This electronic thesis or dissertation has been downloaded from Explore Bristol Research, http://research-information.bristol.ac.uk

Author: Arena, Gaetano
Title: Adaptive Compliant Structures for Fluid Flow Control
A ‘catastrophic’ approach

General rights
Access to the thesis is subject to the Creative Commons Attribution - NonCommercial-No Derivatives 4.0 International Public License. A copy of this may be found at https://creativecommons.org/licenses/by-nc-nd/4.0/legalcode This license sets out your rights and the restrictions that apply to your access to the thesis so it is important you read this before proceeding.

Take down policy
Some pages of this thesis may have been removed for copyright restrictions prior to having it been deposited in Explore Bristol Research. However, if you have discovered material within the thesis that you consider to be unlawful e.g. breaches of copyright (either yours or that of a third party) or any other law, including but not limited to those relating to patent, trademark, confidentiality, data protection, obscenity, defamation, libel, then please contact collections-metadata@bristol.ac.uk and include the following information in your message:

• Your contact details
• Bibliographic details for the item, including a URL
• An outline nature of the complaint

Your claim will be investigated and, where appropriate, the item in question will be removed from public view as soon as possible.
Adaptive Compliant Structures for Fluid Flow Control

A ‘catastrophic’ approach

By

GAETANO ARENA

Department of Aerospace Engineering
UNIVERSITY OF BRISTOL

A dissertation submitted to the University of Bristol in accordance with the requirements of the degree of DOCTOR OF PHILOSOPHY.

JANUARY 2019

Word count: ca. 30000
AUTHOR’S DECLARATION

I DECLARE that the work in this dissertation was carried out in accordance with the requirements of the University’s Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate’s own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

SIGNED: .................................................... DATE: ..........................................

i
“Il tempo degli eventi è diverso dal nostro.” — Eugenio Montale

To my parents. An intrinsic source of pure and unconditional love.
I would like to gratefully acknowledge my mentor and friend Alberto Pirrera for his incomparable supervision, Rainer Groh for his constant motivation and technical support and Raf Theunissen for contributing to the project with his special knowledge of aerodynamics. A special thank you to Paul Weaver for supporting me with excellent advice and engaging enthusiasm. I would also like to thank William Sholten, Darren Hartl and Travis Turner for the fulfilling co-operation during the project with NASA on the tailoring of a composite slat-cove filler. I also wish to acknowledge the Engineering and Physical Sciences Research Council (EPSRC) for their financial support through the Doctoral Training Partnership with the University of Bristol.

Finally, I extend my gratitude to my family and friends, who have significantly contributed to the completion of this long and arduous journey.

Bristol, January 2019

G.A.
ABSTRACT

Structural adaptivity is an enabling concept when an engineering system requires trade-offs between stiffness, strength, weight and functionality. Adaptive devices, that sense and react to their environment, allow the system to change geometry and/or material properties in response to external stimuli. As such, they have huge potential for a broad range of engineering applications. Mechanical instabilities and elastic nonlinearities are one of the emerging engineering design methods to facilitate shape-adaptation and multistability. Although nonlinear adaptive structures have captured the interest of the research community, they have not yet found their way into industry. The complexity of nonlinear systems and lack of controlled actuation and precise design guidelines, contribute to practical skepticism towards the application of these kind of morphing structures.

This thesis illustrates a ‘catastrophic’ approach, in the sense of based on Catastrophe Theory principles, for the development of passively adaptive structures. Shape-changing devices are obtained by embracing elastic instabilities deriving from buckling of pre-compressed panels. A taxonomy of nonlinear post-buckling behaviours is defined and exploited as a design framework to obtain geometrical changes and multistability. The introduction of asymmetries into buckled morphing components enables tailoring of fold and cusp catastrophes, as well as the identification of regions of stability and instability. The definition of design spaces delimited by folds and cusps permits an intuitive design approach for embedding multistability and shape-adaptation into structures.

This design method is used to develop a morphing inlet that can morph and adapt shape in response to external stimuli and varying environmental conditions. In particular, the inlet is conceived to be open at low airspeeds, such that air can flow freely into a connected duct. When the airspeed increases above a pre-defined critical value, the adaptive component—a glass-fibre composite plate—snaps passively and closes the inlet to the duct, such that air ceases to flow into the channel. Experimental results from wind tunnel tests replicate design features such as folds and cusps, validating the usefulness of the ‘catastrophic’ approach. In particular, the location of the cusp catastrophe in the post-buckling regime is of interest to the designer, as it defines the onset of the hysteretic and snapping behaviours used for shape adaptation.

The general applicability of this design framework is further demonstrated by applying the same technique to the optimisation of an aircraft high-lift device component—a leading edge slat-cove filler for airframe noise reduction. The slat cove filler is designed to passively deploy and retract into the gap created by the high-lift device, in a manner that considerably
reduces induced noise, while maintaining the benefits of increased lift.

Given the generality of the underlying physical principles, the system’s parameters can be tailored to fit specific operating conditions, and consequently, are suitable for a wide range of applications where shape adaptation and passive actuation are desired.
Some of the ideas reported in this Ph.D. dissertation have been published as journal and conference publications.

**Journal Articles**


**Conference Contributions**


PLAGIARISM DECLARATION

I DECLARE that some of the text used in this dissertation has been taken from the published journal and conference papers indicated below, and that the text utilised was written by myself.


SIGNED BY THE FIRST AUTHOR: .................................... DATE: ............................

SIGNED BY THE SENIOR AUTHOR: ................................. DATE: ............................

xi
# Table of Contents

<table>
<thead>
<tr>
<th>List of Tables</th>
<th>xvii</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>xix</td>
</tr>
</tbody>
</table>

## 1 Introduction

1.1 An introduction to morphing structures ................................................. 3
1.2 Buckling: from aberration to design tool ............................................. 5
1.3 Thesis scope and aims .............................................................................. 6
1.4 Thesis outline ......................................................................................... 7

## 2 Literature review and background

2.1 Introduction .............................................................................................. 9
2.2 Morphing and adaptive structures .......................................................... 10
   2.2.1 Bio-Inspiration ............................................................................... 10
   2.2.2 Shape-changing methodology classification .................................... 12
2.3 Morphing structures for flow control ...................................................... 24
2.4 Catastrophe Theory .................................................................................. 26
2.5 Buckling and multistability for shape-adaptation ...................................... 28
   2.5.1 Buckling of a thin strut ................................................................... 28
   2.5.2 Equilibrium paths and dynamic snap-through .................................. 29
2.6 Fluid-structure interaction ....................................................................... 33
# TABLE OF CONTENTS

2.6.1 Couple Eulerian-Lagrangian approach ........................................... 33
2.7 Conclusions ....................................................................................... 37

3 Buckling failure as a means for shape-adaptation 39

3.1 Introduction ....................................................................................... 40
3.2 Geometry and boundary conditions ..................................................... 40
  3.2.1 Parametric nonlinear beam structural analysis .............................. 40
3.3 Results .............................................................................................. 43
  3.3.1 Multistability and snap-through .................................................... 43
  3.3.2 Taxonomy of nonlinear behaviours .............................................. 44
3.4 Conclusions ....................................................................................... 47

4 Adaptive air inlet: Design and Fluid-structure interaction simulation 49

4.1 Introduction ....................................................................................... 50
4.2 Model definition ................................................................................ 51
  4.2.1 Inlet geometry, materials and structural analysis .......................... 51
  4.2.2 Fluid-structure interaction .......................................................... 52
4.3 Results .............................................................................................. 54
  4.3.1 Snap-through analysis of the inlet morphing component ............. 54
  4.3.2 Aerodynamic actuation ............................................................... 55
4.4 Discussion ......................................................................................... 57
4.5 Conclusions ....................................................................................... 58

5 Proof of concept: wind tunnel experiments 59

5.1 Introduction ....................................................................................... 60
5.2 Design of multistable component ...................................................... 60
  5.2.1 Methods .................................................................................... 60
  5.2.2 Results ..................................................................................... 62
5.3 Manufacturing of the fiberglass component ........................................ 66
# Table of Contents

## 5.4 Design of test rig

## 5.5 Wind tunnel test

## 5.6 Results: Wind tunnel experiments

## 5.7 Discussion

## 5.8 Conclusions

## 6 A superelastic composite slat-cove filler for airframe noise reduction

### 6.1 Introduction

### 6.2 Model development

#### 6.2.1 Finite element structural model

#### 6.2.2 Constraints and requirements

### 6.3 Results

#### 6.3.1 Structural analysis of tailored SCF designs

### 6.4 Conclusions

## 7 Conclusions and future works

### 7.1 Future work

#### 7.1.1 Adaptive air inlet

#### 7.1.2 Slat-cove filler

#### 7.1.3 New ideas of adaptive devices for fluid flow control

## A Additional considerations on the design of the adaptive air inlet

### A.1 Stability analysis of elastica with vertical displacement at one end

### A.2 Effect of boundary conditions on equilibrium manifolds

### A.3 Preliminary FSI study with Abaqus ALE co-simulation

#### A.3.1 Model

#### A.3.2 Results

## B Technical drawings, rig sizing and stress analysis
TABLE OF CONTENTS

B.1 Technical drawings of air inlet assembly parts . . . . . . . . . . . . . . . . . . . . . . 123
B.2 ABAQUS structural analysis on test rig . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 131

Bibliography 133
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Sets of values used for the structural boundary conditions in the parametric analyses.</td>
<td>42</td>
</tr>
<tr>
<td>4.1 E-Glass 913 layer properties.</td>
<td>51</td>
</tr>
<tr>
<td>6.1 Stiffness distribution of Composite- and SMA-based SCFs.</td>
<td>86</td>
</tr>
<tr>
<td>6.2 E-Glass 913 layer properties and maximum values at failure.</td>
<td>87</td>
</tr>
<tr>
<td>6.3 SMA material properties.</td>
<td>91</td>
</tr>
<tr>
<td>6.4 Comparison of design with different material solutions.</td>
<td>98</td>
</tr>
<tr>
<td>B.1 Mechanical properties of assembly components.</td>
<td>132</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>The requirement triangle.</td>
</tr>
<tr>
<td>1.2</td>
<td>Buckling failure of rails.</td>
</tr>
<tr>
<td>1.3</td>
<td>Catastrophic collapse of the Hubert H. Humphrey Metrodome in 2010.</td>
</tr>
<tr>
<td>1.4</td>
<td>Schematic representation of adaptive air inlet.</td>
</tr>
<tr>
<td>2.1</td>
<td>The Venus flytrap.</td>
</tr>
<tr>
<td>2.2</td>
<td>NASA’s Morphing Aircraft vision.</td>
</tr>
<tr>
<td>2.3</td>
<td>CF-SMP composite skin’s deployment.</td>
</tr>
<tr>
<td>2.4</td>
<td>Flight demonstrator with morphing piezoelectric surfaces.</td>
</tr>
<tr>
<td>2.5</td>
<td>SMA phase transformations and stress strain curve.</td>
</tr>
<tr>
<td>2.6</td>
<td>NASA Super-elastic Tire.</td>
</tr>
<tr>
<td>2.7</td>
<td>Effect on Slat Cove Filler on flow streamlines.</td>
</tr>
<tr>
<td>2.8</td>
<td>Passive twisting of a composite beam structure by elastic instabilities.</td>
</tr>
<tr>
<td>2.9</td>
<td>Tow-steering manufacturing technique for bistable flaps.</td>
</tr>
<tr>
<td>2.10</td>
<td>Posite and negative Poisson’s ratio of honeycombs.</td>
</tr>
<tr>
<td>2.11</td>
<td>Relaxed and actuated 3D-printed auxetic structure</td>
</tr>
<tr>
<td>2.12</td>
<td>Use of elastic instabilities in research publications.</td>
</tr>
<tr>
<td>2.13</td>
<td>Buckling-induced morphing applications.</td>
</tr>
<tr>
<td>2.14</td>
<td>Morphing structures in fluid control applications.</td>
</tr>
<tr>
<td>2.15</td>
<td>The cusp catastrophe.</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

2.16 Buckling and equilibrium manifold of Euler beam. .............................. 29
2.17 Fundamental and bifurcation equilibrium paths. ............................... 31
2.18 Interface reconstruction methods .................................................. 35
2.19 ALE Abaqus/standard-explicit co-simulation (left) compared with CEL method. 35

3.1 Schematic representation of adaptive air inlet. ................................. 41
3.2 Parameters governing the elastic stability of a representative post-buckled beam. 42
3.3 Multistability and snap-through behaviour of a clamped-clamped elastic strip. 44
3.4 Bifurcation and force-deflection diagrams of a clamped-clamped elastic beam. 46

4.1 Composite tailoring of the inlet morphing component for FSI analysis. ....... 52
4.2 Portion of the computational domain for the FSI model. ....................... 53
4.3 Snap-through behaviour of bistable and super-elastic monostable adaptive inlets. 55
4.4 FSI results of passive actuation of a bistable adaptive air inlet. ................ 56

5.1 Adaptive air inlet demonstrator ....................................................... 60
5.2 Composite layups for the multistable component of the air inlet prototype. .... 62
5.3 Schematic 1D representation and dimensions of air inlet aperture. ............ 62
5.4 Finite Element buckling analysis of a multistable composite plate of the air inlet prototype. ......................................................... 63
5.5 Changing of locus of limit points with applied load. ........................... 65
5.6 Comparison of equilibrium manifolds for Lay-up 1 and 2. ..................... 67
5.7 Vacuum bagging of UD fibreglass plate. .......................................... 67
5.8 Manufactured UD fibreglass plate. ................................................ 68
5.9 Air inlet test rig details .............................................................. 70
5.10 Exploded view of CAD design of the air inlet test rig. ......................... 71
5.11 Assembly view of CAD design of the air inlet test rig. ....................... 72
5.12 Schematic of the adaptive inlet and six pressure taps. ........................ 73
5.13 Open and closed configuration of the air inlet demonstrator during a wind tunnel experiment. 75
5.14 Relative static pressure measurements. 76
5.15 Cups like behaviour of the air inlet during wind tunnel experiments. 79

6.1 Effect of Slat Cove Filler on flow streamlines. 84
6.2 Structural FEA model of 2D SCF assembly. 88
6.3 SCF composite layup. 88
6.4 Static mechanical properties of SMA and E-Glass 913. 92
6.5 Retraction process of $SMAM$ SCF. 95
6.6 SMA and composite based SCF structural behaviour. 96
6.7 Composite layup effect on SCF structural behaviour. 99

7.1 Deflection of SCF components under aerodynamic loading. 112
7.2 Adaptive cavity shape optimization of wind turbine blade. 114

A.1 An elastica with vertical displacement at one end loaded by a horizontal force. 117
A.2 Effect of ends rotations on equilibrium manifolds. 118
A.3 Effect of one end vertical displacement on equilibrium manifolds. 119
A.4 ALE FSI analysis of snap-through of a monostable structure due to an applied pressure field. 120
A.5 ALE simulation showing the snap-through of the monostable air duct. 121

B.1 Technical drawings of trapezoidal lead-screw Nut, trapezoidal lead-screw and support unit. 124
B.2 Technical drawing of HIWIN linear carriage. 125
B.3 Technical drawing of HIWIN linear rail and bottom support plate. 126
B.4 Technical drawing of outlet channel and front and back wall supports. 127
B.5 Technical drawing of Connecting support. 128
| B.6  | Technical drawing of Top plate and adjustable cover tap. | 129 |
| B.7  | Technical drawing of later wall and morphing component. | 130 |
| B.8  | FE structural analysis on air inlet prototype.          | 131 |
INTRODUCTION

Some people don’t like change, but you need to embrace change if the alternative is disaster.

Elon Musk

Generally, engineering systems are designed to meet multiple requirements that derive from: (i) the functionalities that the system is meant to fulfill; and (ii) the expected operating conditions/environment. When viewed in isolation, individual requirements can drive designs in opposing directions. The goal of classical design philosophies is to find the best compromise between competing design drivers. The disadvantage of such design philosophies is that a system’s performance will be suboptimal in most, if not all, of the individual operating conditions. In structural engineering, one possible refinement to this traditional design approach is the use of the so-called morphing and adaptive technologies, which allow structures to change geometry and/or material properties in response to external stimuli [1]. Particularly attractive from a weight and minimal design philosophy perspective are passively actuated adaptive structures. While active adaptive morphing
apparatuses rely on external actuation—be it mechanical, thermal or piezoelectric [1, 2], as e.g. the bistable air inlet designed and manufactured by Daynes et al. [3]—a passively adaptive structure does not rely on external mechanisms for actuation, but rather responds to changing environmental or operating conditions to drive actuation, either by modifying internal constitutive parameters or by exploiting fluctuating external forces. Consequently, these technologies promise less stringent compromises between load-carrying capability and functionality, while shedding the additional mass of external mechanisms [4].

Historically, structural instabilities and the multistability derived thereof, have been considered as catastrophic and detrimental phenomena [5]. On the other hand, mechanical instabilities and elastic nonlinearities are emerging engineering means for designing shape-changing devices and have placed the utility of instabilities into new relief [6–10]. This dissertation contributes to the progression of the aforementioned trend, by looking at the investigation and exploitation of a taxonomy of post-buckling behaviours of a fibreglass panel, with the final aim of designing an adaptive air inlet for fluid flow regulation. Sensing, actuation and control are entirely governed by the characteristics of the post-buckled component.

In this thesis, the development stages of an adaptive compliant air inlet are chronicled. Firstly, the buckling ‘failure’ of a fixed elastic beam loaded in compression is explored. In particular, a parametric study of the effect of various boundary conditions on the structure’s stability and multistability is carried out. This investigation provides fundamental insight for the design of a controlled multistable element, and the derived principles are then used to design the deformable portion of the adaptive air inlet. Based on this insight, multistable fibreglass buckled panels are designed that exploit mechanical instabilities and allow passive transition, at specific airflow speeds, between open and closed inlet configurations. The inlet’s passive-adaptive behaviour is investigated by means of fluid-structure interaction (FSI) analyses. These simulations aim to investigate the possibility of using pressure fields, generated by air flowing around the structure, as an actuating
1.1. AN INTRODUCTION TO MORPHING STRUCTURES

stimulus for passive shape adaptation. Subsequently, an air inlet prototype and a test rig were produced for experimental validation. The development of the prototype device involved: application of the concepts of Catastrophe Theory for the design of the multistable glass-fibre component; manufacturing of the composite panel and assembly of the test rig; experimental validation of the inlet’s adaptive behaviour in a wind tunnel.

Following the work on the inlet, the ‘adaptation-by-instability’ approach is then also applied to tailor a morphing component known as a slat-cove filler (SCF) [11]. SCFs are currently being investigated by NASA to reduce the airframe noise generated by unsteady airflow in the cove between the leading-edge slat and the main wing during low speed manoeuvres of approach and landing of transport aircraft. The SCF is designed to deploy and retract purely by interacting with the main wing as the slat is actuated. In its retracted configuration, contact between the main wing and the slat forces the SCF to conform to the space between the slat and the main wing. This deformed state is statically unstable under zero load, meaning that the SCF automatically deploys and snaps out into the cove as the slat moves away from the main wing. To tailor this actuation process and control the stability of the structure, a structural evaluation is carried out by studying the nonlinear ‘snap-through’ behaviour occurring during the retraction-deployment process.

1.1 An introduction to morphing structures

The continuous necessity for improving the performance, reliability and environmental safety of engineering systems requires a sustained stream of innovative solutions. Currently, morphing structures are identified as a viable option in a wide range of engineering fields.

In 2002, the United States’ Defence Advanced Research Projects Agency (DARPA), defined a morphing aircraft as ‘an adaptable, time-variant airframe, whose changes in geometry influence performance’ [12]. In other words, a smart structure that, by exploiting advanced materials, actuators and mechanisms, can adapt its shape to varying operating conditions. More recently, Brinkmeyer refined the definition of morphing as ‘a structure
Figure 1.1: The requirement triangle. A morphing structure lies at the centre of this triangle as it can be light, flexible and load-carrying. Reproduced from [4].

which undergoes large and seamless configurational changes in shape to continuously adapt to its environment’ [13], thereby excluding complex hinged mechanisms.

With this denotation, morphing structures are identified at the same time as shape-adaptable, lightweight and load-bearing. This trifecta therefore resolves the renowned engineering paradox illustrated by the requirement triangle in Fig. 1.1 and proposed in 2005 by Campanile [4], for which most engineering systems can be designed to fulfil only two out of the illustrated requirements. For instance, a common structure can be produced to be simultaneously light and stiff to withstand specific load conditions, but, if shape-adaptivity and flexibility are also desired, the system consequently requires additional shape-changing mechanisms, which necessarily add weight. On the other hand, a morphing structure can be designed to be load-carrying and to change configuration without needing hinge jointed frames, thereby minimising the total weight and existing in the centre of Campanile’s triangle.

Furthermore, morphing structures can be more specifically classified as adaptive when designed to modify and rearrange their behaviour and/or properties in response to varying
1.2 Buckling: from aberration to design tool

Engineering structures are normally designed to withstand loads that could cause buckling failure. Particularly in the case of thin-walled and light-weight structures, buckling instabilities can lead to collapse [14], as illustrated by the failure mode of rails depicted in Fig. 1.2 or the collapse of the roof of the Humphrey Metrodome in 2010, shown in Fig. 1.3. In these
cases, the sudden and *unexpected* loss of stiffness is obviously seen as catastrophic failure.

However, buckling instabilities, as well as gradual stiffness transitions, can be exploited to achieve quieter and more controlled *catastrophes*—where, this time, the term is used with the meaning attributed to it in the realm of Catastrophe Theory. As a matter of fact, in the last decade engineers have started to consider elastic instabilities and catastrophic transitions as design tools, causing a trend reversal towards *Well-Behaved Nonlinear Structures* [15–18] or, in Reis’ words, from *Buckliphobia* to *Buckliphilia* [19]: *when mechanical instabilities are carefully studied and consequently predicted, the deriving temporary loss of structural stiffness is no longer perceived as a catastrophic aberration, but rather exploited as a means for adaptation and multifunctionality*. With this new design paradigm, it follows that it would be possible to take advantage of concepts such as multistability and elastic ‘snap-through’ instabilities to create repeatable, ‘well-behaved’ shape changes.

In this dissertation a ‘catastrophic’—again, meaning based on ‘Catastrophe Theory’—approach is chosen to design the nonlinear multistable behaviour of adaptive structures for fluid control. This methodology is generally used to understand stability and predict sudden instabilities of any discontinuous and divergent phenomena, not necessarily only structural.

### 1.3 Thesis scope and aims

This thesis introduces conceptual design principles for a novel class of adaptive structures that provide both flow regulation and control. While of general applicability, these design principles, which revolve around the idea of using the instabilities and elastically nonlinear behaviour of post-buckled panels, are exemplified through a case study: the design of a shape-adaptive air inlet. The inlet, schematically shown in Fig. 1.4, comprises a deformable post-buckled member that changes shape depending on the pressure field applied by the surrounding fluid, thereby regulating the inlet aperture. By tailoring the stress field in the post-buckled state and the geometry of the initial, stress-free configuration, the deformable section can snap through to close or open the inlet completely. Owing to its inherent ability
1.4 Thesis outline

The work carried out during this PhD project is described in seven chapters and is summarized as follows.

Chapter 2 summarizes the literature on state-of-the-art shape-changing devices and their actuation methods. A brief theoretical background on the buckling of elastic beams,
catastrophe theory and FSI is also provided.

Chapter 3 concerns the effect of various boundary conditions on slender buckled beams. The results are used to create a taxonomy of post-buckling behaviours exploitable as a means for shape-adaptation.

Chapter 4 illustrates the FSI model of an air inlet, designed exploiting the information gathered from the previous chapter. The model is used to investigate the inlet’s morphing capabilities through passive aerodynamic actuation.

Chapter 5 provides experimental validation of the concepts introduced thus far. A multistable glass-fibre air inlet is designed, manufactured and wind tunnel tested.

In Chapter 6 mechanical instabilities are exploited further for the investigation and validation of a slat-cove filler (SCF) for airframe noise reduction.

Chapter 7 draws the thesis to a conclusion with a summary and review of the results and a discussion on potential future research and development directions.
LITERATURE REVIEW AND BACKGROUND

Share your knowledge. It is a way to achieve immortality.

_Dalai Lama_

2.1 Introduction

SIMILAR to shape-memory alloys, piezoelectric materials and polymers, mechanical instabilities and elastic nonlinearities are emerging engineering means for designing shape-changing devices. Since the 2000s, morphing and adaptive structures are increasingly using the above mentioned tools to achieve extreme deformations and higher flexibility during operation. This PhD project focuses on the exploitation of buckling and post-buckling instabilities for the design of adaptive devices for fluid flow control. Consequently, this chapter provides a background to morphing and adaptive structures and their means of shape-adaptation, as well as to theoretical concepts of elastic instabilities and to the fluid-structure interaction approach.
2.2 Morphing and adaptive structures

Engineering systems are generally designed to meet multiple requirements that derive from (i) the functionalities that a system is meant to fulfil and (ii) the expected operating conditions/environment. When viewed in isolation, individual requirements can drive designs in opposing directions.

The goal of classical design philosophies is to find the best compromise between competing drivers. The disadvantage of such design philosophies is that a system’s performance will be sub-optimal in most, if not all, of the individual operating conditions. In structural engineering, one possible refinement to this traditional design approach is the use of the so-called morphing and adaptive technologies, which allow structures to change geometry and/or material properties in response to external stimuli [1]. Specifically, morphing and adaptive structures promise to enable less stringent trade-offs between stiffness, strength, weight and functionality [4, 20].

2.2.1 Bio-Inspiration

The application of bio-inspired adaptivity in engineering, particularly in the aerospace sector, promises significant improvements in performance—and, in turn, environmental sustainability—by making structures more efficient over a broader range of operating conditions.

*Biomimetics* is defined as the imitation of nature and application of biological systems’ principles to science and engineering [22, 23]. A good example of its application in the aerospace sector dates back to 1891, when Prussian aviator Otto Lilienthal built and flew a series of gliders, using findings from the study of birds’ flight mechanics [24]. Similar approaches are used today, as the vast majority of the current research efforts on shape-changing technologies take their inspiration from nature, where many organisms use compliance to adapt to changing environmental or operating conditions [21, 25, 26]. Many organisms use adaptively compliant structures (*e.g.*, muscles) to adapt to changing
operating conditions. A fascinating example of shape-adaptability is given by the Venus flytrap (shown in Fig. 2.1), whose rapid transition from an open to a closed configuration to capture its prey occurs following a snap-buckling instability. Specifically, the Venus flytrap uses an instability, akin to the sudden kinking of a drinking straw bent between two hands, to enable a temporary loss of structural stiffness, thereby creating the means for quick shape reconfiguration [21].

The adaptive aerodynamic shape of bird wings is used in aerospace engineering as the archetypical example of evolution-optimised morphing. Two relevant governmental research programs proved the importance of bio-inspired studies for morphing systems. The NASA Morphing Aircraft Project [27], conducted from 1994 to 2004, enabled and incorporated research from bio- and nano-technology to adaptive structures and optimisation, in order to achieve a futuristic aircraft concept, shown in Fig. 2.2. This aircraft brought together most of the earlier morphing concepts to mimic the way a bird readjusts its profile to changing flight conditions. Although this morphing aircraft can be identified as one of the symbols that contributed to the beginning for the development of morphing technologies in aerospace
CHAPTER 2. LITERATURE REVIEW AND BACKGROUND

Figure 2.2: NASA morphing air vehicle concept. Reproduced from [27].

engineering, it remained purely a concept as it was remotely practically implemented.

Between 1995 and 2001, DARPA’s Morphing Aircraft Structures (MAS) Program aimed to improve the aerodynamic and aeroelastic performance of military aircraft by developing smart materials for surface control [28].

Recently, the Air Force Research Laboratory (AFRL) and FlexSys Inc., successfully tasted a shape-changing wing for next generation aviation. During the test, a seamless wing could morph its trailing-edge flap through angles ranging from −2 degrees up to 30 degrees [29]. Additionally, NASA also applied a new shape-memory alloy (SMA) that permits aircraft to fold their wings while airborne without the additional weight of a heavy hydraulic system [30].

2.2.2 Shape-changing methodology classification

An adaptive structure is defined as a system that can modify its shape and/or material properties to adapt to changing operating and/or environmental conditions [1].

The shape-changing process can be active, which means that its actuation requires external stimulus, or passive, where the structure adapts itself in response to varying
working conditions. Consequently, passively adaptive structures do not rely on separate, and often heavy, actuation devices for shape reconfiguration making them particularly attractive from a weight and minimal-design philosophy [8, 31–36].

In this section, a review of the different methods and/or tools used as either actuators or means of shape-adaptivity are presented. Several examples of adaptive structures are shown in which repeatable shape changes are obtained by either tailoring the properties of the constituent materials or designing specific stress fields into the structure.

### 2.2.2.1 Material properties

**Polymers.** Elastic and viscoelastic properties of polymeric materials have been widely exploited to achieve shape-changing components. Due to weak secondary chemical bondings between polymeric chains, these materials normally present particularly low elastic modulus, deformations up to 1000% of their initial length and complete elastic recovery, thus showing behaviours that can be associated with the shape-memory effect (SME) [2]. In addition, polymeric materials can be exploited for their reversible thermodynamic transition at the glass temperature ($T_g$), which delimits a distinct change in stiffness. Below $T_g$, the material is brittle and glassy, while at higher temperatures it becomes rubbery and viscous [38].

Yu *et al.* [37] fabricated a carbon fibre reinforced (CF) shape-memory polymer (SMP) to develop deployable wings for morphing aircraft, as shown in Fig. 2.3. The wing is conceived to be in a compressed and curled configuration before flight. An heating system is then used
to increase the temperature above $T_g$, thereby reducing material stiffness to unlock and deploy the wing.

Electro-active polymers (EAP) are also used for shape-changing applications as they produce high strains when electrically stimulated and present the advantage of a quick actuation response [39]. Pelrine et al. used dielectric elastomers, such as silicones, to make high-speed electrical actuators. They were able to achieve strains up to 215% and response times faster than those of natural muscle, making them suitable for artificial muscles [40]. Bar-Cohen [41] applied EAPs as artificial muscles in biomimetic robots to actuate their movements pneumatically. In 2007, Michael et al. [42] successfully applied EAPs in an aerospace application by testing a small air vehicle prototype where EAPs were used to control rudders and elevators [42].

Brinkmeyer et al. [43] exploited the viscoelastic properties of elastomers to obtain time-dependent pseudo-bistable morphing structures. They showed that viscoelasticity can temporarily lead to a loss of bistability and therefore cause an inverted configuration to snap back to its undeformed state.
2.2. MORPHING AND ADAPTIVE STRUCTURES

![Diagram](image)

Figure 2.5: (a) SMA phase transformation due to stress and strain. (b) Super-elastic behaviour of an SMA subjected to external load. Reproduced and modified from [22].

**Piezoelectric materials.** Piezoelectric crystals can produce an electrical charge in response to a mechanical load. Vice versa, strain (maximum 0.1% [45]) can be obtained by applying a voltage to the material. In morphing technology, these crystals are used to compose actuators that need to produce output forces and high-frequency responses. These characteristics are found particularly useful in the design of hydraulic pumps. For instance, O’Neil and Burchfield [46] developed a piezoelectric ceramic actuator to drive a hydraulic pump for powering a morphing wing during a remotely controlled flight demonstrator. Bilgen et al. [44] fabricated a piezocomposite (micro-fiber composite) actuator to change the control surfaces of an aircraft prototype. A micro-fiber composite (MFC) is composed of unidirectional piezoceramic fibers embedded in a thermoset matrix. An MFC, compared to a piezoelectric ceramic, has the advantage of being flexible, making it more practical for direct shape control [44]. In this research, Bilgen et al. tested a solid-state piezocomposite actuated morphing vehicle for the first time (Fig. 2.4). They assessed the effect of actuated morphing surfaces on the aerodynamic control during a complete flight.

**Shape-memory alloys.** Shape-memory alloys (SMA) exhibit a shape-changing mechanism based on a martensitic phase transformation driven by changing temperature and an
applied external load (Fig. 2.5). Figure 2.5b illustrates the typical static elastic response of an SMA when a load is applied. The graph clearly highlights the super-elastic characteristic of the SMA. The alloy behaves linearly until the applied stress reaches a critical value, \( \sigma_{Ms} \). At this level, a microstructural transformation from austenite to martensite occurs, causing a large deformation without incurring significant plasticity. When the transformation of the detwinned martensite is complete, the materials returns to behaving elastically. Load removal causes a linear-elastic recovery up to another critical value, \( \sigma_{As} \), which is lower than \( \sigma_{Ms} \), thereby showing hysteresis. Further load reduction leads to phase transformation from martensite to austenite and complete recovery [47].

Therefore, by purely considering the constitutive behaviour of the material, the super-elastic properties of SMAs seemingly lend themselves to shape-changing applications where large deformations and high load bearing is desired. SMAs are for these reasons largely used in morphing applications. Williams et al. [35] designed and characterised an adaptive vibration absorber with tunable natural frequency by exploiting the temperature-dependent elastic modulus of an SMA. Researchers at NASA’s Glenn Research centre used SMAs, instead of elastic materials, to develop a compliant tire [48]. These super-elastic tires, shown in Fig. 2.6, eliminate the possibility of failure and do not need an inner frame.

SMAs are also used for airframe noise reduction. Mabe et al. used SMAs to actuate a variable geometry jet nozzle and chevrons [49, 50]. The authors carried out a full scale flight test showing that the morphing chevrons significantly improved the mixing of the free-stream air and fan-stream to diminishing airframe noise.

Scholten et al. [51] developed SMA-based slat-cove fillers (SCFs). The SCF is a morphing component that reduces airframe noise produced by unsteady flow in the cove between the leading-edge slat and the main wing during low speed maneuvers of approach and landing of transport aircraft [52–57], as shown in Fig. 2.7. In its deployed state, the SCF protrudes outward from the slat, filling the cove to reduce noise. To eliminate auxiliary actuators and maintain mechanical simplicity, the SCF is designed to deploy and retract purely by
interacting with the main wing as the slat is actuated. In its retracted configuration, contact between the main wing and the slat forces the SCF to conform to the space between the slat and the main wing. This deformed state is statically unstable under zero load, meaning that the SCF automatically deploys and snaps out into the cove as the slat moves away from the main wing.
Limitations. Previous paragraphs presented a summary of the state-of-the-art research on the development of material-based morphing technologies. Some of these so-called ‘Smart Material’ systems have successfully been applied in the medical field [58], but their use in other industries is still progressing with difficulty. This is mainly due to the additional requirements that must be met. Shape changing materials have to withstand higher loads, support faster and larger geometrical deformation and operate within wide ranges of temperature and humidity. As examples, SMA and piezoelectric materials can perform differently depending on temperature and humidity conditions. Moreover, SMAs and SMPs are well known for their hysterical behaviour that allows super-elastic effects, but suffer from inherent limitations, such as thermo-mechanical fatigue, that cause deterioration of their performances with cycles [59]. Additionally, shape-memory materials require reaction times for the actuation that can be longer than those desired [60]. SMPs exploit glass transition phase to allow shape adaptation, not assuring high stiffness at all the stages of the performance. Similarly, piezoelectric materials present brittle behaviour that might not be suitable in applications where extreme deformations are required. In this regard, relying solely on the properties of smart materials as a means for shape adaptation can be limiting. For this reason, academic research is now also focusing on alternative solutions that exploit more traditional elastic deformations. In this case, shape-adaptation can be achieved by tailoring a structure enlarging its design space to incorporate stiffness couplings or elastic nonlinearities, thereby allowing the use of a wider range of more traditional and dependable materials.

2.2.2.2 Variable stiffness

In aerospace engineering, load-carrying is one of the most important performance considerations for a structure. Consequently, it is essential to select high stiffness materials while designing strong aerospace structures. At the same time, morphing devices need to be flexible and deformable without losing rigidity [4]. A structure with variable stiffness can guarantee
2.2. MORPHING AND ADAPTIVE STRUCTURES

Figure 2.8: Passive twisting of a composite beam by elastic instabilities. Geometry of beam and manufactured beam with buckling field introduced by shear load. Reproduced from [36].

Figure 2.9: (a) and (b) Tow-steering manufacturing technique for (c) bistable flaps. Reproduced from [61].

at the same time both high stiffness in the load direction and shape-adaptation, as it needs minimal actuation force to change shape in the softer direction [2].

Anisotropy. Adjustable stiffness of a structure can be obtained in multiple ways. The use of composite materials, for instance, permits tailoring by variations of thickness and fibre orientation. Additionally, the advantage provided by a morphing composite structure is the simplicity with which manufacturing techniques lend themselves to tailoring the structural behavior. Techniques such as layup vacuum-bagging and tow-steering [62, 63] allow local tailoring of the material stiffness, which is essential for the design of anisotropic shape-adaptive components.
Figure 2.10: (a) Positive Poisson’s ratio honeycomb and (b) negative Poisson’s ratio auxetic honeycomb. Reproduced from [65].

On the subject of stiffness adaptation, Runkel et al. [36] report of a thin-walled composite wing box that twists upon aerodynamic bending only past a prescribed load, shown in Fig. 2.8, for attaining passive shape adaptation. The phenomenon is induced by means of a variable (load-dependent) torsional stiffness. In particular, ‘on-demand’ twisting of the beam structure is enabled by the onset of non-catastrophic buckling in one of the webs of the box section. The approach permits the sudden onset of bend-twist coupling, where the onset of web buckling is controlled by tailoring the fibre angle orientations in the web.

Panesar and Weaver [61] used two tow-steered composite laminate configurations (Fig. 2.9) to manufacture a bistable flap with maximum angle of attack increment and maximum out-of-plane displacement. Similarly, Murugan and Friswell [64] studied the effect of curved fibre paths on the flexibility of a morphing wing. The results showed that with a tow-steering manufacturing technique it is possible to achieve a relevant increase in flexibility compared to the same plate with straight fibres.

**Auxetic honeycombs.** Auxetic honeycombs structures exhibit negative Poisson’s ratio as they become wider if stretched and narrower if compressed. This behaviour is achievable by designing the internal structure of the material with re-entrant unit cells [65], as illustrated in Fig. 2.10. By varying the geometry of the unit cells it is possible to optimize the structural
2.2. MORPHING AND ADAPTIVE STRUCTURES

Figure 2.11: Relaxed (a) and actuated (b) 3D-printed auxetic structure. Reproduced from [66].

behaviour and obtain changes in overall bending curvature that can be exploited for shape adaptation. In this regard, Theodoros et al. [66] developed an iterative method to define the morphing behaviour of an auxetic 3D-printed structure, which would change curvature when pneumatically actuated, as shown in Fig. 2.11. In aerospace engineering, auxetic materials can be applied to achieve aerodynamically clean surfaces to prevent boundary layer fluid separation, which is a limitation in performances of high-lift aerofoils. A common solution to this problem is the application of static vortex generation (VG). Michael et al. [67] produced a 2D auxetic lattice surface for a compliant VG that can be actuated to increase the prevention of fluid separations compared to a static VG. Neville et al. [68] developed a demonstrator of a Kirigami three-dimensional re-entrant auxetic honeycomb for volumetric deployment whose actuation is obtained by using SMPs.

**Elastic instabilities.** Figure 2.12 summarises a common thread found amongst recent research publications: the use of elastic instabilities in shape-changing and energy related applications is constantly increasing. Independent of the design features generating nonlinearity, adaptation can be realised through structural instabilities. This feature corroborates
CHAPTER 2. LITERATURE REVIEW AND BACKGROUND

Figure 2.12: Use of elastic instabilities in research publications. Reproduced from [6].

Figure 2.13: (a) Bistable airfoil flap composed by six prestressed GFRP laminates. Reproduced from [69]. (b) Bistable micro accelerometer. Reproduced from [70]. (c) Deployable sphere 'Buckliball'. Reproduced from [71]. (d) Trailing edge composed by two outer and inner skins able to achieve different configurations. Reproduced from [72]. (e) High sensitive mechanical nonlinear sensor. Reproduced from [73].
2.2. MORPHING AND ADAPTIVE STRUCTURES

the fundamental idea behind the work presented in this thesis and mentioned in Section 1.2: 
elastic instabilities or, in other words, the temporary loss of stiffness should no longer be 
considered as a catastrophic aberration [5], but rather exploited as a means for adaptation 
and multifunctionality [6, 19]. With this new design paradigm, it follows that it would be 
possible to take advantage of concepts such as multistability and elastic ‘snap-through’ 
instabilities [74] to create repeatable, ‘well-behaved’ shape changes.

One possibility of designing with instabilities is to induce initial stress fields. For instance, 
Daynes et al. [3, 7] manufactured a flap and an air inlet capable of snapping between two 
stable configurations. Both devices utilise prestressed composite elements for changing shape. 
For the former in particular, multistability is obtained by locking in self-equilibrated stress 
fields during the manufacturing process. A direct load is applied to the edges of a symmetric 
glass-fibre reinforced polymer (GFRP) before curing in an autoclave. This process causes a 
residual stress field making the composite plate bistable. The same authors combined six of 
these prestressed laminates to manufacture a bistable airfoil flap able to deflect by 10° [69], 
as shown in Fig. 2.13a.

In Runkel’s beam [36], as already described above and shown in Fig. 2.8, passive twisting 
is enabled by controlling buckling-induced instabilities. In a subsequent work, Runkel et al. [75], incorporated the beam as part of a wing box. The buckling-induced twisting de-
formation of the web leads to change in spanwise angle of attack, which can be exploited 
when load alleviation is desired. With this approach, buckling instabilities are applied to 
attain passively actuated and highly controlled linear reversible deflections. Conversely, 
the methodology presented in this thesis uses elastic instabilities to achieve sudden and 
fast change of configurations to adapt structures in response to varying environmental 
conditions.

Micro-electromechanical systems (MEMS) can also exploit the benefits of elastic instabil-
ities. Hansen et al. [70] developed a multistable micro latching accelerometer (Fig. 2.13b) 
where the switching thresholds varies between two stable states making electrical connec-
Shim et al. [71] designed a multistable buckling-induced continuum shell structure—the Buckliball in Fig. 2.13c—that shrinks and folds in an encapsulated stable configuration when subjected to pressure loading.

Wildschek et al. [72], conceived a trailing edge flap composed by two outer skins and two inner middle skins able to slide relative to each other (Fig. 2.13d). By pushing/pulling the inner skins independently, the trailing edge can achieve different configurations for various flight conditions and is capable of withstanding aerodynamic loads at high deflections.

In a recent work, An et al. [73] combined nonlinear dynamics and precision measurement to develop a force sensor for the detection of weak mechanical vibrations. As shown in Fig. 2.13e, the sensor involves a buckled cantilever tip which becomes softer near the bifurcation point (BP). The transition from the softened tip to the flipped configuration permits sensitive detection of noise without noticeable wear of the tip.

2.3 Morphing structures for flow control

In this work, passively adaptive morphing structures for fluid flow applications are designed and optimized by investigating buckling and post-buckling behaviour of composite structures. The following section highlights a few recent examples of shape-changing structures for flow control applications where shape-adaptation is also achieved by exploiting nonlinear instabilities.

Figures 2.14a-c show the carbon fibre reinforced polymer (CFRP) air inlet by Daynes et al. [3], which can switch between two stable equilibria. Its ‘open’ configuration is set when air flow needs to be directed inside the scoop, for instance, to cool an engine. When this is not necessary, snap-through of the scoop can be actuated to achieve a ‘closed’ state that makes the structure flush with the external aerodynamic surface, thereby reducing drag. The stable states do not require a holding force to withstand aerodynamic loads, but external actuation

24
2.3. MORPHING STRUCTURES FOR FLOW CONTROL

Figure 2.14: Bistable air scoop in its open (a) and closed (b) configurations. (c) Lateral profiles of the two configurations. Courtesy of Daynes et al. [3]. (d) Viscous flow through a channel containing a flexible wall. A thin elastic strip, buckled into an arch, initially constricts part of a channel (red shape). At higher flow rates, the arch rapidly snaps through (blue shape); the flow is then unconstricted and the channel’s conductivity increases. The three-dimensional view shows the finite channel depth. Red dashed curves show the shapes of the arch during a snapping experiment. Reproduced from [76]. (e) Schematic showing the components of soft valve with ‘open’ and ‘closed’ states. The snap-through between the two configuration is actuated by pressure difference. Reproduced from [77].
is required for bidirectional operation of the device. As mentioned in the previous section, bistability is here induced by applying prestress and stiffness tailoring.

Gomez et al. [76] showed the potential use of elastic instabilities in microfluidic applications. They demonstrated passive control of a viscous fluid flowing in a channel that involves an elastic arch, as illustrated in Fig. 2.14d. The snap-through of the arch, in this case, is passively actuated by the viscous fluid.

Rothemund et al. [77] conceived and fabricated a soft, bistable valve which acts as an air flow controller. Fig. 2.14e shows two stable states of an elastic dome. Snapping between the downward and upward configurations is actuated by two different pressure values. Also, both states are self-equilibrated, such that actuation energy is required only to switch between the states.

### 2.4 Catastrophe Theory

Catastrophe Theory refers to a branch of mathematics, introduced by Rene Thom in 1972 [78], that deals with discontinuous and divergent phenomena. It analyses degenerate critical points—i.e. points where first and higher-order derivatives are equal to zero—of a potential function. The theoretical framework is particularly effective in situations where nonlinear phenomena are characterized by sudden shifts in behaviour arising from small changes of certain parameters [79]. Consequently, such methodology is largely applied in structural applications [80].

Catastrophe Theory is fundamentally a branch of topology as the ‘catastrophes’ of the fold, cusp, etc., are all singular quantities of a multi-dimensional surface when projected onto a lower-dimensional manifold. The reason why catastrophe theory is so useful in structural mechanics is that systems in equilibrium can be described by smooth equilibrium surfaces parametrised by one or multiple parameters. A catastrophe, or instability, occurs when a topological singularity is traversed on the equilibrium surface. The possible shapes of the equilibrium manifolds are described in terms of a few archetypal forms, the so-called
elementary catastrophes [79]. When a function depends on a maximum of four factors, there are just seven elementary catastrophes.

In this work, the nonlinear behaviour of an elastic beam is exploited as a design tool. These behaviours, as it will be shown in the following Section 2.5 and in Chapter 3, are all examples of the ‘cusp catastrophe’. The potential function of a cusp-like catastrophe is of the form

\[ V(x) = x^4 + ax^2 + bx \]

where \( a \) and \( b \) are the control parameters respectively called ‘splitting’ and ‘normal’ factors.

Figure 2.15 shows a behaviour surface typical of a cusp catastrophe. Top and bottom
sheets fold over a middle curve—the fold curve. The projection of the fold on the space of control parameters \((a, b)\) results in a cusp point, which defines the threshold value for the bifurcation set, and a region, where sudden changes—\(i.e.\) catastrophes—occur. As long as the system remains outside or on the cusp, the function is smooth and continuous. When the control point \((a, b)\) is located inside the bifurcation set, two modes of behaviour are possible and connected by the fold curve, which represents an inaccessible region. The two accessible behaviours are connected through catastrophic jumps (\(e.g.\) snap buckling in the case of a structure).

2.5 Buckling and multistability for shape-adaptation

In the work presented in this thesis, nonlinear instabilities of elastic systems are investigated to be used as a design tool for fluid control devices. The following section provides a brief theoretical background to buckling, post-buckling and the important characteristics pertaining to shape-adaptation and the nonlinear concepts upon which the fluid flow control devices are based.

2.5.1 Buckling of a thin strut

The basic concepts of buckling ‘failure’ can be explained by considering the pin-jointed Euler strut illustrated in Fig. 2.16a. Here, the application of a compressive axial displacement, \(u\), initially shortens the structure in the flat configuration, but for levels of compression greater than the critical ‘buckling’ load, the strut suddenly deflects out-of-plane by \(\delta\), into one of the two mirror-symmetric curved configurations. Fig. 2.16b shows the equilibrium branches of a perfectly symmetric system in transverse deflection vs compressive load space (often referred to as a ‘pitchfork’ bifurcation due to its shape). The introduction of initial imperfections that break the symmetry along the arc-length of the strut also ‘breaks’ the pitchfork (Fig. 2.16c). In this case, as the compressive load is applied, the beam naturally follows a stable primary branch, deflecting into a preferred, bent configuration. A stable
2.5. BUCKLING AND MULTISTABILITY FOR SHAPE-ADAPTATION

Figure 2.16: Buckling failure of a pin-jointed beam and corresponding bifurcation diagrams. (a) A pin-jointed beam bows sideways when subjected to a compressive force greater than the buckling load. (b) An idealized symmetric beam with no geometric or loading imperfections features a symmetric pitchfork bifurcation diagram in load vs displacement space. For small levels of compression the beam remains straight. This equilibrium destabilizes at the buckling load and the structure deflects into one of two mirror-symmetric configurations. (c) Conversely, the bifurcation graph related to a beam with symmetry-breaking geometry and/or loading is characterized by a ‘broken pitchfork’. It shows a primary stable branch and a secondary equilibrium branch. By applying a compressive load, the structure naturally follows the primary branch. The second configuration can be reached by application of a transverse force.

A post-buckled structure is defined to be multistable when it can take two or more equilibria for the same set of external loading conditions [74]. As an example, consider the Euler strut in the inset of Fig. 2.17a, buckled into its first stable configuration (1). For given combinations of compression and symmetry-breaking defects, such a post-buckled structure can exhibit dynamic snap-through behaviour between the stable states, when subjected
to an external transverse load, $F$. These configurations are connected by an equilibrium path, shown graphically in Fig. 2.17a, where the magnitude of a centrally applied force, $F$, is plotted against the respective deflection at the midpoint, $\delta$. The structure initially deflects in a stable manner before reaching a maximum limit point, when the strut dynamically snaps through a region of instability (2) onto the segment of the equilibrium branch to which its second stable configuration (3) belongs. Figure 2.17b shows how the equilibrium path in the $F$ vs $\delta$ plane connects the two stable branches of the broken pitchfork. Figure 2.17b also shows how the maximum and minimum limit points of the $F$ vs $\delta$ equilibrium path change as a function of the compressive displacement, $u$, by means of the black foldline.

Specifically, the foldline tracks the two limit points with respect to changes in the compressive displacement, $u$, thereby illustrating the border between stable and unstable equilibria. By reducing $u$, the two limit points of the equilibrium path in Fig. 2.17a gradually collide in a cusp singularity. This cusp singularity therefore determines the critical value of compression, $u$, at the onset of dynamic snap-through behaviour [17, 81].

The investigation of equilibrium paths and limit points provides an essential tool for using elastic instabilities as a means of shape reconfiguration [15, 17, 81]. Indeed, referring again to the canonical Euler strut, depending on the value of compression, $u$, three distinct types of post-buckling behaviour can be observed when the transverse load, $F$, is applied [15, 81]:

1. For values of compression, $u$, greater than the limit point on the broken-away pitchfork branch, the structure snaps from its first stable shape to its second configuration, traversing the region of instability delimited by the foldline. A self-equilibrated second configuration exists (stable even when $F$ is removed). The structure is said to be bistable.

2. Reducing the compression, $u$, into the region between the limit point on the broken-away pitchfork branch and the cusp singularity, allows the beam to traverse a region
2.5. BUCKLING AND MULTISTABILITY FOR SHAPE-ADAPTATION

Figure 2.17: (a) Equilibrium path of force ($F$) vs central deflection ($\delta$). (b) Compression-central deflection-force space showing a broken pitchfork and the locus of limit points of the snap-through curve. Panels (c) and (d) show the locus of limit points in central deflection-compression and compression-force space, respectively. (All curves computed using the generalised path-following framework of Groh et al. [17])
of instability when $F$ is applied, thereby still exhibiting snap-through behaviour. However, at these values of compression, the structure does not have a second stable configuration for $F = 0$. This means that when the external force is removed, the strut snaps back to its primary state. In other words the structure shows ‘super-elastic’ monostability. This terminology is introduced first by Danso et al. [81] to describe the nonlinear snap-through dynamic behaviour of a monostable post-buckled structure. Generally, a super-elastic behaviour refers to intrinsic SMA materials response when subjected to high stresses. In fact, SMAs, after an initial linearly elastic response, can reversibly withstand extreme deformations. Since post-buckled structures that have only one stable configuration, but still show dynamic snap-through, can similarly exhibit reversible extreme deflections, the terminology ‘super-elastic’ is used in this thesis to describe a monostable snap-through behaviour and distinguish it from bistability and simple elastic stability.

3. By decreasing the level of compression, $u$, even further, the structure deforms non-linearly, displaying stiffness adaptation but without snap-through. The structure is elastically monostable or simply stable.

The mechanical instabilities of any structure can be investigated in order to define such a taxonomy of possible nonlinear behaviours, and this design space then exploited for shape-adaptivity. The control of geometrical parameters, material properties and/or boundary conditions can be used to tailor the equilibrium manifolds in order to modify and adapt multistability of the system to specific working and environmental conditions [15, 17]. As a novelty, this understanding of the relation between shape adaptivity, elastic instabilities and the principles of Catastrophe Theory, is exploited in this thesis as a means to design novel shape-adaptive well-behaved nonlinear structures.
2.6 Fluid-structure interaction

Fluid-structure interaction (FSI) is the study of the effect of a fluid on a deformable structure, and vice versa. In this kind of problems, temperature, pressure and velocity of the fluid influence, and are influenced by, the displacements, temperature and electrical field of the structure [82].

This project involves FSI simulations for the investigation of the influence of varying air flow conditions on the air inlet passive actuation. Due to its post-buckling behaviour, the morphing component in the inlet undergoes extreme deformations. As such, a coupled Eulerian-Lagrangian (CEL) approach is chosen for the FSI simulations presented herein, for the reasons explained in the following section.

2.6.1 Couple Eulerian-Lagrangian approach

Abaqus/CEL is a recently released extension of Abaqus/Explicit. With this software package, the interaction between fluid and structure is solved by means of contact constraints. On the other hand, the more commonly used arbitrary Lagrangian Eulerian (ALE) formulation takes advantage of the coupling of Abaqus/CFD and Abaqus/Standard solvers [83], and is therefore also known as a ‘co-simulation’.

The ALE method simulates the fluid dynamics by means of the Abaqus/CFD solver, which solves the incompressible form of the Navier-Stokes equations. Conversely, the CEL method uses an equation of state (EOS), assumed for relative pressure ($p$) as a function of density ($\rho$) and specific energy ($E_m$)

\[ p = g(\rho, E_m), \]

in combination with the compressible form of the Navier-Stokes equations [83].

The two approaches use the same Eulerian material tracking method, the so-called volume of fluid (VOF) method [84], which is based on the concept of fractional volume of fluid per mesh cell. Considering a mesh cell, the Eulerian volume fraction (EVF) can be defined as
a function $f \in [0,1]$, where $f$ is equal to 1 at any point occupied by fluid and to 0 anywhere else. The average of $f$ in the cell therefore indicates the fractional volume of the fluid in that cell. Subsequently, $\forall t \in [t_0, \infty)$:

$$
(fV)_{i,j,k}(t) = \int_{V_{i,j,k}} f(x,y,z,t)dV \iff f_{i,j,k} = \frac{\int_{V_{i,j,k}} f(x,y,z,t)dV}{V_{i,j,k}}
$$

where $f$ is the volume fraction and $V$ the cell volume, index $i,j,k$ indicate the mesh cell position. The volume of fluid method uses the function $f$ to identify mesh cells that contain fluid.

In a Lagrangian mesh, $f$ remains constant in each mesh cell. Conversely, in a Eulerian mesh $f$ must be updated to calculate its evolution and give the new fluid configuration at each time increment. Conservation of $f$ derives from the conservation of volume. By necessity of the divergence theorem, the net $f$ out of a control volume through its bounding surface, has to be equal to the time rate of decrease of $f$ inside the control volume:

$$
\frac{\partial}{\partial t} \int \int \int_V f dV = - \int_S f\mathbf{v} \cdot \mathbf{n} dS,
$$

where $\mathbf{v}$ and $\mathbf{n}$ are the velocity vector and the outward pointing unit normal field of the boundary $dV$ of the mesh cell, respectively.

In this case, VOF uses an Eulerian grid which makes $V$ time independent. Therefore, from the divergence theorem applied on the right hand side, it follows:

$$
\int \int \int_V \left[ \frac{\partial f}{\partial t} + \nabla \cdot f\mathbf{v} \right] = 0
$$

which is the integral form of the advection equation [85].

For an incompressible fluid:

$$
\int \int \int_V \left[ \frac{\partial f}{\partial t} + \nabla f \cdot \mathbf{v} \right] = 0
$$

Consequently, the evolution of $f$ is known but a reconstruction method is required to calculate where exactly the fluid is located in each mesh element. Abaqus/CEL uses a
second-order accurate interface reconstruction method PLIC (i.e. piecewise linear interface calculation) [86], shown in Fig. 2.18.

While the structural behaviour is analysed by means of a Lagrangian formulation in both cases, the FSI interface tracking methods are different. As shown in Fig. 2.19, the fluid mesh in the ALE formulation tracks the structural deformation, which can lead to highly distorted fluid elements when extreme structural deformations occur. The CEL formulation, on the other hand, is an ‘immersed boundary’ technique [26], where a Lagrangian structure is immersed in, free to move within, and deform through a fixed Eulerian fluid mesh. For this reason, in Abaqus/CEL, the fluid-structure interfaces are considered to be free boundaries, surfaces on which discontinuities exist in one or more variables. In fact, the influence of the structure is transferred to the surrounding fluid with interpolating functions and no
real interface exists in the fluid domain. Cells where $f$ has values between zero and one contain a free surface—e.g. fluid-structure interface. With the VOF approach, a mesh cell $(i, j,k)$ with a free surface is defined as a cell with a non-zero value of $f$ and with a minimum of one neighbouring cell, $(i \pm 1, j,k), (i, j \pm 1,k)$ or $(i,j,k \pm 1)$, with $f = 0$ [84]. Abaqus/CEL applies contact constraints between fluid and structure when, at the specific interface node, the EVF arithmetic mean of the surrounding elements is higher than 0.5. If the mean EVF < 0.5, contact is not imposed. The disadvantage of this approach is that, in some cases, a Eulerian material can penetrate through the Lagrangian contact surface [83]. This can lead to ‘leakage’ and inauthentic acceleration of the fluid [87].

In the ALE framework, on the other hand, the fluid mesh deforms following the movement/deformation of the solid body and kinematic and dynamic constraints at the fluid-structure boundaries are applied to guarantee the equivalence of the fluid and solid grid velocity at the interface. These constraints enforce equilibrium of the surface tractions

$$\sigma^f \cdot n = \sigma^s \cdot n \quad (2.7)$$

where $\sigma^f$ is the traction vector in the fluid, $\sigma^s$ is the traction vector in the solid, and $n$ is the vector normal to the boundary. Furthermore, the ‘no slip’ condition

$$u^f = \frac{\partial d^s}{\partial t} \quad (2.8)$$

must hold, where $u$ and $d$ are velocity and displacement vectors, respectively. In this way, the structural geometry within the fluid mesh is defined exactly. With this method, when large deformations of the structure occur, the re-meshing of the fluid domain is necessary.

For the problem considered here, the CEL approach was chosen for the following reasons:

- The ALE co-simulation requires a perfect match between the structural geometry in Abaqus/Standard and the structural profile within the fluid domain in Abaqus/CFD. For this reason, it is not possible to start from a previously defined orphan mesh (i.e. Lagrangian mesh imported from a structural-only analysis where the panel is
forced into its post-buckled state) of the buckled air inlet. The pre-compression step, required to induce multi-stability, must be performed within an ALE co-simulation. The CEL method, however, allows a buckled orphan mesh from a previous analysis to be imported.

- The ALE co-simulation is computationally very expensive due to the extreme deformation of the structure and the re-meshing of the fluid volume.

- Contact between different structural parts is not possible in the fluid mesh of the ALE model, as this would cause fluid elements to collapse. The adaptive air inlet studied here requires contact between the buckled multi-stable part and the inlet cover.

Even though the CEL method was chosen as more appropriate for this study, it is important to note that the fluid flow analysis carried out with this method is generally considered to be a less accurate implementation of CFD simulations [83]. For this reason, the FSI results shown in Chapter 4 are considered here as a qualitative demonstration of the passive actuation of the air inlet. Additionally, the adaptive behaviour of the structure is investigated experimentally and shown in Chapter 5.

## 2.7 Conclusions

Governmental programs and research interests show that morphing structures are not just futuristic concepts, but realistic solutions for performance improvement of engineering systems. Multiple methods and materials have been used and furthered to achieve the best trade-off between stiffness, flexibility and lightweight. In particular, as demonstrated by the growing research interest, elastic instabilities are clearly becoming an important aspect in the design of shape-changing structures. In fact, the advantage of exploiting a temporary loss of stability and stiffness of post-buckled structures allows to attain a precise and controllable tool for the design of a wide range of applications where structural dynamic and/or smooth shape-adaptive behaviours are required.
In the next chapter, a buckling and post-buckling study of a simple fixed beam is presented. Results show the potential use of the snap-through behaviour in adaptive applications.
I always tried to turn every disaster into an opportunity.

John D. Rockefeller

Abstract

Conceptual design principles for a novel class of adaptive structures are investigated. The nonlinear characteristics of a post-buckled clamped-clamped beam are studied parametrically in order to evaluate the influence of varying boundary conditions and the characteristics of stability of the structure, including snap-through and multistability. Results show a taxonomy of nonlinear behaviours that can be exploited to adaptive design multistable structures.
3.1 Introduction

This chapter presents the concepts upon which the design of the adaptive, variable-geometry device presented in the following chapters is based. The underlying working principle relies on the structurally nonlinear characteristics of a post-buckled beam. While of general applicability and interest, these design principles are applied in this work and exemplified through the design of the said shape-adaptive air inlet. With reference to Fig. 3.1, shown previously in Section 1.3 but reproduced here for convenience, the inlet comprises a deformable insert, set between rigid components, and a cover. These elements are arranged to form a channel that diverts part of the external flow to an outlet downstream. The deformable component morphs in response to the pressure field caused by the fluid flow. In particular, increasing air speeds create areas of low pressure that actuate the deformable component towards the cover, thereby closing the inlet. The morphing air inlet can therefore snap back and forth between an ‘open’ and ‘closed’ configuration, purely in response to air flowing at different speeds over the curved geometry. The kinematic characteristics of shape adaptation depend on the nonlinear structural mechanics of the post-buckled member.

Throughout this chapter, the exploration and successive exploitation of buckling ‘failure’ for the design of the adaptive structure is shown. In engineering parlance, the term buckling refers to a symmetry-breaking bifurcation, whereby a particular structural equilibrium (or state—often referred to as the fundamental state) becomes unstable causing a static or dynamic transition to a secondary configuration [88].

3.2 Geometry and boundary conditions

3.2.1 Parametric nonlinear beam structural analysis

In order to develop the insight required to design the morphing inlet, the geometrically nonlinear elasticity of a representative post-buckled clamped beam is investigated paramet-
3.2. GEOMETRY AND BOUNDARY CONDITIONS

Figure 3.1: Schematic representation of adaptive air inlet. (a) The air inlet, with the morphing component (in red) in its open configuration, can be actuated (b) and closed (c) by the pressure field imposed by a fluid flowing at a certain speed over the curved structure. The multistable properties of the structure and the fluid’s boundary conditions at the end of the duct dictate whether the inlet remains closed or opens again when the air speed reduces (d).

rically, as shown in Fig. 3.2. The results obtained permit a general understanding of how pre-loading and boundary conditions affect the post-buckling behaviour and its relationship with multistability. Specifically, with reference to Fig. 3.2, the influence of parameters such as end rotations ($\alpha$ and $\beta$), end transverse displacement ($w$), compressive displacement ($u$) and thickness variation along the length of the beam (obtained by linearly increasing the thickness towards the right end) is studied. Each displacement condition—$\alpha$, $\beta$, $w$ and $u$—is applied sequentially, as illustrated in Fig. 3.2 with values as per Table 3.1. The structure is then snapped into its inverted shape by means of a force-controlled arc-length method using the commercial finite element code ABAQUS [83].

The design of the adaptive air inlet is based on the parametric study of the buckling and post-buckling behaviour of a representative one-dimensional beam of unit length and circular cross section. Structural analyses were run for all possible combinations of the values indicated in the sets shown in Table 3.1. Equilibrium manifolds in transverse central deflection versus compression space are traced numerically by means of the ABAQUS implementation of the arc-length method based on Riks’ formulation [89, 90]. Load-displacement curves are traced using the same formulation with load as the arc-length parameter. In
Figure 3.2: Parameters governing the elastic stability of a representative post-buckled beam. The buckling and post-buckling behaviour of a clamped-clamped beam (a) was studied by investigating the effects on elastic stability of different boundary conditions parametrically. In particular, we varied the beam angle at the extremities (b), the vertical displacement of one end (c), the compressive shortening (d), and the variation of stiffness (thickness) over the length. The beam is then snapped to the other side via vertical point force.

Table 3.1: Sets of values used for the structural boundary conditions in the parametric analyses.

<table>
<thead>
<tr>
<th>Transverse displ.</th>
<th>Left end rotation</th>
<th>Right end rotation</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$ (mm)</td>
<td>$\alpha$ (deg)</td>
<td>$\beta$ (deg)</td>
<td>$d$ (cm)</td>
</tr>
<tr>
<td>{1, 2.5, 5, 7.5, 10}</td>
<td>{0, $-1.25, -2.5, -3.75, -5$}</td>
<td>{0, 1.25, 2.5, 3.75, 5}</td>
<td>{1.5, lin. var.*}</td>
</tr>
</tbody>
</table>

*d linearly increases from from the left end of 1.2 cm toward the right end of 1.5 cm.

In particular, a transverse force is applied at the mid point (i.e. the central node along the arc length) of the beam to make the structure transition towards the inverted equilibrium state. These calculations are performed for three different values of compression, $u = \{1, 3, 6\}$ mm.

For the finite element simulations, the beam is discretised with 2-node linear beam elements of the type B31. A total of 140 elements ensures a converged mesh. An isotropic elastic material with Young’s modulus of 70 GPa is used for the beam.
3.3 Results

3.3.1 Multistability and snap-through

Prior to presenting the results of the parametric investigation, multistability and dynamic snap-through instabilities, i.e. the concepts upon which the inlet design is based on, are formally defined and demonstrated by means of a toy model, as shown in Fig. 3.3. Figures 3.3a, b and Fig. 3.3d, e show the post-buckled configurations of a clamped-clamped strut with different boundary conditions. The stable states of the strut in Fig. 3.3a are connected by the equilibrium branch in Fig. 3.3c, demonstrating bistability. Similarly, Fig. 3.3f depicts the load-displacement diagram for the strut in Fig. 3.3d, demonstrating a monostable behaviour with snap-through instability. An immediate physical consequence of this difference in behaviour is that, although similar in shape, the structural configuration in Fig. 3.3b is in a state of stable equilibrium, whereas that in Fig. 3.3e must be held into position.

In summary, depending on pre-compression and boundary conditions, a post-buckled, clamped-clamped, slender strut presents the following nonlinear behaviours (see Fig. 3.4 for reference):

**Super-elastic monostability:** A region of instability under load control is observed in the load-displacement diagram. This region is confined between a local maximum and a local minimum. The strut is able to snap-through and so reach a distant inverted configuration. The inverted state is a stable equilibrium only under application of an external transverse load. The structure snaps back to the initial configuration when the transverse load is removed, because the load-displacement curve does not intersect the displacement axis other than at the origin (See Fig. 3.3f for reference).

**Bistability:** The strut exhibits two stable states upon application of a transverse load. When the applied force reaches a critical value, the structure snaps from the first stable state into the second configuration, traversing a region of instability. When the transverse
CHAPTER 3. BUCKLING FAILURE AS A MEANS FOR SHAPE-ADAPTATION

Figure 3.3: Multistability and snap-through behaviour of a clamped-clamped elastic strip. (a) The application of a transverse load to a bistable clamped strip in its first stable configuration causes snap-through into the inverted stable shape (b). (c) The applied load increases until it reaches a critical value. At this point the beam snaps through a region of instability, where applied load decreases, reaching a second stable branch. Upon load removal the structures settles on the secondary stable state. Similarly, a monostable buckled structure snaps from its first (d) to its second inverted configuration (e) when a transverse load is applied, but, as shown in (f), load removal causes snap back to the original unloaded equilibrium (d).

load is removed the structure remains in the inverted position at the point where the load-displacement curve intersects the displacement axis. Hence, a second stable unloaded configuration has been found (See Fig. 3.3c for reference).

3.3.2 Taxonomy of nonlinear behaviours

Figure 3.4 summarises key findings from the parametric analysis. Figure 3.4a shows a cut-set of the equilibrium manifold in central deflection versus compression space, for different amounts of vertical displacement of the representative beam’s right end. All equilibrium loci are symmetric unbroken pitchforks, as suggested by the fact that application of $w$ does not break the longitudinal (that is along the curvilinear domain) symmetry of the structure's displacement field. The greater the vertical displacement, the greater the required compressive displacement to buckle the beam. In fact, the buckling load scales approximately
3.3. RESULTS

with

\[ P_c \propto \left[ 1 - \frac{a^2}{8} \cos \alpha \right]^{-1} \quad (3.1) \]

where \( \cos \alpha = \sqrt{1 - \frac{w^2}{L^2}} \) and \( L \) is the undeformed length of the beam (see Appendix A for details).

For a given \( w \), Fig. 3.4b shows the influence of end rotation (\( \alpha \) and \( \beta \)) and the effect of longitudinal thickness variation on the equilibrium manifold. The rotational and thickness parameters introduce asymmetries that break the symmetric pitchfork, thereby resulting in two disconnected equilibrium branches. The specific shape of the broken pitchfork depends on the combination of the parameters’ variations. For example, the rotation \( \beta \) at the right end of the beam breaks the pitchfork and produces a large distance between the primary stable branch and the limit point on the secondary branch. Comparatively, this separation is smaller when an angle \( \alpha \) is applied or the beam thickness is varied along the length.

The degree of separation of the equilibrium branches is in fact crucial for design purposes, because the portion of the curve on the primary branch before the limit point of the secondary branch corresponds to a region of ‘super-elastic’ monostability, which is of particular interest for the inlet design. More detailed results on the effect of single boundary conditions on the equilibrium manifold are described in Appendix A.

The findings of the parametric post-buckling study provide general guidelines for designing the adaptive air inlet. Figure 3.4c shows the solution manifold for the beam with applied \( w = -5 \, \text{cm}, \alpha = 0 \, \text{deg} \) and \( \beta = 2.5 \, \text{deg} \). The graph illustrates the relationship between buckling and multistability by means of a two-dimensional equilibrium manifold presented in compression vs central deflection vs central transverse force space. This solution manifold therefore superimposes the load-deflection curves of transverse force, \( F \), versus central deflection on the bifurcation pitchfork diagrams, denoting the snap-through behaviour from one post-buckling configuration to another. For the beam with same conditions, the individual load-deflection snap-through curves are shown by means of an orthographic projection.
Figure 3.4: Bifurcation and force-deflection diagrams. (a) Bifurcation diagrams showing that imposition of a transverse displacement, \( w \), at one end does not break the symmetry of the pitchfork equilibrium manifolds, but increases the compression at which buckling occurs. (b) End rotations, \( \alpha \) and \( \beta \), and thickness variation along the beam length break the equilibrium pitchforks by breaking the symmetry of the structure. Two separate equilibrium branches appear: A primary stable branch and a secondary branch with a limit point, corresponding to a change of stability. The application of \( \beta \) produces a larger separation between the limit point and the primary stable branch than the application of \( \alpha \) or stiffness variations. (c) shows the relationship between buckling and multistability via a two-dimensional equilibrium manifold in compression vs central deflection vs central transverse force space. The equilibrium cut-sets in force vs deflection space demonstrate that, depending on the value of compression \( u \), three different scenarios are possible: if the compression is greater than the limit point on the secondary equilibrium branch, the structure snaps through to a second stable state (red curves); by decreasing the compression, the structure is monostable but still exhibits snap-through (blue curves); at a certain point snap-through is no longer possible and the structure is simply stable (green curves). The specific conditions are \( w = -5 \text{ cm}, \alpha = 0 \text{ deg} \) and \( \beta = 2.5 \text{ deg} \). (d) shows an orthographic projection of the equilibrium manifold in force-central deflection space.
in Fig. 3.4d, where three distinct types of post-buckling behaviours are observed. This observations allows a taxonomy of behaviours to be defined:

**Bistability.** For values of compression greater than the limit point on the secondary equilibrium branch, the structure shows the typical snap-through curves that intersect the central displacement axis at three points. These curves correspond to configurations where, when $F$ is applied, the structure snaps from its first stable shape to its second configuration, which is stable even when $F$ is removed. The structure is said to be bistable.

**Super-elastic monostability.** For smaller compressions, the beam becomes monostable but exhibits snap-through behaviour when subjected to $F$. In this case the structure does not have a second stable configuration when the external force is removed. Instead the system snaps back to its primary state upon load removal. The structure is said to be monostable with snap-through behaviour or to exhibit super-elastic monostability.

**Simple stability.** By decreasing the level of compression even further the structure deforms nonlinearly displaying stiffness adaptation, but not featuring any snap through. No instabilities are observed. In other words, the structure is simply stable.

### 3.4 Conclusions

In conclusion, the multistability of a clamped-clamped beam can be predicted in a straightforward manner simply by inspecting the bifurcation pitchfork diagrams. Therefore, to design an adaptive air inlet for a specific application and operational envelope, it is sufficient to produce and then study the equilibrium manifold of the morphing member. In the case of a symmetric structure, the post-buckled beam can only act in a bistable manner. When at least one symmetry is broken, the structure may also exhibit snapping monostability [91]. The size of the region where monostability is observed can be controlled by changing the boundary conditions, as detailed in Fig. 3.4. The observed taxonomy of behaviours can be used to design
adaptive inlets with different operational envelopes. It is important to note that during the design of a shape-adaptive structures, material limits (e.g. ultimate strains, delamination of composite materials, etc.) must be considered as additional design parameter. Specifically, the described dynamic snap-through behaviours observed in response to external transverse loads and the achievable extreme deformations can be limited by material performances.

While this chapter focused on studying the fundamentals of multistable beams under mechanical loading, the next chapter will delve deeper into an in-depth aeroelastic analysis of actuation by means of FSI simulations. In particular, a multistable air inlet is presented that exploits the design concept illustrated in this chapter and analysed by means of numerical simulations. The investigation focuses on the snap-through actuation achievable by taking advantage of the aerodynamic loads deriving from the interaction between the fluid and the buckled structure.
Adaptive air inlet: design and fluid-structure interaction simulation

Those who flow as life flows know they need no other force.

Lao Tzu

Abstract

Mechanical instabilities can be exploited as a means for shape adaptation. This chapter shows the design process for the composite morphing component of an adaptive air inlet. Results show the realisation of the taxonomy of nonlinear behaviours allowing the structure to respond to external stimuli as simply elastic, super-elastic or bistable, depending on different design parameter settings. An FSI study is carried out in order to demonstrate the passive response of the inlet to varying fluid flow conditions. Simulations prove the validity of using aero/fluid-dynamic pressure fields for actuation.
4.1 Introduction

The design principles investigated in Chapter 3 and the taxonomy on nonlinear behaviours unveiled provide a novel tool to design morphing structures. This chapter illustrates the use of elastic instabilities for the development of the multistable morphing component to be used within the adaptive air inlet design.

Let us consider the inlet in Fig. 3.1 of Chapter 3, which was conceived to be fully open at low air speeds. An increase of fluid velocity generates an area of low static pressure over the adaptive component. This pressure field is equivalent to a transverse load and can be used to actuate the component and to cause its shape to change to close the inlet. A morphing inlet with simple elastic stability would thus deform to decrease the inlet aperture. In this case, the extent of flow regulation is proportional to air speed and limited by the stiffness and maximum displacement of the adaptive member. Bistable or super-elastic inlets can be designed to decrease the inlet's aperture for increasing air speeds and close it completely upon snap-through at a critical fluid velocity. In this respect, these two designs are similar, but they feature a fundamental difference as air speed decreases: super-elastic inlets are able to snap back to the open configuration autonomously, bistable ones are not. A super-elastic device would be a preferred solution in applications where a ‘valve-like’ behaviour is required. A passively actuated bistable inlet can be exploited in applications where for safety reasons a channel must be suddenly closed when a threshold value of pressure or fluid velocity is exceeded.

The passive actuation of these inlets is investigated in this chapter, firstly with nonlinear structural analysis of the post-buckled structure and, subsequently, through an FSI study with a CEL approach.
4.2 MODEL DEFINITION

### 4.2.1 Inlet geometry, materials and structural analysis

The material of choice for the adaptive part of the inlet is a uni-directional glass fiber epoxy resin composite, E-Glass 913 (material properties as shown in Table 4.1). An isotropic elastic material with Young’s modulus of 70 GPa is used for the inlet cover and rigid components. The composite member measures 150 mm in length and 11.2 mm in width. The bistable design is composed of eight layers of glass fiber for a total thickness of 1.04 mm. A 15 mm vertical displacement and 1.5 mm of compression are applied to one end of the inlet, with no rotations at either side. The lay-up and boundary conditions are chosen following a preliminary FSI study (shown in Appendix A), carried out to ensure that the resulting stiffness of the morphing component would ensure passive snap-through actuation within the airflow velocity range 0-80 m/s (i.e. velocity achievable in the wind tunnel facility of the University of Bristol). The monostable design is derived from the bistable one altering geometry and pre-load. In particular, the thickness is varied along the beam length as shown in Fig. 4.1 and the longitudinal compression is lowered to 1.3 mm. All other properties and boundary conditions are kept unchanged. As a preliminary study, the nonlinear snap-through behaviour in the absence of air is evaluated, again using an arc-length Riks algorithm [89].

For both the mono- and bi-stable structure, an 8-node linear brick element (type C3RD8R), with reduced integration and enhanced hourglass control, is selected. Brick elements are required for ALE FSI analysis, while in a CEL FSI simulation, for the lagrangian part, both shell and brick elements can be used. However, preliminary study showed that a better
fluid-structure general contact interaction as minimized fluid penetrations is obtained with brick elements. A fine mesh with 568 elements is used along the beam length, which is sufficient to ensure convergence of the nonlinear post-buckling behaviour. One element for every two layers is used in the thickness direction; further fidelity is not necessary because the composite layup is uni-directional.

4.2.2 Fluid-structure interaction

The effect of the aerodynamic loads on the adaptive air inlet is studied by means of fluid-structure interaction (FSI) simulations. Due to extreme deformations of both the structural and fluid domains, a Coupled Eulerian-Lagrangian (CEL) approach is chosen [83, 93]. A preliminary FSI study carried out with Adaptive Lagrangian Eulerian (ALE) method is shown in Section A.3 Appendix A. An explicit integration scheme is used, with an automatic, adaptive time-step. Figure 4.2 shows a diagrammatic representation of the FSI model. Numerically, the structure is represented by a Lagrangian orphan mesh imported from a structural-only analysis where the inlet is forced into its post-buckled state. In the FSI simulations, post-buckling stresses are applied as a pre-defined field. The Lagrangian inlet is then ‘immersed’ in an Eulerian domain completely filled with air to ensure a Eulerian Volume Fraction (EVF) of fluid consistently equal to 1 everywhere. The air volume is modelled as a Newtonian fluid with standard properties at 25 °C and atmospheric pressure, i.e. density $\rho_a = 1.205 \text{kg/m}^3$ and viscosity $\mu_a = 1.915 \times 10^{-5} \text{Pa} \cdot \text{s}$ [94]. CEL simulations require the speed
4.2. MODEL DEFINITION

Air velocity, $V_{in}$

Pressure, $P_{outlet}$

No flow

150 mm - $u$

67 - 70 mm

350 mm

Figure 4.2: A portion of the computational domain for the Fluid-Structure Interaction model. Air flows from left to right. Pressure boundary conditions are applied at the outlet, together with a no flow condition below the structure.

of sound in air to be defined ($c = 346\text{m/s at } 25\, ^\circ\text{C}$) as a material input. This value is used to calculate elastic bulk modulus as $\rho_a c^2$, which is a measure of the fluid compressibility. A gauge relative pressure, $P = 0\text{Pa}$, is assigned beneath the air inlet where no fluid is assumed to flow. When using the CEL method, by default, the boundaries of the Eulerian domain with set pressure values reflect pressure waves [83], which affect the numerical solution adversely. This problem is avoided by assigning a uniform initial velocity field throughout the entire Eulerian domain. Additionally, a sufficiently long duct must be used and/or a negative gauge relative pressure, $P_{out} = -100\text{Pa}$, and non-reflecting boundary condition need to be imposed at the outlet to avoid flow reversal. Negative values are used here to represent applications in which the duct connects to interfacing components that allow the air to naturally flow out. Both the bi- and monostable inlets are designed for a snap-through velocity of about 60 m/s. Aerodynamic loads at this air speed are found in preliminary FSI studies, reported in Section A.3 of Appendix A, to be sufficiently high for the snap-through actuation and therefore imposed as the initial fluid velocity for the Eulerian mesh.

A default penalty contact method manages the solid-solid interaction, while the so-called volume of fluid (VOF) method [84] is used for the fluid-solid interface tracking. The VOF method enforces the contact constraints and no-slip conditions, when, at the specific interface nodes, the arithmetic mean of the EVF of the surrounding elements is higher
than 0.5 [83]. Only one element is used through the width of the domain (into the page) as three-dimensional effects are neglected. The size of the fluid domain is chosen to be large enough (0.5 × 0.4 m) to minimise boundary effects. For accuracy and convergence, the fluid mesh is refined homogeneously resulting in 700,000 8-node linear Eulerian brick element of type EC3D8R.

4.3 Results

4.3.1 Snap-through analysis of the inlet morphing component

The parametric buckling and post-buckling study outlined in Chapter 3 provides the insight required to design bistable or super-elastic monostable beams. This insight is used to design the deformable portion of the adaptive air inlet shown in Fig. 4.2—one with bistable and one with super-elastic characteristics.

Figure 4.3 shows the force-displacement curves for these bistable (Fig. 4.3a) and super-elastic (Fig. 4.3b) devices, which match the characteristics of the curves depicted in Fig. 3.3. Both structures exhibit snap-through behaviour, but with different loading-unloading paths. As desired, the bistable structure has a second stable configuration in its unloaded state, whereas the super-elastic beam snaps back upon removal of the transverse load. Figure 4.3 compares the snap-through behaviour of the bistable (Fig. 4.3a) and super-elastic (Fig. 4.3b) structures in reaction to a point force at the central nodes of the arch and a uniformly distributed vertical transverse load across the entire, left half, right half and middle upper surface of the structure. For both the cases, the type of loading affects the snap-through behaviour quantitatively in that the total load acting on the beam at snap-through changes. On the other hand, stability characteristics remain unchanged regardless of whether the load is distributed or not and in which region it is applied. In fact, as the transverse loading conditions are modified, the equilibrium paths tracking the locus of maxima and minima related to the curves in Fig. 4.3a and b, change according to the loading conditions, thereby
4.3. RESULTS

Figure 4.3: Snap-through behaviour of bistable and super-elastic monostable adaptive inlets. (a) The force versus central deflection of the bistable inlet—obtained by applying a compressive load greater than the buckling load and maintaining the symmetry of the structure—intersects the central deflection axis three times, indicating bistability. (b) Breaking the axial symmetry of the structure by varying thickness along the length, results in monostable snap-through behaviour.

4.3.2 Aerodynamic actuation

Both the bistable and monostable morphing inlets are immersed into a 60 m/s airflow, which is sufficiently fast to cause snap-through. With reference to Fig. 4.2, a ‘no flow’ condition is imposed below the structure. The gauge pressure at the outlet, i.e. at the end of the duct, is set to be lower than the static pressure of the external air. This fluid boundary condition is application-specific. Low downstream pressures are representative of applications in which the air diverted by the inlet joins faster and/or cooler flows. It is important to note that the outlet pressure is a fundamental parameter, which drives the fluid-structure interaction
Figure 4.4: Passive actuation of a bistable adaptive air inlet. A 60 m/s air flow above the bistable inlet, cause a pressure field that actuates snap-through from the initially open state to the closed state. The bistable configuration holds its closed configuration even when the air flow ceases due to its structural characteristics. The monostable inlet (not shown) shows similar behaviour, but the closed configuration is not stable with respect to decreasing air speeds. Coloured arrows represent the velocity vector field, with minimum and maximum magnitude speeds indicated in blue and red, respectively. From open to closed state the snap-through takes of the order of 10 ms.

and, therefore, influences the structural design of the adaptive component. The behaviour of the inlets when submerged in a fluid flow is shown in Fig. 4.4. Fluid-structure interaction simulations demonstrate that the airflow induces a pressure field over the inlet that actuates the deformable structure to snap into the second ‘closed’ configuration. The coloured vector contours in Fig. 4.4 indicate air velocity, with low speeds in blue and higher speeds in red. As expected, the closed state of the bistable inlet, shown in Fig. 4.4d, is stable even when the air ceases to flow. The super-elastic monostable device exhibits a similar snapping response, but the inlet opens again at reduced air speeds, thus showing valve-like opening-and-closing behaviour. Preliminary observations suggest the existence of conditions under which high frequency oscillations of opening and closing cycles can be induced. However, the stability
of the closed state can also be controlled by the outlet pressure, meaning that, a specific outlet pressure can be prescribed to keep the monostable inlet in its closed state upon snap-through. The oscillating behaviour seems to be achievable by exploiting a combination of pressure conditions at the inlet's outlet and airflow reversal due to the sudden closing of the inlet. In this way, after the snap-through of the morphing post-buckled component, air flows backward causing the snap-back and re-opening of the inlet. At this stage, air flows again above the structure and inside the inlet, thereby actuating one more time the snap-through. This behaviour can continue cyclically and an oscillating response can be obtained.

4.4 Discussion

The insight gained from the parametric study of Chapter 3 on the effect of different boundary conditions on the post-buckling behaviour of a clamped-clamped beam (Fig. 3.4) shows that multistability of a one-dimensional morphing component can be readily controlled by varying the end conditions, such as longitudinal compression, transverse displacements and rotations. Indeed, in Chapter 3, we have derived a taxonomy for nonlinear behaviours demonstrating that the desired multistability and snap-through behaviour of a morphing component for a specific application can be evaluated simply by plotting and studying the equilibrium manifold of the structure. In particular, it was demonstrated that by introducing asymmetries it is possible to transition from the bistable behaviour of the classical elastica, to a monostable design that still exhibits snap-through and therefore the much desired large deformations. In deflection versus compression space, the specific region of super-elastic monostability, situated between the limit point of the secondary stable branch and the primary equilibrium curve of a broken pitchfork, can be controlled by varying the parameters of the system, i.e. compression, end rotations and vertical end deflections. This insight is important for design purposes as it allows the design of both binary inlets, statically stable in both open and closed configurations, and valve-like inlets, that can passively transition
between open and closed states. The fluid-structure interaction simulations presented in this chapter show that these devices can be actuated passively simply by exploiting the changes in the pressure field caused by air flowing over the curved inlet (Fig. 4.4). For the particular example studied herein, snap-through is activated at a velocity of 60 m/s, at which point the inlet closes within a time frame of approximately 10 ms so that air can no longer flow into the duct.

The actuating flow speed can be tailored to specific applications by varying parameters such as material properties and structural boundary conditions. The appropriate design can be chosen by conducting a parametric post-buckling analysis and studying the resulting equilibrium manifolds.

4.5 Conclusions

Chapter 3 described a framework for exploiting structural instabilities as an engineering design tool. In this chapter, the multistability of simple one-dimensional beam structures is then used to design a passively actuated variable geometry air inlet. The passive actuation mechanism renders the proposed concept as a lightweight solution that is not subject to weight and volumetric restrictions. As a result, the proposed morphing air inlet promises to benefit many applications where fluid-structure interactions and shape adaptation are key, e.g. in the biomedical, automotive, and aerospace industries.

In the previous chapter the multistable behaviour of a beam with respect to mechanical loads was analysed. This chapter further developed this analysis by studying the multistable behaviour of a representative morphing panel under aeroelastic loads. In the following chapter, the design, manufacturing and testing of a prototype are carried out for validation and to demonstrate its potential for use in industrial applications.
PROOF OF CONCEPT: WIND TUNNEL EXPERIMENTS

Everything is energy and that's all there is to it. Match the frequency of the reality you want and you cannot help but get that reality.

Albert Einstein

Abstract

Previous chapters focused on FE analyses and FSI simulations carried out to validate the use of elastic nonlinearities as a means for shape adaptation and to achieve passive actuation of a morphing air inlet. In this chapter, all these concepts are brought together to develop an inlet prototype for experimental validation. This involved manufacturing of the inlet’s morphing composite component and assembly of a test rig for experiments in a wind tunnel facility. The results show the expected series of the device’s adaptive responses to varying airflow conditions, providing confirmation of the underlying physical principles and validation of the design and analyses methodology.
5.1 Introduction

The taxonomy of nonlinear snap-through behaviours—explored via extensive finite element and fluid-structure interaction analysis in Chapters 3 and 4—is now adopted to design and manufacture the adaptive air inlet, shown in Fig. 5.1. This chapter focuses on the study and tailoring, for subsequent manufacture, of the dynamic response of a glass-fibre composite panel that can dynamically snap between two configurations and be used as the morphing component of a passively adaptive air inlet. By varying the layup sequence and boundary conditions of the composite panel, any of the three behaviours discussed in Section 3.3.1 can be achieved (simply stable, super-elastic monostable, and bistable). The nonlinear behaviour of the morphing component and aerodynamic response of the air inlet to various fluid flow conditions are tested in a sub-sonic wind tunnel available at the University of Bristol. Snap-through and snap-back trends are compared with numerical results obtained from ABAQUS simulations.

5.2 Design of multistable component

5.2.1 Methods

A finite element model of the adaptive component of the air inlet is constructed. The material chosen is a UD (uni-directional) glass fibre epoxy resin composite, E-Glass 913, with material
5.2. DESIGN OF MULTISTABLE COMPONENT

properties as shown in Table 4.1. In order to induce not only bistability, but also super-elastic monostable snap-through behaviour, the symmetry of the structure is broken by changing the thickness along the length of the panel, as shown in Fig. 5.2. Two layups (layup 1 and layup 2 in Fig. 5.2) are considered to investigate the effect of stacking sequences on the composite panel’s multistability. In Chapter 4 the layup was chosen purely to break the symmetry of the structure and achieve super-elasticity. In this case, with reference to the schematic side view of the demonstrator shown in Fig. 5.3, the layups are also chosen to obtain, parametrically, a buckled configuration that is flush with the external aerodynamic surface of the air inlet and to maximize the duct aperture. Additionally, layup 1 is chosen to have same thickness at the two extremities, whereas layup 2 has a more substantial difference in order to increase asymmetry and consequently achieve a wider region of super-elasticity in the bifurcation diagrams. The structures are therefore composed of a minimum of three and a maximum of eight layers, where each layer has a thickness of 0.13 mm, a maximum length of 450 mm and a width of 150 mm. Vertical displacement and precompression are applied to the last 50 mm of the right end of the composite plate. The vertical displacement is kept at 50 mm, while precompression varies from 0 to 10 mm. In the finite element model, the second equilibrium configuration is reached by application of concentrated transverse forces acting on the central nodes across the width of the plate. In order to investigate the effect of varying applied transverse loads on the multistability of the component, the second configuration is reached also by applying a distributed load: on the entire panel surface; and on the left and right halves.

Four-noded doubly curved shell elements with reduced integration S4R and enhanced hourglassing control are selected for the ABAQUS simulations. The thickness discontinuity is created by assigning to the part conventional shell composite layups. A fine mesh with 6720 elements is required to ensure convergence in the nonlinear post-buckling behaviour. Equilibrium manifolds in transverse central deflection vs compression space, as well as transverse load vs transverse central deflection snap-through curves are traced numerically by means
CHAPTER 5. PROOF OF CONCEPT: WIND TUNNEL EXPERIMENTS

Figure 5.2: Composite layups for the multistable component of the air inlet prototype. Blue and green lines represent composite layers. Step changes in thickness cause stiffness variations and the structural asymmetry required for monostable behaviour with snap-through.

Figure 5.3: Schematic 1D representation and dimensions of air inlet aperture.

of the ABAQUS implementation of the arc-length method based on Riks’ formulation [89].

5.2.2 Results

Figure 5.4 summarizes the results obtained from the buckling and post-buckling study of the glass-fibre structure with layup 1. As shown in Fig. 5.4a, the composite plate is clamped at its left extremity. The application of 50 mm of vertical displacement and horizontal compression to the right extremity produces the first buckled stable configuration. The second state
Figure 5.4: Finite Element buckling analysis of a multistable composite plate with layup 1 of the air inlet prototype. (a) First and second buckling configurations of the morphing component. (b) Broken pitchfork in compression-central deflection space and related dynamic behaviour. (c) Snap-through curves of a bistable, super-elastic monostable and simply stable structure. (d) Locus of limit points in compression-force space.
is reached loading the structure with a transverse load. Depending on $u$, the horizontal compression, this configuration can be either stable or unstable. Because the composite plate contains asymmetries in its geometry, the pitchfork equilibrium manifold is broken, as shown in Fig. 5.4b. When $u$ is applied, the structure naturally follows the primary equilibrium branch and at $u \approx 5$ mm it deflects into its initial “open” configuration with negative central deflection. Note that before compression, the central deflection is approximately $-25$ mm. This is due to the vertical displacement of the right extremity of the plate, which is applied as a initial loading step before compression.

The findings of the post-buckling study provide general guidelines for designing the adaptive air inlet. The dynamic response and multistable capabilities of the composite panel are summarised in Figs. 5.4b, c and d. Starting from a constant value of $u = 6.4$ mm, which is beyond the limit point on the broken-away pitchfork branch of the bifurcation diagram of Fig. 5.4b, the snap-through equilibrium curve in Fig. 5.4c shows that the structure snaps through into its second configuration when $F$ reaches $0.7$ N. If the transverse load is removed, the structure remains in the inverted position at the point where the load-displacement curve intersects the zero-force axis. A force of $-0.15$ N is required to snap back to the first state. In conclusion, for values of compression greater than the limit point of the broken-away pitchfork branch, the structure behaves in a bistable manner. In this configuration, the midpoint of the composite can deflect up to $50$ mm for $u > 10$ mm.

By reducing the compressive displacement to $u = 5.9$ mm, the structure lies in the super-elastic zone between the limit point and the cusp (see Fig. 2.17 in Section 2.5.2 for reference). With reference to Fig. 5.4c, the green fundamental path shows that after snap-through at around $F = 0.3$ N, the structure reaches a second configuration. This state is unstable and the composite panel automatically snaps back upon load removal. Switching between bistable and super-elastic configuration have effect on the maximum deflection, $\delta$, achievable with the snap-through. Consequently, the opening of the channel can change and, in order to assure a completely closed inlet, the cover must be either adjustable to compensate the reduction
5.2. DESIGN OF MULTISTABLE COMPONENT

Concentrated force on panel midline
Pressure on entire surface
Pressure on left half surface
Pressure on right half surface

Figure 5.5: Changing of locus of limit points with applied load. (a) Load distribution influences the locus of limit points, thereby modifying the borders of stability and instability regions and consequently the dynamic of the snap through. (b) Threshold between bistability and monostability is not affected by the load conditions, while the cusps of limit points can vary.

in opening or designed to allow a complete closing in all the configurations. However, this might results in more severe impacts between the morphing component and the top cover when the structure is set to a configuration that permits higher deflections. Therefore, during the design of a adaptive structures, it is important to take into account these trade-off within the functionality provided.

Figure 5.4d shows the cusp singularity on the locus of limit points on snap-through deflection ($\delta$) vs load ($F$) curves. The red curve represents the forces at which the panel snaps through (locus of maxima) onto the second equilibrium, while the black curve shows the forces at which the panel snaps back (locus of minima), both as a function of $u$. As previously described in Section 2.5.2, the cusp defines the onset of snapping behaviour. Consequently, no snap-through is observed if the system is configured to have a compressive displacement below the cusp value. The region of super-elastic monostability is confined between the cusp and the intersection of the snap-back curve with the compression axis $F = 0$.

Note that the deflection-load fundamental paths in Fig. 5.4c are obtained by applying
a transverse load on the central nodes of the composite panel. If the load is uniformly distributed rather than concentrated, the shape of the snap-through curves will change, and so will the limit points and cusps, but the qualitative aspects of the bifurcation diagrams themselves will remain unchanged. Figure 5.5 compares the loci of limit points and the position of the cusp catastrophes as a function of the distribution of transverse load. Figure 5.5a clearly shows how the region of instability is reduced when distributed, rather than concentrated, forces are applied. This feature has consequences for the dynamics and rapidity of the snap-through event as the higher potential energy required to actuate the snap-through is then transferred into dynamic release of kinetic energy, thereby leading to a more severe snapping event. Figure 5.5b confirms that the threshold value between bistability and monostability is not influenced by the nature of the transverse load because these equilibria are inherent properties of the system that exist in the unloaded state. However, the position of the cusp—i.e. the first value of pre-compression to create snap-through dynamics—changes, influencing the region of super-elasticity, in this specific case, by up to 0.1 mm of applied pre-compression.

The nonlinear behaviour of the two layups is compared in Fig. 5.6. As expected, both layup 1 and layup 2 are characterized by a broken pitchfork. However, the onset for bistable behaviours—i.e. limit point of the secondary stable branch—for layup 1 requires pre-compression of 6.2 mm, whereas layup 2 needs up to 9.1 mm of applied compressive displacement. Consequently, the composite plate with layup 2 requires loads for snap-through actuation ten times greater than layup 1. For this reason, even though layup 2 shows a wider region of super-elasticity, only a panel with composite layup 1 is tested in the wind tunnel.

5.3 Manufacturing of the fiberglass component

Following the information gathered from the buckling and post-buckling studies described in Section 5.2, fibreglass panels are manufactured with layup stacking sequences 1 and 2.

The choice of using a composite material stems from the advantage of obtaining a thin,
Figure 5.6: Comparison of equilibrium manifolds for layup 1 and 2. (a) Broken pitchforks show that region of bistability for layup 1 require lower compressive displacement (6.2 mm) compared to layup 2 (9.1 mm). (b) Loci of limit points shows that, while the onset of dynamic behaviours is around same pre-compression (5.8 mm), layup requires higher loads for the snap-through actuation.

Figure 5.7: Vacuum bagging of UD fibreglass plate.
Figure 5.8: Cured UD fibreglass plate with layup 2. Twelve holes each side are drilled to connect the panel with the rest of the prototype. End tabs and pressure taps are added on the bottom side.

lightweight structure. In addition, composite manufacturing techniques, such as layup vacuum-bagging and potentially, in the future, tow-steering [62, 63], allow local tailoring of the material stiffness and the ensuing equilibrium manifold.

The composite plate is composed of three to six/eight plies of conventional E-Glass 913 reinforced epoxy matrix UD prepreg (HexPly 913G-E-5-30), with 0.13 mm of nominal cured thickness, and mechanical properties as show in Table 4.1. UD panels, with ply sequences as per the scheme of Fig. 5.2, were cured with a vacuum bagging technique at a temperature of 125 deg C and pressure of 1 bar for 60 min. A heat-up rate of 2 deg C/min was chosen.

The lamination and vacuum bagging techniques, as shown in Fig. 5.7a, involved the following tools and materials:
5.4. DESIGN OF TEST RIG

- aluminium plate as flat open mold;
- release film to provide easy release between mold and cured prepreg;
- prepreg layers stacked as shown in Fig. 5.2;
- cork tape surrounding the laminate plate in order to avoid resin flow;
- aluminium thin sheet to provide uniformly distributed pressure and fine surface finishing;
- release film to allow easy release between the aluminium sheet and the breather;
- breather layer, which allows the vacuum to be uniformly distributed all over the composite surface;
- vacuum-bag layer covering the entire laminate and sealed to the mold with sealant tape.

The cured panels were cut with a diamond blade to obtain a width of 150 mm and length of 450 mm, of which 50 mm each side are clamped to the air inlet mechanism shown in Fig. 5.9, resulting in a final length, $L$, of 350 mm.

Figure 5.8 shows the cured fibreglass panel (layup 2). The thickness variations are clearly visible along the panel length. Twelve holes are carefully drilled with a composite drill bit in order to allow connections with other components of the prototype. End tabs are added to the bottom side extremities to protect the composite material from damage by the grips that clamp the specimen ends. Pressure taps are also installed as is described in Section 5.5.

5.4 Design of test rig

A test rig (shown in Fig. 5.9) for proof of passive actuation via airflow was designed to fit the wind tunnel facilities at the University of Bristol. The available low turbulence wind tunnel has a Polymethylmethacrylate (PMMA) plate bolted on the floor that is exploited as the top cover of the air inlet.
Figure 5.9: The air inlet placed in the wind tunnel. (a) One of the composite panel's extremities is clamped to the PMMA plate on the wind tunnel floor. Air flows through the duct to the bottom of the tunnel. (b) The full test rig is shown with details in (c) and (d). The mechanism is composed of: (1) morphing composite inlet; (2) PMMA plate; (3) adjustable cover tap; (4) aluminum bottom plate; (5) handwheel; (6) PMMA lateral walls; (7) trapezoidal lead-screw; (8) pressure taps; (9) aluminum right support; (10) bronze trapezoidal flanged nut; (11) Hiwin carriage; (12) encoder for linear measuring system; (13) Hiwin linear rails; (14) outlet channel.

AUTODESK INVENTOR and ABAQUS structural models were used to design and size the air inlet prototype for experimental validation. The CAD exploded (Fig.5.10), 3D and side (Fig. 5.11) views of the prototype illustrate the designed mechanism and connected components used to apply vertical displacement and pre-compression necessary to buckle the composite panel and achieve the desired post-buckling behaviour: one extremity of the composite plate is clamped to the PMMA plate (10 mm thick), which is bolted on the floor of the wind tunnel. The other end of the inlet is connected through an opening in the PMMA plate to an H shaped aluminum support placed immediately underneath the wind tunnel floor. With this arrangement, the height difference between the extremities of the composite
## 5.4. Design of Test Rig

### Parts List

<table>
<thead>
<tr>
<th>Item</th>
<th>Description</th>
<th>Material</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Connecting support for horizontal displacement</td>
<td>Aluminum</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>HIWIN carriage</td>
<td>Steel, Plastic</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Bronze 20x4 trapezoidal flanged nut</td>
<td>Steel, Cast</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Front wall</td>
<td>Aluminum</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>FF17 supported side unit</td>
<td>Steel, Plastic</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>20x4 trapezoidal lead-screw</td>
<td>Steel, Cast</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>Top plate</td>
<td>PMMA</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>Side wall</td>
<td>PMMA</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Outlet channel</td>
<td>PMMA</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>Morphing component</td>
<td>UD fibreglass</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>Adjustable cover</td>
<td>Aluminum</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>HIWIN linear rails</td>
<td>Steel, Plastic</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>Bottom plate</td>
<td>Aluminum</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>Pressure vessel</td>
<td>Steel, Cast</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>Rod</td>
<td>Steel, Plastic</td>
<td>1</td>
</tr>
</tbody>
</table>

![Exploded view of CAD design of the air inlet test rig](image-url)
Figure 5.11: (a) 3D view and (b) side view of CAD assembly of the test rig.
Figure 5.12: Schematic of the adaptive inlet and six pressure taps (PT) located along the composite plate.

Panel is exactly 50 mm. Compression is imposed by a simple linear mechanism: a handwheel applies rotation and therefore torsion to a TR 20 × 4 D steel trapezoidal lead-screw that converts rotational motion to linear displacement by means of a KSM 20 × 4 bronze flanged trapezoidal nut fixed to the aluminum support of the composite panel. A controlled and smooth linear movement is guaranteed by the HIWIN HGW20CCZ linear carriages and HGR20RH-500 rails on which the support is fixed. Compressive displacement is measured by the encoder of an HIWIN MAGIC PG positioning measurement system. PMMA lateral walls abut the composite panel on both sides in order to drive the airflow above its surface and to allow the air to flow out and through an outlet channel of 200 mm in length. The manufactured air inlet, whose assembly is shown in Fig. 5.9, is placed on the floor of the working section of the wind tunnel. The lead-screw, nut, support bearings and HIWIN linear systems were purchased from MOORE INTERNATIONAL LTD. Additional technical drawings with the specifics of each component and ABAQUS simulations are reported in Appendix B.

### 5.5 Wind tunnel test

Alongside the ABAQUS FE model, experiments in the low turbulence wind tunnel at the University of Bristol are carried out to investigate the inlet response to varying fluid flow conditions. The wind tunnel has an octagonal working section of 0.8 m × 0.6 m × 1 m and
can attain speeds up to 88 m/s while maintaining turbulence intensity levels in the order of 0.09% due to its 12:1 contraction ratio.

During wind tunnel testing, the inlet is initially set into its open configuration. This stable post-buckled configuration is achieved by applying a compressive displacement at the lower edge of the plate. A range of displacements from 3.6 to 6 mm is applied and, for each value, the test is repeated 5 times. This provides the best data independency with uncertainty levels (at 95% confidence) less than 10% of the velocity and pressure at snap-through.

The initially stable shape of the composite plate corresponds to the maximum inlet aperture. The morphing panel then snaps onto its second equilibrium as the fluid velocity is increased and reaches a critical value, thereby closing the inlet. The second state can either be self-equilibrated, meaning that the inlet will remain closed even when the air ceases to flow (bistability), or super-elastic and simply stable, i.e. the plate returns to its open configuration once the fluid flow speed is reduced.

The structural behaviour of the morphing component is studied by recording the static pressure profile induced by air flowing over the structure. Six pressure taps are placed on the back side of the composite plate (Fig. 5.12) and connected to a MicroDaq Pressure Scanner. The pressure taps for the pressure distributions were made from 1.6 mm diameter brass tubing with 0.4 mm pinholes with angle perpendicular to the surface of the composite plate to avoid aerodynamic interference between the pressure taps. Each of these taps measure pressure relative to the atmospheric value, for 50 s, at a frequency of 1000 s\(^{-1}\). Additionally, total and static pressure taps of a Pitot tube, placed inside the wind tunnel, were connected to the pressure scanner. Using Bernoulli’s equation,

\[ P_{\text{tot}} - P_{\text{stat}} = \frac{1}{2} \rho_a v^2 \]

the pressure data from the Pitot tube was used to calculate the instantaneous airflow velocity in the wind tunnel, \( v \), for a specific density \( \rho_a = 1.225 \text{ kg/m}^3 \) of the fluid, and where \( P_{\text{tot}} \) and \( P_{\text{stat}} \) are total and static pressure, respectively.
5.6. RESULTS: WIND TUNNEL EXPERIMENTS

Figure 5.13: Adaptive air inlet demonstrator in its (a) open and (b) closed configuration during a wind tunnel experiment.

5.6 Results: Wind tunnel experiments

The results illustrated in Section 5.2.2 show the taxonomy of dynamic snap behaviour of the UD glass-fibre composite panel with layup 1. As shown in Chapter 4, an air inlet, designed using this taxonomy, can be actuated by aerodynamic loads when air flow reaches a critical speed. The fluid flow conditions above the curved morphing component applies a field of negative relative static pressure distributed across the composite surface, which pulls the structure up and actuates the snap-through.

In this section, we present the experimental validation of this design principle for the development of shape-adaptive devices. Specifically, the inlet’s dynamic response to varying airspeed is tested by means of wind tunnel experiments.

The air inlet was designed to remain open for slow fluid flows and to close immediately when the airspeed reaches a critical value. For this reason, the composite component is designed to always buckle into the ‘open’ configuration upon compression, such that it can then be snapped into the ‘closed’ state.

It is important to note that the experimental values of compression reported here are generally lower than those obtained with the FE investigation (Fig. 5.4), because the cali-
Figure 5.14: Measurement of relative static pressure of PT1 and PT6 and airspeed for (a) bistable and (b) monostable configurations.

Bistable configuration

Monostable configuration

Figure 5.13a shows the adaptive inlet in its open configuration. For values of \( u \geq 4.4 \) mm, the structure shows bistability. In particular, for airspeeds greater than \( v = 31.7 \) m/s, and relative pressure (pressure difference with respect to the atmospheric one), \( P = -202.0 \) Pa, the structure snaps into its closed configuration (Fig. 5.13b), which remains stable even when the velocity is lowered below these values. When the applied compressive displacement is reduced to values below 4.4 mm, the second closed configuration becomes unstable. Hence, the composite panel snaps back, and opens the inlet again, once the airspeed falls below a critical value.

In order to quantify the critical velocities and pressures at snap-through and snap-back,
relative static pressure was measured along the composite panel, as shown in Fig. 5.12. An example of the extrapolated results is illustrated in Fig. 5.14a and Fig. 5.14b, for a bistable and monostable configuration, respectively. A negative pressure implies the local static pressure to be lower than the atmospheric pressure, essentially indicating the presence of a force normal to the composite panel and directed into the airstream. The figure shows the static pressure at the leading (PT6) and trailing (PT1) edge of the composite plate, as a function of the measurement time. Both pressure curves clearly indicate when the structure snaps between open and closed states, thereby providing the critical pressure and critical velocity at snap-through and snap-back. The blue curves related to PT6 show that, with an increase in air velocity, the pressure smoothly decreases until snap-through occurs, modifying the curvature of the structure and causing a sudden jump in pressure. PT1 also indicates an initial decrease in relative pressure. In this case, on the left hand side of the critical snap-through speed, PT1 records more random pressure changes. This is because the trailing edge of the composite plate vibrates just before snap-through, contributing to the formation of turbulent flow and alternating pressure changes. Nonetheless, snap-through is still clearly visible from PT1 measurements because, as the inlet closes, air ceases to flow through the air duct and the relative pressure instantaneously drops to zero. When the inlet is configured to be bistable, decreasing the air velocity does not alter the relative pressure at PT1 (which is shielded from the airflow in the duct), while PT6 shows that the relative pressure reduces without any discontinuities associated to snap-back. Conversely, Fig. 5.14b illustrates clearly that when airspeed is reduced below a critical value, the actuation of the snap-back causes the inlet to re-open. Consequently, the relative pressure in the proximity of PT1 reduces again and the PT6 curve shows an additional jump. Critical values of pressure and velocity are therefore indicated by jumps in the pressure measurements.

At snap-through, the red curves in Fig. 5.14 show a peak of positive pressure. This can be related to a fluid hammer [95], similar to those commonly experienced in pipelines. Due to the dynamic closing of the inlet, the air flow is suddenly forced to stop, causing a pressure
wave that, in this case, is dissipated without influencing the structural behaviour of the composite component.

The dynamic response of the adaptive device is summarized in Fig. 5.15, where critical pressures and velocities at snap-through and snap-back are plotted as a function of the applied compressive displacement. Error bars correspond to the 95% confidence level based on a Student’s t-probability distribution for the five test repetitions considered. For values of compression between 4.4 and 6 mm, velocities between 31.7 and 40.3 m/s (or, equivalently, pressures between $-202.0$ and $-320.3$ Pa) are required to actuate snap-through and close the air inlet. In this configuration, the inlet is bistable and maintains its closed state when airspeed is decreased to 0 m/s. Lowering the compression to 4.35 mm, the inlet starts to show super-elastic monostability. In particular, at this level of compression, airflow has to reach a velocity of 31.5 m/s in order to actuate snap-through and close the inlet, whereas the velocity must be reduced to 7.2 m/s to open it again. The lower the compression, the lower the velocity (or pressure) gap between snap-through and snap-back.

With reference to Fig. 5.4d, the curves in Fig. 5.15 show a qualitatively similar trend. The loci of maxima and minima seem to coalesce in a cusp singularity. This cannot be seen in the experimental results as the structure is subjected to different loading scenario during at the snap-through and snap-back. In fact, in the ABAQUS simulations, applied force is distributed on the same surface both for the snap-through and snap-back. Conversely, during the wind tunnel experiments, when the inlet is open, the aerodynamic loads are distributed over the entire morphing component, whereas when the inlet is closed, forces are applied on a smaller portion of the composite due to the presence of the inlet cover. Therefore, the limit points for the snap-through and snap-back belong to different scenarios and cannot coalesce in the same cusp. This is also the reason why the regions of super-elasticity calculated from the ABAQUS structural analysis (Fig. 5.4d) and from the wind tunnel experiments (Fig. 5.15) show different range of applied compression, $u$.

The results obtained from the wind tunnel experiments demonstrate the flexibility of
5.7. DISCUSSION

this device. By simply changing one design parameter, in this case the compression, the device can be adapted to a wide range of working conditions.

5.7 Discussion

Structural adaptivity is an enabling concept when an engineering system requires trade-offs between stiffness, strength, weight and functionality. Mechanical instabilities are emerging as an effective and promising means for shape adaptation. Furthermore, current research is focusing on identifying different types of nonlinear structural behaviour from the study of equilibrium manifolds that can be used for novel functionality.

In the previous chapters, the influence of boundary conditions and geometrical parameters on the multistability and snap-through responses of a simple beam were investigated. A taxonomy of nonlinear dynamic behaviours was classified, which is used in this chapter to design and test a shape-adaptive air inlet. The device is able to morph in response to external stimuli. In particular, the inlet is conceived to be open at low airspeeds, such that air can flow freely into a connected duct. When the airspeed increases above a predefined
critical value, the adaptive component—a glass-fibre composite plate—snaps passively and closes the inlet to the duct, so that air ceases to flow into the channel. Depending on the boundary conditions applied on the composite panel and its stiffness profile, the air inlet can either remain in its closed state or autonomously snap back and open again when the airspeed reduces. External mechanisms are therefore not required for the snap-through actuation as this is driven by external stimuli instead.

The first stage of the inlet development begins with an FE analysis of the buckling and post-buckling behaviour of the glass-fibre composite plate. The choice of composite material stems from the advantage of obtaining a thin, lightweight structure. In addition, composite manufacturing techniques, such as layup vacuum-bagging and potentially, in the future, tow-steering [62, 63], allow local tailoring of the material stiffness and the equilibrium manifold.

Wind tunnel tests have been performed to validate the proposed design concept and investigate a variety of responses to changing airspeeds. The information gathered from the FE study is used to manufacture a test rig. The taxonomy of adaptive behaviours is verified by investigation of the critical velocity and static pressure at snap-through and snap-back as a function of the applied compressive displacement on the shape-changing composite panel. The control of this parameter, through a system comprising a lead-screw and linear rails, successfully permit the reconfiguration of the characteristic of elastic stability of the inlet. The system can be set to be bistable, super-elastic or simply stable.

Both experimental and theoretical results indicate the presence of a cusp singularity in the inlet’s force vs displacement vs compression space, as well as its importance on the inlet’s qualitative behaviour. Specifically, the cusp forms because the locus of snap-through and snap-back critical velocities and pressures coalesce as the panel compression in decreased. In general, the greater the applied compression, the higher the airspeed required to actuate the snap-through. For the design tested in this work, the critical snap-through velocities range between 26.7 and 40.3 m/s. Conversely, snap-back can be seen only when
the structure behaves super-elastically, with compression between 3.55 and 4.35 mm. Under these circumstances, the higher the degree of compression, the lower the airspeed to actuate snap-back. Given the generality of the underlying physical principle, the system’s parameters can be tailored to fit specific operating conditions.

5.8 Conclusions

Chapter 3 introduced the idea of using post-buckling behaviours of a structure as a means of shape-adaptation. It is possible to create and control a taxonomy of nonlinear behaviours and multistability by working on boundary conditions and/or stiffness tailoring of the component. Subsequently, Chapter 4 illustrated the possibility of using the forces generated by airflow to passively actuate the snap-through of the air inlet’s morphing component. The pressure fields above the structure, at specific air speed, act as a transverse load, thereby closing the inlet. Finally, in this chapter elastic instabilities have been exploited as a design tool to design and manufacture a passively adaptive air inlet with the goal of validating the design concepts and demonstrate the flexibility of the method. The design framework proposed facilitates the reconfiguration and control of stability and dynamic snap-through characteristics of the inlet, making the device suitable for a wide range of applications where shape adaptation and passive actuation are key, e.g. artificial valves and drug delivery in biomedical applications [26], NACA ducts in automotive and aerospace industries, and steering devices for sport yachts.

So far the concepts of exploiting well-behaved nonlinear deformations, and the advantages achievable by creating and exploiting a taxonomy of nonlinear behaviours based on a structure’s nonlinear equilibrium manifold, have been applied both theoretically and experimentally to the specific case of an adaptive air inlet. To further demonstrate the applicability and generality of these concepts, and the underlying physical principles, in Chapter 6, the design approach is applied to a different structure: a slat-cove filler.
CHAPTER 6

A SUPERELASTIC COMPOSITE SLAT-COVE FILLER FOR AIRFRAME NOISE REDUCTION

Carl Fredricksen: Hey, let's play a game. It's called “See who can be quiet the longest.” Russell: Cool! My mom loves that game!

Disney Pixar movie “Up”

Abstract

The design approach used for developing the air inlet is applied here to evaluate and tailor a morphing component used to reduce airframe noise, known as a slat-cove filler (SCF). During the phases of take-off and landing of an aircraft, the vortices in the cove between the deployed leading edge slat and the main wing are among the most important sources of airframe noise. The nonlinear structural behaviour of the SCF during the process of retraction and deployment is studied in order achieve a desirable ‘snap-through’ behaviour. The dynamic behaviour of a composite SCF is compared with similarly tailored SMA-based SCF and a reference, uniformly thick superelastic SMA-based SCF. Results show that by exploiting elastic nonlinearities, both the tailored
6.1 Introduction

Previous chapters showed the application of elastic instabilities as a design option to achieve the large displacements and multiple stable configurations required for morphing applications. Tracing equilibria deep in the post-buckling regime revealed nonlinear behaviours that were used to design multistable snap-through responses for the morphing component of an adaptive air inlet.

In this chapter\(^1\), the proposed adaptation-by-instability approach is used to design and tailor the behaviour of slat-cove fillers (SCFs). The SCF is a morphing component that reduces airframe noise produced by unsteady flow in the cove between the leading-edge slat and the main wing during low speed manoeuvres of approach and landing for transport aircraft [52–57] (Fig. 6.1). In its deployed state, the SCF protrudes outwards from the slat, filling the cove to reduce noise. To eliminate auxiliary actuators and maintain mechanical

---
\(^1\)The work reported in this chapter was carried out within a research project in collaboration with NASA’s Langley Research Center and the College of Engineering at Texas A&M University.
simplicity, the SCF is designed to deploy and retract purely by interacting with the main wing as the slat is actuated. In its retracted configuration, contact between the main wing and the slat forces the SCF to conform to the space between the slat and the main wing. This deformed state is statically unstable under zero load, meaning that the SCF automatically deploys and snaps out into the cove as the slat moves away from the main wing. Due to the extreme deformations the SCF undergoes, an investigation of the structural nonlinear response is required to achieve optimum performances. Stiffness of the SCF is tailored to control the actuation process and the stability of the structure, with the aim of minimising: (i) the energy required for deployment through a snap-through event; (ii) the severity of the snap-through event, as measured by kinetic energy, and (iii) mass a desirable ‘snap-through’ behaviour is achieved.

The adaptation-by-instability approach is demonstrated in this chapter by comparing three SCF designs: 1) a tailored shape memory alloy (SMA-based SCF); 2) a tailored laminated composite SCF; and 3) a uniformly thick (monolithic) SMA SCF, which serves as a baseline for comparison being NASA’s current best design candidate. Traditional laminated composite materials are considered in this work because ply stacking strategies can be exploited to create stiffness tailoring advantages. The tailored designs follow the previously mentioned principles of geometrically nonlinear elasticity for the purpose of tailoring its behaviour during retraction and deployment. To further demonstrate the capabilities that composite materials offer in conjunction with nonlinear elasticity, two additional tailored composite SCF designs are presented.

6.2 Model development

The baseline geometry for all models is a freestream-aligned section of the Boeing-NASA Common Research Model (CRM) [97] in a high-lift configuration with an additional SCF profile. Previous work [56] considered a 6.25% scale version of the geometry for wind tunnel tests, whereas the present work utilises a full scale version of the geometry (for reference,
### Table 6.1: Ply properties, stacking sequences and thickness distribution along SCF arc-length.

<table>
<thead>
<tr>
<th>Sample Ply thickness (mm)</th>
<th>Section 1a</th>
<th>Section 1b</th>
<th>Section 2a</th>
<th>Section 2b</th>
<th>Section 3a</th>
<th>Section 3b</th>
<th>Tailored SMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>CORE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Composite 1</td>
</tr>
<tr>
<td>[0/90/0/0/0/0][0/90/0/0/0/0][0/90/0/0/0/0]</td>
<td>-</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>0.13</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Ply properties, stacking sequences and thickness distribution along SCF arc-length.
### Table 6.2: E-Glass 913 layer properties and maximum values at failure.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Tensile Modulus $E_1, E_2$ (GPa)</th>
<th>Shear Modulus $G_{12}$ (GPa)</th>
<th>Poisson’s Ratio $\nu_{12}$</th>
<th>Tensile Failure $X_1^*, Y_1^{**}$ (MPa)</th>
<th>Compressive Failure $X_c^*, Y_c^{**}$ (MPa)</th>
<th>Shear Strength $S$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Glass 913</td>
<td>1900$^{\dagger}$</td>
<td>$38.7^{\ddagger}$, $8^{\ddagger}$</td>
<td>$4.3^{\ddagger}$</td>
<td>$0.28^{\ddagger}$</td>
<td>$1548^{\ddagger}$, $65.5^{\ddagger}$</td>
<td>$1000^{\ddagger}$, $150^{\ddagger}$</td>
<td>$40^{\ddagger}$</td>
</tr>
</tbody>
</table>

* Fibers direction 0 deg. ** Fibers direction 90 deg. $^\dagger$Based on Hexcel data. $^\ddagger$Based on Ref. [92, 96].
Figure 6.2: Structural FEA model of 2.5D SCF assembly.

Figure 6.3: Composite 1 layup of the fibre-glass SCF. Lines represent composite layers. Step changes in thickness cause stiffness variations and the structural asymmetry required for nonlinear behaviour tailoring and stability control.

the chord of the retracted configuration is 5.1 m).

6.2.1 Finite element structural model

Figure 6.2a shows the structural finite element model of the SCF. The model is built using the commercial finite element suite Abaqus/Standard [83]. The model consists of the slat, main wing, SCF in its unstressed profile and a hinge for assisting SCF stowage. All parts in the model have a 16 mm spanwise length. It is assumed that the main wing, slat, and hinge are much stiffer than the SCF and are therefore modeled as rigid bodies. Positioning of the hinge axis (18.2 mm from the cove wall) and length of the hinge-arm (34.6 mm) are based on the hinge for the scaled wind tunnel model, which was found in previous work [56] to be the design that minimised actuation force. Although it is possible that a different hinge
position and length could benefit the optimised SCF designs shown here, variations in the hinge position and length are not considered in this study.

For this work, the extremities of the SCF are tied, using a tie constraint, to the trailing edge of the slat and the hinge. The SCF is initially unstressed and partitioned into four sections (see Fig. 6.2b): the SCF-hinge arm (projection of the hinge length unto the SCF) and three equally long sections used in the tailoring process. The mesh for the SCF consists of 259 general shell elements of type S4R along the SCF curve. The SCF (as well as all other parts) is one element wide in the spanwise direction. Symmetry conditions are implemented on the SCF edges aligned with the X-Y plane, modelling an infinitely-long spanwise wing. Contact between the SCF and the other components is modelled using surface-to-surface contact in Abaqus with linear penetration in the normal direction and frictionless behaviour in the tangential direction.

Composite SCFs are modelled using the composite laminate feature in Abaqus to create a stack of plies with fibres oriented in different directions. Three different stacking sequences (Composite 1, Composite 2, and Composite 3) are considered, as shown in Table 6.1. These stacking sequences are selected to demonstrate that different snap-through behaviours can be obtained by means of composite tailoring. Additionally, Fig. 6.3 schematically shows ply orientations for Composite 1, where each layer has a thickness of 0.13 mm. The material chosen is a uni-directional (UD) glass fibre epoxy resin composite, E-Glass 913, with material properties as shown in Table 6.2. In order to obtain an optimised snap-through behaviour, the composite stacking sequence was tailored along the SCF length with the stacking sequences shown in Table 6.1. The composite solutions are selected starting from a core ([C]) stacking sequence which satisfies material failure requirements (see Section 6.2.2) and tailored parametrically by adding and removing on the six SCF subsections layers at 0 and 90 deg of fibre orientation. The tailoring process must also meet the additional constraints of maximum displacement of the SCF under aerodynamic loads (see Section 6.2.2).

For SMA-based SCFs, the super-elastic effect of the SMA material needs to be considered
due to the isothermal operational environment. In this work, the super-elastic material behaviour of SMAs is modeled with the constitutive model developed by Auricchio and coworkers [98]. The model is an Abaqus-native user-material (UMAT) subroutine, making it particularly useful for collaborative work. Material properties for the model are calibrated using data from tensile tests of SMA dogbone specimens made from material used for a scaled SMA-based SCF [57]. Calibrated properties are shown in Table 6.3. The structural behaviour during the retraction-deployment process of the tailored composite SCF is compared with a monolithic SMA-based SCF, with constant thickness of 1.9 mm, and a tailored SMA-based SCF with thickness of 1.48 mm (Section 1&2) and 1.8 mm (Section 3).

Figure 6.4 shows the static mechanical properties of SMA and composite glass fibre materials. The graph clearly highlights the super-elastic intrinsic material characteristic of the SMA. The alloy behaves linearly until the applied stress reaches a critical value. At this level, a microstructural transformation from austenite to martensite occurs, causing a large deformation without incurring significant plasticity [47]. Conversely, the composite material behaves as a classic linear elastic material. Therefore, by purely considering the constitutive behaviour of the two materials, the super-elastic properties of the SMA material seemingly lend themselves more naturally to facilitating large deformations and snapping behaviour. However, the additional or individual occurrence of geometrically nonlinear elastic structural instabilities per se can lead to similar structural super-elastic behaviour. Extreme deformations can be achieved by exploiting also super-elastic structural behaviour rather than, or in addition to, super-elastic SMAs material properties. The advantage of elastic instabilities is that they lend themselves more naturally to tailoring. A nonlinear super-elastic structural behaviour can be designed as desired by working on the component profile, introducing geometrical imperfections and/or tailoring material stiffness [15, 17]. Hence, due to constraints on the SCF profile, the snap-through response is modified only by stiffness tailoring throughout this chapter.

Evaluation of the SCF during the tailoring process considers two load cases: 1) static
Table 6.3: SMA material properties. See [98] for definition of parameters.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Elastic Properties)</td>
<td></td>
</tr>
<tr>
<td>$E_A, E_M$</td>
<td>44.9 GPa, 26.4 GPa</td>
</tr>
<tr>
<td>$\nu_A = \nu_M$</td>
<td>0.33</td>
</tr>
<tr>
<td>(Phase Diagram Properties)</td>
<td></td>
</tr>
<tr>
<td>$\sigma^{Ms}, \sigma^{Mf}$</td>
<td>422 MPa, 425 MPa</td>
</tr>
<tr>
<td>$\sigma^{As}, \sigma^{Af}$</td>
<td>247 MPa, 231 MPa</td>
</tr>
<tr>
<td>$C^A = C^M$</td>
<td>7.12 MPa/K</td>
</tr>
<tr>
<td>(Transformation Strain Properties)</td>
<td></td>
</tr>
<tr>
<td>$H$</td>
<td>5.15%</td>
</tr>
<tr>
<td>(Other Properties)</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>6480 kg/m$^3$</td>
</tr>
</tbody>
</table>

aerodynamic loading, and 2) retraction and deployment of the slat. The two load cases are separately assessed. The static aerodynamic loading of the SCF is conducted using a static analysis. For this load case, the slat is initially fixed in its fully deployed configuration. The pressure distribution for the loading is extracted from a CFD analysis of the wing at 6° angle of attack in Mach 0.2 flow and implemented as a point cloud in Abaqus. The slat/SCF articulation is analysed using an implicit dynamic solver with a quasi-static approach. Aerodynamic loading is not considered for this load case. The articulation of the slat is controlled by applying rotational displacements to a reference point [83] assigned to the rigid body. Full retraction of the slat occurs with a rotation of 25.2°.

The snap-through responses of all the SCFs are studied by plotting $F$ vs $\delta$ curves, where $F$ is the sum of the reaction forces at the leading and trailing edges of the SCF, while $\delta$ is the magnitude of the deflection in the node between Section 1 and Section 2.
Figure 6.4: Stress-strain response of SMA using the Auricchio [98] model and of Glass 913 at various orientations.

6.2.2 Constraints and requirements

Tsai-Wu failure criterion

To assess the likelihood of static failure of the composite SCF, the stress-based Tsai-Wu failure criterion (TWFC) [99] is implemented. The TWFC is chosen because it accounts for interaction between different stress components and provides good failure predictions within the current engineering requirements [100]. Furthermore, the TWFC has been identified as one of the five best-performing failure criteria assessed in the recent World-Wide Failure Exercise and is recommended for uni-directional laminae under combined loading [101].

For a state of plane-stress in the through-thickness direction, the failure index $I_F$ according to the orthotropic TWFC is

\[
I_F = F_1 \sigma_{11} + F_2 \sigma_{22} + 2F_{12} \sigma_{11} \sigma_{22} + F_{11} \sigma_{11}^2 + F_{22} \sigma_{22}^2 + F_{66} \sigma_{12}^2,
\]

where $\sigma_{11}$ and $\sigma_{22}$ are the stress components longitudinal and transverse to the fibre direction, respectively, and $\sigma_{12}$ is the in-plane shear stress. The coefficients $F_{ij}$ of the
orthotropic TWFC in Eq. (6.1) are given by

\[
F_1 = \frac{1}{X_t} - \frac{1}{X_c}, \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}
\]

(6.2)

\[
F_{11} = f^* \sqrt{F_{11} F_{22}}, \quad F_{12} = \frac{1}{X_t X_c}
\]

\[
F_{22} = \frac{1}{Y_t Y_c}, \quad F_{66} = \frac{1}{S^2}
\]

where \(X_t, X_c, Y_t\) and \(Y_c\) are experimentally determined material failure strengths, shown in Table 6.2, in uniaxial tension (\(t\)) and compression (\(c\)) in the directions longitudinal \(X\) and transverse \(Y\) to the fibres, respectively. The in-plane shear strength is given by \(S\) and all five experimental strengths in Eq. (6.2) take positive signs. The magnitude of the normalised interaction term \(f^*\) is difficult to ascertain experimentally and a value of \(f^* = -0.5\) is chosen here based on suggested ranges in the literature [100].

The TWFC is applied to each ply in the laminate separately, bearing in mind that the stresses within each SCF ply are functions of location (SCF arc-length \(s\)) such that \(I_F = I_F(s)\). Thus, to prevent local failure over the whole laminate domain, the failure index related to each ply is required to satisfy

(6.3) \[
\max_{k \in [1,N]} I_F^k(s) < 1
\]

for a total of \(N\) plies within the SCF laminate.

**Displacement from aerodynamic loading**

A major requirement for all SCF designs is that the SCF cannot significantly displace while under aerodynamic loading. Significant deflections can lead to a loss of noise mitigation and potential oscillation of the SCF, generating additional noise. Previous work [102] considered a deflection constraint of 2.54 mm for a 75%-scale SCF. As this work uses a full-scale version of the SCF, the constraint is scaled to 3.4 mm. For the monolithic SMA-based SCF, a thickness of 1.85 mm is the minimum size required to satisfy the constraint with a maximum displacement of 3.3 mm.
6.3 Results

6.3.1 Structural analysis of tailored SCF designs

Figure 6.5 shows the retraction process of the monolithic SMA SCF driven by the main wing pushing into the SCF, where the red dot indicates the tracked node for which deflection, $\delta$, is plotted against force, $F$, in Fig. 6.6. As the main wing pushes the SCF from its fully deployed shape (Fig. 6.5b and c), the SCF deforms and deflects towards the slat cove. Because the trailing edge of the SCF is sliding over and being pushed by the main wing, the leading edge of the SCF temporarily becomes unstable and autonomously separates from the wing (Fig. 6.5d) until contact with the slat wall restabilises the SCF. Finally, the wing deforms the SCF sufficiently to cause snap-through into the inverted and retracted (stowed) configuration, as shown in Fig. 6.5e–f. At this point, the SCF is stowed in the space between the main wing and the slat. During deployment, as the main wing retracts by sliding along the SCF, the point of contact with the main wing moves progressively towards the top boundary of the SCF (Fig. 6.5h) until, at some point, the SCF becomes unstable and is free to snap-out into its deployed shape (Fig. 6.5i to l). The retraction and deployment of the SCF is therefore governed by the constraint applied by the contact between the main wing and the SCF. In summary: (i) during retraction, the main wing pushes the SCF into its stowed configuration aided by the nonlinear mechanics of the SCF that progressively reduces the SCF’s rigidity; while (ii) during deployment, the main wing slides along the SCF until the restraint provided by the main wing is no longer sufficient to prevent the SCF to snap-out into the cove.

Figure 6.6 illustrates the nonlinear behaviour deriving from the monolithic SMA, tailored SMA and tailored Composite 1 SCFs retraction-deployment process showed in Fig. 6.5 as well as related released kinetic and stored strain energies. Figures 6.6a and b plot the deflection of the node indicated by the red dot in Fig. 6.5 vs the sum of reaction forces at the edges of the SCF during retraction and deployment, respectively.

With reference with the red curve of the monolithic SMA in Fig. 6.6, as soon as the SCF
Figure 6.5: Retraction process of SCF. (a) The SCF is initially fully deployed, the red dot indicates the node used for plotting displacements in the figures below. As contact between the SCF and main wing begins, the SCF deflects towards the slat cove, (b)–(c). In (d), contact between the SCF and main wing is momentously lost and then re-established in (e). In (f) the SCF configuration snaps-through and then rests in the retracted (stowed) shape (g). During redeployment, the main wing slides over the SCF (h) until the main wing can no longer support the SCF in its stowed configuration (i) and the SCF snaps-out autonomously via a dynamic snap to (j). Finally, the wing loses contact with the SCF in (k) and (l).
establishes a point of contact with the main wing, force $F$ increases until the SCF sliding over the wing allows an increase of contact surface, thereby favoring the deflection with decreasing force. In fact, $F$ gradually reduces until, with reference to Fig. 6.5d, around 0.45 m of deflection, it suddenly drops to a minimum of 4.7 N due to the temporary loss of stability of the SCF, which moves autonomously towards the slat that eventually stops the spontaneous deflection creating a new point of contact (Fig. 6.5d). Figure 6.6c show that, in correspondence with such force drop, strain energy $SE$ reduces its slope. Once the contact with the wing is re-established, force increases again while $SE$, in the case of the $SMA_m$, does not. Here, with reference to Fig. 6.5e and f, the main wing actuates the
SCF snap-through, which is identified in the $F_{us}\delta$ curves of Fig.6.6a with a jump of $F$ at 0.89 s and 0.90 s of simulation time for the composite and the SMA, respectively. Here, a displacement turning point is present but it is not visible from a dynamic implicit analysis. The snap-through causes a further drop of strain energy and a sudden peak of kinetic energy (Fig.6.6c and d, respectively), as the SCF releases the energy stored so far. Figure 6.6b shows that during the SCF re-deployment, after an initial drop of $F$, force and strain energy (Fig. 6.6c) remain constant, as the SCF stays stable in its retracted configuration (see Fig.6.5h-l for reference) held by a high surface of contact. Subsequently, the interaction with the main wing moves to the right edge of the SCF, causing first a reduction of $F$ and the a spontaneous snap-back. This again relates to a drop of strain energy and maximum release of kinetic energy.

All cases show a monostable snap-through response (see Fig. 3.3 for reference), which is essential for the desired autonomous re-deployment. However, the $F$ $vs$ $\delta$ curve in Fig. 6.6a, related to the monolithic SCF, has a minimum force trough of 4.7 N, i.e. very close to intersecting the zero-force axis. This suggests that potential uncertainties in the intrinsic super-elastic properties caused by thermo-mechanical fatigue [59] could modify the stability of the structure and make it bistable. By stiffness tailoring, the minimum point of the equilibrium curve can be increased for both SMA-based and fibreglass composite SCFs, thereby controlling the monostability of the SCF component.

The SCF undergoes considerable amounts of local curvature change before finally snapping through. In the case of the monolithic SMA-based design, the intrinsic super-elastic properties of material softening facilitate snap-through, thereby reducing the degree of local curvature (bending strains) and resulting in an energetically convenient actuation process.

By tailoring the stiffness of the SMA along the arc-length of the component, the degree of local curvature is reduced, thereby minimizing the internal stresses and altering significantly the shape of the equilibrium curve, indicating a more gradual retraction and smoother transitions toward the slat. This is demonstrated by a decrease of 19.8 % in the force at
Table 6.4: Comparison of design with different material solutions.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Max KE (J)</th>
<th>Max SE (J)</th>
<th>Force at Snap-through $F$ (N)</th>
<th>Max Deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monolithic SMA</td>
<td>6480</td>
<td>2.6</td>
<td>5.3</td>
<td>73.2</td>
<td>3.31</td>
</tr>
<tr>
<td>Tailored SMA</td>
<td>6480</td>
<td>1.4</td>
<td>3.4</td>
<td>58.7</td>
<td>3.27</td>
</tr>
<tr>
<td>Tailored Composite 1</td>
<td>1900</td>
<td>1.5</td>
<td>3.8</td>
<td>61.7</td>
<td>3.24</td>
</tr>
<tr>
<td>Tailored Composite 2</td>
<td>1900</td>
<td>1.2</td>
<td>3.8</td>
<td>60.9</td>
<td>3.04</td>
</tr>
<tr>
<td>Tailored Composite 3</td>
<td>1900</td>
<td>0.9</td>
<td>3.3</td>
<td>60.7</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Snap-through and a 35.9\% reduction in peak strain energy, as shown in Table 6.4 and Fig. 6.6a and c. A softer and more controlled dynamic retraction and re-deployment, as achieved by the tailored SMA, also leads to a less severe dynamic snap, as shown by the reduction in the release of kinetic energy (Fig. 6.6d), of up to 46\%. The reduction in peak kinetic energy of the SMA-based SCF is beneficial as it reduces the vehemence of potential collisions with the wall of the slat cove, and the deflection and oscillations of the SCF profile at snap-back. Similarly, the reduction in peak strain energy lowers the actuation power requirements.

The black curves in Fig. 6.6 illustrate the structural behaviour of the Composite 1 SCF compared to the SMA designs. The Composite 1 dynamic response is similar to the tailored SMA. The material choice and stiffness tailoring cause a reduction in kinetic and strain energy, as well as a more drawn-out monostable behaviour. Consequently, these results, summarised in Table 6.4, suggest that elastic instabilities can be used when ‘super-elastic’ characteristics are required, regardless of the intrinsic properties of the material. Furthermore, the structural mass and cost of the fibreglass solution also improve on the SMA design. An additional advantage of a composite design is the simplicity with which manufacturing techniques lend themselves to tailoring the structural behaviour. Techniques such as layup vacuum-bagging and tow-steering [62, 63] allow local tailoring of the material stiffness, and hence the equilibrium manifold, which is essential for the design of the described retraction-deployment process. On the other hand, although the analysed composite solutions satisfy the Tsai-Wu failure criterion, with a maximum index $I_F = 0.86$, due to the extreme deforma-
6.3. RESULTS

Figure 6.7: Snap-through (a) and snap-back (b) of Composite 1, 2 and 3 SCFs. On the bottom the related stored strain energy (c) and released kinetic energy (d) are plotted against time.

Snap-through, snap-back and the related kinetic and strain energies are shown in Fig. 6.7. The retraction curve of Composite 2 (Fig. 6.7a) does not significantly differ from Composite 1, whereas Fig. 6.7b shows that the $F$ vs $\delta$ curve of Composite 2 becomes flatter.
for a smoother redeployment. As a result, while there are no significant changes in strain and kinetic energies at snap-through, at snap-back Fig. 6.7d shows a decrease in released kinetic energy of 20% for Composite 2 relative to Composite 1. The effect of tailoring is even more pronounced for Composite 3, where stiffness is increased softly, from left to right hand side, in three steps by adding a 90 deg plies in Section 2 and 3 of the SCF part. This allowed a reduction the loss of structural stiffness once the critical load is reached. In this way, the structure responds to the external load via a smoother and less severe snap, which results in both a lower release of kinetic energy (40%) and reduction in internal strain energy (13%), compared to Composite 1.

6.4 Conclusions

This chapter showed the dynamic response of an SCF during the retraction and re-deployment process actuated by the interaction with the main wing. Abaqus simulations were carried out to study the effect of materials selection and stiffness tailoring on the snap-through nonlinear behaviour of the SCF. Monolithic and tailored SMAs and three fibreglass tailored composites with different lay-up stacking sequences were investigated. In all cases, the SCF shows monostable snap-through behaviour. However, the tailored designs affect the dynamic behaviour of the structural response resulting in a lower required force and strain energy required for actuation of the retraction and deployment processes. Additionally, the amount of kinetic energy released at snap-through and snap-back is also reduced up to 40%. These results suggest that the constitutive superelastic response of the SMA can be replaced by elastic instabilities and elastic tailoring of a composite SCF for portions of the airframe where deformation requirements are sufficiently low to permit the use of laminated composite materials. Comparisons between three analysed lay-up solutions highlighted the flexibility and efficiency of composite tailoring. In fact, results show that depending on the stacking sequence, it is possible to modify one or more aspects of the SCF nonlinear dynamic behaviour.
In conclusion, also for the case of the SCF, elastically nonlinear behaviours have been successfully employed to expand the structure’s design space and enhance functionality and performance.
CHAPTER 7

CONCLUSIONS AND FUTURE WORKS

Times and conditions change so rapidly that we must keep our aim constantly focused on the future.

Walt Disney

The continuously growing research interest in morphing and adaptive systems is being sustained by the development and progress of smart materials and new design approaches. Multiple methods are now available to achieve the best trade-off between stiffness, flexibility and light weight. Specifically, elastic instabilities of post-buckled structures are becoming an important aspect in the design of shape-changing structures for aerospace engineering and fluid flow control applications. Instabilities and transitions can indeed be exploited to achieve safe and controlled catastrophes. As a matter of fact, such elastic instabilities and catastrophic transitions have recently been reevaluated as design tools, causing a trend reversal towards well-behaved nonlinearities or from Buckliphobia to Buckliphilia [19]: when mechanical instabilities are carefully studied and consequently predicted, the deriving temporary loss of structural stiffness is no longer perceived as a catas-
trophic aberration, but rather exploited as a means for adaptation and multifunctionality.

With this new design paradigm, this thesis takes advantage of concepts such as multistability and elastic snap-through instabilities to create repeatable, ‘well-behaved’ shape changes to design the nonlinear multistable behaviour of adaptive structures for fluid control.

Chapter 3 shows the parametric buckling and post-buckling investigation on the effect of different boundary conditions of a clamped-clamped beam. Its multistability can be predicted in a straightforward manner by inspecting the bifurcation pitchfork diagrams. It is sufficient to produce and then study the equilibrium manifold of the morphing member. In the case of a symmetric structure, the post-buckled beam can only act as bistable. When at least one symmetry is broken, the structure may also exhibit superelastic monostability. The controlled changing of boundary conditions and/or geometrical parameters can be used to modify and tailor the severity of a snap-through response, but also the multistability of a morphing component. In fact, locus of limit points and the shape of broken pitchforks consequently affect the regions of stability and instability as well as the onset of dynamic snap-through responses.

Linear structures are typically designed to specifications that define a particular geometry and loading environment. In an optimisation study the material properties can, for example, be tailored to guarantee safe operation for minimum mass. In the case of multistable nonlinear structures the specifics of the loading environment are initially not as important. In contrast, it is critically important to ensure that the structure can snap and be mono- or bistable. The particulars of the loading environment change the shape of the snap-through equilibrium curves (limit points and cusps), but the qualitative aspects of the bifurcation diagrams themselves remain unchanged. Hence, the characteristics of stability of the system are not influenced by the type of load used to switch between stable configurations, because these equilibria are inherent properties of the system that exist in the unloaded state. This feature allows a different design approach to be followed that focuses on the qualitative aspects of the structure first (without focusing on details of the
load), with subsequent design refinement afterwards (including details of the load). This is particularly attractive for this study, as most of the optimisation can be performed in a quasistatic setting first, and then the design can be refined in a more computationally intensive fluid-structure interaction step.

The observed taxonomy of behaviours is successively used to design adaptive inlets with different operational envelopes. With reference to the inlet in Fig. 3.1 in Section 3.1, the post-buckled adaptive component, initially in its ‘open’ configuration, exploits as transverse load the increase of fluid velocity and the consequently generated low static pressure. The closing of the morphing component is thereby actuated passively without the need of any kind of external actuators. An adaptive inlet with simple stability would thus deform to decrease the inlet aperture. In this case, the extent of flow regulation is proportional to air speed and limited by the stiffness and maximum displacement of the adaptive member. Bistable or superelastic monostable inlets can be designed to decrease the aperture for increasing air speeds and close completely upon snap-through, when a critical speed is reached. In this respect, monostable inlets are able to snap back to the open configuration autonomously when air speed is decreased beneath a critical value. Conversely, bistable ones do not need external inputs to hold the closed, inverted shape.

The passive actuation of the adaptive post-buckled component of an air inlet is proven in Chapter 4, which delves deeper into an in-depth aeroelastic analysis of actuation by means of FSI simulations. In particular, according to the results achieved in the previous chapter, a fibreglass composite plate is stiffness-tailored in order to introduce asymmetries in the geometry and, as a consequence, in the equilibrium manifold. By simply varying the magnitude of pre-compression applied on the post-buckled panel, its dynamic multistability transits from bistable to a superelastic monostable. This insight is important for design purposes as it allows the achievement of both binary inlets, statically stable in both open and closed configurations, and valve-like inlets, that can passively transition between open and closed states. The fluid-structure interaction results show that these devices can be actuated
passively simply by exploiting the changes in the pressure field caused by air flowing over the curved inlet (Fig. 4.4). Specifically, snap-through is activated at a velocity of 60 m/s, at which point the inlet closes within a time frame of approximately 10 ms so that air can no longer flow into the duct.

The proposed design concept and passive actuation are validated experimentally by tests in a wind tunnel facility. The information gathered from the FE study is used to manufacture a test rig. The taxonomy of adaptive behaviours is verified by investigation of the critical velocity and static pressure at snap-through and snap-back as a function of the applied compressive displacement on the shape-changing composite panel. The control of this parameter, through a system comprising a lead-screw and linear rails, successfully permits the elastic characteristics of the inlet to be reconfigured. The system can be set to be bistable, superelastic or simply stable. By controlling such parameter, a variety of responses can be achieved for changing airspeeds.

Theoretical and experimental results on the buckling and post-buckling behaviour of the inlet’s morphing component indicate the presence of a cusp singularity in the force vs displacement vs compression space, as well as its importance on the inlet’s qualitative behaviour. Specifically, the cusp forms because the locus of snap-through and snap-back critical velocities and pressures coalesces as the panel compression is decreased. In general, the greater the applied compression, the higher the airspeed required to actuate snap-through. For the design tested in this work, the critical snap-through velocities range between 26.7 and 40.3 m/s. Conversely, snap-back can be seen only when the structure behaves superelastically, with compression between 3.55 and 4.35 mm. Under these circumstances, the higher the degree of compression, the lower the airspeed to actuate snap-back. Given the generality of the underlying physical principle, the system’s parameters can be tailored to fit specific operating conditions.

The nonlinear design framework defined herein, describes the reconfiguration and control of the dynamic snap-through characteristics of the inlet, making its multistability tailorable
7.1. FUTURE WORK

and suitable for a wide range of applications across industry sectors where shape adaptation and passive actuation are key, e.g. biomedical, automotive, aerospace, marine. In fact, the design framework is also employed for the evaluation and optimisation of the structural behaviour of a slat-cove filler for airframe noise reduction. Chapter 6 describes the effect of materials selection and stiffness tailoring on the snap-through nonlinear behaviour of the SCF. Mechanical instabilities are studied to control the multistability of the adaptive component and to improve the energy efficiency of its structural response during the retraction and re-deployment process. Monolithic and tailored SMAs and three fibreglass tailored composites with different lay-up stacking sequences were investigated. In all cases, the SCF shows monostable snap-through behaviour. However, the tailored designs affect the dynamic behaviour of the structural response resulting in a lower snap-through load and lower amounts of energy required for actuation. Additionally, the amount of kinetic energy released at snap-through and snap-back is also reduced up to 65%. These results suggest that the constitutive superelastic response of the SMA can be replaced by elastic instabilities and elastic tailoring of a composite SCF for portions of the airframe where deformation requirements are sufficiently low to permit the use of laminated composite materials. Comparisons between three analyzed lay-up solutions highlighted the flexibility and efficiency of composite tailoring. In fact, results show that depending on the stacking sequence, it is possible to modify one or more aspects of the SCF snap-through behaviour.

The state-of-the-art gathered in this thesis and the results achieved, reinforce the thriving trend reversal towards well-behaved nonlinearities and from Buckliphobia to Buckliphilia. A ‘catastrophic’ approach is used to build confidence in designing and applying well-behaved shape-changing devices in aerospace applications.

7.1 Future work

This PhD thesis proves the simplicity and flexibility of using elastic instabilities as a design methodology for shape-changing structures. This approach allows to design engineering
CHAPTER 7. CONCLUSIONS AND FUTURE WORKS

devices with an optimum trade-off between adaptability, lightweight and load-bearing cap-
pabilities, thereby contributing in making a step forward toward the resolution of the
engineering paradox, as discussed in Section 1.1 of Chapter 1, illustrated by the Campanile’s
requirement triangle [4], for which most systems can fulfil only two out of the aforementioned
requirements. In fact, the proposed adaptation-by-instability approach clearly permits to
improve the flexibility of a designed component as it can adapt itself to changing in envi-
ronmental conditions by passively switching between different multistable configurations.
Additionally, the lightweight requirement is fulfilled, not only because of passive actuation
that allows to avoid external mechanisms and motors otherwise required for active actuation,
but also thanks to the possibility of using lightweight materials. In fact, the design tool
relies only on geometrical instabilities and not on intrinsic material properties.

Theoretically, any materials can be used to achieve post-buckled configurations. On
the other hand, high strains and extreme deformations can cause material failure and
consequently limit the general applicability of the design methodology. Depending on the
final application, materials required and specific boundary conditions, a morphing structure
might fulfil only partially the remaining requirement of load-carrying capabilities. For
this reason, future work on morphing structures must focus on improving load bearing
capabilities of shape-changing structures.

7.1.1 Adaptive air inlet

The adaptive device manufactured during this PhD project, reached a Technology Readiness
Level (TRL) [103] of 4 as, not only analytical and experimental critical function were proven
(TRL 3), but also development and validation of the technology in a relevant environment
was achieved. In order to advance the TRL of both the inlet and its design methodology
future work will possibly include:

• A detailed study on the failure, fatigue and creep behaviour of the morphing component.

In fact, the component is subjected to high strains, cyclic loads and constant compression
for long times. Although applied loads do not induce sufficiently high strains to cause premature failure, they must be investigated in order to quantify the expected fatigue life. Materials performances must be considered as it is identified as a primary limitation of the design tool.

- FSI simulations for the optimisation of the aerodynamic profile of the air inlet. Its geometry must be based on the NACA duct common form that allows air to be drawn into the aircraft with minimal flow disturbance and flow separation [104]. Generally, shape optimisation must be considered in the design methodology. A specific final shape is of course identified as a constrain of the design flexibility but it does not impede the achievement and control of multistability.

- Expand current design and modelling capabilities to include: (a) the effects of environmental factors, such as temperature [9] and moisture [105], which will affect performance; and (b) smart materials, e.g. piezoelectrics and shape memory alloys, for embedded actuation. In this way it will be possible to achieve full design and control capabilities of the passive actuation processes for shape-adaptive air inlets.

- Explore industrially convenient production manufacturing techniques.

- Include in the design methodology generalised path-following frameworks [17] in order to efficiently analyse post-buckled structures and elastic instabilities.

Additionally, it is necessary to undertake an industry consultation and market investigation to assess the potential for exploitation of the proposed technology and the most promising end-user applications across different engineering sectors. This will bridge the gap between academic fundamental research and industrial application by directing future fundamental researches to maximising the potential for impact and industrial take-up of nonlinear morphing technologies.
7.1.2 Slat-cove filler

Chapter 6 showed how mechanical instabilities were exploited to evaluate and tailor the structural response of a SCF component during its retraction-deployment process. Preliminary FSI analysis—more details on the FSI analysis methodology can be found in reference [106]—were also conducted on the SCF components (monolithic SMA, tailored Composite 1, and tailored SMA) to assess their behaviour in flow with flight condition of Mach 0.2 at an angle of attack of 6 deg.

It is expected that the SCF displacement will initially oscillate at the start of the analysis but eventually decay to a constant value, perhaps with small oscillations about the mean. Figure 7.1 shows the displacement of the SCF in time at two points along its curve where the displacement is greatest. The first point (Node 1) is in the third tailoring section near the trailing edge, whereas the second point (Node 2) is in the first tailoring section near the hinge. For the monolithic SMA-based SCF (Fig. 7.1a), the displacement at both points decays, but not to a constant value by the end of the analysis. A longer analysis is expected to lead to further reduction of the displacement. Additionally, the oscillation of the displacements at both nodes is of similar value. For the tailored composite SCF (Fig. 7.1b), there is some decay initially but at the end of the analysis, the displacement at both points has nearly constant oscillations. The tailored composite design also exhibits a much larger oscillation at Node 2 than Node 1. This may be due to the reduced thickness (and thus reduced stiffness) in the first and second sections of the SCF. It is also interesting to note that the displacement at Node 2 reaches a maximum when the displacement at Node 1 reaches a minimum (and vice versa), suggesting that the SCF may be oscillating about a central point between Nodes 1 and 2. This behaviour is also exhibited in the monolithic SMA-based SCF, but it is not as obvious. Additionally, when the displacement at Node 2 is at a maximum, it is higher than the Node 1 displacement. This shift in location for the overall maximum displacement and an increase in amplitude of the displacement near the end of analysis may suggest that the SCF is close to becoming unstable or may become unstable under a slightly higher flow...
velocity. The tailored SMA-based SCF (Fig. 7.1c) exhibits many of the same displacement characteristics as the tailored composite SCF, however, they are more severe and suggest that the tailored SMA-based SCF is unstable. By the end of the analysis, the oscillations have not diminished significantly and the displacement at Node 2 exceeds the Node 1 displacement for longer portions of the analysis. Unlike the previous two SCF designs, the average displacement of both nodes are of similar value for the tailored SMA-based SCF. Additionally, the tailored SMA-based SCF appears to more clearly exhibit multiple modes contributing to the dynamic response. However, since 1.5 s did not result in full decaying of the response for the monolithic SMA SCF to an approximately steady value, longer FSI analyses are needed to fully understand the behaviour of the tailored SCF designs in flow.

All three SCF designs satisfied the displacement constraint of 3.4 mm under a static aerodynamic loading extracted from the CFD analysis used in the initialization of the FSI analysis. However, in the FSI analysis, all considered SCFs have displacements that do not meet the constraint. Furthermore, the dynamic FSI response may suggest that the SCF structures designed to the steady load are marginal or even unstable when the fully coupled response is considered. Figure 7.1d shows the displacement of Node 1 for the monolithic SMA-based SCF in the FSI analysis plotted against the displacement when the SCF is subjected to static aerodynamic loading using the averaged pressure distributions extracted from the FSI analysis and the initial CFD analysis. It can be seen that the mean value of the displacement from the FSI analysis is significantly greater than the static displacement due to the steady load from the CFD analysis. The static displacement due to the average pressure distribution from the FSI analysis is also slightly lower than the mean dynamic displacement. These observations suggest that nonlinear coupling may be present in the FSI response that tends to destabilize the system, i.e., positive feedback may be present between the displacement of the SCF and the pressure distribution. As any SCF deforms, the pressure distribution acting on it changes in such a way that leads to further displacement.

Similar behaviours are exhibited for the other SCF designs. These results suggest that
Figure 7.1: Deflection of multiple SCF designs when immersed in airflow with flight conditions of Mach = 0.2 and 6 deg angle of attack.
the coupling between the displacement and aerodynamic loading (i.e., FSI) may need to be taken into account during the tailoring process. The current process evaluates the dynamic behaviour of the SCF under retraction/deployment cycles with no aerodynamic loading while satisfying a displacement constraint from a static load. The interaction between the structure and flow in the FSI analysis may be resulting in dynamic behaviours not captured when tailoring SCF response to retraction/deployment only. Additionally, displacement constraints from aerodynamic loading either need to be reduced or used for more severe flow conditions (such as faster flow speeds) than the operational environment. Both changes will produce a stiffer structure more resistant to the feedback mechanism.

Future work will focus on incorporating more flight conditions (angles of attack and flow speeds) to tailor the SCF to a wide range of loadings. Current FSI models and results will be investigated and improved upon to ensure accuracy. FSI analysis will be conducted for longer periods of time to observe further SCF behaviour in flow. Additionally, adverse flight conditions above operational use will need to be considered to tailor SCFs that can meet deflection constraints in a dynamic analysis. Behaviour of the SCF in flow during retraction and deployment of the slat will also be considered.

7.1.3 New ideas of adaptive devices for fluid flow control

Fatehi et al. [107], conceived a passive way to enhance airfoil performances by modifying its profile including a shape-optimized cavity. As illustrated in Fig. 7.2a, results show that such a cavity can trap vortices to control the stall margin and increase the lift-to-drag ratio. However, this advantage is achieved only above a threshold value of angles of attack. Below a specific one, as shown in Fig. 7.2b, the performances of the airfoil are higher without the use of the cavity.

In this regard, a bistable structure could be applied into the cavity and actuated only when required in order to close and be flush with the external aerodynamic surface. Figure 7.2c outlines a potential component in its stable open (in green) and closed (in red)
Figure 7.2: Adaptive cavity shape optimization of wind turbine blade. (a) Comparing streamlines around the airfoils with and without the optimized cavity. (b) Lift to drag ratio versus angle of attack at Reynolds number of $10^5$. (c) Application of a bistable device able to open and close the cavity when required. Reproduced and modified from [107].

configurations. The bistable capability of this device would require actuation only to switch between the two stable states.
A.1 Stability analysis of elastica with vertical displacement at one end

The stability of the clamped elastica shown in Fig. A.1 is analysed using an energy approach. The Lagrangian of the system, $\Pi$, is given by the difference between the internal strain energy, $U$, and the work done by the external force, $V$. For the initially straight elastica inclined at an angle $\alpha$ to the global $x$-axis shown in Fig. A.1a,

\begin{align}
\Pi &= U - V \quad \text{(A.1)} \\
\Pi[\theta] &= \frac{1}{2} \int_0^L EI \theta'^2 \, ds - P \cos \alpha \left( L - \int_0^L \cos \theta \, ds \right) \quad \text{(A.2)}
\end{align}

where $EI$ is the bending rigidity of the elastica, $\theta$ is the bending rotation with respect to the elastica axis $s$, $P$ is the force in the global $x$-direction, and $L$ is the length of the elastica. The comma notation is used throughout to denote differentiation.
APPENDIX A. ADDITIONAL CONSIDERATIONS ON THE DESIGN OF THE AIR INLET

The second variation of this functional with respect to \( \theta \) is

\[
\Pi''[\theta]\theta_1\theta_2 = \int_0^L EI\theta_1,\theta_2, s ds - P \cos \alpha \int_0^L \theta_1\theta_2 \cos \theta ds,
\]

where \( \theta_1 \) and \( \theta_2 \) are the first and second variation of \( \theta \), respectively.

According to Budiansky [108], the criticality condition is given by \( \Pi''\theta_1^2 = 0 \). Solving for \( P \) gives

\[
P_C = \frac{\int_0^L EI\theta_1^2, s ds}{\cos \alpha \int_0^L \theta_1 \cos \theta_C ds}.
\]

At the critical point, the elastica is assumed to be in its flat state, i.e. the elastica only compresses uniaxially in the pre-buckling regime and \( \theta_C(s) = 0 \). The first variation \( \theta_1(s) \) is a small perturbation from this uniaxially compressed state with boundary conditions \( \theta_1(0) = \theta_1(L) = 0 \). Thus, we assume \( \theta_1(s) = ||\theta_1|| \sin \left( \frac{2\pi s}{L} \right) \). Substituting these assumptions into (A.4) above gives

\[
P_C = \frac{4\pi^2 EI}{L^2 \cos \alpha}.
\]

with \( \cos \alpha = \sqrt{1 - \frac{w^2}{L^2}} \), where \( w \) is the imposed vertical displacement.

Alternatively, Fig. A.1B features the same elastica at an angle \( \alpha \) to the global x-axis but with an initial imperfection, \( \theta_0(s) \), that arises because the elastica is forced to be parallel to the global x-axis at the two extremities. This scenario more closely resembles the imposition of a vertical displacement shown in Fig. 3.2 of Chapter 3. For simplicity, it is here assumed that the stresses induced by imposing this vertical displacement are negligible.

The energy expression now reads,

\[
\Pi[\theta] = \frac{1}{2} \int_0^L EI(\theta_1 - \theta_0, s)^2 ds - P \cos \alpha \left( \int_0^L \cos \theta ds \right).
\]

The expression for the criticality condition \( \Pi''\theta_1^2 = 0 \) of (A.4) remains unchanged as \( \theta_0 \) vanishes in the second variation. However, the angle \( \theta_C \) is no longer zero along the axis of the elastica. Given the boundary conditions of the initial shape \( \theta_0(0) = \theta_0(L) = -\alpha \), we assume that \( \theta_0 = -\alpha \cos \left( \frac{2\pi s}{L} \right) \). Furthermore, we assume that any changes in the original
A.2. EFFECT OF BOUNDARY CONDITIONS ON EQUILIBRIUM MANIFOLDS

In this section more detailed results with regards to the parametric study discussed in Chapter 3 are shown.

With reference to Fig. 3.2 shown in Chapter 3, Fig. A.2 shows the bifurcation diagrams related to the equilibrium solutions of the clamped-clamped beam when one of the extremities is subjected to vertical displacement $w = 5\text{ cm}$ and both the ends are rotated of $\alpha$ and $\beta$. Figures A.2a and b indicate that the application of $\alpha$ and $\beta$ separately breaks the symmetry.
of the pitchfork, thereby creating a region of superelastic monostability, as discussed in Section 3.3.2. However, the limit point of the secondary stable branch is more sensitive to the application of only $\beta$. Consequently, if on one hand higher compressive load is required to achieve bistability, on the other hand the region of superelasticity can be controlled and expanded with more flexibility. Figures A.2c and d confirm the influence of $\beta$ and underline how effect of $\alpha$ is stronger on the shape of the primary stable branch.

Figures A.3a-c show in more detail the effect of vertical displacement $w$ together with ends rotations and thickness variation. It can be noticed that the use of $\beta$ combined with
A.3 Preliminary FSI study with Abaqus ALE co-simulation

Passive actuation of the snap-through for the air inlet morphing component is demonstrated by FSI analysis as shown in Chapter 4. As discussed in Section 2.6 a CEL method is preferred...
Figure A.4: ALE FSI analysis of snap-through of a monostable structure due to an applied pressure field. Blue contours represent negative pressure, while red positive pressure. (a) $t=0\text{s } v=40\text{m/s}$, structure in its stable first configuration. (b) $t=0.025\text{s } v=85\text{m/s}$, snap-through triggered. (c) $t=0.16\text{s } v=135\text{m/s}$, structure in its second configuration, held in position by negative pressure. (d) $t=1\text{s } v=40\text{m/s}$, structure snaps-back into the first configuration.

for the investigation of the passive actuation due to airflow. However, some preliminary analyses are also carried out with ALE Abaqus/CFD and Abaqus/Standard co-simulations.

A first FSI co-simulation focused on the adaptive response of a monostable structure to high-speed airflow. In this initial study, the presence of the inlet cover was ignored in order to prove the feasibility of designing a morphing air inlet passively driven by intrinsic nonlinear behaviour.

A.3.1 Model

The FE model of Abaqus/Standard is the same used for the CEL analysis, see Section 4.2 for reference. In the ALE Abaqus/CFD model a static relative pressure $P = 0\text{Pa}$ is assigned beneath the air inlet, where no fluid is assumed to flow. The inlet velocity was increased from
40 to 150 m/s to detect the critical velocity that initiates the snap-through of the structures. As the minimum critical velocity for both the bi- and mono-stable inlets was found to be around 60 m/s, this velocity was applied as inlet velocity boundary condition for the CEL method. Only one element was used through the width of the domain. The size of the fluid domain was chosen to be large enough (0.5 x 0.4 m) to minimise any boundary effects. In order to achieve better convergence properties, a more refined mesh was used around the air inlet where extreme structural deformation occurs. A second FSI co-simulation is also carried out with the presence of a cover. As contact between the air inlet and the cover is not possible in this type of analysis due to excessive mesh distortion, the model was not run to completion. The fluid-structure co-simulation is achieved by imposing the boundary kinematic and dynamic constraints and a “no-slip” condition, Equations 2.7 and 2.8, respectively.
A.3.2 Results

Results in Fig. A.4 show the different steps in the snap-through process actuated by airflow. The initial inlet velocity is 40 m/s and this induces a relative pressure field over the inlet that does not affect its configuration (Fig. A.4a). Hence, the airflow is not strong enough to induce a pressure field that actuates the structure to snap into its second configuration. The coloured contour indicates this pressure field with negative quantities in blue and positive quantities in red. When air velocity reaches 85 m/s, relative pressure is strong enough to trigger a snap-through response (Fig. A.4b). As long as the velocity is higher than 85 m/s, the second configuration of the monostable inlet is held by the pressure field (Fig. A.4c). Once the velocity is reduced to 40 m/s, snap-back to the original configuration occurs (Fig. A.4d).

A second model was created to study the adaptive behaviour of the inlet with a cover, i.e. the realistic configuration introduced in Fig. 1.4. Therefore, an upper cover was added to the model. However, its presence introduces numerous mesh complexities and leads to element collapse during the snap-through process. Nevertheless, the results obtained before the analysis aborts can still be used to assess whether such an adaptive valve will indeed respond to a specified flow condition, and can also be used to validate the CEL model. Figures A.5a-c show the adaptive behaviour of the inlet with a cover. At low air speeds, the structure is stable in its open state and air is allowed to flow into the duct. When the velocity increases between 50 and 60 m/s, the structure is actuated by the pressure field and starts to snap from the open to the closed state. However, as shown Fig. A.5d, the inlet cannot be closed completely due to the occurrence of element collapse.
B.1 Technical drawings of air inlet assembly parts

With reference to the CAD assembly of the air inlet prototype, shown in Fig. 5.10 of Chapter 5, specifics of each component are reported herein.
Figure B.2: HIWIN linear carriages that, through support (Part 1) and linear rails, allow application of axial pre-compression to the composite plate.
Figure B.3: Technical drawing of HIWIN linear rail and bottom support plate. Rails are connected to the aluminium plate through 18 M9 screws. The bottom plate geometry is designed to allow perfect alignment of the rails and sliding of lateral walls (Part 8).
Figure B.4: Technical drawing of outlet channel and front and back wall supports. Front and back walls are connected through the endless lead-screw.
Figure B.5: The aluminum support connects HIWIN carriages with the lower extremity of the composite plate.
Figure B.6: Top plate and flush with the wind-tunnel floor. The cover tap is designed to adjust the inlet opening when required.
Figure B.7: Technical drawing of later wall and morphing component.
Figure B.8: FE structural analysis on air inlet prototype. Contour-plots indicate (a) stresses and (b) strains in the mechanism.

### B.2 ABAQUS structural analysis on test rig

ABAQUS structural analysis are carried out for the design of the wind tunnel rig. The assembly components’ materials and dimensions are chosen according to stress and strain
Table B.1: Mechanical properties ($\sigma_f$ and $\epsilon_f$) of assembly components and maximum stresses and strains experienced during FE analysis ($\sigma_m$ and $\epsilon_m$).

<table>
<thead>
<tr>
<th>Part</th>
<th>Stress at failure $\sigma_f$ (MPa)</th>
<th>Strain at failure $\epsilon_f$</th>
<th>Stress max $\sigma_m$ (kPa)</th>
<th>Strain max $\epsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead-screw</td>
<td>550 – 600†</td>
<td>0.12 – 0.25†</td>
<td>601.0</td>
<td>6.7 x 10^{-6}</td>
</tr>
<tr>
<td>Lead-screw nut</td>
<td>600 – 700†</td>
<td>0.35 – 0.40†</td>
<td>220.6</td>
<td>3.0 x 10^{-6}</td>
</tr>
<tr>
<td>PMMA front plate</td>
<td>70‡</td>
<td>0.025‡</td>
<td>7.3</td>
<td>3.0 x 10^{-3}</td>
</tr>
<tr>
<td>Moving support</td>
<td>240 – 300†</td>
<td>0.12 – 0.25†</td>
<td>153.0 x 10^3</td>
<td>2.0 x 10^{-3}</td>
</tr>
<tr>
<td>HIWIN linear rail</td>
<td>400 – 500†</td>
<td>0.20†</td>
<td>165.8</td>
<td>2.3 x 10^{-5}</td>
</tr>
<tr>
<td>HIWIN carriage</td>
<td>400 – 500†</td>
<td>0.20†</td>
<td>232.5</td>
<td>7.3 x 10^{-5}</td>
</tr>
</tbody>
</table>

†Based on Ref. [109]. ‡Based on Ref. [110]


BIBLIOGRAPHY


