



Salih, H., McCutcheon, W., Hance, J., & Rarity, J. (2018). Do the laws of physics prohibit counterfactual communication?. Manuscript submitted for publication <https://arxiv.org/abs/1806.01257>

Early version, also known as pre-print

[Link to publication record in Explore Bristol Research](#)  
PDF-document

This is the submitted manuscript (SM). The accepted author manuscript version (AAM) is available online via Arivx at <https://arxiv.org/abs/1806.01257>. Please contact the author for any terms of use of the publisher.

## University of Bristol - Explore Bristol Research

### General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available: <http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

# Do the laws of physics prohibit counterfactual communication?

Hatim Salih,<sup>1,\*</sup> Will McCutcheon,<sup>2</sup> Jonte Hance,<sup>2</sup> Paul Skrzypczyk,<sup>3</sup> and John Rarity<sup>2</sup>

<sup>1</sup>Quantum Technology Enterprise Centre, HH Wills Physics Laboratory,  
University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, UK

<sup>2</sup>Quantum Engineering Technology Laboratory, Department of Electrical and Electronic Engineering,  
University of Bristol, Woodland Road, Bristol, BS8 1UB, UK

<sup>3</sup>HH Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, UK

(Dated: September 27, 2018)

It has been conjectured that counterfactual communication is impossible, even for post-selected quantum particles. We strongly challenge this by proposing exactly such a counterfactual scheme where—unambiguously—none of Alice’s photons that make it has been to Bob. We demonstrate counterfactuality experimentally by means of weak measurements, as well as conceptually using consistent histories. Importantly, the accuracy of Alice learning Bob’s bit can be made arbitrarily close to unity with no trace left by Bob on Alice’s photon.

*Introduction.*— The prospect of communicating a message deterministically without exchanging physical carriers [1, 2], particles of light for example—apart from being utterly mind-boggling—raises deep questions about the nature of physical reality. What carried the message across space? And, for the case of transmitted information itself being quantum [3, 4]: Did quantum bits vanish from one point in space only to discontinuously appear elsewhere? No wonder, many prominent physicists were deeply skeptical—most recently Griffiths [5], and for different reasons, Vaidman [6–8], among others [9].

Recently, a proof [10, 11] of counterfactuality of Salih et al.’s protocol for communicating a classical message without the exchange of physical particles [1], and its first generalization to communicating quantum bits [3, 4], has been constructed based on a consistent histories approach. We noted in that proof that the actual Michelson implementation, which employs a polarization degree of freedom, allows for a proof of counterfactuality that the (pedagogical) Mach-Zehnder version does not. Our current proposal follows naturally.

*Setup.*—Our aim here is not to construct an efficient communication protocol, but rather to construct a communication protocol where 1) Alice can determine Bob’s bit choice with arbitrarily high accuracy, and 2) It can be shown unambiguously that Alice’s post-selected photons have never been to Bob.

Consider our proposed experimental setup of Figure 1, which consists of the equivalent of one cycle of Salih et al.’s (Michelson-type) protocol laid-out sequentially in time, as in [10]. The two underlying principles here are interaction-free measurement [12] and the quantum Zeno effect [13]. Here’s how the setup works. Alice sends a H-polarized photon from photon source S, whose polarization is then rotated by the action of polarization rotator HWP1, before polarizing beam splitter PBS passes the H part along arm A, while reflecting the V part along arm D. (All PBS’s pass H while reflecting V.) The V-polarized component in D then encounters a series of polarization rotators HWP2, each affecting a small rotation, and po-

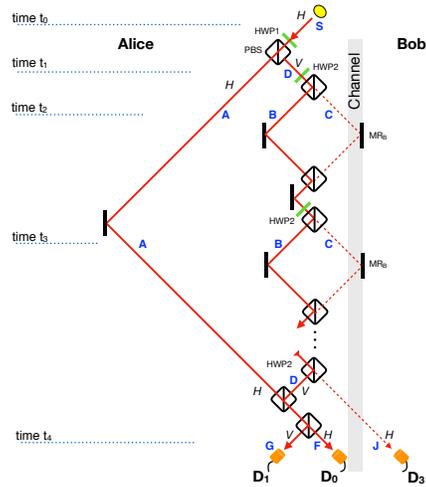


FIG. 1. Schematic setup. We want to know if photons detected at Alice’s  $D_0$  have been to Bob on the right-hand side.

larizing beamsplitters PBS, whose collective action is to rotate polarization from V to H, if Bob does not block the transmission channel. In this case, this component is passed straight towards detector  $D_3$ . If the photon is not lost to  $D_3$  then we know that it has traveled along arm A instead, in which case it passes through two consecutive PBS’s on its way to  $D_0$ . What happens if Bob blocks the transmission channel? Provided that the photon is not lost to Bob’s blocking devices, the part of the photon superposition that was in arm D at time  $t_1$  is now, after last HWP2, in arm D, V polarized. It is then reflected by two consecutive PBS’s on its way towards detector  $D_1$ . A click at detector  $D_1$  corresponds uniquely to Bob blocking the channel. But there’s a chance that the photon component that has traveled along A causes detector  $D_0$  to incorrectly click. For example, given that Alice had initially rotated her photon’s polarization after time  $t_0$  such that it is in arm A with probability  $1/3$ , and in arm D with probability  $2/3$ , and given a large number of HWP2’s such that the chance of losing the photon to

Bob's blocking device is negligible, then it is straight forward to calculate that the accuracy of detector  $D_0$  is 75%, in contrast to 100% accuracy for  $D_1$ , with half the photons being lost on average. Importantly, accuracy can be made arbitrarily close to 100%, by HWP1 initially rotating the photon's polarization closer to V, at the expense of more photons being lost.

*Proof of Counterfactuality.*—Now let's turn to the question of whether Alice's post-selected photon has ever been to Bob. It is accepted that for the case of Bob blocking the transmission channel Alice's photon could not have been to Bob. It is the case of Bob not blocking the channel that is interesting.

We first consider the question from a consistent histories (CH) viewpoint, building on the analysis in [10]. See [5] for a thorough explanation of CH. By constructing a family  $\mathcal{Y}$  of consistent histories (we will shortly explain what this means) between an initial state and a final state, that includes histories where the photon takes path C, we can ask what the probability of the photon having been to Bob is. Our setup allows us to do just that.

$$\begin{aligned} \mathcal{Y} : & S_0 \otimes H_0 \odot \{A_1 \otimes I_1, D_1 \otimes I_1\} \odot \\ & \{A_2 \otimes I_2, B_2 \otimes I_2, C_2 \otimes I_2\} \odot \\ & \{A_3 \otimes I_3, B_3 \otimes I_3, C_3 \otimes I_3\} \odot F_4 \otimes H_4 \end{aligned}$$

where  $S_0$  and  $H_0$  are the projectors onto arm  $S$  and polarization  $H$ , respectively, at time  $t_0$ .  $A_1$  and  $I_1$  are the projectors onto arm  $A$  and the identity polarization  $I$  at time  $t_1$ , etc.. The curly brackets contain different possible projectors at that given time. A history then consists of a sequence of projectors, at successive times. This family of histories therefore consists of a total of 18 histories. For example, the history  $(S_0 \otimes H_0) \odot (A_1 \otimes I_1) \odot (A_2 \otimes I_2) \odot (A_3 \otimes I_3) \odot (F_4 \otimes H_4)$  has the photon traveling along arm  $A$  on its way to detector  $D_0$ . Each history has an associated chain ket, whose inner product with itself gives the probability of the sequence of events described by that particular history. Here's the chain ket associated with the history we just stated,  $|S_0 \otimes H_0, A_1 \otimes I_1, A_2 \otimes I_2, A_3 \otimes I_3, F_4 \otimes H_4\rangle = (F_4 \otimes H_4)T_{4,3}(A_3 \otimes I_3)T_{3,2}(A_2 \otimes I_2)T_{2,1}(A_1 \otimes I_1)T_{1,0}|S_0H_0\rangle$ , where  $T_{1,0}$  is the unitary transformation between times  $t_0$  and  $t_1$ , etc. By applying these unitary transformations and projections, we see that this chain ket is equal to, up to a normalization factor,  $|F_4H_4\rangle$ .

A family of histories is said to be consistent if all its associated chain kets are mutually orthogonal. It is straight forward to verify that for the family  $\mathcal{Y}$  above, each of the other 17 chain ket is zero. For example, the chain ket  $|S_0 \otimes H_0, D_1 \otimes I_1, C_2 \otimes I_2, C_3 \otimes I_3, F_4 \otimes H_4\rangle = (F_4 \otimes H_4)T_{4,3}(I_3 \otimes I_3)T_{3,2}(C_2 \otimes I_2)T_{2,1}(D_1 \otimes I_1)T_{1,0}|S_0H_0\rangle = (F_4 \otimes H_4)T_{4,3}(I_3 \otimes I_3)T_{3,2}(C_2 \otimes I_2)T_{2,1}|D_1V_1\rangle = (F_4 \otimes H_4)T_{4,3}(I_3 \otimes I_3)T_{3,2}|C_2H_2\rangle = (F_4 \otimes H_4)T_{4,3}(|C_3H_3\rangle +$

$|B_3V_3\rangle) = (F_4 \otimes H_4)(|G_4V_4\rangle + |J_4H_4\rangle)$ , up to a normalization factor. Because projectors  $F$ ,  $G$ , and  $J$  are mutually orthogonal, this chain ket is zero.

Family  $\mathcal{Y}$  is therefore consistent, which means we can ask the question of whether the photon has been to Bob. CH gives a clear answer: Since every history in this family, except the one where the photon travels along arm  $A$ , is zero, we can conclude that the photon has never been to Bob.

We now ask the same question in the weak measurement language. Weak measurements [14], as the name suggests, consists of making measurements so weak that their effect on individual particles are smaller than uncertainty, and is therefore indistinguishable. These weak measurements are performed on pre- and post-selected states, meaning only particles that start in a particular initial state and are found in a particular final state are considered. By looking at a large enough number of such particles, these measurements acquire definite predictable values.

An elegant way of predicting the outcome of weak measurements, at least as a first order approximation, is the two state vector formulation, TSVF [15]. If the initial state evolving forward in time overlaps at a given point with the final state evolving backward in time, then a weak measurement made at that point is nonzero.

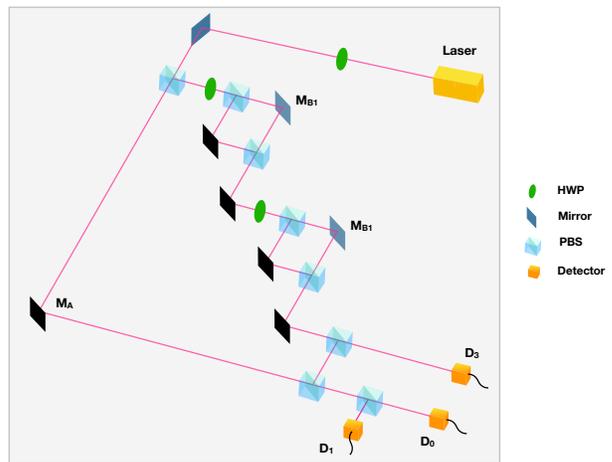


FIG. 2. 3D depiction of our experimental setup.  $M_A$ ,  $M_{B1}$  and  $M_{B2}$  are MEMS mirrors oscillating at different frequencies. The idea is that if a frequency associated with a given mirror is absent from the power spectrum at detector  $D_0$ , then we know that photons detected at  $D_0$  have not been near that mirror.

Let's apply this to our setup. The pre-selected state is that of the photon in arm  $S$ ,  $H$ -polarized. And the post-selected state, for the case in question of Bob not blocking, is that of the photon in  $F$ , also  $H$ -polarized. Consider weak measurements where Bob's mirrors,  $MR_B$ 's, are made to vibrate at specific frequencies, before check-

ing if these frequencies show up at a detector  $D_0$  capable of such measurement [16–18]. The forward evolving state from S is clearly present at Bob’s, because of the photon component directed by the action of HWP1 and PBS along arm D. What about the backward evolving state from F? A H-polarized photon traveling from F will pass through the two consecutive PBS’s along arm A, away from Bob. Since, the forward evolving state and the backward evolving state do not overlap at Bob, a weak measurement, at least as a first order approximation, will be zero.

We now show that any weak measurement at Bob will be zero—not just to a first order approximation. Consider a weak measurement where Bob vibrates one or more of his mirrors. This disturbance will cause the part of the photon superposition in arm D, after the last HWP2, which can only be V polarized, to be nonzero. This small V component will be reflected by two PBS’s towards  $D_1$ , and crucially, away from detector  $D_0$ . Bob’s action has no way of reaching Alice’s post-selected state: The photon has never been to Bob.

This experiment was performed, using a version of the setup in Fig 1., with two inner M-Z interferometers within the outer cycle of Salih et al.’s protocol (see Fig 2). In this setup, the single photon source is replaced by a 630nm laser, and three of the mirrors are MEMS - Alice’s mirror ( $M_A$ ) oscillates at 30Hz, Bob’s first mirror ( $M_{B1}$ ) at 40Hz, and his second mirror ( $M_{B2}$ ) at 50Hz. This oscillation is rendered sufficiently weak (0.01mm movement detected over a 5mm beam diameter at the detectors). As can be seen from Figure 3, in  $D_0$  we see the oscillations from Alice’s mirror, but not from either of Bob’s, proving that the weak measurement at Bob is zero.

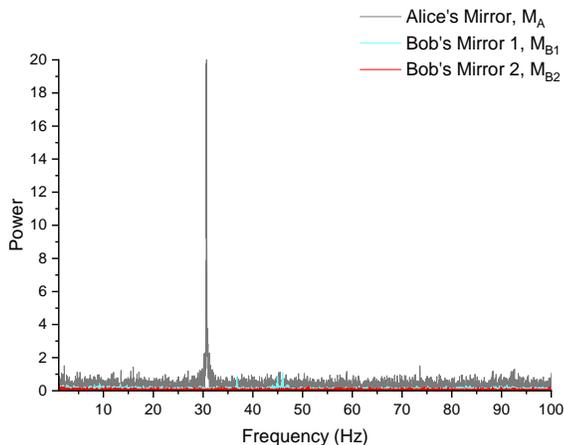


FIG. 3. Fourier transform of observed oscillation in position of light beam incident on detector  $D_0$ .

*Success Probability.*— The experimental setup is depicted in Figure 1. In each round of the proposed ex-

periment Bob chooses a bit,  $X$ , he would like to communicate to Alice. He blocks (does not block) his channel when  $X = 0$  ( $X = 1$ ). Alice then prepares a single photon, passes it through the system and it is either detected in one of the detectors  $D_0$ ,  $D_1$  and  $D_3$ , or is lost to Bob’s blocking device. If Alice detects the photon in either  $D_0$  or  $D_1$ , then the round was successful and Alice assigns the estimated values  $X_{est} = 0$  and  $X_{est} = 1$  to detections in  $D_0$  and  $D_1$  respectively. If the round was not successful another round is performed until she obtains a successful outcome. The post-selected data she obtains displays clear communication from Bob despite the fact that, as we have shown, the postselected photons *never* passed through the communication channel to Bob. Furthermore, by turning the initial half-wave plate, the system can be tuned to achieve a postselected success probability arbitrarily approaching unity, at the expense of decreasing the post-selection probability.

We explore the success of the scheme in terms of the free parameter  $P$ , the raw probability the photon would be found in the right half of the setup, determined by the setting of HWP1.

The raw conditional probabilities of detection in each of the detectors given Bob was Blocking (B) and Not Blocking (NB), for the infinite inner cycle version of the protocol, are presented in Table I.

TABLE I. Raw conditional probabilities

	<b>Blocking</b>	<b>Not Blocking</b>
$D_0$	$(1 - P)$	$(1 - P)$
$D_1$	$P$	0
$D_3$ (lost)	0	$P$

Consider for now the limit in which the probability of losing the photon to Bob’s blocking apparatus vanishes. Since Bob’s strategy for not blocking only succeeds with probability  $P(D_0|NB) = (1 - P)$ , Alice must perform these tests many times to get a successfully postselected event, so the raw probability that Bob was Blocking is greater than a half  $P_B \geq 1/2$ . This leads to apparent communication in the postselected data since the conditional probability of Blocking given detection events at  $D_0$  decreases.

We assume that on average Bob successfully sends as many zeros as he does ones. We have  $\langle X \rangle = 1/2$ , leading to the normalized postselected probability for not blocking  $\tilde{P}_B = P_B/N = 1/2$ , where the normalization factor is  $N = P_B + (1 - P_B)(1 - P)$ .

We then find that the probabilities of the postselected detection events are given by those in Table II, and the total probability of loss is given by  $P_L = P/(2 - P)$ . This corresponds to a postselected probability of correct outcome of  $P_c = (1 + P)/2$ .

We see that in the limit  $P \rightarrow 1$  the protocol becomes

TABLE II. Normalized Postselected Probabilities

	Blocking	Not Blocking
$D_0$	$(1 - P)/2$	$1/2$
$D_1$	$P/2$	$0$

deterministic, however the probability of postselection vanishes.

In Figure 4 we plot the overall probability of successful postselected outcome and postselection probability, and for different values of the probability  $P$ . Notably, for  $P = 1/2$  postselection succeeds with  $2/3$  probability, and is correct with  $3/4$  probability. Increasing  $P$  to  $2/3$ , the likelihood of successful postselection drops to  $1/2$  whilst the probability of being correct increases to  $5/6$ .

Finally, let's illustrate our findings using a somewhat amusing scenario. Imagine an outcome-obsessed lab director in charge of this experiment, who is quite happy firing Alice and Bob if a single run of the experiment fails, replacing them with a fresh pair of experimentalists, to start all over, also nicknamed Alice and Bob. The task for Alice and Bob is to communicate a 10-bit message, one bit at a time. Assume the experiment is set up such that the chance of any given run failing is one half. Therefore, in order to successfully communicate a 10-bit message, the lab director has to, on average, go through just over a thousand pairs of experimentalists—which the director in fact secretly enjoys. Each new pair of experimentalists is provided with a new message. Eventually, a lucky Alice and Bob manage to communicate their message. (Bit accuracy will be above eighty percent on average.) Now the question for the successful pair is: Has any of Alice's photons been to Bob? The answer, as we have shown, is an emphatic no.

In summary, we have shown that, given post-selection, sending a message without exchanging any physical particles is allowed by the laws of physics. What carries information, however, remains an open question.

While preparing this manuscript, the posting of a relevant manuscript was brought to our attention, arXiv:1805.10634 [19]. We will comment on its approach and results, which are distinct from ours, in due course.

*Acknowledgments.*— HS thanks Li Yu-Huai for useful feedback on the basic concept, received in autumn 2016. The authors thank David Lowndes for the components used during the experiment, Henry Webb and Holly Caskie for help with 3D drawing of the experimental setup, and Huxley Sessa for help with experimental prototyping. This work was supported by the UK's Engineering and Physical Sciences Research Council (EP/PS10269/1 and EP/L024020/1).

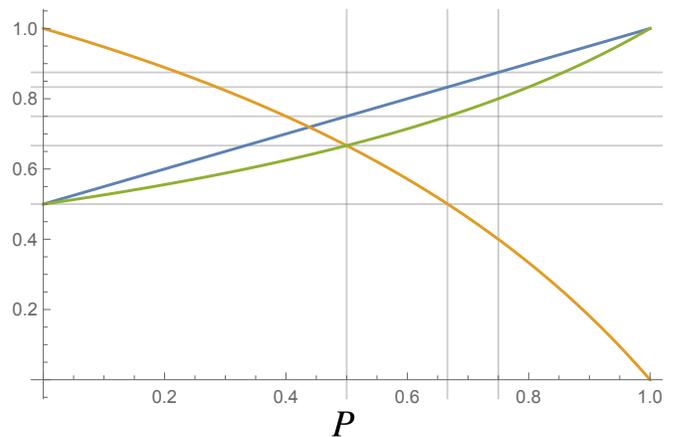


FIG. 4. The probability of postselected correct outcome,  $P_c$  (Blue), postselected correct outcome for when a photon is detected at detector  $D_0$  (green), and total postselection success probability (orange), for a given bit sent from Bob to Alice in the infinite-cycle case of the protocol—plotted against  $P$ , the probability of the photon entering inner interferometer.

\* salih.hatim@gmail.com

- [1] H. Salih, Z.-H. Li, M. Al-Amri, and M.S. Zubairy. Protocol for direct counterfactual quantum communication. *Physical Review Letters*, 110(17):170502, 2013.
- [2] Quantum mechanics: Exchange-free communication. *Nature*, 497(7447):9–9, 5 2013.
- [3] Hatim Salih. Protocol for Counterfactually Transporting an Unknown Qubit. *arXiv:1404.2200*, 4 2014.
- [4] Hatim Salih. Protocol for Counterfactually Transporting an Unknown Qubit. *Frontiers in Physics*, 3:94, 1 2016.
- [5] Robert B. Griffiths. Particle path through a nested Mach-Zehnder interferometer. *Physical Review A*, 94(3):032115, 9 2016.
- [6] L. Vaidman. Counterfactuality of 'counterfactual' communication. *Journal of Physics A*, 48(46), 2015.
- [7] Lev Vaidman. Comment on "Protocol for Direct Counterfactual Quantum Communication". *Physical Review Letters*, 112(20):208901, 5 2014.
- [8] H. Salih, Z.-H. Li, M. Al-Amri, and M.S. Zubairy. Salih et al. Reply. *Physical Review Letters*, 112(20):208902, 2014.
- [9] D. R. M. Arvidsson-Shukur, A. N. O. Gottfries, and C. H. W. Barnes. Evaluation of counterfactuality in counterfactual communication protocols. *Physical Review A*, 96(6):062316, 12 2017.
- [10] H. Salih. Comment on "particle path through a nested Mach-Zehnder interferometer". *Physical Review A*, 97(2):026101, 2018.
- [11] Robert B. Griffiths. Reply to "Comment on 'Particle path through a nested Mach-Zehnder interferometer'". *Physical Review A*, 97(2):026102, 2 2018.
- [12] Avshalom C. Elitzur and Lev Vaidman. Quantum mechanical interaction-free measurements. *Foundations of Physics*, 23(7):987–997, 7 1993.
- [13] B. Misra and E. C. G. Sudarshan. The Zeno's paradox in quantum theory. *Journal of Mathematical Physics*,

- 18(4):756–763, 4 1977.
- [14] Yakir Aharonov, David Z. Albert, and Lev Vaidman. How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. *Physical Review Letters*, 60(14):1351–1354, 4 1988.
- [15] Yakir Aharonov and Lev Vaidman. Properties of a quantum system during the time interval between two measurements. *Physical Review A*, 41(1):11–20, 1 1990.
- [16] A. Danan, D. Farfurnik, S. Bar-Ad, and L. Vaidman. Asking Photons Where They Have Been. *Physical Review Letters*, 111(24):240402, 12 2013.
- [17] Hatim Salih. Commentary: “Asking photons where they have been” - without telling them what to say. *Frontiers in Physics*, 3:47, 6 2015.
- [18] Ariel Danan, Dmitry Farfurnik, Shimshon Bar-Ad, and Lev Vaidman. Response: Commentary: “Asking photons where they have been” - without telling them what to say. *Frontiers in Physics*, 3:48, 6 2015.
- [19] Yakir Aharonov and Lev Vaidman. Modification of “Counterfactual communication protocols” which makes them truly counterfactual. *arXiv:1805.10634*, 5 2018.