



Park, I. U., & Panagopoulos, A. (2019). Patent Protection, Startup Takeovers, and Open Innovation. *International Journal of the Economics of Business*, 26(1), 39-64.
<https://doi.org/10.1080/13571516.2018.1553285>

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Patent Protection, Startup Takeovers, and Open Innovation*

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June 8, 2018

Abstract

Open-innovation largely relies on startup innovators transferring their R&D to incumbent firms. Yet such innovators are at a disadvantage when faced with incumbents holding patent portfolios, raising the question why do such *Lilliputian* firms choose to innovate? In view of this we study the impact of patent protection on the innovation incentives of startup firms in a dynamic model where an incumbent faces a sequence of potential startups and the incumbent's chance of winning an infringement lawsuit increases with the size of its patent portfolio. It is shown that open-innovation style takeover deals generate extra benefits for the incumbent via its enhanced future bargaining positions, a part of which accrues to the current startup as an increased bargaining share, justifying R&D activity that would not have taken place otherwise.

Keywords: Open innovation, startup takeovers, patent portfolios.

JEL No: O31, O34, D21, L26

*We would like to thank Philippe Aghion, Jim Bessen, Vincenzo Denicolò, Stuart Graham, Dietmar Harhoff, Josh Lerner, Gerard Llobet, Eric Maskin, Manolis Petrakis, Bob Rothschild, and the participants at the EPIP conference and the ZEW conference on the economics of innovation and patenting for their helpful comments and suggestions. All errors remain our responsibility. This work was supported by the ESRC small grant program (RES-000-22-2240) and by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2014S1A5A8016037).

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1 Introduction

Open innovation acknowledges the latest industry trends where startups are increasingly credited with developing innovations (Chesbrough, 2003). In open innovation the firm stops being the engine of innovative activity. Instead the firm is seen as a coordinator of efforts carried out “out-of-house”, quite often by startup firms.¹ In this sense, the firm is able to utilize an array of capabilities that frequently border beyond its core competencies (Panagopoulos, 2016). Internal company characteristics related to demographics such as employee characteristics (Harison and Koski, 2010), firm size (Bianchi *et al.*, 2011), strategic orientation (Lichtenthaler and Ernst, 2009) and product cycles (Lee *et al.*, 2010) have been recognized as drivers of open innovation (Huizingh, 2011). The most obvious external context characteristics driving open innovation seems to be the industrial sector the firm is located in (Poot *et al.*, 2009), globalization, technology intensity, technology fusion, new business models, and knowledge leveraging (Gassman, 2006). At the same time, it has lately been understood that open innovation seems to be influenced not only by external and internal trends but by business strategy as well (Pitelis, Desylas, Panagopoulos, 2018). In this paper we explore firm strategy along a dimension that has failed to attract attention in the open innovation literature, intellectual property, and how it can lead to technology outsourcing by utilizing patent portfolios.

Specifically, the latest industrial trend followed by high tech firms is to build patent portfolios through takeovers and acquisitions. For example, Google bought 24,500 patents and patent applications from Motorola, while Microsoft acquired an equally extensive patent portfolio from Novell. At the same time, as of 2017, Google has acquired more than 200 firms (mostly startups) with Apple, Cisco, Microsoft, Yahoo, G.E. and Siemens been equally prolific. This drive for portfolio building has largely coincided with the open-innovation paradigm. At first look portfolio building by dominant incumbents and open-innovation are at odds because portfolios discourage potential competitors from innovating out of fear that their innovation will be found as infringing on the portfolio’s patents. This is especially so for startups, which lack such portfolios (Lanjouw and Schankerman, 2004), raising the question: *why do such Lilliputian firms choose to innovate?* In this paper we argue that patent portfolios allow startups to share inter-temporal gains stemming from takeover negotiations under the threat of infringement litigation. Specifically, the anticipated future bargaining benefits from an enlarged portfolio can help the portfolio builder to bolster open-innovation style takeover deals, which may incentivize startups to

¹Frequent examples of firms resting on open innovation include Intel, IBM, Google, Facebook, Proctor & Gamble and Sun Microsystems.

engage in innovation efforts that would not have taken place otherwise.

Building on the idea that patents are endowed with tradable bargaining power that is best facilitated via patent portfolios (Panagopoulos and Park, 2018), we analyze a model in which an incumbent faces a sequence of potential startups and, similar to Panagopoulos and Park (2018), the incumbent’s chance of winning an infringement lawsuit increases with the size of its patent portfolio. Allowing the R&D cost of the startup to be probabilistic we show that takeovers can motivate a startup’s innovation activities even when the startup’s R&D is relatively high. This is because the incumbent capitalizes on the enhanced bargaining position that the current takeover, unlike other forms of settlements, will bring in all potential future deals by incorporating the current startup’s patented ideas to its own patent portfolio. Since this prospect of future surplus for the incumbent hinges on the current takeover, a part of the surplus accrues to the current startup, enlarging the startup’s bargaining share.

Even though we show that this dynamic effect of intellectual property (IP) protection can motivate startup R&D activities we emphasize, however, that for maximum effect the level of IP protection should be selected carefully at a moderate level because, excessive IP protection would accumulate the incumbent’s bargaining power too quickly, killing off the innovation incentives for startups prematurely, leading to closed innovation. Equally, too little IP protection would fall short of incentivizing startups due to the lack of the aforesaid dynamic effect feeding into a startup’s bargaining share, forcing incumbents to rely on closed innovation. Bearing in mind that IP protection refers to immaterial property, defining moderate strength is elusive. Yet, in this context, “strength” eventually translates into the ability of IP to shape the size of patent portfolios. As such, moderate protection defines portfolios that are neither too small, nor overreaching as to curtail the dynamic process.

To recapitulate we have modeled a repetitive cycle of startup innovation that is followed by open innovation styled acquisitions of the startup’s technology by incumbents. What drives open innovation in this setting is an external factor, intellectual property. As long as IP strength is “balanced”, it shapes the firm’s strategy from both the supply and the demand side. From the supply side it allows innovative startups a larger than otherwise payoff, inducing the availability of startup innovations and R&D. From the demand side it empowers firms in employing and building patent portfolios that can be used in facilitating the transfer of technology through the threat of infringement litigation. In short, IP has the capacity to shape firm strategy in terms of the firm’s choice between closed and open innovation. To illustrate the issue, in the discussion section, we explain how patent donations by established incumbents with excessive patent portfolios can be

used in promoting open innovation by stimulating startup entry.

The rest of the paper is organized as follows. Section 2 offers a brief literature review that provides some background for our analysis. Section 3 presents a stage game as a benchmark. Section 4 provides the main analysis of the dynamic model and characterizes the unique equilibrium. Section 5 discusses some implications and features of the dynamic analysis and equilibrium, partly based on the simulation results reported in Section 6. Section 7 contains some concluding remarks.

2 Literature review

Open innovation is a relatively new theme in the study of innovation. In the past, scholars of innovation conceptualized R&D as a “closed process”. It was closed in the sense that firms attained a competitive advantage through exclusive IP ownership, restricting all R&D activities within the firm’s boundaries, opting for limited interaction with external entities (Cainelli, Evangelista, and Savona, 2004). In the past years, organizations began adopting a more “open” approach to innovation. Collaboration with external stakeholders became the new norm. Such collaborations span beyond the firm’s boundaries and involve a continuous exchange of ideas, technology, and resources. Even though this lacuna in our understanding of this “open” approach to innovation attracted the attention from early on (Powell, Koput, and Smith-Doerr, 1996; von Hippel, 1988), the first to propose that firms can and should use external ideas as well as internal ideas in advancing their technologies was Chesbrough (2003).

Chesbrough understood organizational boundaries under open innovation as being permeable rather than closed (Randhawa, Wilden and Hohberger, 2016). As such they allow innovation to be moved outside the firm’s organizational structure and to include external partners (Bogers and West, 2012). This porous structure enables innovation to emerge through purposeful inflows and outflows of ideas, which are distributed among a network of actors (Enkel, Gassmann and Chesbrough, 2009). In doing so, open innovation facilitates the integration and commercialization of complementary resources and capabilities that allow the firm to capture value from innovation (Laursen and Salter, 2006).

What drives open innovation is a research theme that filled a lot of journal pages. In their literature review of open innovation West and Bogers (2014) categorize open innovation as being the result of a) how firms leverage external sources of innovation, b) the organization’s particular research design, c) its business model, d) the way the firm interacts with other actors, e) the firm’s need to obtain innovation from external sources

and f) the firm’s absorptive capacity in intergrading the newly acquired knowledge into its structure. We differ from the above strands of literature because we introduce a new driver for open innovation, firm strategy. The strategy we examine is one of purposefully amassing a patent portfolio as to facilitate the acquisition of technologies. Even though this is the first paper to propose a strategy link between IP and open innovation, we are not the first to emphasize the role of licensing agreements and their strength under varying degrees of IP protection (Ili *et al.*, 2010; Laursen, Leone, and Torrisi, 2010). In terms of this literature, our contribution lays in articulating a link between IP protection and firm strategy through a specific channel: the use of infringement allegations that is leveraged by patent portfolios.

To understand infringement allegations one needs to come to terms with patents and their use in protecting ideas. Patents are monopoly-grants that hold for a time period of 20 years. During this period no one, apart from the patent holder, may freely make use of the technology embodied in the patent’s claims. Nevertheless, occasionally new ideas arrive, whose technological domain may well rest in a technological territory vaguely entrenched by the patent’s claims, in which case the issue of possible infringement arises. This infringement differs from a direct copying and re-branding of one’s patented ideas, in as much as it progresses the prior art. The question of how and by how much the novel idea progresses prior art finds no equivalent in other forms of material-property. This is because, asserting property rights on one’s ideas is far from simple.

An idea, contrasting land, can never be fully barricaded or entrenched. Defining boundaries of IP is inherently imperfect and as a result, disputes are inevitable and the court plays an active role in the way the patent system operates. The effects of patent litigation have been studied by Meurer (1989), Choi (1998), Aoki and Hu (1999), Crampes and Langinier (2002), and Llobet (2003), however these authors mostly dealt with a single patent to protect and thus, their foci of analysis differ from ours: In a model where the incumbent’s patent portfolio can evolve over time, we examine the feedback effect that the prospect of such evolution may have on the startup’s innovation incentives.

Notwithstanding the above, litigation is an easier and less costly path to follow for firms with large patent portfolios. For example, Lanjouw and Schankerman (2004) find that having a larger portfolio of patents reduces the probability of filing a suit on any individual patent in the portfolio. Furthermore, as they indicate, large firms (with large patent portfolios) have the experience and the ability to settle disputes by pooling or trading intellectual property. In the same vein, Galasso and Schankerman (2010) identify “comparative advantage in patent enforcement” as a novel source of gains from patent trade, that can be obtained by reallocating patent ownership to “*entities that are more*

effective at resolving disputes over these rights without resorting to the courts.”

In this paper we focus on cumulative/sequential innovation that builds on prior art. In such environments, due to the inevitable conflict in providing incentives to current and future innovators, the dynamic effects of patent systems can be fundamentally different from the conventional wisdom, as argued by Bessen and Maskin (2009) in a dynamic game between rival innovators, and by Hopenhayn *et al.* (2006) from the social planner’s optimal patent design perspective. The current paper contributes to this debate by furthering our understanding of the impact of different levels of IP protection on the long-run innovation dynamics of startup firms.

The existing literature on cumulative innovations also includes Scotchmer (1996), Green and Scotchmer (1995), and Chang (1995), which focus on a single follow-on innovation; and O’Donoghue, *et al.* (1998) on multiple sequential innovations. The main differentiating feature of our paper is that we explicitly model the uncertain nature of court rulings in infringement suits, based on how dissimilar firms (incumbents *vs.* startups), having amassed patent portfolios in different depths, defend their IP rights.

The tenet has been advocated in the literature that restrained IP protection is needed to promote cumulative innovations, based on the core idea that excessive protection would suffocate derivative innovations, which in turn would depress the incentives for earlier innovations below the socially desirable level: Green and Scotchmer (1995) and O’Donoghue, *et al.* (1998), among others, study optimal protection levels in their respective models of cumulative innovation. In a different context, Bessen and Maskin (2009) argue for weak protection in order not to impede imitations by producers of differentiated products, which would permit innovative complementarities and thereby, improve prospects for future innovation.² We focus on a different dynamic mechanism pertinent to startup innovators: moderate protection promotes transfer of future innovation benefits to early innovators via the incumbent’s strengthened future bargaining position when it enlarges its patent portfolio by takeovers.

The model at hand is based on Panagopoulos and Park (2018). Panagopoulos and Park (2018) examine how a patent portfolio that recursively increases via acquisitions affects a firm’s decision to patent her technology instead of keeping it a secret. The recursive increase in patent portfolios is an attribute this model shares as well. However, whereas in this model a startup firm tries to establish the conditions that will allow it to innovate in view of its particular R&D cost, which is probabilistic, in Panagopoulos

²In a model of a single, multistage R&D race between firms with asymmetric R&D abilities at different stages, Fershtman and Markovich (2010) also examine when weak protection that allows free imitation is desirable.

and Park (2018) two different modes of technology protection are recursively compared in terms of their capacity to sponsor innovation. To use a sports metaphor, in Panagopoulos and Park a patent and a trade secret race in order to reach a predefined finish point, while in this model a startup races alone and tries to reach a finish point which is not clearly visible and depends on the startup’s ability to do R&D at a low cost. It is the startup’s ability to innovate at a certain cost that frames the model’s dynamics, because if a startup finds it costly to innovate the recursive sequence of events collapses in which case patent portfolios cannot aid the incumbent firm in its licensing negotiations and equally startups firms cannot hope from the additional payoff stemming from future buyouts. In short, our emphasis on a probabilistic finish line is what inevitably drives the dynamic analysis of the model and the simulation.

3 A static benchmark

We consider two firms in the same industry, operating under a single line of patented, cumulative technology. Firm 1 is an established incumbent, holding an extensive patent portfolio. Firm 2, a potential entrant/startup, first decides whether to invest in an R&D project that costs c . We assume that c is a random variable whose value is 0 or $C > 0$ with probabilities $\eta \in (0, 1)$ and $1 - \eta$, respectively. The realized value of c is private information of the startup but η is common knowledge.³ This is in line with Bessen and Maskin (2009) and amounts to assuming the existence of very innovative startups that, contrasting their high-cost counterparts, innovate with minimum cost.

If firm 2 decides against investing in R&D, the market stays unchanged and the game ends with payoffs normalized as 0 for both firms. If firm 2 invests c , it develops a promising new technology and obtains one single-claim patent as a testimony to its innovativeness.⁴ Given the cumulative nature of technology, however, firm 2’s patent will be perceived as potentially infringing on one or more of the incumbent’s patents.

If firm 2 was to commercialize the new technology on its own it would generate a profit of $V > 0$. On the other hand, if firm 1 were to commercialize it as the sole patent holder,

³The lower value of c being 0 is for purely expositional ease. Our results are intact so long as c is less than $\bar{s}_2(1)$ to be defined shortly in equation (2). In order to illustrate our core insight with clarity, we abstract from additional informational asymmetries and risk aversion. For the purpose of this paper, adding such features would complicate the analysis without much extra insights to be gained.

⁴Even though we live in an era where startups are aided by venture capitalists, incubators and technology-transfer specialists, *ex ante* scientists often lack a clear perception of R&D costs. Acknowledging this fact, our results extend straightforwardly to the cases that investing c leads to an innovation with a known probability less than (rather than equal to) 1.

it could generate a total profit of $V^* \geq V$ using its greater marketing experience. There are three options that firm 1 may take at this point as described below.

The first option is to file a suit alleging that 2's technology is infringing on its patents, the outcome of which is uncertain. We model this as follows: the court finds firm 2's technology infringing (firm 1 wins) with a commonly known probability $p \in (0, 1)$, invalidating the startup's single-claim patent and thus, depriving its ownership of the ideas in the revoked patent claim; or finds it non-infringing (firm 2 wins) with probability $1 - p$, confirming the startup's sole legal right to commercialize the new technology.⁵ Regardless of the outcome, going through the legal battle is costly for both parties involved, which we reflect by assuming that litigation incurs a monetary equivalent cost of $\ell > 0$ for both the plaintiff and the defendant.

As to the eventual outcome ensuing the court's decision, we assume that the payoff to the losing party is zero (not counting the litigation cost ℓ) and the winner solely commercializes the new technology. In the case that the defendant/startup wins, this means that it reaps a profit of V . If the incumbent wins, it does not always mean that the winner can fully appropriate and commercialize the ideas contained in the revoked patent, which are of merit to users yet judged failed to fulfil the substantial requirements of patentability. Such ideas are effectively in the public domain and any third party can free ride on them. Moreover, there could be residual tacit knowledge not apparent in the document yet needed for realization of the full value of the ideas. Consequently, it is unlikely that firm 1 will be able to capture the whole value of V^* . On account of the above, we assume that firm 1 reaps a profit of $bV^* \leq V^*$ upon winning the litigation, where $b \in [0, 1]$ is an exogenous value. As will be explained later in conjunction with numerical simulations (in Section 6), different values of b do not alter our main results.

Apparently, the above description of post-litigation outcomes falls short of fully re-

⁵Here, we abstract from two possible court rulings. One is the ruling that the startup's technology *per se* does not infringe upon the incumbent's patents, but in order for the underlying technology to be commercialized it would require the licensing of essential patented technologies from the incumbent (i.e. the incumbent's portfolio includes what is commonly referred to as "blocking patents"), so that neither firm may commercialize the new technology without a license from the other. Allowing this possibility would not alter our main results, because the only effect it would have on our analysis is to change the threat points of the Nash bargaining between the two firms (via changing the expected payoffs from litigation). The other is invalidation of the incumbent's patents as a result of the defendant's counter accusation of invalidity (see Choi, 1998), which if allowed in our model, would reduce the value of filing a lawsuit for firm 1 due to the added possibility of the court finding some of 1's patents invalid. Unless this possibility is so large that the incumbent's expected payoff from filing a lawsuit is negative, the substance of our analysis and results remain unchanged.

flecting various post-litigation dealings that may take place depending on the specific circumstances, such as transfer of “soft knowledge” from the losing party to enhance the profit of the winning party, or even a sale of the validated patent by the startup upon winning the lawsuit. These possibilities will increase the post-litigation profits of both the incumbent and the startup, but the combined profits cannot exceed V^* in any case. In the presence of the litigation cost $\ell > 0$, therefore, both parties will prefer an out-of-court agreement to be reached in the manner explained below (because they will share any under-realized profits and also save ℓ), regardless of the details of possible post-litigation dealings. Hence, such details do not change our main results.

The second option is for firm 1 to seek a technology sharing agreement with 2 (i.e., a takeover deal or a licensing/cross-licensing agreement),⁶ which we model, in line with literature,⁷ as a Nash bargaining where the disagreement/threat points are the expected surpluses when firm 1 files an infringement lawsuit. We present our analysis presuming that the two firms have equal bargaining powers, but our main qualitative results remain intact for a wide range of unequal bargaining powers.⁸ ⁹ If an agreement is reached, firm 1 commercializes the technology as the sole patent holder and thus, reaps the full value V^* as described earlier.

The third option is for firm 1 to do nothing, in which case firm 2’s payoff is V from commercializing the technology and firm 1’s payoff is 0. As will be verified shortly, this option is worse than pursuing a technology agreement *à la* Nash bargaining. The structure of the game is common knowledge.

The main exogenous variable of our model is p (and its determinants in a dynamic

⁶Forming a patent pool will have the same effect as a cross-licensing agreement in so far as both firms have access to all the patents in the pool but the ownership is separated.

⁷E.g., Green and Scotchmer (1995), Chang (1995), and Crampes and Langinier (2002).

⁸Our core results remain valid unless the incumbent has all the bargaining power, i.e., makes a take-it-or-leave-it-offer. In practical terms, this latter case implies double ultimata that seem to depart from contemporary norms. Specifically, it suggests that unless the startup accepts the offer (the first ultimatum) the incumbent will issue a second ultimatum by filing the case in a court of law. Considering current legal practice (and the use of juries) the use of such a “public” threat must be detrimental to one’s chances of winning a court battle. More importantly, and in relation with the residual tacit knowledge that the incumbent is interested in, firms like Cisco (the champion of buyouts) do not issue ultimatums but bargain. That is, given that the tacit knowledge the acquiring firms are usually interested in takes the form of a network of clients whose needs the startup knows and understands, an ultimatum is not conducive to the startup’s voluntarily disclosure of such knowledge. In addition, such an ultimatum would hurt the acquirer’s reputation that may be valuable for future takeover deals.

⁹The model operates under the assumption that the cost of out-of-court settlement is not excessively large as to *a priori* impose litigation as the only available choice.

setting analyzed in the next section) where we interpret a higher p as reflecting a stronger stance of the court toward IP protection. The focus of analysis is on the extent to which p affects innovation incentives via its impact on the bargaining outcome.

In the stage game described above, a strategy of firm 2 is whether to invest in R&D or not, and that of firm 1 is whether to file a suit, to seek a Nash bargaining outcome, or to do nothing. The subgame-perfect equilibrium of this game is obtained straightforwardly from the Nash bargaining outcome as explained below. Specifically, conditional on firm 2 having developed a new technology, the disagreement/threat points of firms 1 and 2 are, respectively, $d_1 = pbV^* - \ell$ and $d_2 = (1 - p)V - \ell$. Since V^* is the maximum possible industry profits from the technology, the Nash bargaining set is defined as $B = \{(\bar{s}_1, \bar{s}_2) \in \mathfrak{R}_+^2 \mid \bar{s}_1 + \bar{s}_2 \leq V^*\}$ where \bar{s}_i denotes the bargaining share of firm $i = 1, 2$ (the bar above s_i is designatory of the static framework). Since B is compact and convex, there is a unique Nash bargaining outcome (\bar{s}_1, \bar{s}_2) that solves $\max_{(\bar{s}_1, \bar{s}_2) \in B} (\bar{s}_1 - d_1)(\bar{s}_2 - d_2)$, expressed as the following functions of p where $r = V/V^* \in [0, 1]$:

$$\bar{s}_1(p) = \frac{V^* + d_1 - d_2}{2} = \frac{1 + p(b + r) - r}{2} V^* \quad (1)$$

$$\bar{s}_2(p) = \frac{V^* - d_1 + d_2}{2} = \frac{1 - p(b + r) + r}{2} V^*. \quad (2)$$

Note that $\bar{s}_1(p) > 0$ because $V^* + d_1 - d_2 = (1 + pb)V^* - (1 - p)V \geq (pb + p)V > 0$. Thus, the strategy of doing nothing is strictly dominated by that of pursuing Nash bargaining for the incumbent. For this reason, the strategy of doing nothing will not be discussed further in the sequel.

Since $\bar{s}_1(p) > d_1$ and $\bar{s}_2(p) > d_2$ hold, both firms will find it optimal to pursue a technology sharing agreement *à la* Nash bargaining, instead of litigation, once an innovation has taken place. Anticipating such an agreement, a startup always innovates if $\bar{s}_2(p) \geq C$, but it innovates only when $c = 0$ if $\bar{s}_2(p) < C$.

The next proposition summarizes the findings in the static setting. Since the ownership of the patent *per se* does not alter the maximum value of the technology, V^* , the bargaining agreement does not need to take the form of a takeover, as it can equally well be attributed to licensing.

Proposition 1: *In the stage game, the Nash bargaining over a startup's innovation splits the total surplus V^* into $\bar{s}_1(p)$ for the incumbent and $\bar{s}_2(p)$ for the startup, as expressed in (1)-(2). Hence, a startup always invests in R&D if $\bar{s}_2(p) \geq C$, but it invests only when $c = 0$ if $\bar{s}_2(p) < C$. Stronger IP protection, i.e., a higher p , therefore, decreases (increases) the share of the startup (incumbent) via weakening (strengthening) its bargaining position, reducing the startup's innovation incentives.*

In offering support for the dynamic model that we will introduce in the next section, let us note that social optimality dictates that even the high-cost startup innovates so long as its cost, C , falls short of the total social surplus from the new technology. According to Proposition 1, however, this would not happen in the static setting if C exceeds V^* (yet falls short of the total social surplus) because $\bar{s}_2(p)$ obtains the maximum value of V^* when $p = 0$ and thus, the high-cost startup would not innovate for any p . In the dynamic setting, on the other hand, the high-cost startup can be induced to innovate even if $C > V^*$ when the level of IP protection is chosen appropriately, as we will show in the next section. As mentioned in the Introduction, this is due to the positive feedback effects that the incumbent's enhanced future bargaining positions from a takeover deal have on the current startup's bargaining share.

Before proceeding, we note that some simplifying assumptions have been made for expositional ease, but these do not affect the main results: We do not consider preliminary injunctions which may enhance the plaintiff's negotiating power,¹⁰ however such an effect can be captured in our model via a greater p . We also assume that justice is swift,¹¹ which allows us to abstain from elaborating on the details of the damages that the losing party needs to pay.¹²

4 A dynamic approach

In this section, building on the main intuition provided by Panagopoulos and Park (2018), we elaborate on the issues arising from the cumulative nature of technology by extending the model to infinite periods. In each period the static game of Section 3 is played as the stage game between a long-lived incumbent (firm 1) and a new potential startup (firm 2) that arrives at the market. The key difference from the stage game is that the incumbent's patent portfolio may grow via takeovers.

If the incumbent acquires new patents through takeover deals, the technological territory covered by its patent portfolio expands and thus, as argued in the Introduction,

¹⁰Lanjouw and Lerner (2001) and Lemley and Shapiro (2007) explain how the plaintiff's bargaining is enhanced through preliminary injunctions.

¹¹In reality court cases can go on for prolonged periods, incurring costs to both parties. This reality also strengthens our result by rendering out-of-court agreements more attractive.

¹²The yardstick used by courts in deriving damages is either the accumulated royalties resulting from a hypothetical licensing agreement (this is usually a per-period payment of 1-2% of the product's value), or the foregone profits from the sale of the infringing good. Both of these are minimal if justice is swift. Specifically, since a final product is yet to be developed there are no foregone profits, and any foregone royalty payments cannot be central to the paper's argument because they have yet to accumulate.

the likelihood increases that it will prevail in patent-infringement suits. In this regard, we assume that legal power increases as the portfolio size gets bigger, but at a decreasing rate. That it increases at a decreasing rate is attested by Bessen and Meurer (2005) who observe decreasing returns to scale between the size of a software firm's patent portfolio and the probability of winning a patent litigation suit. Moreover, it is also largely a logical consequence of the fact that the chance of prevailing in court is bounded above by 1.

To capture this we re-define p , the probability of firm 1 winning an infringement suit, as a function of the degree of IP protection, denoted by $z \in (0, 1)$, and the size of firm 1's patent portfolio, measured by the number of patents in its portfolio. In particular, an increase in z (which can be considered as patent breadth) implies a tougher stance on infringement, increasing p .

To facilitate presentation, we make two indexing conventions. First, since the continuation game from any period is fully described by the size of firm 1's portfolio at the beginning of that period, with slight abuse of terminology we index the period by the size of firm 1's portfolio. Second, since what matters in the analysis is the accumulation of patents on top of the incumbent's initial portfolio, we index the size of the initial portfolio as the base size of 1, and each patent added to it increases the portfolio size by one. Hence, period 1 designates the initial period (of the base portfolio size of 1) and period $t > 1$ designates any period prior to which firm 1's portfolio size has reached t but no higher, i.e., firm 1 has added $t - 1$ patents to its initial portfolio. In other words, period t denotes any period in which the incumbent starts with a stock t of patents in its portfolio. So long as firm 1 has added one patent every period from the initial period, our indexing coincides with the natural indexing of periods by natural numbers.¹³ Two consecutive periods are indexed the same, however, if the incumbent's portfolio did not grow in the first of the two periods. Thus, p is a function of z and t , which we denote as $p_z(t)$. As discussed earlier, we assume that

$$\partial p / \partial z > 0, \quad \partial p / \partial t > 0, \quad \text{and} \quad \partial^2 p / \partial t^2 < 0.$$

Note that the incumbent's portfolio may not grow from one period to the next (in which case we index the two periods the same as explained above) either because the startup did not innovate in the first period,¹⁴ or because it did but the incumbent did

¹³We do not model expiration of patents for expositional clarity. The effect of patent expiration is straightforward and can be seen easily when we have characterized the equilibrium, as briefly discussed at the end of this section.

¹⁴In the context of sequential innovation literature where innovation $n + 1$ cannot be introduced until innovation n has been, this means that the introduction of innovation n is delayed at least by one period (rather than innovation n is skipped).

not buy the innovation. In the sequel, our use of indexing convention and $p_z(t)$ described above pertain to the former case because the latter case does not arise in equilibrium as will be shown in the analysis to follow.

To avoid the replacement effect, as in Bessen and Maskin (2009), we assume that V and V^* , the profits that can be generated from commercializing the new technology, are incremental values. We stress here that, as will be illustrated in Section 6, our results are not driven by the disparity between V^* and V but rather by the additional bargaining power that expanding patent portfolios allow for.

To recap, the order of moves in each period t is as follows. First, firm 2 arrives and decides whether to innovate or not contingent on its R&D cost which is 0 and C with probabilities η and $1 - \eta$, respectively. If 2 does not innovate, nothing happens until the next period starts. If 2 innovates, 1 decides whether to file a suit or pursue a technology-sharing agreement *à la* Nash bargaining. If a suit is filed, both parties incur a legal cost of ℓ and, as a result, with probability $p_z(t)$ firm 1 wins and gets a surplus of bV^* while with probability $1 - p_z(t)$ firm 2 wins and gets a surplus of V . The losing party has a surplus of 0. If an agreement is pursued, the Nash bargaining outcome results over the total producers surplus of V^* , plus, in case of a takeover, the additional benefits that would accrue to firm 1 in future deals due to its enlarged portfolio. We present our main analysis presuming that any technology-sharing agreement takes the form of a takeover, then explain later how the results change when licensing agreements are allowed. The startup in each period maximizes its expected surplus of that period, net of innovation cost when relevant. The incumbent maximizes the expected present value of the stream of its profits with a discount factor $\delta \in (0, 1)$.

Our core argument starts with the observation that the benefits of a takeover for firm 1 venture beyond its deal over the current startup's innovation, as the added bargaining power (caused by the expansion of 1's portfolio) may mean better future deals. This suggests that the total surplus to bargain over can be larger than that in the stage game. Consequently, firm 2 may rationally anticipate a larger bargaining share, suggesting that dynamic incentives may induce innovations that would not have been possible in a static setting. This dynamic argument implies that, unlike in the stage game, a takeover may be preferred to licensing because licensing (or cross-licensing) does not allow for the extra surplus in future deals caused by the expansion of 1's patent portfolio.

Prior to proceeding with a formal analysis, we note that our model may be extended in at least two directions. First, the effect of an enlarged patent portfolio of the incumbent, which we model as an increased probability of winning an infringement lawsuit, may alternatively be modelled as an enhanced bargaining power of the incumbent in a

generalized Nash bargaining model. As both have the same effect of increasing incumbent's bargaining share in forthcoming takeover deals, our results are not sensitive to this change. Second, the incremental commercial value of each innovation may be modelled as a random variable with mean V^* and V when commercialized by the incumbent and the startup, respectively, rather than a fixed constant. This does not change our analysis insofar as the startup makes an investment decision before the value of this random variable is realized. In addition, if V^* increases (decreases) as the incumbent's portfolio grows, the dynamic feedback effect that boosts startup innovation will be prolonged (curtailed).

We now present a formal analysis of the dynamic model and characterize the (subgame-perfect) equilibrium.¹⁵ As we will show, there is a unique equilibrium and it exhibits the basic features elucidated above, namely, that the takeover deal *à la* Nash bargaining provides innovation incentives for high-cost startups (i.e., those with $c = C$), during an early stage of innovation dynamics at least. Such dynamic effects of inducing high-cost innovations would be best illustrated if a high-cost innovation was never possible in the static situation. Hence, we first present our analysis in such environments, and then discuss other environments.

Thus, first we consider the case that $\bar{s}_2(p_z(1)) < C$, or equivalently,

$$\bar{s}_2(p_z(t)) < C \quad \text{for all } t \geq 1 \quad (3)$$

where $\bar{s}_2(\cdot)$ is firm 2's bargaining share in the stage game as defined in (2). Note that a low-cost startup (i.e., one with $c = 0$) always innovates because its bargaining share exceeds its R&D cost regardless of $p_z(t) \in (0, 1)$. Let T denote, in an arbitrary equilibrium, the last period in which a high-cost startup innovates with a positive probability, allowing for the possibility that $T = 0$, i.e., a high-cost startup never innovates. T is our point of departure in the analysis, and for notational purposes, in the sequel a hat on top of a variable is designatory of all $t \leq T$ periods, and absence of a hat denotes all $t > T$ periods. Given that $T < \infty$ exists (indicating that in equilibrium a high-cost startup would not innovate indefinitely) as is proved in Proposition 3 below, for $t \geq T + 1$, let $X(t)$ denote the value of firm 1 at the beginning of period t . Then,

$$X(t) = (1 - \eta)\delta X(t) + \eta(s_1(t) + \delta X(t + 1)) \quad (4)$$

because, a) if a high-cost firm ($c = C$) arrives with probability $1 - \eta$, there is no innovation and firm 1's value in the next period is the same as that in the current period (i.e. $X(t)$)

¹⁵Since firm 1 decides to litigate or bargain without knowing the R&D cost of firm 2, technically speaking the subgame-perfectness does not require rationality of the incumbent's decision. However, the R&D cost of the startup is sunk at this point, so it does not affect the continuation game. Hence, in the spirit of subgame-perfectness, we require that each choice of the incumbent be optimal in the continuation game.

and, b) if a low-cost firm arrives with probability η , firm 1 captures the bargaining surplus over the current innovation, $s_1(t)$, plus its value in the next period which is $X(t+1)$.

Focusing on equation (4), for $t > T$ the total surplus that a startup's innovation generates is maximized when firm 1 commercializes it, adding it to its portfolio. The total surplus it brings forth in this case is $V^* + \delta(X(t+1) - X(t))$, which is the size of the pie on the bargaining table. If the case is litigated, since both parties must accept the court's decision, there is no takeover deal. Therefore, the threat points are the court outcomes minus the legal costs, i.e., $d_1 = p_z(t)bV^* - \ell$ and $d_2 = (1 - p_z(t))V - \ell$. Since the Nash bargaining set in this case is $B(t) = \{(s_1, s_2) \in \mathfrak{R}_+^2 \mid s_1 + s_2 \leq V^* + \delta(X(t+1) - X(t))\}$, the Nash bargaining outcome (s_1, s_2) that solves $\max_{(s_1, s_2) \in B(t)} (s_1 - d_1)(s_2 - d_2)$ is calculated as,

$$s_1(t) = \frac{1 + p_z(t)(b+r) - r}{2}V^* + \frac{\delta(X(t+1) - X(t))}{2} \quad (5)$$

$$s_2(t) = \frac{1 - p_z(t)(b+r) + r}{2}V^* + \frac{\delta(X(t+1) - X(t))}{2} \quad (6)$$

where $r = V/V^* \in [0, 1]$. Plugging $s_1(t)$ back into equation (4) and rearranging, we get

$$X(t+1) - X(t) = \frac{2(1-\delta)}{3\delta\eta}X(t) - \frac{1 + p_z(t)(b+r) - r}{3\delta}V^*, \quad (7)$$

a difference equation that characterizes the sequence $X(t)$ for $t > T$. Since the value of additional patent diminishes to 0 as $t \rightarrow \infty$, it turns out that this sequence increases and converges, as formalized in the next result. Although $X(t)$ is pertinent for $t > T$, it proves useful to treat it as a function defined for all natural numbers $t \geq 1$.

Proposition 2: *The sequence $\{X(t)\}$ defined by (7) is unique, monotonically increases at a decreasing rate, i.e., $X(t) - X(t-1) > X(t+1) - X(t) > 0$ for all $t > 1$, and converges to*

$$X(\infty) = \frac{1 - r + p_z(\infty)(b+r)}{2(1-\delta)}V^*\eta \quad \text{as } t \rightarrow \infty. \quad (8)$$

Proof: See Appendix.

Proposition 3: *If (3) holds, in any equilibrium there exists an earliest period $T < \infty$ such that a high-cost startup does not innovate in any period $t > T$.*

Proof: See Appendix.

Next, we determine the equilibrium for $t \leq T$. Let $\hat{X}(t)$ denote firm 1's value at the beginning of period t for $t \leq T$. Provided that a high-cost startup innovates for sure in period T , firm 1's value at the beginning of T is

$$\hat{X}(T) = \hat{s}_1(T) + \delta X(T+1) \quad (9)$$

where $\hat{s}_1(T)$ denotes the bargaining share that it derives over the current innovation. Since the total surplus to bargain over would be $V^* + \delta(X(T+1) - \hat{X}(T))$, equation (9) would indeed determine the equilibrium value of $\hat{X}(T)$ if the bargaining share of firm 2 thereof, denoted by $\hat{s}_2(T)$, actually covered C , because this would justify the presumption that a high-cost startup innovates for sure in period T .

However, this may not always be the case. To see this, note that $\hat{X}(T) > X(T)$ because $\hat{X}(T)$ includes the surplus from high-cost innovation presumed to take place in period T while $X(T)$ does not include it (by definition of the sequence $X(\cdot)$). This means that the extra value of an enlarged portfolio from the current takeover is lower when high cost innovation were to take place in the current period, T , than when it were not, i.e., $X(T+1) - \hat{X}(T) < X(T+1) - X(T)$, and consequently, the startup's bargaining share is lower when high cost innovation were to take place in period T than when it were not, i.e., $\hat{s}_2(T) < s_2(T)$ where $s_2(T)$ is obtained by evaluating (6) at $t = T$. Thus, even if $s_2(T) > C$ is implied by the definition of $X(T)$ (for otherwise we would have $\hat{s}_2(T) < s_2(T) \leq C$, contradicting T being the last period in which a high-cost startup innovates), $\hat{s}_2(T) \geq C$ may not hold.

If, indeed, $\hat{s}_2(T) \geq C$ does not hold in equilibrium, a high-cost startup must innovate with a positive probability less than 1, say $\hat{a} \in (0, 1)$, in period T . As \hat{a} decreases from 1 to 0, the value of $\hat{X}(T)$ decreases all the way to $X(T)$ because the prospect of current innovation by a high-cost startup dwindles to 0. This, in turn, increases the extra future value of current takeover, $X(T+1) - \hat{X}(T)$, all the way to $X(T+1) - X(T)$, and thereby, the bargaining share of the startup all the way up to $s_2(T) > C$ where the inequality is asserted above. Therefore, there is a unique value of \hat{a} for which the startup's bargaining share is exactly equal to C , justifying a mixed strategy of a high-cost startup between investing in innovation and not in period T . This determines the unique equilibrium probability of innovation by a high-cost startup in period T . By applying an analogous argument recursively as needed, we can uniquely determine the equilibrium probability of high-cost innovation for $t = 1, 2, \dots, T$. This completes characterization of the unique equilibrium for cases that satisfy (3): namely, high-cost startups innovate with a strictly positive probability in every period $t \leq T$, but do not innovate in later periods. (See the proof of Proposition 4 in Appendix for a formal proof of this characterization.)

We now discuss the alternative case that $\bar{s}_2(p_z(1)) \geq C$. There are two subcases to consider, namely, $\bar{s}_2(p_z(\infty)) < C$ and $\bar{s}_2(p_z(\infty)) \geq C$. When $\bar{s}_2(p_z(\infty)) < C$ it is straightforward to see that there exists a unique equilibrium analogous to the one characterized above. Specifically, high-cost startups innovate for sure in all periods t such

that $\bar{s}_2(p_z(t)) \geq C$, because $\hat{X}(t)$ increases in t and consequently,

$$\hat{s}_2(t) = \bar{s}_2(p_z(t)) + \frac{\delta(\hat{X}(t+1) - \hat{X}(t))}{2} > C. \quad (10)$$

Then, since the increase in $\hat{X}(t)$ slows down and, for some t , $\bar{s}_2(p_z(t))$ will eventually dip below C , high-cost startups stop innovating from a certain period. If $\bar{s}_2(p_z(\infty)) \geq C$, on the other hand, a high-cost startup innovates in every period in the unique equilibrium of the dynamic model because $\hat{s}_2(t) > \bar{s}_2(p_z(t))$ as per (10) and $\bar{s}_2(p_z(t)) > \bar{s}_2(p_z(\infty))$. Now we characterize the unique equilibrium of the dynamic model in the next theorem.

Proposition 4: *The dynamic model has a unique equilibrium. If $\bar{s}_2(p_z(\infty)) \geq C$, in this equilibrium a startup innovates for sure in every period regardless of its R&D cost; If $\bar{s}_2(p_z(\infty)) < C$, on the other hand, there is a critical period $T < \infty$ such that a high-cost startup innovates with a positive probability in every period $t \leq T$ but not in periods $t > T$, while a low-cost startup innovates for sure in every period. In either case, when there is an innovation the incumbent reaches a takeover deal with the innovator.*

Proof: See Appendix.

5 Discussion

In this section we discuss some implications that the dynamic analysis and equilibrium above lend on IP protection policies and the incumbent firm's open innovation style management strategy of its patent portfolio. Then, we also discuss the manner in which we modelled unequal status of the startup relative to the incumbent, the key feature of our approach, and how they may be extended for additional insights.

Optimal IP Protection: An interesting policy-relevant question naturally arises from our analysis: what is the optimal level of IP protection, z , that induces innovation by a high-cost firm for the longest possible duration? It is hard to obtain an algebraic answer because of the recursive nature of the solution and the discontinuity of $X(t)$ at $t = T$. Nevertheless, the core logic of our analysis clearly points to the following intuition: If z is excessive, the marginal protective power that an extra patent brings to the incumbent is large initially but quickly dwindles as a result of accumulating its power too rapidly, killing off the positive effect on startup innovation prematurely; If z is feeble, on the other hand, the marginal protective power of an extra patent can be too small and its impact on the startup's innovation incentives limited. Consequently, the optimal level of IP protection tends to be at a moderate level. We confirm this intuition by way of

conducting computer simulations for various parameter values to find the levels of z under which the high-cost innovations are induced for longest. The simulation results, including comparative statics, are summarized in Section 6.

Licensing, Patent Donation, and Optimal Patent Length: We have carried out our main analysis presuming that any technology-sharing agreement is restricted to a takeover deal, i.e., licensing was not considered. Since licensing (lacking the added advantages accruing to firm 1 from future dealings) fails to increase the innovation's total value beyond V^* , one can see that the Nash bargaining outcome of a licensing deal is the same as the stage game's bargaining outcome, $\bar{s}_1(p_z(t))$ and $\bar{s}_2(p_z(t))$. Thus, if $\bar{s}_2(p_z(1)) < C$ licensing in any period t would not cover C for the startup, allowing only low-cost innovations, while if $\bar{s}_2(p_z(\infty)) \geq C$ it would allow high-cost innovations in every period t . As we have demonstrated above, with takeover deals firm 1 anticipates a larger surplus due to its enhanced future bargaining positions and, furthermore, a part of this extra surplus accrues to the current startup firm (at the expense of the startups in future deals). As a result, takeover deals may motivate some high-cost startups to innovate even when $\bar{s}_2(p_z(1)) < C$. Therefore, allowing licensing does not change equilibrium outcome because the takeover deals will prevail as the dominant form of technology-sharing agreement anyway.

If $\bar{s}_2(p_z(1)) \geq C > \bar{s}_2(p_z(\infty))$, on the other hand, there is room for licensing. For example, after expanding its portfolio via takeovers to the largest size, say \bar{t} , such that $\bar{s}_2(p_z(t)) \geq C$, the incumbent may obtain access to new technology through licensing in all subsequent periods, so that startups innovate forever regardless of their R&D cost. Relative to when licensing is disallowed, this would bring about more innovations but the bargaining share of the incumbent would be smaller for the periods in which licensing deals will be reached: This is because \bar{t} is smaller than T , the largest portfolio size consistent with high-cost innovation as explained above, and the portfolio size will not grow after \bar{t} .

In fact, the incumbent may be able to do better by expanding its portfolio size to T via takeovers, then selectively releasing some of the patents in its portfolio to maintain the portfolio size at an optimal level, and thus avoid discouraging startup innovations by becoming too powerful a potential plaintiff. Note that, even when $\bar{s}_2(p_z(1)) < C$, this strategy works in the same manner to induce innovation in every period regardless of the startup's cost. Furthermore, relative to the strategy of licensing after \bar{t} , this strategy has the added advantage of keeping the ownership of patents away from other, potential rival firms. Indeed, this practice is reminiscent of the recent trend of patent donations: in the last few years firms such as DuPont, Lubrizol, Eastman Chemicals, and General Motors have given away patents with an estimated value of hundreds of millions of dollars. An

interesting alternative interpretation of T , therefore, is the optimal patent length that, by constraining the incumbent's portfolio size below a threshold, would allow for the arrival of high-cost innovations *ad infinitum*.

Unequal Status between the Incumbent and Startups: As noted earlier, a novel aspect of our study is to recognize the unequal status that startups face relative to the industry incumbents, in delineating the impact of IP protection on the innovation incentives of the startups. We clarify below the manners in which we reflected the unequal status in our model and their roles in our analysis.

Note that we do not explicitly model the possibility that a startup tries to build up its own portfolio via takeover deals with future startups. Allowing such a possibility would be sensible if we were to analyze a market in which the incumbent's dominance in technological territory is relatively weak. However, if the history evolved in such a way that the incumbent has accumulated an extensive portfolio of patents to wield power in the industry, as in the cases of Google or Cisco, for example, the *ex-ante* value of a startup from pursuing such a route would be low because the startup will have to compete against the powerful incumbent in the product market as well as in future takeover deals, both of which will reduce the expected surplus of its own as well as that of the incumbent. The reward from such a strategy may materialize, if at all, only after a long streak of successive takeovers by the startup, which is a very unlikely event given the incumbent's strong dominance. Thus, the startup would prefer a takeover deal because then the incumbent would expect higher surpluses both in the product market and future takeover deals due to the reduced competition, a portion of which accruing to the startup as its bargaining share.¹⁶

Another difference in our model between the startups and the incumbent is that in each period only the startup may engage in R&D by investing c , but this was made for expositional ease and our insights equally apply when the incumbent and a startup may engage in an R&D race in every period. In particular, in a model where the probabilities with which the incumbent and the startup win the race, q_i and q_s , respectively, are

¹⁶Admittedly, occasionally startups become dominant firms. Genentech and Intel are examples of former startups that rose to power and at some point pursued a strategy of buyouts, although not when they were still considered as startups. These firms benefited from inventing a new and disruptive technology (that lacked substitutes), which allowed them to create and monopolize new markets, rather than "win" the existing market from established incumbents. Furthermore, the disruptive nature of their technology allowed them to operate without the real threat of litigation. Hence, these firms are in contrast with the startups we model, which create incremental innovations that build on an already existing technology.

identical, it is relatively straightforward to see that the dynamic effects of a takeover deal (in case the startup wins the race) work in the same manner to support our main message, namely, that the future benefit of a takeover deal can motivate the startup's innovation activity that would not take place otherwise, but such effect will be short-lived if the IP protection was excessive. Furthermore, if $q_i < q_s$ then the incumbent may find it optimal to save its own R&D cost and pursue a takeover deal of the startup's innovation instead.

6 Simulation and comparative statics

We have established in Section 4 that in the unique dynamic equilibrium high-cost startups innovate with a positive probability until a critical period, denoted by T , but in no later periods. Since low-cost startups innovate in every period, the innovation incentives are enhanced when T is higher. Although our analysis explains clearly how various aspects of the model interact to determine T , a closed-form solution of T is unobtainable due to the infinite nature of the difference equation underlying the value function $X(t)$ and the discontinuity of $X(\cdot)$ at $t = T$. Consequently, using a computer program we simulated our dynamic equilibrium for various parameter values and report the results in this section. As elaborated below, these results confirm the intuition asserted earlier for the optimal levels of IP protection, and also provide additional useful comparative statics.

For numerical simulations, we need to fix the function $p_z(t)$ that represents the incumbent's probability of winning an infringement lawsuit, as well as other parameter values. We argued earlier that $p_z(t)$ increases at a decreasing rate in the portfolio size, t , and increases in the level of IP protection, z . Although empirical estimates are rather scarce on this measure, such properties are in line with the findings of Lanjouw and Schankerman (2004) that the marginal protective power of portfolio size is positive but slowing down. We capture this by setting $p_z(t) = 1 - (1 - z)^t$.¹⁷

The value of z , the level of IP protection, is our main focus as a determinant of T . Therefore, we conduct our simulation with z as the main variable, with a view to identifying the optimal levels of z for which T is maximized, for various specifications of other parameter values.

For other parameter values, we normalize $V = 1$ and set $r = 1$, $C = 1.0001$ and $b = .5$ for the following reasons. Setting $r = 1$ ensures that our results are not driven by the disparity between V and V^* . Since $\bar{s}_2(p) \leq \bar{s}_2(0) = 1$, by choosing $C > 1$ we ensure that

¹⁷To provide an example (in line with the magnitudes of z we find), when $z = .007$ a firm with a portfolio made up of 100 patents stands a 50% chance of winning its case, and an increase of 1 patent raises this by .34%.

a) innovation by high-cost startups may only be possible in a dynamic model, and b) that IP protection is necessary for such innovation because $p_z(t)$ is constant at 0 for all t if $z = 0$, erasing any dynamic effect. We need to mention that $r = 1$ and $C > V^*$ were chosen to demonstrate these points, and should not be construed as representative of real situations.¹⁸ For more relaxed parameter values, high-cost innovation will be sustained more broadly than our simulation results reported below.

The value of b determines the profits bV^* that firm 1 would reap upon winning the litigation, i.e., a higher b means a higher threat point of the incumbent. This has two opposing effects in the dynamic setting. Firstly, as a direct impact a higher b shifts the bargaining share from the startup to the incumbent. Secondly, it enlarges the size of the pie to bargain over (because an extra patent would increase the incumbent's future bargaining share more when b is higher) and consequently, indirectly increases the current startup's share. These two diverging effects are embedded in the first and second terms of (6), respectively. Hence, the net effect is ambiguous. In our simulation, it turns out that the two effects largely cancel each other and as a result, the value of b does not affect T significantly. For this reason, we report the simulation result for $b = 0.5$ mainly, and also report comparative statics on b at the end of this section.

We now explain the simulation process and the results. For expositional purposes we do this mainly for two combinations of δ and η , namely, for $\delta = .98$ and $\eta = .8$ and for $\delta = .97$ and $\eta = .4$. Then, we report comparative statics on δ and η at the end of the section.

Fixing the parameter values as above, first we simulate the unique sequence $\{X(t)\}$ defined by (7), which converges to (8). Figure 1 plots the simulated sequence of $\{X(t)\}$ for t up to 1000 that shows convergence, for $b = .5$, $r = 1$, $V = 1$, $z = .007$, $\delta = .97$ and $\eta = .4$. From this sequence, through equation (6), is derived a convergent sequence $s_2(t)$. Then, the value of T is obtained by identifying the last period for which $s_2(t) > C = 1.0001$. Figure 2 plots $s_2(t)$ for t up to 50, for $b = .5$, $r = 1$, $V = 1$, $z = .007$, $\delta = .97$ and $\eta = .4$, from which we identify $T = 6$.

To examine how z affects T , we find the values of T for z 's between .001 and .02 in 20 steps of .001. Figure 3 shows how T changes as z increases when $\delta = .97$ and $\eta = .4$ (the lower graph) and when $\delta = .98$ and $\eta = .8$ (the upper graph), keeping C , b , r , and V the same as before.

Both graphs are single-peaked with an interior peak, that is, T initially increases with z , then decreases as z increases further. Specifically, if z is high $X(t)$ converges quickly,

¹⁸Note that $C > V = V^*$ does not mean that high-cost innovation is socially inefficient, because V^* does not capture the entire consumer surplus which may be larger than C .

driving the future benefits from an extra takeover to nil and thus, halting the positive effect on startup innovation prematurely. For small z 's, on the other hand, an extra patent increases $p_z(t)$ and $X(t)$ only marginally, failing to sufficiently increase $s_2(t)$ as to allow for a high T . This confirms the aforementioned insight that for maximum dynamic incentives for startup innovations, the level of IP protection should be carefully selected at a moderate level. Needless to say, the precise relationship between z and T changes as other details of the specification change. However, in all our simulations the key feature prevails that T initially increases monotonically and then decreases monotonically in z , so long as $\bar{s}_2(0) < C$ and high-cost innovation is possible at all in the dynamic model. In addition, although both firms are assumed to have equal bargaining power in our analysis, endowing different bargaining powers would lead to different levels of T , e.g., increasing the bargaining power of the incumbent will lead to a smaller $s_2(t)$ and a lower T .

As explained in Section 4, the sequence $X(\cdot)$ is pertinent only for $t > T$ because it was constructed presuming that high-cost innovations do not take place. (Note that, nonetheless, this sequence allows us to identify the level of T .) For $t \leq T$, therefore, we simulate $\hat{X}(t)$ and $\hat{s}_2(t)$ as explained in Section 4, together with the probability of high-cost innovation in period t , denoted by $\hat{a}(t)$, in those periods t in which a high-cost startup innovates with a probability less than 1. The exact formulae used for this simulation are explained in detail in the proof of Proposition 4 in Appendix. The equilibrium values of $\hat{a}(t)$ are reported in Figure 4 for each of the 20 different values of z (i.e., for z 's between .001 and .02 in 20 steps of .001) for $\delta = .97$, $\eta = .4$, $C = 1.0001$, $b = .5$, $r = 1$ and $V = 1$. Due to the discreteness, a range of z provides the highest $T = 6$ as shown in Figure 4. Among these values of z , those with higher values of $\hat{a}(t)$ induce more startup innovation on average in period $t \leq T$. In Figure 4, of the values z that produce the highest $T = 6$, those with higher $\hat{a}(t)$ for one t tend to have higher values of $\hat{a}(t)$ for other t 's as well, although no single value of z is pinned down as having the highest $\hat{a}(t)$ for all $t \leq T$.

Finally, we report the comparative statics that capture how η , δ and b affect T . Starting with δ , Figure 5 plots T as we vary δ from .9 to .99 in 10 steps for various values of z as before (i.e. for z 's between .001 and .02 in 20 steps of .001), for $\eta = .8$, $C = 1.0001$, $b = .5$, $r = 1$ and $V = 1$. Figure 5 indicates that T increases as δ increases. Similarly, for the same values of z , Figure 6 plots T as η varies from .1 to 1 in 10 steps, for $\delta = .98$, $C = 1.0001$, $b = .5$, $r = 1$ and $V = 1$. Figure 6 indicates that T increases (at a decreasing rate) as η increases. Again for the same values of z , Figure 7 plots T as b varies from .1 to 1 in 10 steps for $\delta = .98$, $\eta = .8$, $C = 1.0001$, $r = 1$ and $V = 1$. The effect of b is small: for each value of z considered, the value of T remains the same regardless of b . This underlies our earlier observation that the two opposing effects of varying the value

of b largely cancel each other out.

[Figures 1–9 about here]

7 Conclusions

It is commonly argued that if patents were perfect then, since the borders of a technology will be accurately described by a patent –diminishing infringement, the patentee can capture the full value of her innovation. The full appropriation of the value of an innovation should thus be particularly appealing to fledgling firms like startups that lack the ability to protect their innovations, especially against established incumbents. Since startups feature prominently in the open innovation paradigm in this paper we revisit this commonly held policy prescription and argue otherwise.

We propose that the perspective lack of infringement offered by perfect patents offers fewer incentives than imperfect patents. This is because the possibility of infringement allows the startup and the incumbent to bargain a settlement that has the capacity to affect similar future negotiations. This counterintuitive policy prescription is driven by the asymmetry in legal strength between startups and established incumbents who hold patent portfolios.

This asymmetry allows for two predictions that accord with open innovation: a) there is a continuous supply of startup innovators that choose to enter the market arena despite having to compete against behemoths, and b) startups frequently transfer their technology via takeovers. In particular, the prospect of a takeover is shown to be appealing to startups because the benefits of a takeover venture beyond the current invention via strengthening the incumbent’s bargaining position in future takeover deals due to an enlarge patent portfolio. As a result takeovers, contrasting licensing, can incentivize startup R&D under the threat of infringement litigation.

Appendix

Proof of Proposition 2: First, note that $X(t)$ is bounded below (by 0) and above because maximum surplus in each period is bounded and $\delta < 1$. If $X(t + 1) \leq X(t)$, then the right hand side of equation (7) would be non-positive and, furthermore, its value would strictly decrease when evaluated for $t + 1$ because $X(t + 1) \leq X(t)$ and $p_z(t + 1) > p_z(t)$. This would mean that $X(t + 2) - X(t + 1) < X(t + 1) - X(t) \leq 0$.

Applying the same argument repeatedly, we deduce that if $X(t+1) \leq X(t)$ then the sequence should decrease forever at an increasing rate after t , which is a contradiction because the sequence is bounded below. Hence, we conclude that $X(t+1) - X(t) > 0$ for all t . Since the sequence is bounded above, it further follows that it must converge. The limit value, $X(\infty)$ in (8), is obtained by setting $X(t+1) = X(t)$ and $p_z(t) = p_z(\infty)$ in equation (7) and solving for $X(t)$.

To show uniqueness, suppose to the contrary that there are two sequences, $\{X(t)\}$ and $\{X'(t)\}$, that satisfy (7), such that $X'(t') = X(t') + \gamma$ for some $\gamma > 0$ and t' . By (7), we have $X'(t'+1) = X(t'+1) + (1 + \frac{2(1-\delta)}{3\delta\eta})\gamma > X(t'+1) + \gamma$ and by repeating the same calculation, $X'(t) > X(t) + \gamma$ for all $t \geq t'$. This is impossible because both sequences should converge to the same limit as proved above, proving the uniqueness.

Finally, to show that $X(t) - X(t-1) > X(t+1) - X(t)$, note from equation (7) that

$$X(t+1) - X(t) - (X(t) - X(t-1)) = \frac{2(1-\delta)}{3\delta\eta}(X(t) - X(t-1)) - \frac{(p_z(t) - p_z(t-1))(b+r)}{3\delta}V^*. \quad (11)$$

If $X(t+1) - X(t) \geq X(t) - X(t-1)$ for some t , it would follow from equation (11) that $X(t+2) - X(t+1) \geq X(t+1) - X(t)$ because $0 < p_z(t+1) - p_z(t) < p_z(t) - p_z(t-1)$ due to the assumption that $\partial^2 p / \partial t^2 < 0$. Furthermore, $X(t+1) - X(t)$ would increase in t by repeated application of the same argument. This is impossible because the sequence $X(t)$ converges as shown above, hence we conclude that $X(t) - X(t-1) > X(t+1) - X(t)$. *Q.E.D.*

Proof of Proposition 3: To reach a contradiction, suppose to the contrary that in an equilibrium there is an arbitrarily large t such that a high-cost startup innovates with a positive probability in period t . Note that a takeover deal will be reached if an innovation takes place in period t , for otherwise a high-cost startup would not innovate because $\bar{s}_2(p_z(t)) < C$ by (3). Let \hat{X}_t denote firm 1's value at the beginning of period t , and let α_t denote the probability that an innovation takes place in period t . Since a low-cost startup always innovates, $\alpha_t = \eta + (1 - \eta)a_t \geq \eta$ where a_t is the probability that a high-cost startup innovates in period t . Then, $\hat{X}_t = \alpha_t(\hat{s}_{1t} + \delta\hat{X}_{t+1}) + (1 - \alpha_t)\delta\hat{X}_t$ where $\hat{s}_{1t} = \frac{1+p_z(t)(b+r)-r}{2}V^* + \frac{\delta(\hat{X}_{t+1}-\hat{X}_t)}{2}$, so that

$$\hat{X}_t = \frac{\alpha_t}{1 - \delta + \frac{3}{2}\delta\alpha_t} \left(\frac{3}{2}\delta\hat{X}_{t+1} + \bar{s}_1(p_z(t)) \right). \quad (12)$$

If $\hat{X}_t \geq \hat{X}_{t+1}$, firm 2's bargaining share in period t , $\hat{s}_{2t} = \bar{s}_2(p_z(t)) + \frac{\delta(\hat{X}_{t+1}-\hat{X}_t)}{2}$, is less than C because $\bar{s}_2(p_z(t)) < C$ by (3), and consequently, $\alpha_t = \eta$. Since $\alpha_{t+1} \geq \eta$ and

$p_z(t+1) \geq p_z(t)$, therefore, $\hat{X}_t \geq \hat{X}_{t+1} < \hat{X}_{t+2}$ would imply

$$\frac{\alpha_t}{1 - \delta + \frac{3}{2}\delta\alpha_t} \left(\frac{3}{2}\delta\hat{X}_{t+1} + \bar{s}_1(p_z(t)) \right) < \frac{\alpha_{t+1}}{1 - \delta + \frac{3}{2}\delta\alpha_{t+1}} \left(\frac{3}{2}\delta\hat{X}_{t+2} + \bar{s}_1(p_z(t+1)) \right),$$

contradicting the presumption that $\hat{X}_t \geq \hat{X}_{t+1}$ according to (12). Hence, we deduce that if $\hat{X}_t \geq \hat{X}_{t+1}$ then $\alpha_t = \eta$ and $\hat{X}_{t+1} \geq \hat{X}_{t+2}$, and by repeatedly applying the same logic, $\alpha_{t'} = \eta$ for all $t' > t$. Since this contradicts to the supposed equilibrium, we conclude that $\hat{X}_t < \hat{X}_{t+1}$ for all t . Since firm 1's value is bounded above, it further follows that $\hat{X}_{t+1} - \hat{X}_t \rightarrow 0$ as $t \rightarrow \infty$, which in turn implies that $\hat{s}_{2t} \rightarrow \bar{s}_2(p_z(\infty))$ as $t \rightarrow \infty$, contradicting the presumption that a high-cost startup innovates with a positive probability indefinitely. *Q.E.D.*

Proof of Proposition 4: In Section 4 we proved all claims of Proposition 4 apart from proving that firm 1's value function for $t \leq T$, denoted by $\hat{X}(t)$, is well-defined and unique. Below we provide a proof of this part.

We start with period T . Given that the total surplus to bargain over is $V^* + \delta(X(T+1) - \hat{X}(T))$ and the threat points are $d_1 = p_z(T)bV^* - \ell$ and $d_2 = (1 - p_z(T))V - \ell$ in period T , we calculate the Nash bargaining outcome $(\hat{s}_1(T), \hat{s}_2(T))$ as

$$\hat{s}_1(T) = \frac{1 + p_z(T)(b+r) - r}{2} V^* + \frac{\delta(X(T+1) - \hat{X}(T))}{2} \quad \text{and} \quad (13)$$

$$\hat{s}_2(T) = \frac{1 - p_z(T)(b+r) + r}{2} V^* + \frac{\delta(X(T+1) - \hat{X}(T))}{2}. \quad (14)$$

Plugging $\hat{s}_1(T)$ into equation (9), we derive $\hat{X}(T)$ in terms of $X(T+1)$ as

$$\hat{X}(T) = \left(1 + \frac{\delta}{2}\right)^{-1} \cdot \left(\frac{3\delta}{2}X(T+1) + \frac{1 + p_z(T)(b+r) - r}{2}V^*\right). \quad (15)$$

Furthermore, rearranging equation (7) for $t = T$ we get

$$X(T) = \left(1 - \delta + \frac{3\delta\eta}{2}\right)^{-1} \cdot \left(\frac{3\delta}{2}X(T+1) + \frac{1 + p_z(T)(b+r) - r}{2}V^*\right)\eta. \quad (16)$$

Since $1 - \delta + \frac{3\delta\eta}{2} > (1 - \delta)\eta + \frac{3\delta\eta}{2} = (1 + \frac{\delta}{2})\eta$, it follows from (15) and (16) that $\hat{X}(T) > X(T)$, hence $\hat{s}_2(T) < s_2(T)$. Note that $s_2(T) > C$ must hold for otherwise a high-cost startup would not innovate in period T because $\hat{s}_2(T) < s_2(T)$ would imply $\hat{s}_2(T) < C$. Also, $s_2(T+1) \leq C$ must hold for otherwise a high-cost startup would innovate in period $T+1$. Since $s_2(t)$ monotonically decreases in t by (6) and Proposition 2, therefore, we deduce that

$$T = \max\{t \mid s_2(t) > C\}. \quad (17)$$

If $\hat{s}_2(T) \geq C$, then a high-cost startup would innovate in period T as presumed. But, it is also possible that $\hat{s}_2(T) < C < s_2(T)$, in which case a high-cost startup would not innovate in period T . This problem is resolved when mixed strategies are considered: if a high-cost startup invests with an appropriate probability, $\hat{X}(T)$ gets reduced, pushing up $\hat{s}_2(T)$ to a level equal to C so that the startup is indifferent between investing and not. Specifically, if $\hat{s}_2(T) < C < s_2(T)$ we redefine $\hat{X}(T)$ and $\hat{s}_1(T)$ as $\hat{X}(T, a)$ and $\hat{s}_1(T, a)$ that solve

$$\hat{X}(T, a) = (\eta + a)(\hat{s}_1(T, a) + \delta X(T + 1)) + (1 - \eta - a)\delta \hat{X}(T, a), \quad (18)$$

$$\hat{s}_1(T, a) = \frac{1 + p_z(T)(b + r) - r}{2} V^* + \frac{\delta(X(T + 1) - \hat{X}(T, a))}{2}, \text{ and} \quad (19)$$

$$\hat{s}_2(T, a) = \frac{1 - p_z(T)(b + r) + r}{2} V^* + \frac{\delta(X(T + 1) - \hat{X}(T, a))}{2}, \quad (20)$$

for some $a \in (0, 1 - \eta)$ so that, in particular,

$$\hat{X}(T, a) = \left(1 - \delta + \frac{3\delta(\eta + a)}{2}\right)^{-1} \cdot \left(\frac{3\delta}{2} X(T + 1) + \frac{1 + p_z(T)(b + r) - r}{2} V^*\right) (\eta + a). \quad (21)$$

As a increases from 0 to $1 - \eta$, $\hat{X}(T, a)$ increases from $X(T)$ of equation (16) to $\hat{X}(T)$ of equation (15). Analogously, $\hat{s}_2(T, a)$ decreases from $s_2(T)$ to $\hat{s}_2(T)$. Since $\hat{s}_2(T) < C < s_2(T)$, it follows that there exists a unique value of $a \in (0, 1 - \eta)$, denoted by $\hat{a}(T)$, such that $\hat{s}_2(T, \hat{a}(T)) = C$. Thus, if a high-cost startup were to invest C with probability $\frac{\hat{a}(T)}{1 - \eta}$ in period T , its bargaining share would be $\hat{s}_2(T, \hat{a}(T)) = C$, ensuring that a high-cost startup is indifferent between innovating and not and thus, justifying the mixed strategy.

We have specified above the unique equilibrium behavior in period T , according to which a high-cost startup innovates with a positive probability. This, however, does not warrant that an innovation takes place for sure in all preceding periods $t < T$ because, as before, the prospect of sure innovation in period t may increase the value $\hat{X}(t)$ too much and thereby, reduce the marginal value of an additional patent ($\hat{X}(t + 1) - \hat{X}(t)$) too low a level to generate large enough a bargaining share for a high-cost startup to recoup its R&D cost C . In such periods, by recursively applying the same logic as above to periods $t = T - 1, T - 2$, and so on, we obtain a unique equilibrium strategy in which a high-cost firm innovates with a probability strictly between 0 and 1. Since this process is analogous to finding the probability $\hat{a}(T)$ explained above, we refer the details to a previous version of this paper (2008). Consequently, in the unique equilibrium high-cost startups innovate with a strictly positive probability in all periods $t \leq T$, but do not innovate in later periods. This completes characterization of the unique equilibrium for the cases that satisfy (3). *Q.E.D.*

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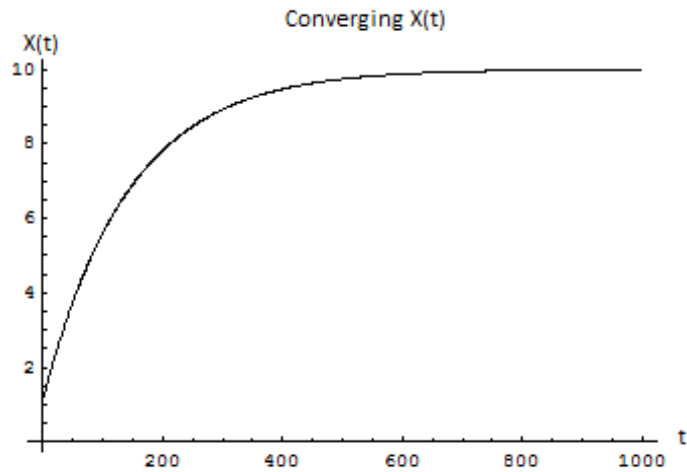


Figure 1: The converging $\{X(t)\}$ for 1000 consecutive startup innovations for $b = .5$, $r = 1$, $V = 1$, $z = .007$, $\delta = .97$ and $\eta = .4$.

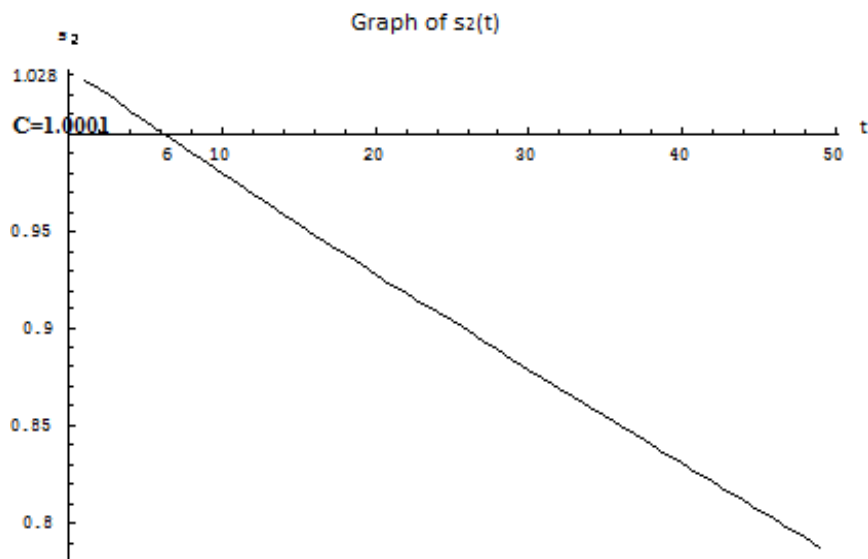


Figure 2: The simulated $s_2(t)$ shows that the last period for which $s_2(t) \geq C$ is $T = 6$ for $C = 1.0001$, $b = .5$, $r = 1$, $V = 1$, $z = .007$, $\delta = .97$ and $\eta = .4$. The vertical axis shows $s_2(t)$, and the horizontal axis shows $C = 1.0001$ and t .

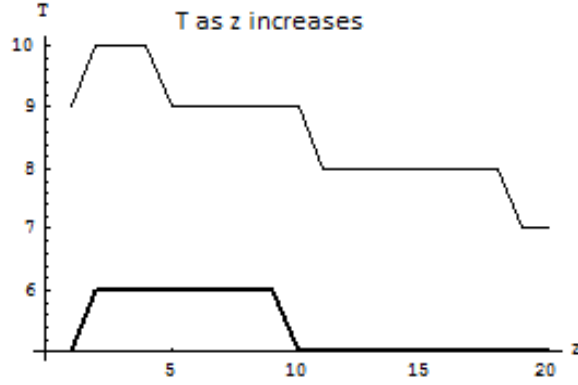


Figure 3: The simulated effect of z on T , calculated for z 's between .0001 and .02 in 20 steps of .001, for $C = 1.0001$, $b = .5$, $r = 1$ and $V = 1$. The lower graph plots the T 's for $\delta = .97$ and $\eta = .4$, while the upper graph does the same for $\delta = .98$ and $\eta = .8$.

z	T	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
0.001	5		0.54721	0.25398	0.10757	0.02161	
0.002	6			0.58337	0.27716	0.12410	0.03410
0.003	6			0.59185	0.27810	0.12302	0.03241
0.004	6			0.58478	0.27182	0.11782	0.02807
0.005	6			0.57365	0.26374	0.1116	0.02311
0.006	6			0.56106	0.25504	0.10509	0.017974
0.007	6			0.54788	0.24614	0.09848	0.01280
0.008	6			0.53449	0.23721	0.09189	0.00766
0.009	6			0.52111	0.22835	0.08538	0.00258
0.01	5			0.50811	0.21978	0.07910	
0.011	5			0.49557	0.21155	0.07308	
0.012	5			0.48322	0.20346	0.06717	
0.013	5			0.47107	0.19552	0.06137	
0.014	5			0.45913	0.18772	0.05569	
0.015	5			0.44741	0.18008	0.05012	
0.016	5			0.43592	0.17260	0.04466	
0.017	5			0.42466	0.16526	0.03931	
0.018	5			0.41361	0.15807	0.03408	
0.019	5			0.40279	0.15103	0.02895	
0.02	5			0.39219	0.14414	0.02393	

Figure 4: The equilibrium value of $\hat{a}(t)$ for each relevant $t \leq T$, calculated for the 20 different values of z that lay in the interval between 0.001 and 0.02, for $C = .0001$, $b = .5$, $r = 1$, $V = 1$, $\delta = .97$ and $\eta = .4$.

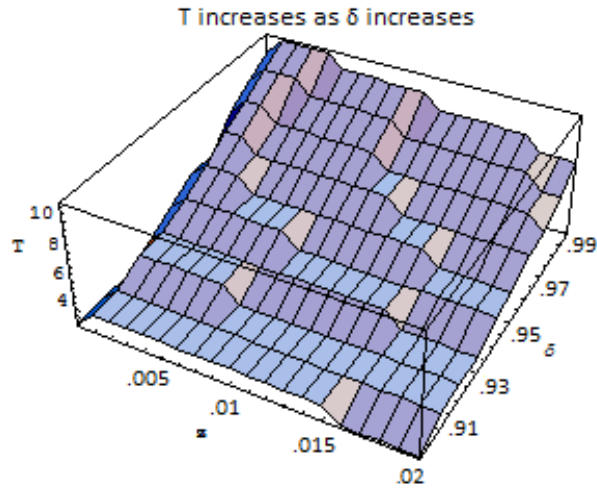


Figure 5: The simulated T that we derive by varying δ from .9 to .99 (in 10 steps of .01) and z from .001 to .02 in 20 steps of .001, for $\eta = .8$, $C = 1.0001$, $b = .5$, $r = 1$ and $V = 1$

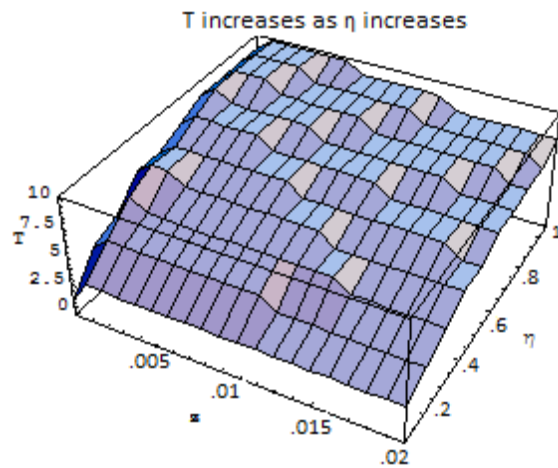


Figure 6: The simulated T that we derive by varying η from .1 to 1 (in 10 steps of .1) and z from .001 to .02 in 20 steps of .001, for $\delta = .98$, $C = 1.0001$, $b = .5$, $r = 1$ and $V = 1$.

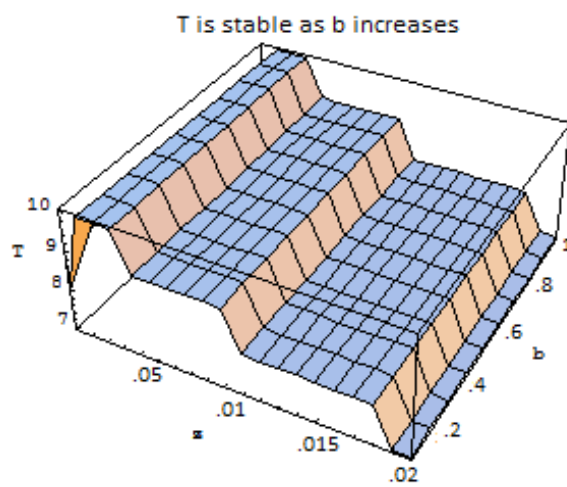


Figure 7: The simulated T that we derive by varying z from .001 to .01 in 20 steps and b from .1 to 1 in 10 steps, for $\delta = .98$, $\eta = .8$, $C = 1.0001$, $r = 1$ and $V = 1$.