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On the role of localised post-buckling equilibria in axially compressed cylinders

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Boston, 8 March 2019
Motivation

• Cylinder buckling governed by a subcritical bifurcation
  - leads to imperfection sensitivity
• Empirical (safe) design threshold, e.g. NASA SP-8007
  - too conservative for modern manufacturing methods
  - difficult for small satellite launchers
• Need less conservative knockdown factors

Small satellite launchers
e.g. Skyrora, Scotland
Smaller launchers are less
efficient
Need less conservative
knockdown factors

Illustration of perfect and imperfect bifurcation curves. For the unperturbed system, empirical (safe) design threshold, imperfect, and end shortening.
Approach

1. Study the equilibrium path of the buckling mode observed (most commonly) in experiments—the single dimple

2. Describe the deformation patterns that emerge as compression is varied

3. Derive a buckling threshold based on the level of compression for which the single dimple exists as an edge state

Eßlinger & Geier (1972)
Generalised Path-Following

- “Riks” path-following with additional capabilities:
  - equilibrium is stable or unstable
  - pinpoint critical points
  - branch switch at bifurcation points
  - track critical points with other parameters

- Run Yamaki’s (1984) longest cylinder:
  - $R = 100$ mm, $L = 160.9$ mm, $t = 0.247$ mm
  - $E = 5.56$ GPa (Mylar), $v = 0.3$
  - Both ends clamped + axial compression
  - Quarter model using nonlinear finite elements with large rotations
Single-dimple Snaking — Odd Nr Buckles

![Graph showing equilibrium paths and mode shapes](image)

- **A**: Fundamental path
- **SD**: Single-dimple saddle (mountain pass)
- **PB**: Post-buckling solution

**Figure 3**: (a) Equilibrium path of a single-dimple post-buckling solution growing sequentially via snaking. The single-dimple snaking solution (in red/blue) growing sequentially via homoclinic snaking (or cellular buckling). (b) The radial deformation mode shapes over the domain of the cylinder for different points O–V in (a). (c) Equilibrium path of a four-dimple post-buckling solution. The additional red/blue equilibrium segment shown in Figure 3c, the snaking equilibrium path of odd buckles, is once again delimited by a turning point close the first critical point on the fundamental path (see point O in Figure 3a). Point O describes two sets of two dimples located at a quarter-circumference to the left and right of the single-dimple solution (see Figure 3d). (d) The radial deformation mode shapes over the domain of the cylinder for different points O–E in (c).
Single-dimple Snaking — Even Nr Buckles

Snaking solutions of odd and even number of buckles are intertwined
Four-dimple Snaking

Figure 3: (a) Equilibrium path of a single-dimple post-buckling solution growing sequentially around the cylinder circumference through a series of destabilisations and restabilisations known as homoclinic snaking (or cellular buckling). (b) The radial deformation mode shapes over the domain of the cylinder for different points 0–V in (a). (c) Equilibrium path of a four-dimple post-buckling solution (in red/blue) growing sequentially via snaking. The single-dimple snaking solution from (a), shown in grey, connects to this red/blue path at a pitchfork bifurcation (see point D in inset B). (d) The radial deformation mode shapes over the domain of the cylinder for different points O–E in (c).

In Figure 3c, the snaking equilibrium path of odd buckles (Figure 3a) is shown in light grey for reference. The additional red/blue equilibrium segment shown in Figure 3c, which branches off point PB in Figure 3a, is once again delimited by a turning point close the first critical point on the fundamental path (see point O in Figure 3c). Point O describes two sets of two dimples located at a quarter-circumference to the left and right of the single-dimple solution (see Figure 3d). With decreasing end-shortening the two sets of two dimples grow in profile and through a series of de- and restabilisations grow from four to eight and, finally, to ten buckles. The mode shapes corresponding to various points on the equilibrium path of Figure 3c are shown in Figure 3d and clearly depict the progressive growth of the additional buckles. The two equilibrium paths shown...
Four-dimple Snaking

Yoshimura-like deep post-buckling pattern
Single-dimple Tracking

- From pre-buckling to stable diamond pattern via the single dimple
- When is the single dimple an unstable saddle?
- Use bifurcation tracking capability
for shells manufactured using modern manufacturing techniques. We therefore attempt to derive a less conservative
mode in experiments. Figure 10: (a) Critical boundaries denoting the evolution of the single-dimple limit point SD with respect to geometric
behaviour observed in experiments.

Single-dimple curve is a function of the Batdorf parameter

\[
\frac{P}{P_{cl}} = 1.48 \frac{R^{0.16}t^{0.16}}{L^{0.32}}
\]

\[
= 1.48\eta
\]

\[
Z/\sqrt{1-\nu^2} = \frac{L^2}{Rt}
\]

\[
= \eta^{-6.25}
\]
• Outlier by Hutchinson et al. (1971) is an axisymmetric dimple with relatively large amplitude

• Also, many data points from NASA SP-8007 exist below single-dimple curve, e.g. Donnell (1934)

• Asymptote NASA: $P/P_{cl} \sim 0.2$ and SD: $P/P_{cl} \sim 0.35$
  - previous manufacturing technology less controlled

• Buckling below the threshold is possible, but not likely with modern engineering process control

• Less conservative design guideline than NASA SP-8007 for cylinders manufactured to tight dimensional control
On the role of localised post-buckling equilibria in axially compressed cylinders

Thank you for your attention!
Single-Dimple Imperfections

Throughout this additional snaking sequence, the end-compression increases considerably until the entire cylinder is eroded to a post-buckled state corresponding to the well-known Yoshimura pattern (see mode (4) in Figure 3(b)).

Most experimental studies confirm this wave-form deep in the post-buckling range, and our analysis suggests that it lies on a continuous path beginning from a single dimple as the axial compression is increased.

As the equilibrium path is traced beyond this limit point, the stable single-dimple mode increases in size until it loses stability at a further limit point denoted by point (1) in Figure 3(a).

Previously we suggested that the small energy barrier around the pre-buckling well associated with the single-dimple imperfection at the cylinder mid-length and loading the cylinder in compression from the unloaded state. The equilibrium localization could readily be eroded by initial imperfections. We test this hypothesis by imposing an initial single-dimple imperfection which corresponds exactly to the figure reported previously by Kreilos & Schneider [11].

As the equilibrium path is traced beyond this limit point, the stable single-dimple mode increases in size until it loses stability at a further limit point denoted by point (1) in Figure 3(a). At this point, the dimpled post-buckling curve doubles back on itself and shows the winding path of a homoclinic snaking sequence. The inset in Figure 3(a) shows a feathered equilibrium manifold with five protruding dominant spikes. Each of these spikes corresponds to a unique odd-numbered mode of increasing frequency. As we proceed along the snaking path (in the general direction of decreasing load), the single dimple grows in the sequence of 1, 3, 5, 7 and 9 waves until an entire ring of buckles around the cylinder is filled. At point A (shown in the inset) the snaking curve takes a total of 10 circumferential waves.

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