
Early version, also known as pre-print

Link to published version (if available):
10.1063/1.4729474

Link to publication record in Explore Bristol Research
PDF-document

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The following article appeared in Rev. Sci. Instrum. 83, 063705 (2012) and may be found at http://scitation.aip.org/content/aip/journal/rsi/83/6/10.1063/1.4729474.

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Balanced Detection for Interferometry with a Noisy Source

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Optical properties of nanostructures depend on size, shape, material, and local environment. These characteristics can be probed interferometrically, given a broadband source. However, broadband supercontinuum sources are intrinsically noisy, limiting the measurement sensitivity. In this article we describe the application of an auto-balancing technique to reduce the noise in a broadband supercontinuum source, thus increasing the signal to noise ratio. We show a noise reduction of 41dB allowing optical powers as small as 0.01pW to be interferometrically detected with a 5ms integration time.

I Introduction

Measuring the optical properties of individual nanostructures is challenging due to their small absorption and scattering cross sections. Interferometry, where the weak optical signal from an individual nanostructure is amplified by interference with a strong optical reference beam, allows such small signals to be detected1,2,3. In fact interferometric detection is limited only by the shot noise, allowing sensitive detection of small optical signals4,5. In addition, by using modulation techniques, information can be provided on both phase and amplitude changes induced by the nanostructure1,6.

Interferometric techniques to detect individual nanostructures typically operate at a single wavelength. However, the optical properties of nanostructures have a spectral dependence which varies greatly with size, shape, material and local environment, suggesting the need for broadband detection methods7. Recently, Pearson et. al.8 used a dual-color common path interferometer to distinguish between gold and silver nanoparticles by utilising their different spectral response of their plasmon resonances. Broadband interferometry with a supercontinuum white light source for illumination has been used to record plasmonic spectra of metallic
nanoparticles. Although supercontinuum light sources produce a broadband visible light spectrum they are intrinsically noisy – typically exhibiting intensity noise greater than 1%. Since in broadband interferometry the small optical signal is being multiplied by a reference beam from the supercontinuum source, the noise in the stronger reference beam will quickly dominate over the signal. This effect set the lower limit of detection in the measurements described by Lindfors et. al.

Balanced detection is an established method to reduce intensity noise intrinsic to the laser source. By using two detectors and ensuring that on subtraction the two detected signals have exactly equal excess noise, noise common to both detectors should be completely removed. Hence, balanced detection can also be used to detect a small signal change and this has even been used to detect the absorption of single molecules at room temperature. A combination of balanced detection and heterodyne interferometry could therefore be an attractive approach to the measurement of small optical signals with a noisy source.

Combining balanced detection with interferometry has the added advantage that on recombination of reference and signal, in this case at a beam splitter, the two interferometric outputs will have opposite phase and will therefore sum with the same sign in a subtractive circuit. This phase difference arises between the reflected and transmitted beams at the beam splitter and combining the signal with the reference will, in the ideal case of a 50:50 beam splitter, increase the detected signal by a factor of 2. This is made use of regularly in Optical Coherence Tomography. In practice the exact increase in the interference signal depends on the split ratio of the beam splitters.

To achieve complete noise subtraction it is essential that the optical power, and therefore common mode noise, is equal on both detectors. Balancing the optical signals can be done manually, but this requires constant adjustment and rarely gives a better noise suppression than 20dB. In addition, in a measurement over a larger spectral range, where a limited range of frequencies is selected sequentially from the broadband source, the relative irradiance measured on the detectors will change during the sweep due to the wavelength dependent split ratio of the beam splitters. The latter can be compensated for by using a digital balancing method which has the advantage that it operates at high bandwidth but this method will suffer from digitisation noise. Hobbs describes an all electronic auto-balancing detector for noise reduc-
tion, which is also the basis of a commercial device\textsuperscript{22, 23}. In this approach, the laser power incident on one detector is made larger than the other and the generated photocurrent is split by a bipolar junction transistor (BJT) pair controlled by a feedback loop. By continuously equating the photocurrents at subtraction complete noise suppression is achieved, limited only by the optical shot noise. This circuit would also automatically adjust for a wavelength dependent split ratio.

While supercontinuum sources emit high powers over a broad wavelength range, their spectral power density is relatively low. Once an appropriate wavelength range and polarisation have been selected this power is reduced to no more than a few hundred micro Watts and so any balancing method needs to work within these limitations. On construction of Hobbs’ basic noise canceller\textsuperscript{12} we found like Lindsay et al.\textsuperscript{24} that there was poor noise cancellation at low laser powers. On investigation of the performance of the circuit we found that the poor cancellation was due to the reduced bandwidth response of the BJTs at low collector currents\textsuperscript{12, 23}.

Here, we describe an auto-balancing method where the optical power on one detector is adjusted using a feedback loop to change the transmission through an acousto optic modulator (AOM). This approach retains the key advantages of the electronic balancing scheme described by Hobbs, namely that no additional noise is introduced before subtraction and that balancing at a relatively low frequency results in noise cancellation over a large bandwidth. However, by balancing the optical powers directly, our method avoids the reduced bandwidth at low collector currents of the BJTs used in the electronic balancing scheme and therefore allows noise cancellation at low laser powers. In this paper we discuss the application of our auto-balancing scheme to interferometry using beam splitters with an uneven split ratio. We show noise reduction of 41dB over a limited wavelength range from a supercontinuum light source allowing interferometric detection of optical powers as small as 0.01pW with a 5 ms integration time, a speed compatible with typical scanning confocal integration times. The noise reduction and resulting improvement in the signal to noise (SNR) makes the method sensitive enough to enable spectrally resolved detection of individual nanostructures.
II Interferometry and Balanced Detection

In a Mach-Zehnder interferometer (figure 1) the incident laser beam with electric field $E_i = E_i \exp[i(kr - wt + \phi)]$ is split by a beam splitter (BS1) into a signal $E_s = \sqrt{(1 - T_{r1}^2)} T_{s1} E_i$ and reference $E_r = T_{r1} E_i$ branch. $E_s$ contains reflection $\left(\sqrt{(1 - T_{r1}^2)}\right)$ and transmission $(T_{s1})$ terms since, due to the delay line, the signal branch has traversed BS1 twice. The light in the signal branch interacts with the sample, represented by transfer function $H = He^{i\phi_H}$, and recombines with the reference branch at the second beam splitter (BS2). The field transmission $T_{pq}$ of each beam splitter is different, and we assume perfect beam splitters so that light that is not transmitted is reflected. The subscript $q = 1, 2$ refers to the transmission of each beam splitter, and subscript $p = r, s$ refers to the branch for which the transmission is measured. The direction of transmission or reflection is treated differently to incorporate factors such as differences in the angle of incidence on each beam splitter, and differences between the coatings on the different surfaces. The outputs of the interferometer are collected on photodiodes A and B.

In our analysis we choose to consider only amplitude noise, that is intensity fluctuations, which we write as $E_i = \overline{E_i} + \Delta E_i(t)$. Balanced detection schemes only remove amplitude noise\cite{12}, and phase noise will only appear in the interference term when the path lengths of the reference and signal branch are not equal. Here equal path lengths, within the coherence length of the laser, are required for the supercontinuum to interfere. Assuming plane waves, the irradiance on detector A is:

$$I_A = |\overline{E_i} + \Delta E_i(t)|^2 \left[ \left\{ T_{r1}^2 T_{r2}^2 \right\} + \left\{ (1 - T_{r1}^2) T_{s1}^2 |H|^2 (1 - T_{s2}^2) \right\} \right. \right.$$

$$- \left\{ T_{r1} \sqrt{(1 - T_{R1}^2)} T_{s1} T_{r2} \sqrt{(1 - T_{s2}^2)} 2mH \cos(kr_r - kr_s + \phi_r - \phi_s + \Delta \phi_H) \right\} \right].$$

(1)

Where $m$ is a factor correcting for the quality of the interference, which depends on spatial and temporal coherence and the alignment of the setup. To measure $m$ we calculate the expected values of maximum and minimum interference based on the detected irradiance, and compare these with the experimentally determined maximum and minimum. The subscripts $r$ and $s$ on the path length $r$ and phase $\phi$ refer to the reference and signal branch respectively. The measured irradiance on detector B is:
\[ I_B = T_{AOM}^2 |E_i + \Delta E_i(t)|^2 \left\{ T_{r1}^2 (1 - T_{r2}^2) \right\} + \{ (1 - T_{r1}^2) T_{s1}^2 T_{s2}^2 |H|^2 \}
+ \left\{ T_{r1} \sqrt{(1 - T_{r1}^2)} T_{s1} \sqrt{(1 - T_{r2}^2)} T_{s2} 2 m H \cos(k r_s - k r_r + \phi_s - \phi_r + \Delta \phi_H) \right\} \] (2)

The irradiance on each photodiode consists of three terms. The first two are proportional to the optical power of the reference beam \( P_r \) and the signal beam \( P_s \) respectively, and the third term is the interferometric term. Upon (ideal) balancing the interferometric term is the only term remaining. The interferometric signal is modulated by modulating the path length at frequency \( \Omega \) over less than half a wavelength so that \( r_s = r'_s + \delta r_s \sin(\Omega t) \). Since the coherence length of the supercontinuum is low we assume equal path lengths \( r'_s = r_r \). Substituting \( r_s \) into the cosine term of either irradiance (equation 1 or 2), expanding using Bessel functions \( J_n(x) \) and keeping only terms linearly dependant on \( \Omega \), we find:

\[ \cos(k \delta r_s \sin(\Omega t) + \phi_s - \phi_r + \Delta \phi_H) = const + \eta \sin(\Omega t) \] (3)

where analytically \( \eta = 2 J_1(k \delta r_s \sin(\phi_s - \phi_r + \Delta \phi_H)) \). Unlike heterodyning this method of phase modulation does not directly extract the amplitude and phase information. Here, the phase information on the sample is contained in \( \eta \). Experimentally \( \eta \) is determined as the fraction of the modulation depth we observe compared to the maximum achievable modulation depth, given the quality of interference, \( m \), and the powers of the reference and signal beams.

As a result of an uneven split ratio in the beam splitters the irradiance and noise measured at each photodiode is different. The photocurrents, which are proportional to the irradiance, need to be made equal before subtraction if noise cancellation is to be achieved. Here we achieve this by placing an AOM in the stronger beam and adjusting the transmission of the zeroth order to change the optical irradiance present on the detector. In practice the power in the reference is much greater than the power in the signal, \( P_r \gg P_s \), and so the terms corresponding to the irradiance of the signal and interferometric terms are small compared to the irradiance of the reference. The required AOM transmission, \( T_{AOM} \), is therefore:

\[ T_{AOM}^2 = \frac{T_{r2}^2}{(1 - T_{r2}^2)} \] (4)
which only depends on the split ratio of the second beam splitter. Assuming perfect balance, and \( P_r \gg P_s \), the balanced interference term is:

\[
I_A - I_B = -|E_i + \Delta E_i|^2 T_{s1} T_{r1} \sqrt{(1 - T_{r1}^2)} \\
\left\{2T_{r2} \left[ \sqrt{(1 - T_{s2}^2)} + \frac{T_{r2} T_{s2}}{\sqrt{(1 - T_{r2}^2)}} \right] \right\} mH [\text{const} + \eta \sin(\Omega t)].
\] (5)

The relative size of the balanced interferometric signal, compared to the interferometric signal on a single detector (equation 1) depends on the transmission of the second beam splitter. For a 50:50 beam splitter, where \( T_{p2} = \frac{1}{\sqrt{2}} \), the balanced interferometric signal is twice as large as the interferometric signal on a single detector. The auto-balancing method requires a split ratio different from 50:50, and in addition, when measuring over a wide wavelength range the beams splitters will in practice have a wavelength dependant split ratio that deviates from 50:50. In this case the increase in signal is smaller. Note, however, that there is no corresponding increase in the signal to noise ratio beyond the removal of the noise from the reference beam. Since the measured signal of interest \( H \) has been multiplied by the noisy reference the signal, equation 5, will still contain some of the amplitude noise of the source. Multiplicative noise of this type cannot be reduced by subtractive cancellation methods which only reduce additive noise. Nevertheless, cancellation of the dominant amplitude noise in the reference beam, irrespective of beam splitter ratio, will provide the major advantages of balanced interferometric detection with broadband or tuneable sources.

### III Results and Discussion

For complete suppression of the amplitude noise the optical power on each photodiode needs to be equal. We balanced on the DC component of the interference signal which is essentially the reference beam which contains most of the intensity noise. The generated photocurrents are subtracted and the difference converted to a voltage (figure 2a). To achieve automatic balancing a feedback loop (figure 2b) takes the difference signal and applies a voltage \( V_{AOM} \) to the control input of an AOM, adjusting and then maintaining the difference signal at the dual detector \( V_{OUT} \) at zero.
To demonstrate the ability of this approach to remove noise common to both detectors we added amplitude noise to a Helium Neon laser beam (633nm). Figure 3a shows how we used a second AOM, AOM$_N$, to add a sinusoidal intensity modulation and AOM$_T$, as before, to balance. We assume that this artificial amplitude noise is the only noise present. We use a lock in detector to measure the rms irradiance of the artificial sinusoidal noise detected on the photodiode. We add noise from 1kHz to 250kHz (the maximum frequency of our lock in detector), and measure the ability of the circuit to reduce the added intensity noise. An example of noise reduction at 7kHz is shown in figure 3b. The laser powers on photodiode A and B before balancing were measured as 45$\mu W$ and 66$\mu W$ respectively, comparable to the powers available with the supercontinuum source. The rms amplitude of the added amplitude noise was measured without balancing (with photodiode B blocked) and the inset shows the balanced result. The noise reduction from the voltage ratio was calculated to be $53 \pm 6$ dB. The slight offset observed in the balanced signal (figure 3b) is due to electronic cross-talk between the two AOMs. The method was tested with powers a factor of 8 smaller and a similar noise reduction within the frequency range was observed. This clearly demonstrates that our auto-balancing approach is capable of reducing common mode noise that has been added to the source even when photocurrents in the detectors are low.

To test whether this method of auto-balancing is capable of shot noise limited detection in an interferometric measurement we follow a similar approach to Stierlin et. al.\textsuperscript{13} and use optical neutral density filters to gradually reduce the intensity in the signal branch until a signal can no longer be distinguished. First a low noise Helium Neon laser source (633nm) is used which we assume to be an ideal source containing no noise other than shot noise. By using a low noise source and comparing to theoretical values we expect to get shot noise limited results. When using a noisy source we expect to see a real reduction in the noise floor, but initial use of a low noise source allows us to determine the practical limitations of the balancing method. The measurements were taken using a lock in amplifier (bandwidth = 37.5 Hz) at a modulation frequency of $\Omega = 7$kHz, which is above the 1/f noise regime, and below the resonance frequency of the piezo mirror. Interferometric detection of a decreasing optical signal power for a single and balanced detection system is shown in figure 4. The expected peak to peak signal of the modulated interference $P_{\text{MOD}}$ is:
\[ P_{\text{MOD}} = 4\eta m \sqrt{P_r P_s}. \]  

From equation 1 the laser powers on detector A are
\[ P_r = |T_{r1}T_{r2}E_i|^2 \]
and
\[ P_s = |HT_{s1}\sqrt{(1 - T_{r1}^2)}\sqrt{(1 - T_{s2}^2)}E_i|^2, \]
where \( H \) is now the field transmission of the optical neutral density filter. The transmissions of the second beam splitter were measured as: \( T_{r2}^2 = 0.320 \pm 0.003 \), and \( T_{s2}^2 = 0.353 \pm 0.001 \). The relative size of the interference for single and balanced detection is dependent on the split ratio of beam splitter 2 only, i.e. the terms in the curly brackets of equation 5. For the single detector this is equal to \( 0.910 \pm 0.004 \) (first term in the curly brackets) and for the balanced detector this is \( 1.371 \pm 0.007 \) (both terms in the curly brackets). Thus the balanced interference is \( 1.51 \pm 0.01 \) times bigger. The rms electrical power as measured by the lock in amplifier is given by:

\[ P = \frac{1}{R_{IN}} \left( \frac{SR_{FB}P_{\text{MOD}}}{2\sqrt{2}} \right)^2 \]

Where \( S = 0.43 \text{AW}^{-1} \) is the photodiode sensitivity, \( R_{FB} = 100k\Omega \) is the feedback resistance of the transimpedance amplifier, and \( R_{IN} \) is the input impedance of the lock in amplifier (1 M\( \Omega \)). The noise floors were measured with the lock in amplifier. The shot noise for the single measurement (figure 4a) was measured by blocking photodiode B (thin, solid line), and shows a good agreement with the calculated shot noise (thin, dashed line)\(^4,5\), which is calculated from the laser power incident on photodiode A. The theoretical shot noise of the balanced measurement (figure 4b) is twice that of the single measurement. The background electronic noise was measured by blocking both detector inputs (thick, solid) line and shows good agreement with the theoretical Johnson noise of the feedback resistor (thick, dashed line)\(^4,5\).

Laser powers of the reference beam (experimental schematic shown in figure 1) were measured as \( 45\mu W \) on photodiode A and before balancing \( 61\mu W \) on photodiode B. \( m \) and \( \eta \) were measured as 0.481 and 0.642 respectively. The data from the single detector is in good agreement with the linear log-log relationship between optical power of the signal beam and the predicted rms electrical power of the interference, as is the data from the balanced detector. With the single detector (figure 4a) we were able to measure down to approximately a factor of 10 above the noise floor. This suggests that other noise was present that appears mainly in the interferometric term, for instance due to the pointing stability of the laser. With balanced detec-
tion, this or other noise that is uncorrelated on the two detectors will not cancel on subtraction. We therefore attribute the difference between theory and experiment of the final data point from the balanced detector (figure 4b) to uncorrelated noise. The laser noise floor is also higher by a factor of 2, since the shot noise present on both detectors is statistically independent it does not subtract. With a 50:50 split ratio, this is exactly offset by the increase in the interferometric signal, but for all other split ratios the increase in the interferometric term is smaller than the increase in shot noise. With a low noise source as the HeNe laser, balancing therefore offered no real benefit although the interferometric signal has been enhanced. However, when intensity noise is present in the source this will be common on both detectors, and so will subtract when balanced as discussed below.

We repeated the interferometric measurement using the experimental set up of figure 1 with a supercontinuum laser source. To allow a comparison with the measurements taken using the Helium Neon laser, the wavelength range of the supercontinuum was limited with a bandpass filter (central wavelength = 630nm, FWHM = 10nm). In a real experiment to perform spectrally resolved interferometry the wavelengths selected from the supercontinuum source will be swept through the wavelength range of interest, thereby also having a limited optical bandwidth for each measurement point. The noise reduction was critically dependent on the polarisation of the source, and inserting a Glan-Taylor prism at the start of the set up greatly improved noise reduction. The reference and signal paths of the interferometer need to be equal to within the coherence length of the source. To achieve interference at each measurement point the delay stage was adjusted to compensate for the change in optical path length due to the different thicknesses in of neutral density filters used.

As shown in figure 5, the noise floor is reduced by 41dB when comparing the single and balanced measurements. The measurement points, were taken as above using a lock in detector (bandwidth = 37.5 Hz). The laser power of the reference were measured on photodiode A as 43\(\mu\)W and, before balancing, on photodiode B as 63\(\mu\)W, comparable to the measurements with the Helium Neon laser. The noise floors were measured and calculated as above and the theoretical rms electrical power determined as above from equations 6 and 7. \(m\) and \(\eta\) were measured as 0.728 and 0.530 respectively. The interference is measured to be slightly less than expected compared to the theoretical values and there is some spread in the data.
compared to the measurements using the HeNe laser. We attribute the larger spread in data to difficulties in positioning the delay stage so that the signal branch is within the coherence length of the reference branch. This, together with the fact that the two branches are not dispersion matched explains the lower values for the measured interference. The final bracketed point was hard to quantify as the noise was comparable to the signal level. This was also the case with the HeNe laser, and also here we attribute this to laser noise in addition to shot noise that is uncorrelated on both detectors and therefore does not cancel on subtraction. However, even with the additional uncorrelated noise, using this method of balanced detection we were able to interferometrically measure signals as low as 0.01pW with a 5ms integration time by reducing the noise floor by 41dB, well below the level of the single detector. Comparing the measurements with the HeNe laser the noise floor with the supercontinuum is still higher than the shot noise. We discount phase noise for reasons discussed previously, but there are likely to be other sources of noise present in the signal such as polarization dependent noise. The method cannot balance this noise effectively due to the polarization dependent split of the beam splitters. We can however measure signals as close to the noise floor as with the HeNe laser.

IV Conclusions

Using an auto-balancing method, we have reduced noise by 41dB over a limited wavelength range from a supercontinuum source, which allowed detection of optical powers as small as 0.01pW with a 5ms integration time. Our method relies on two photodiodes in a subtractive transimpedance amplifier circuit and a feedback loop adjusting the laser power and noise incident on one of the photodiodes to make the intensities of the two detectors equal. The continuous adjustment of the laser power in the auto-balancing method would allow for compensation of a wavelength dependent split ratio of the beam splitter that occurs when sequentially selecting different wavelength ranges from the broadband source. This together with the noise reduction and increased sensitivity suggests that auto-balancing can be used for broadband interferometric detection and characterisation of nanostructures.
Acknowledgements

The authors would like to thank Steve Simpson for fruitful discussions. J. Trägårdh and H. Gersen acknowledge support from EPSRC (Grant Ref: EP/G022291/1), and I. D. Lindsay acknowledges the support of EPSRC via an Advanced Research Fellowship (EP/E054536/1). The work was carried out with the support of the Bristol Centre for Nanoscience and Quantum Information.

References


FIG. 1: Schematic diagram of a Mach-Zehnder interferometer with each output collected on a photodiode (A or B). The power incident on photodiode B can be adjusted by changing the transmission through an acousto optic modulator (AOM). It contains beam splitters (BS1, BS2) with different transmission coefficients $T_{pq}$. $H e^{i\phi_H}$ represents the phase and amplitude changes induced by the sample. The length of the sample branch is modulated at frequency $\Omega$ by a piezo mirror, and translated longitudinally on a delay stage to allow path matching of the two branches.
FIG. 2: a) Balanced detector with transimpedance amplifier circuit – photodiodes A and B are Hamamatsu S2386 - 5K. b) Feedback circuit consisting of an integrator (1), a half wave rectifier (2), a clamp (3), and a buffer (4) \(^{26}\). The photodiode subtractive circuit can be used without the feedback loop.

FIG. 3: a) Optical set up used to test the ability of the of the balancing method to reduce intensity noise added to a low noise source using AOM\(_N\). AOM\(_T\) is used to adjust the optical power on photodiode B to be equal to the optical power on photodiode A. Photocurrents are subtracted in the amplifier (figure 2a) and transmission through AOM\(_T\) is adjusted with a feedback loop (figure 2b). b) Example of noise reduction at 7kHz. Traces are measured with an oscilloscope and the rms magnitude with a lock in amplifier (bandwidth = 37.5Hz) showing a noise reduction of 53 ± 6 dB.
FIG. 4: The electrical power of the rms of the interference signal versus measured optical signal power. The optical set up (figure 1) was used with a HeNe laser source. a) shows measurements taken with a single detector (photodiode B blocked) and b) shows measurements with the balanced detector. Measurement points (circles) were taken with a lock in amplifier (bandwidth = 37.5 Hz) and compared with expected values, for single and balanced, from equation 6 (thick line). The noise floors are: measured laser noise (thin solid line), theoretical shot noise (thin dashed line), background electronic noise (thick solid line), calculated Johnson noise (thick dashed line).
FIG. 5: The electrical power of the rms of the interference signal versus measured optical signal power. The optical set up (figure 1) was used with a limited wavelength range from a super-continuum source (central wavelength = 630nm, FWHM = 10nm). The measurement points of the interference on photodiode A alone, with photodiode B blocked, are represented by +, while filled circles indicate auto-balanced measurements using both photodiodes. Measurement points were taken with a lock in amplifier and compared with expected values, for single and balanced, from equation 6 (thick lines). The noise floors are: measured laser noise (thin solid line), theoretical shot noise on detector A (thin dashed line), background electronic noise (thick solid line), calculated Johnson noise (thick dashed line).