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Counterfactuality, Definiteness, and Bell's Theorem

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Counterfactual definiteness is shown from analysis of Bell's Theorem to be the factor separating classical from quantum theories. From this, it is shown that, by replacing it with 'counterfactual semi-definiteness', the definiteness of possible options available after a measurement event, some apt analysis of possible states can be kept. While not as solid as that forbidden by the EPR paradox and Bell's Theorem, it allows us to start investigating the physical implementation of possible states in a way that has rarely been done. Working from this, the idea of counterfactuality, and interaction between counterfactual possibilities, is developed further.

INTRODUCTION

Counterfactual definiteness is the commonsensical idea that, were something to be done a different way, it is meaningful to talk about there being one 'definite' way in which the world would change. It forms a key part of our day-to-day life, to imagine that if there were a small change, the universe would be affected in a small, but measurable, and ultimately deterministic way, as ultimately that means that if everything were repeated in exactly the same way, the world would run as it has. However, since the advent of quantum mechanics, this has come under doubt. The inherently probabilistic nature of quantum collapse removes the security of the idea that, were you to repeat an experiment in exactly the same way, you would get the same result.

In this paper, we will investigate the arguments put forward that the only way to make sense of a quantum universe is to remove counterfactual definiteness, before then looking at just how strong this exclusion needs be. This will be done by evaluation of the necessity of the loss of counterfactual definiteness to various interpretations of quantum mechanics, and the credibility and usability of the resulting interpretation. Through this, we hope that at least something of our notion of common sense may be preserved when dealing with the quantum world, and that a more useful tool for dealing with apparent counterfactual definiteness may be sourced.

EPR AND BELL'S THEOREM

The EPR Paradox

EPR and Uncertainty

Einstein made use of the notion of conjugate quantities, for which the uncertainty in one must increase as that in the other decreases [1]. Between this concept, and that fact that quantum mechanics allows the states of two particles to become correlated in such a way as they can-

not be rendered independent of one another (the state cannot be written as a tensor product of two or more other states), Einstein came to a paradox - that by measuring one of a set of conjugate variables of one of a pair of two correlated particles, the conjugate variable for the correlated particle becomes uncertain.

For example, consider two particles (labelled 1 and 2), each described by their own (non-independent) wavefunction (ψ_1 and ψ_2). Were we to try and measure the position, x_1, of particle 1, we would apply the position operator $\hat{X}$ to it, allowing us to get an exact position for it. This would be at the expense of then becoming uncertain as to what it's momentum would be, were we to apply the momentum operator, $\hat{P}$, as the uncertainties of the two are related by the equation $\Delta x \Delta p \geq \frac{\hbar}{2}$. However, through taking the position of particle 1, we would also have to become uncertain of the momentum of particle 2, as otherwise from that we could derive particle 1's momentum using the system's initial conditions. But, before taking the measure of particle 1's position, we could have freely applied the momentum operator to particle 2, and obtained its momentum. Therefore, instantaneously, without making any direct contact with it, we are changing particle 2's state.

To Einstein, this problem, that you could go from something supposedly definitely existing to being indefinite - in his words, this loss of the element of reality corresponding to the momentum of the second particle - seemed a nonsense, and showed that, in his mind, the quantum mechanical treatment of reality was incomplete.

Bohm's Reformulation

However, the above example somewhat over-convolutes things by introducing the idea of conjugate variables. In his 1951 book, Quantum Theory, Bohm provides a simplification, doing away with non-commuting observables and focusing on the idea of a joint state which cannot be written as the tensor product of a single state per particle [2]. Bohm rendered this simplified case of Einstein, Podolsky and Rosen's argument into an immediately ob-

vious form by imagining the joint spin state, $|\psi\rangle$ of two electrons in such an 'entangled' set-up, written in Dirac notation as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle + |\downarrow\rangle \otimes |\uparrow\rangle) \qquad (1)$$

where the first ket applies to the first electron, and so on. As this joint state cannot be written as a product state, these two electrons cannot have their spin states described independently of one another. This means, when the measurement of one of the electron's spins collapses that state, it forces the overall state to collapse, pushing the other electron into its corresponding spin - both instantaneously and without any direct mechanism. Not only this, but the initial state into which the first electron collapses is random. This means there is no way the second electron can be said to be 'pre-prepared' into the corresponding state, to avoid this occurrence being both instantaneous and mechanism-free. This gets right to the heart of the paradox Einstein was talking about when he referred to 'spooky action at a distance' - information supposedly passing from one place to another, instantly, without mechanism.

Compared to what had appeared before, this result seemed deeply disturbing - gone is the stable, deterministic universe of classical physics, with the bounding limit for data-sending of special relativity. Instead, in its place, is something probabilistic, non-local, and all together less palatable than what came before. To reconcile this to the world around us, something previously assumed impossible by classical physics and special and general relativity must be allowed. The first possibility is to remove locality - to assume that, in these quantum cases, an instantaneous signal can be sent, and moreover sent without any visible mechanism, to allow the second particle to align with the result of the first. This obviously didn't appeal to Einstein, given it would mean nature violates Special Relativity, something previously considered fact. The second possibility is that in all previous experiments, all attempts to remove pre-existing correlations have failed, and that whatever causes the first particle to prefer one option forces the second into its corresponding state - the result was never random, but one of two similar-looking but ultimately superdetermined options. This, while initially promising, is just either claiming that all evidence so far for quantum theory is the result of undetectably small correlations, which grows less and less likely as information is gathered, or that we live in a fully superdetermined universe. The third option is to bite the metaphorical bullet, and deny that the state of the particles is counterfactually definite - to say that the universe is probabilistic, that repeating the measurement process exactly has no certainty of giving the same result, and that these simultaneous joint probability states (e.g. Eq.1) truly exist. While the previous two options violate

physical laws or experimental evidence, this one, while mathematically more allowable, seems to strike at the heart of what was viewed to be the rational assumptions of scientific theory - repeatability, rationality, and determinism.

To that end, Einstein, and later to a lesser degree Bohm, advocated that, rather than accept any of these three seemingly nonsensical options, quantum mechanics must be incomplete, and so there must be some local hidden variable that is governing the quantum phenomena in a way that at the least respects the basic tenets of our understanding of reality (locality/no superluminality, counterfactual definiteness, and the fact that all events can be causally and deterministically traced, rather than probabilistic). This debate, as to whether quantum theory was incomplete, and a local hidden variable model could complete it, was the basis of the debate between Bohr and Einstein - but very little could be said to prove one side over the other.

Bell's Theorem

"On the Einstein-Podolsky-Rosen Paradox"

For nearly thirty years, the question of which of these four possible options you believed remained, broadly, a point of interpretation - there was no real physical difference ascribable to either quantum mechanics being an incomplete local hidden variable theory, or being complete but meaning one of a number of weird conclusions for the universe. All this changed, however, with the work of Bell, who showed that, in certain circumstances, there would be a difference between the predictions resulting from a local-hidden-variable model and a fully quantum-mechanical model of the world [3].

Specifically, he proposed that there would be a set of experiments that one could undertake, where there would be an upper bound on the correlations that you would get classically (say, if the universe could be fully described by a local-hidden-variable model), that could be beaten if the problem was rendered into quantum-informational form, as would be valid if quantum mechanics provided a complete picture of reality. This formed the crux of what was later referred to as Bell's Theorem - that these Bell Inequalities, which act as limits when dealing with situations using a local-hidden-variable model, are both demonstrable, and can be demonstrably violated when using quantum mechanics.

CHSH

The most famous of these Bell Inequalities is the CHSH Inequality [4]. Here, we imagine a source that emits pairs of particles, each of which goes into a programmable test

unit. For the left-hand particle, it either is subjected to test a or a', and for the right, to test b or b'. All of these tests have result either $+1$ or -1 (e.g. tests of spin, where a and a' (b and b') are along different axes). While the two tests on each side have no requirement to test orthogonally to each other, the result is most prominent if they do (e.g. a and b testing spin along the x-axis, and a' and b' along the y-axis).

We then derive the quantum correlations, $E(x,y)$, for x being one of the tests on the left-hand particle (a or a'), and y being one of the tests on the right (b or b'). We do this by taking the four coincidence counts for our particular choice of x and y (N_{++}, when both detectors register $+1$, N_{--}, when both register -1, N_{+-}, when the left registers $+1$ and the right -1, and N_{-+}, when the left registers -1 and the right $+1$), and taking

$$E(x,y) = \frac{(N_{++} + N_{--}) - (N_{+-} - N_{-+})}{(N_{++} + N_{--}) + (N_{+-} + N_{-+})} \qquad (2)$$

to get a weighted measure of the quantum correlation of x and y. From this, we then generate S, where

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq 2 \qquad (3)$$

This gives us our Bell Inequality - if a quantum version of the system can get a value for S of greater than 2, then we have our test for which of these two forms the logic of the universe takes. This quantum case we can compare it to, often referred to as Tsirelson's Bound [5], is where we take the sum of the expectation values of the products of the observables that correspond to the tests ($\hat{A}$ for a, $\hat{A}'$ for a', etc...) to get

$$S = \langle \hat{A}\hat{B} \rangle + \langle \hat{A}\hat{B}' \rangle + \langle \hat{A}'\hat{B} \rangle - \langle \hat{A}'\hat{B}' \rangle \leq 2\sqrt{2} \qquad (4)$$

This shows that, in the quantum case, the Bell Inequality for the CHSH set-up can be violated - showing that a local-hidden-variable model fails to account for quantum correlations between the two particles. This shows that an experiment which gives us an S-value of greater than 2, as has just been shown to be possible, will prove Bell's Theorem, and show that the Universe doesn't obey a local-hidden-variable model. As will be seen below, this has been shown experimentally to be the case.

Experimental Verification

A large body of work has been done to experimentally test Bell's Theorem, given the difficulty in closing the loophole presented by trying to ensure that the apparent quantum correlations which allow the Bell inequality violation aren't due to preexisting correlations governing the choice of test. These experimental attempts started with work based on CH74, another Bell Inequality also made by Clauser and Horne [6], and CHSH - of these,

the most notable is that by Aspect et al, which involved the choice of test photons were subjected to being chosen while they were in flight [7, 8]. Since then, there have been many more tests, each with ever more ingenious ways of reducing the likelihood that the results are due to loopholes, to the extent that the most significant recent one, by Hensen et al, effectively closes this loophole of correlation entirely, presenting only that there can be prior correlation governing the choice of experiment if superdeterminism is accepted [9, 10]. This effectively means that, given this proven difference between how any possible local hidden variable theory would behave, and how our universe does behave, we can state once and for all that quantum mechanics isn't incomplete.

COUNTERFACTUALITY AND DEFINITENESS

The Trilemma Revisited

Given that experimental proofs of Bell's Theorem effectively remove the option that Quantum Mechanics is merely the incomplete form of a local hidden-variable theory, we have to move back to the other three options that were proposed, to allow a quantum-mechanical description of reality to match the universe. These options are either that superluminal communication is possible, superdeterminism is in effect in our universe, or there is no counterfactual definiteness. All of these have problems.

Superluminal Communication

The first option allowing the possibility of Bell Inequality violation is that, on the collapse of one particle's state, it can instantaneously send a message to its partner, allowing it to collapse into the corresponding state. This is the most physically palatable of the three options, insofar as it respects standard definitions of causality and definiteness, but it has one major flaw - it violates special relativity.

According to Special Relativity, the fastest that any information can propagate through space is at c, the vacuum speed of light [11]. This result has been repeatedly proven experimentally [12–16], and is regarded as a cornerstone of modern physics. Therefore, denying this, by saying that entanglement is somehow able to uniquely circumvent this limit and allow a causal change in the state of one particle simultaneously to that of its causer, seems implausible - not to mention, given the nature of Lorentz invariance, effectively identical to saying that the collapse of the wavefunction of a particle can cause something to occur which happened **before** it. Such a violation would be eminently observable, and would have had massive implications on the development of our theories on the nature of light - yet this hasn't shown up at all,

despite the massive impact that it would have. This leads us to conclude that such an ability of quantum systems to nonlocally communicate must be nonexistent.

Superdeterminsim

The second possibility is that the universe is superdetermined - that, above and beyond the typical mechanistic determinism caused by universal laws, each and every event is uniquely and independently caused. In such a system, each particle 'knows' in advance which state it will be in, in a way that requires no regular laws or rules to determine. While, on the surface, this approach resolves the issue of the supposed instantaneous, mechanism-free causation of the collapse of one particle based on the other (due to it instead having been separately pre-ordained into which states the particles will collapse), this approach is both epistemically tricky, and presents a number of physical issues.

The reason superdeterminism is considered epistemically difficult is that, given it relies on every single event having been caused independently and separately from any other event, it is impossible to establish a causal relationship between any two things - nothing causes, or is caused by, anything else. This means attempts to fabricate rules as to how certain events cause other events, such as the entire sum of the laws of physics, are rendered to nought - there can be no laws governing causal interactions as there are no causal interactions. Admittedly, such an argument relies on all events being superdetermined, but, given the sheer number of quantum events that would need to be, this doesn't seem too much of a leap. Further, it begs the question what force is predetermining these events? While initially popular as a school of metaphysics with proponents such as Bishop Berkeley, such ideas of grand superdeterminism by, say, a supreme being have fallen out of favour with even the majority of religious philosophers, who view the sheer sum of evidence based on otherwise coincidental repetitions between certain causes and given effects as proof that causality, in which physical events cause other physical events, seems far more parsimonious than everything being caused independently of any other event.

Aside from this more philosophical issue, there is also the more physical one whereby we have no mechanism by which such a full predestination could be 'remembered' by every single object in the universe. Such a choreographing of events would lead to a number of potentially ludicrous scenarios, such as being able to interrogate the future of anything in the universe from any other particle, allowing effective superluminal classical communication, and rendering meaningless all of the conclusions of special relativity. This, if anything, renders superdeterminism even less palatable than the previous option in the trilemma, as at least in the latter case, superluminal communication is restricted only to quantum events.

Counterfactual Definiteness

The final option posited by Einstein is the one about which he was most critical - the idea that the universe, at least when dealing with quantum interactions, lacks counterfactual definiteness. Counterfactual definiteness is where, if a measurement is repeated with no change in the conditions present, that the result will be the same. While the other two forks in the trichotomy grant possibilities whereby the conditions around a measurement may be altered without the measurer being able to do anything about it (either by instantaneous signalling, or vastly widening the number of them), claiming that the universe isn't counterfactually definite removes this property entirely - nature goes from being deterministic to probabilistic.

For what it is worth, it is this idea which Einstein was most unable to accept, repeatedly asserting God "does not throw dice" [17, 18]. However, as per the Born Rule, this is also a well-accepted element nowadays of Quantum Mechanics, and few serious physicists today would consider it an argument for the incompleteness of the theory, as inherently counterintuitive as it is. It does however, beg the question of just how counterfactually indefinite quantum theory actually is, given, barring measurement, wavefunction dynamics is regularly described as being deterministic, insofar as a wavefunction's time evolution can be entirely evaluated from its initial conditions. This leads us to wonder, in such a case, just how counterfactually indefinite quantum theory really is, and if a weakening of this definition can help it look more commonsensical to an outside observer.

Interpretations and Definiteness

The easiest way to evaluate the extent to which the removal of some form of counterfactual definiteness is needed is to look at the interpretations in which it is removed. Given quantum mechanics demands the existence of conjugate variables, and so the ability of a variable to be indeterminate (to allow its conjugate to be precisely known), all quantum mechanical interpretations must permit some element of 'fogginess' about the reality of certain variables. How they implement that fogginess, however, is where they differ.

Copenhagen Interpretation

Originally the standard interpretation, the Copenhagen Interpretation directly rejects questions of the sort

that would define whether the world was or wasn't counterfactually definite - in such an interpretation, a question such as "what would the spin of an electron be, if it was measured in the x-direction rather than the y?" are meaningless. In this almost quietist interpretation, the theorist is banned from even considering counterfactual cases, let alone assessing if they possess counterfactual definiteness. Therefore, this interpretation, alongside the other minimalist interpretations, the Ensemble and the Statistical Interpretation, doesn't contribute much to our discussion.

Everettian Interpretation

Everett's Relative State Interpretation, later extended into the wider-known 'Many-Worlds' Interpretation, posits the idea that different possible quantum states all co-exist - we merely observe one facet of the whole. This is analogised as a variety of alternate worlds, one for each quantum state, with the 'collapse' acting as a branching point from which they split. Therefore, in this interpretation, counterfactual definiteness is avoided by having not one definite counterfactual option, but many simultaneously-acting possibilities [19].

Bohmian Interpretation

In a Bohmian Interpretation, real particles exist, and they have a defined position, momentum, and real values for all other observables, as per classical physics. The key difference, however, which makes it a valid interpretation of quantum mechanics, is that each particle is subject to an expressly non-local quantum potential - thus, the position, and through that according to Bohmian mechanics all other observables, are perturbed into precisely the state that would be expected by quantum mechanics [20]. However, this still leaves the particles with an objectively-existing value for any required observable (in Bohm's theory, Heisenberg's Uncertainty Principle is a limit on our ability to know the value of conjugate variables, rather than an actual uncertainty on their existence), and so counterfactual definiteness exists - the element of the trilemma lost here is locality. To that end, Bohm theory shows potentially a route for the trading counterfactual definiteness in exchange for a weakening of one of the other facets - given that Bohm's non-locality doesn't allow faster-than-light signalling, it should preserve special relativity, despite still ostensibly containing non-local effects [21].

Collapse-Based Interpretations

Collapse-Based Interpretations of quantum mechanics are fairly broad, but have been grouped together here as they have one key feature in common. Whether based on consciousness, or gravity, or some other objective phenomenon, they all involve a process of collapse - at some point, typically measurement, the wavefunction's decay from many states to just one [22]. In such an interpretation, the other states, previously part of the wavefunction, are lost entirely - there is no definiteness associated with them, and so counterfactual definiteness doesn't exist in this theory. However, these theories also have the issue that collapse is difficult to observe, and we still have no idea what mediates it, or any reason for preferring one possibility over any of the others - showing that just removing counterfactual definiteness without any additional loss from one of the other possible options may not be as useful as thought for producing a valid interpretation of quantum theory.

Weakened Definiteness

Given what has been gleaned from looking at other interpretations, we can move on to looking at what is necessary for, and what can be excluded from, our interpretation vis-a-vis counterfactual definiteness.

Uncertainty of Conjugate Variables

The first key area which proscribed, based on above, is the simultaneous reality of conjugate variables. The only interpretation which proposes the real definiteness of both such variables in a pair is Bohm's theory, and it is counted as one of the key weaknesses of that theory. This is for two reasons. Firstly, progressive experiments have made it clearer and clearer that such a conjugate indeterminacy isn't to do with our ability to find out both variables, but down to the real lack of coexistence of them - the indeterminacy is built into the universe, as per Heisenberg's uncertainty relation giving a maximal co-resolution of $\frac{\hbar}{2}$, rather than being due to our apparatus. The second, and possibly more important point is that this indeterminacy seems key to how certain quantum objects interact and behave - such as an electron taking up all of its shell, rather than being solely a point that we struggle determining the exact location of, in order to allow Pauli exclusion to build the outer structures of atoms. While you could simply slide all of these properties to Bohm's quantum potential, eventually that seems simply to make the potential more the 'real' object, and the point-like particle an effectively non-existent marker used solely for the sake of our own comprehension - bringing in uncertainty indeterminacy by the effective

backdoor of the theory. This means, in all interpretations, as per experimental evidence, the uncertainty relation between two conjugate variables, and the indeterminacy caused by this, must be included.

Indeterminacy of 'Counterfactual' Options

Looking now slightly wider, we have to evaluate to what extent we can preserve counterfactual definiteness for those elements of a state that, prior to a point, were in a superposition within the state, but, after measurement occurred, no longer exist within it. Given the range of possible options, from the looser 'simultaneous reality' of all options posited by relative state/'many worlds' interpretations, to the complete indefiniteness of collapse theories, to the quietist ignorance advocated by the Copenhagen interpretation, we aren't ever left with any truly counterfactually definite options, but must find adequate ground to consider just what happens to the 'lost' parts of states post-measurement.

An easy initial way to go about this might be by considering system energy and entropy. The Von Neumann entropy is a quantum measure analogous to the classical Shannon entropy. As a way to consider information lost during a collapse process, by deriving the relative entropy between two states, it allows us to view the effective change during a measurement process. Given, in any collapse-based interpretation, this relative entropy actively counts as the information lost (as opposed to in relative state interpretations where the creation of 'alternate worlds' allows this information to be preserved), it seems far more informationally serendipitous to prefer interpretations where measurement doesn't cause information loss. To that end, by using a theory whereby states all have simultaneous existence, less information is lost, and so we can become at least slightly more counterfactually definite. It does this by positing, if not the sole and definite existence of a counterfactual possibility, at least the existence of multiple counterfactual options.

Counterfactual Semi-Definiteness

Based on this, linking to an Everettian perspective, looking to preserve as much counterfactual definiteness as we can while still respecting experimental evidence, we have stumbled upon this: counterfactual semi-definiteness. Through this, while being unable to attribute a single result to potential measurement as we can do classically with counterfactual definiteness, we are able to, for a finite number of possible states, define and determine the result that each of them would bring, and weigh this up as per the Born rule, allowing us not just to think in terms of possible worlds, but carefully weighted possible-world-ontologies. This

returns some measure of common sense to the discussion - as opposed to Copenhagen-style or collapse-style interpretations, whereby counterfactual possible ontologies are either ignored or treated as non-existent respectively, here we can discuss, evaluate and consider them. This holds use not just for after the point at which a different measure could be made, but before - as will be seen below, possible options already have been shown to be able to interfere to derive results in no way possible classically, which are ontologically even more promising than the simple idea of just one counterfactually definite option.

Shifted Counterfactuality

Looking now at counterfactuality less as what could be done by a person to a system, and more as, within these systems, ways measurement could have collapsed the state to in our world but didn't, we can see a fair bit of similarity, especially if we attribute counterfactual semi-definiteness to all these possible options. Indeed, prior to collapse, by the very nature of the density matrix formulation created by Von Neumann [22], we can see interactions between different modes representing different possibilities for the system to collapse into, which would be impossible under a classical system. Such interactions have been posited to do wondrous things, such as observing counterfactually something without damaging it, as per Elitzur and Vaidman's 'Bomb Detector' [23], or communicating without sending anything between two parties [24], as per Salih et al's counterfactual communicator [25–27] - things deemed impossible classically, and which, without the evaluation of these semi-definite options, would never have been thought of.

All this shows the need for a more open viewpoint on counterfactual options, or, even more broadly, metaphysical elements of our ontologies, than is prohibited by the traditional Copenhagen interpretation. These ideas were only opened up nearly forty years after the Selvig conference, by researchers such as Bell writing in Epistemological Letters [28]. The ability to put Einstein's challenge to this quantum orthodoxy into a testable form, even though it ended with the disproof of his ideal local-hidden-variable viewpoint, led researchers to once again plumb the more philosophical side of quantum mechanics, long-neglected by followers of Heisenberg and Dirac - and hopefully, either through counterfactuality specifically or through wider work in the field, will lead to the next key advances.

CONCLUSION

Through this evaluation, looking initially at the logic of Bell's Theorem, and then the trilemma that it causes, we

have been able to see just how necessary the lack of counterfactual definiteness, one of the most peculiar aspects of quantum mechanics, is to allow even basic quantum phenomena to occur. Alongside that, we have then been able to evaluate what the minimal amount of this loss is so we can still maintain the theoretical underpinnings of the experimental evidence we observe. Between these two sides, the problem of loss of counterfactual definiteness has been presented and evaluated, with the conclusion reached that, while Heisenberg's uncertainty relation prevents us from having the strong counterfactual definiteness of classical mechanics, we can at least obtain a form of counterfactual semi-definiteness. This definiteness of possible options available after a measurement event, while in no way as solid as that forbidden by Einstein, Podolsky and Rosen's paradox and Bell's Theorem, allows us to start investigating the physical implementation of possible states in a way that, until Bell, had rarely been done since the advent of quantum theory. Working from this insight, the myriad of potential areas of development in counterfactuality seems breathtaking, and likely to underpin developments in quantum theory and application for years to come.

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