick (1979) filter and the U.K. Institute of Hydrology (1980) method. The two methods show good agreement. While the choice of filter might have a significant impact on individual catchments, it does not alter the overall results. We continue using the baseflow estimates obtained by using the Lyne-Hollick filter.

3.2. Parameter Estimation and Uncertainty
The Ponce-Shetty parameters are fitted to each individual catchment by means of a nonlinear least squares fitting algorithm, whereby $\lambda_p$ and $\lambda_W$ are restricted to be between 0 and unity (their theoretical limits), and $W_p$ and $V_p$ are restricted to be between 0 mm and an upper limit. We choose an (arbitrary) upper limit of 50,000 mm, which is deemed high enough to not affect the parameter estimation. An even higher limit does not affect the estimated parameter values except for very few catchments with $W_p$ and/or $V_p$ values, which are (almost) at the limit. The problem that some of the obtained parameter values are at the upper limit is discussed in the next paragraph. We can use two values for the wetting $W$ to fit the second partitioning stage. Either the observed $W$ obtained from equation (1) or the modeled $W$ following from the fitted model for the first partitioning stage (equation (6)). Following Sivapalan et al. (2011), we use the modeled $W$ to obtain an internally consistent water balance.

To fit a meaningful parameter set, the catchments should exhibit their functional behavior (Sivapalan et al., 2011). If the vaporization values (wetting values) are far away from the vaporization potential (wetting potential), we will have a roughly linear relationship and hence fitting the functional form is not possible (see Figure 4a). This can be seen especially for $V_p$ in arid catchments (e.g., in the middle of the United States). In these catchments, the obtained potentials are at the specified upper limit (50,000 mm). Similarly, being at the potential all the time does not allow us to fit a functional relationship either; this can be seen especially for $V_p$ in humid catchments (e.g., along the west coast of the United Kingdom). In these catchments the obtained initial abstraction coefficient is unity (see Figure 4b). We remove these catchments from the analysis because the Ponce-Shetty model is unable to describe them adequately.

Table 2 shows overall statistics for the parameter estimation after having removed the catchments described in the last paragraph. The parameter uncertainty (in the form of 95% confidence intervals) is particularly high for extremely large values for either of the potentials ($\gg 10,000$ mm). These large values are consistently uncertain, which coincides with Sivapalan et al. (2011) who found that for some catchments the (apparently very high) potentials could not be properly identified. The confidence intervals for $\lambda_p$ and $\lambda_W$ need careful
interpretation, as these two parameters have heavily skewed distributions (most catchments have parameter values close to 0). We do not remove catchments with high uncertainty from the analysis as a threshold would necessarily be subjective, which leaves us with 545 catchments for the ongoing analysis.

### 3.3. Maps of Ponce-Shetty Parameters and Baseflow Metrics

Figure 5 shows maps of the fitted parameters for CAMELS catchments. The patterns agree well with Sivapalan et al. (2011) who used Model Parameter Estimation Experiment (MOPEX) catchments. High wetting potentials $W_p$ can be seen in the middle of the United States (Great Plains), in the east (southern parts of the Appalachians), south east (around Florida), and in parts of the central north (Michigan). High vaporization potentials $V_p$ can be seen in the middle of the United States (Great Plains) and in all southern regions. The fast flow thresholds $W_p \lambda_p$ are high in the south, the southeast, and in the middle of the United States except for the north. The baseflow thresholds $V_p \lambda_W$ are similarly high in most of these areas and also in some catchments along the west coast. The spatial similarity of the thresholds is reflected by a significant rank correlation of 0.61 between $W_p \lambda_p$ and $V_p \lambda_W$.

### Table 2

<table>
<thead>
<tr>
<th>Parameter Statistics and Uncertainty for All Catchments Used in the Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$W_p$ (mm)</td>
</tr>
<tr>
<td>$\lambda_p$ (-)</td>
</tr>
<tr>
<td>$V_p$ (mm)</td>
</tr>
<tr>
<td>$\lambda_W$ (-)</td>
</tr>
</tbody>
</table>

**Note.** CI 95% denotes the half-width of the 95% confidence intervals. Rel. CI 95% denotes the half-width of the relative confidence intervals that is, the confidence interval normalized by the parameter values. Spearman denotes the Spearman correlation of the relative confidence intervals with the parameter values.
Figure 5. The fitted parameters for CAMELS catchments: wetting potential (a), fast flow threshold (b), vaporization potential (c), and baseflow threshold (d). Crosses denote catchments where some of the parameters could not be identified properly.

Figure 6 shows maps of the fitted parameters for UKBN2 catchments. On average, the values are lower than for the CAMELS catchments, especially for $V_p$, which is consistent with generally lower vaporization intensities (cf. to $E_p$). High wetting potentials $W_p$ can be found in the southwest, the south, the middle (the Midlands), and along the south eastern coast. The vaporization potentials $V_p$ are high in the south, especially in the southeast. High $W_p, \lambda_p$ can be found in the southeast and for a few catchments in the north. High $V_p, \lambda_w$ can be found in catchments scattered throughout the United Kingdom, most notably all along the west coast and in the southeast.

Figure 7 shows maps of $K_b$ and BFI for CAMELS and UKBN2 catchments. Generally, $K_b$ is lower than BFI as it compares $Q_b$ to $\bar{P}$ rather than $Q$, which is always lower than $\bar{P}$. This is reflected in the ranges of values shown in Figure 7. While in some regions both $K_b$ and BFI are rather high (e.g., in the eastern United States and in the south west of the United Kingdom), in other regions BFI can be high while $K_b$ is rather low (e.g., in the southern United States and in the middle of the United States and in the southeast of the United Kingdom), which broadly agrees with Santhi et al. (2008) who found that catchments with high BFI can still
Figure 7. (a) BFI and (b) $K_B$ for CAMELS and UKBN2 catchments. Note that the color scales are different to reflect the range of the values. Crosses denote catchments where some of the parameters could not be identified properly. Note that the maps of the United States and the United Kingdom are not to the same scale.

3.4. Baseflow Variability With Climate Variables

Figure 8 shows how the baseflow fraction varies with the rescaled climate variables. To show the dependence of $K_B$ on both $\tilde{P}$ and $\tilde{V}_p$, we make use of a contour plot (see Figure 8a). We plot $\tilde{P}$ and $\tilde{V}_p$ on the x and y axes, respectively, and use contours to represent the model for $K_B$ (equation (15)) and colored dots to represent the observed $K_B$ values (equation (13)). Figure 8b shows an equivalent plot using the ratio between $\tilde{P}$ and $\tilde{V}_p$ (rescaled aridity index $\tilde{\phi}$) with some example model curves with either fixed $\tilde{P}$ or $\tilde{V}_p$, respectively—this is comparable to common Budyko-type plots. To get a better understanding, it is useful to recall how a contour plot of the rescaled aridity index would look like, which is shown in Figure 8c. The line through the origin represents a rescaled aridity index of unity, above that line (top left) are humid catchments, below that line (bottom right) are arid catchments. Note that we are using rescaled variables, and hence, we are not looking at the common aridity index. $\tilde{P}$ is a relative rainfall amount, and $\tilde{V}_p$ is a relative vaporization potential, both rescaled by their thresholds and the wetting potential of the catchment. The general notion that low $\tilde{\phi}$ indicates humid (energy-limited) catchments and that high $\tilde{\phi}$ indicates arid (water-limited) catchments is still valid.

The contours in Figure 8a start parallel to the line through the origin and thus parallel to the rescaled aridity index. They start to bend for higher values of $\tilde{P}$ (humid side of the plot) and become perpendicular to
Figure 8. (a) Contour plot of $K_B$ as a function of the rescaled vaporization potential $\bar{V}_p$ and rescaled precipitation $\bar{P}$ (equation (15)). The dots indicate the observed values (equation (13)). (b) $K_B$ as function of the ratio between $\bar{V}_p$ and $\bar{P}$ (i.e., rescaled aridity index $\bar{\phi}$). The black and gray lines (solid and dashed) are example model curves with either fixed $\bar{V}_p$ or $\bar{P}$. The dots indicate the observed values. (c) Logarithm of the rescaled aridity index $\bar{\phi}$ as a function of $\bar{V}_p$ and $\bar{P}$. The gray line denotes a rescaled aridity index of unity (log equals zero). (d) Different regions of the $K_B$ contour plot are annotated, a more detailed explanation is given in section 4.

Figure 9 shows how BFI varies with $\bar{P}$ and $\bar{V}_p$. The contours shown in Figure 9a are symmetric around the line through the origin. The BFI is highest for low $\bar{P}$ and $\bar{V}_p$ and gets lower for both higher $\bar{P}$ and $\bar{V}_p$. The observed values agree well with the model contours (median absolute error = 0.05, median relative error = 0.14). Again, this can be expected, since the model has sufficient degrees of freedom to fit the data well. Figure 9b shows that there is no clear relationship between BFI and the rescaled aridity index. This is in agreement with the observed values, which are scattered over most areas of the plot.
4. Discussion

The ranges of the parameter values (see Table 2) are in general agreement with Sivapalan et al. (2011), who also used a nonlinear least squares method, and Harman et al. (2011) who used a Bayesian framework. The high parameter uncertainty for some catchments and problems in parameter identifiability might have two reasons. As described before, it could simply be a consequence of not having sufficient data to meaningfully fit the Ponce-Shetty model. It could, however, also indicate that the Ponce-Shetty model is not adequate for certain catchments. Even a good fit does not necessarily mean that the model is correctly representing the processes, which are arguably very simplified. We assume interannual water storage change as well as other water gains and losses to be negligible. This might not be a valid assumption for every catchment investigated here and hence adds uncertainty to the parameter estimation. To assess the influence of interannual water storage change, we alternatively calculated 3-year averages and calibrated the Ponce-Shetty model to these. This leads to overall similar parameter values (Pearson correlations: \( W_p: 0.86 \), \( \lambda_p W_p: 0.81 \), \( V_p: 0.79 \), and \( \lambda_W V_p: 0.67 \). There are, however, problems associated with averaging. Extreme years, which are especially important to fit the Ponce-Shetty model, are averaged out, and thus, information is lost. Furthermore, by averaging and fitting a nonlinear function, we introduce some bias (“the average of the function will not be the function of the average inputs,” see Rouholahnejad Freund & Kirchner, 2017). This makes it difficult to tell whether interannual water storage change is the cause for the deviations in the parameter values. For now we argue that the model fits our data sufficiently well for the purpose of this work. Being capable of explaining the observed variations in baseflow further corroborates the model’s suitability. For specific places, however, the uncertainty might be very large and conclusions or predictions should therefore be made with care. It would be interesting to see whether more detailed modeling approaches would lead to the emergent behavior inherent in the Ponce-Shetty theory and/or similar parameter values.

From Figure 8 we can see how \( K_p \) varies with \( \bar{P} \) and \( \bar{V}_p \). Generally, \( K_p \) cannot be described by a single Budyko-type curve but by a continuum of curves that depend on the catchment’s (Ponce-Shetty) parameters. \( K_p \) is consistently low for high rescaled aridity values, which can be attributed to relatively high amounts of vaporization (\( K_p \) is dominated by the second partitioning stage, i.e., \( V_p \)). The behavior of \( K_p \) is more complicated for humid catchments. Starting at the origin of Figure 8a and moving along the y axis toward more humid catchments, \( K_p \) first increases, then reaches a peak, and decreases again. This decrease can be attributed to an exhausted wetting potential leading to “saturation excess fast flow” (\( K_p \) is dominated by the first partitioning stage, i.e., \( W_p \)). This was already recognized by Milly (1994) who stated that finite water storage capacity and finite permeability are possible causes for runoff. In such humid catchments, an increase in precipitation thus mainly leads to an increase in fast flow, which agrees with Harman et al.
Figure 10. Scatter plots of mean annual baseflow fraction $\overline{Q}_b/\overline{P}$ versus mean aridity index $\overline{E}_p/\overline{P}$. CAMELS catchments are denoted by circles, and UKBN2 catchments are denoted by triangles. Catchments are highlighted according to their wetting potential $W_p$: (a) low wetting potentials, (b) medium wetting potentials, and (c) high wetting potentials. Darker shading indicates higher vaporization potential $V_p$. All units are in millimeters.

(2011) who found that fast flow elasticities are clearly larger than baseflow elasticities in humid catchments. Similarly, Trancoso et al. (2017) found that “higher precipitation in tropical regions may be generating more overland flow, which tends to reduce the slow component [...]”. Baseflow fraction can hence be low for both arid and humid catchments, but for different reasons. This may help to explain the diversity of results from empirical studies on controls on baseflow.

Figure 9 shows how the BFI varies with $\tilde{P}$ and $\tilde{V}_p$. The magnitude of $\tilde{P}$ and $\tilde{V}_p$ rather than the ratio between them determines the BFI. If both $\tilde{P}$ and $\tilde{V}_p$ are low, BFI is high. That means that at the first partitioning stage precipitation becomes mainly wetting, and at the second partitioning stage this wetting becomes mainly baseflow. If either $\tilde{P}$ and $\tilde{V}_p$ are high, we obtain a lower BFI. In the first case, most of the precipitation becomes fast flow, and thus, the BFI is low. In the second case, most of the precipitation becomes wetting, but most of that wetting evaporates, so that $Q_b$, and thus, the BFI will be rather low. In comparison to $K_B$, BFI is highly variable also for high rescaled aridity. Low amounts of baseflow (compared to precipitation)
can lead to a high BFI if the amount of fast flow is even lower. This explains most of the differences between $K_B$ and BFI (see Figures 5 and 6 and the description in section 3.3).

The results show that $K_B$ (and BFI) is influenced by the magnitude of $\overline{P}$ and $\overline{V}_p$ and not just their ratio. This explains the scatter especially for humid catchments (see Figure 8b). While an aridity index is certainly useful, it can be restrictive in cases where the magnitude of precipitation is important. This agrees, for example, with Berghuijs et al. (2017) who found that runoff is most sensitive to changes in precipitation and this sensitivity is not captured by only looking at the aridity index. Similarly, the ratio between precipitation and the wetting potential ($\approx \overline{P}$) explains most of the variability in baseflow fraction, which the aridity index could not explain (see Figure 8a, especially Region II, and Figure 10).

Especially in humid catchments, the ratio of precipitation to a catchment’s wetting potential can be a major control on baseflow. Given the same climate, a catchment with a higher wetting potential will have a higher baseflow fraction and BFI. This is a possible explanation for the partly inconclusive results found in studies before. Regional studies with similar climate could relate the amount of baseflow to a catchment’s form, mostly soils (Boorman et al., 1995), and geology (Bloomfield et al., 2009; Longobardi & Villani, 2008; Neff et al., 2005). These attributes are parametrized by the Ponce-Shetty parameters (especially $W_p$), yet in a rather abstract way which so far eludes a quantitative linking to landscape characteristics. Continental (Schneider et al., 2007; Trancoso et al., 2017; Van Dijk, 2010) and global studies (Beck et al., 2013, 2015) found catchment form to be less influential and often could not come to conclusive results, as it is neither climate nor form alone that lead to a certain catchment response, but their interaction.

Figure 10 shows the $Q_b/\overline{P}$ versus $\overline{P}/\overline{V}_p$ plot (from Figure 1) with catchments stratified and colored according to their wetting and vaporization potentials, respectively. Three different ranges of $W_p$ are shown, and they form three somewhat distinct point clouds. The remaining variation can be attributed to differences in the thresholds, the rather broadly defined categories, and differences in the magnitude of $\overline{P}$ and $\overline{V}_p$. The cloud with the lowest $W_p$ exhibits the lowest baseflow fraction and vice versa. High values of $K_B$ are usually associated with low values of $V_p$ (indicated by the lightness of the colors). We can also see that CAMELS and UKBN2 catchments do not generally behave differently, but since certain catchment types occur predominately in the United States or the United Kingdom, the CAMELS and UKBN2 point clouds appear to be different. Very humid catchments with rather low $W_p$ are mostly located in the United Kingdom, and they are most clearly deviating from the point cloud representing CAMELS catchments (see also Figure 1).

We did not include catchments with significant snow fraction or lakes. While these catchments might be seen as having an “extended” wetting potential (storage), they represent conceptually different processes, for which additional explanatory variables might be needed. These processes might be added as an additional partitioning stage to the model to make it more universal. Especially the snowy catchments show an increase in $K_B$ for increasing humidity almost up to unity (not shown here), which could explain, for example, why Wang and Wu (2013) used a baseflow Budyko model that approaches unity. Snowy catchments might be considered to have virtually unlimited storage potential as the snowpack can grow continuously, and thus, baseflow fractions in these catchments can get very high.

The Ponce-Shetty parameters are emergent, rather abstract properties and relating them to catchment characteristics might not be straightforward. The Ponce-Shetty parameters are lumping a variety of processes and characteristics, notably soils, geology, vegetation, topography, and climate seasonality. This means that for now, the presented model can only explain and predict annual baseflow variability in gauged catchments where the model was calibrated. It might be used to investigate the effects of a changing climate (e.g., changing precipitation) on baseflow in different types of (gauged) catchments (cf. Buttle, 2018). A transfer to ungauged catchments requires a regionalization procedure. Qualitatively, links between parameters and catchment characteristics can be seen. $V_p$ is correlated with energy availability (comparable to potential evapotranspiration), yet it rather emerges from the interaction of the available energy with vegetation and other catchment characteristics. Large wetting potentials can be seen in moorland and wetland areas (e.g., southwest England and Florida) and in the presence of major aquifers (e.g., Chalk in southern England and Great Plains aquifer). A quantitative linking of the Ponce-Shetty parameters to landscape properties or other regionalization approaches are, however, beyond the scope of this work.
5. Conclusions

The present work shows that there is no single baseflow Budyko curve; that is, in general, baseflow fraction cannot be modeled as a function of an aridity index alone. Even if samples of catchments seem to form a single curve, this might be misleading as many of them might actually sit on different curves (see Figure 9b). The influence of catchment water storage on long-term water balance has long been recognized (e.g., Milly, 1994). The approach employed here incorporates that in a simple way by modeling baseflow fraction as a function of two variables: a rescaled precipitation, which is the ratio between precipitation and a catchment’s wetting potential, and a rescaled vaporization potential. These two variables reflect the two-stage partitioning underlying the Ponce-Shetty model, namely, the partitioning between fast flow and wetting and the subsequent partitioning between slow flow and vaporization. Depending on the climatic regime, one of these partitioning stages dominates. In arid catchments, baseflow fraction is mainly limited by high amounts of vaporization. In humid catchments, baseflow fraction is mainly limited by the storage capacity of a catchment.

The differences between CAMELS (United States) and UKBN2 (United Kingdom) catchments shown in Figures 1b and 10 have two main causes. First, using aridity as a ratio is restrictive. Catchments with a similar aridity index usually have lower precipitation and vaporization intensities in the United Kingdom than in the United States. Second, the wetting potentials in the United Kingdom differ from the ones in the United States. Most of the very humid catchments in the United Kingdom have rather low wetting potentials; that is, they are (almost) fully saturated, and a large fraction of precipitation runs off quickly to the stream. This difference is, however, not a clear distinction as it can be seen from Figure 10. Catchments in the United States and the United Kingdom do not behave fundamentally differently; they rather happen to have predominantly different characteristics.

Baseflow (a catchment function) can be seen as the result of climate interacting with landscape (forcing acting on form, cf. Wagener et al., 2007). To explain baseflow variability in a process-based way, we should try to disentangle forcing and form, knowing that this might only be partially possible as catchment form (and function) may reflect a coevolution with climate forcing. The Ponce-Shetty approach partly disentangles forcing and form, yet in a rather abstract way. Furthermore, the parameters still lump together a variety of processes that are reflecting not only catchment form (e.g., topography, geology, and vegetation) but also climate (e.g., seasonality and storminess). Intraannual climate variability can have a significant impact on such lumped parameters (Berghuijs & Woods, 2016; Roderick & Farquhar, 2011).

Using large samples of catchments allows us to detect and explain (dis-)similarities and patterns and to synthesize already available data (Falkenmark & Chapman, 1989; Harman & Troch, 2014; Sivapalan, 2005). While large sample hydrology arguably neglects many details, synthesizing data to find new theory has proven to be a fruitful approach that—besides improved understanding—might help to constrain models (Shafii et al., 2017), to transfer knowledge to ungauged catchments (Hrachowitz et al., 2013) and to deal with predictions under change (Ehret et al., 2014; Wagener et al., 2010). It is essential to include a variety of catchments, both in terms of climate and landscape characteristics, which is exemplified by the “unexpected behavior” of U.K. catchments in this work. Even more data are needed to corroborate the theory, to understand more of the details (e.g., Ponce-Shetty parameters) or to detect limitations of the presented approach, which eventually advances our understanding.

Simple approaches such as the Ponce-Shetty model are useful as they are easily applied to large samples. They also allow us to better understand the model’s dynamics and stop us from being lost in the calibration stage. We acknowledge that there is a danger in being too simple or simple due to lack of understanding (cf. Schwartz et al., 2017), which might partly be true for the hydrograph separation approach and the Ponce-Shetty model here. We are confident, however, that the chosen methods are appropriate for the present work as they are capable of explaining the observed phenomena and thus help to improve our understanding of how baseflow varies with climate and landscape.
Appendix A

To obtain an equation for the BFI, we make use of another catchment index presented in Sivapalan et al. (2011), the runoff ratio $K_R$:

$$K_R = \frac{Q_f + Q_b}{P - \lambda W V_p}$$  \hspace{1cm} (A1)

$K_R$ can be approximated theoretically by the following:

$$K_R = \frac{P(1 + V_p)}{P + V_p + V_p P}$$  \hspace{1cm} (A2)

We can write the BFI using $K_g$ and $K_R$:

$$BFI = \frac{Q_b}{Q_f + Q_b} = \frac{K_g}{K_R}$$  \hspace{1cm} (A3)

$$BFI = \frac{P(1 + P)^{-1} - P(1 + V_p) - P V_p}{P + V_p + V_p P}$$  \hspace{1cm} (A4)

$$BFI = \frac{1}{(1 + P)(1 + V_p)}$$  \hspace{1cm} (A5)

References


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