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CP asymmetry from hadronic charm rescattering in $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decays at the high mass region

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ABSTRACT

A model for the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay amplitude is proposed to study the large CP violation observed at the high mass region of the Dalitz plane. A short distance $b \rightarrow u$ amplitude with the weak phase γ is considered together with the contribution of a hadronic charm loop and a s-wave $D\bar{D} \rightarrow \pi\pi$ rescattering. In the model, the χ_{c0} appears as a narrow resonant state of the $D\bar{D}$ system below threshold. It is introduced in an unitary two channel S-matrix model of the coupled $D\bar{D}$ and $\pi\pi$ channels, where the χ_{c0} complex pole in $D\bar{D}$ channel shows its signature in the off-diagonal matrix element and in the associated $D\bar{D} \rightarrow \pi\pi$ transition amplitude. The strong phase of the resulting decay amplitude has a sharp sign change at the $D\bar{D}$ threshold, changing the sign of the CP asymmetry, as it is observed in the data. We conclude that the hadronic charm loop and rescattering mechanism are relevant to the broadening of the CP asymmetry around the χ_{c0} resonance in the $\pi\pi$ channel. This novel mechanism provides a possible interpretation for the CP asymmetry challenging experimental result presented by the LHCb collaboration for the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay in the high mass region.

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Experimental results from charmless three-body B decays have shown a rich distribution of CP violation (CPV) within the Dalitz phase-space, the so called Mirandizing distribution¹ [1–3]. Positive and negative CP asymmetry are frequently seen in the same B decay channel and sometimes very close to each other in the phase-space, as have been observed in $B^\pm \rightarrow K^\pm \pi^- \pi^+$ and $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decays at low $\pi\pi$ mass. These particular phenomena can be explained through the interference term between the σ and the $\rho(770)$ resonances [3,5]. Another important source of CP asymmetry comes from the $\pi\pi \leftrightarrow KK$ rescattering, which couples different decay channels, namely, $B^\pm \rightarrow K^\pm \pi^- \pi^+$ with $B^\pm \rightarrow K^- K^+ K^\pm$ and also $B^\pm \rightarrow K^- K^+ \pi^\pm$ with $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ [1–3,5,6]. Between others, these two sources of CP violation were already confirmed for $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ [7,8] and $B^\pm \rightarrow K^- K^+ \pi^\pm$ decay channels [9] through a recent amplitude analysis performed by the LHCb collaboration for the Run I data. There are also strong experimental evidences for CPV in the Dalitz phase-space along the high mass region in all of those charged charmless B three-body decays [3]. Although the source for this CPV is not yet identified, we can assume that the variation of the CP asymmetry in the Dalitz

plane is originated by a running strong phase along the phase-space, since the dominant weak phase contributing to these decays is the CKM phase γ , which must be a constant.

Recently we proposed a new source of strong phase variation, associated with the possible $D\bar{D} \rightarrow K^+ K^-$ rescattering, which couples the $B^\pm \rightarrow D\bar{D} K^\pm$ to $B^\pm \rightarrow K^- K^+ K^\pm$ and the $B_c^\pm \rightarrow D\bar{D} \pi^\pm$ to $B_c^\pm \rightarrow K^- K^+ \pi^\pm$ decay channels [10,11]. Where in the later we have also considered the contribution of the channel $B_c^\pm \rightarrow D\bar{D}_s K^\pm$ through $D\bar{D}_s \rightarrow K\pi$ rescattering. In these studies, we concluded that the long distance hadronic loop originated by the double charm penguin contribution can produce a strong phase that changes along the Dalitz phase-space. The phase starts around -1 radian until the $D\bar{D}$ threshold, then it has a quick phase variation given by a sharp change from negative to positive values. This phase variation can be responsible to change the sign of the CP asymmetry, as observed in experimental data [3],² if the associated amplitude is interfering with another one carrying the weak phase.

More than two decades ago it was predicted a CP violation in the high mass region of the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay phase-space

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¹ CP asymmetry distribution in a Dalitz plot [4].

² See LHCb-PAPER-2014-044 supplemental material at <https://cds.cern.ch/record/1751517/files/>.

due to the presence of χ_{c0} resonance [12]. χ_{c0} would be produced from $b \rightarrow c\bar{c}d$ transition at tree level, without weak phase, and it can interfere with $b \rightarrow u\bar{u}d$ tree diagram amplitude, with weak phase, leading to a strong CP asymmetry in the phase-space, with the possibility to extract the CKM weak phase γ [12,13]. One expects the χ_{c0} would be finally observed soon with the Run II LHCb data. This conclusion is based on counting the number of events already seen in the Cabibbo allowed $B^\pm \rightarrow K^\pm \chi_{c0}$ decay, in the $B^\pm \rightarrow K^- K^+ K^\pm$ and $B^\pm \rightarrow K^\pm \pi^- \pi^+$ decays. Amplitude analysis performed by the Babar collaboration found a fit fraction of 1% for these three-body final states [14] and [15], respectively. From that one can do a simple relation with these decays and the Cabibbo suppressed $B^\pm \rightarrow \pi^\pm \chi_{c0}$ ($\frac{\sin^2\theta}{\cos^2\theta} \approx 0.05$) to estimate the number of events expected in LHCb Run II for the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay, arriving up to a few hundred events involving this scalar charm-onium resonance.

Although LHCb did not find yet contribution from the $B^\pm \rightarrow \pi^\pm \chi_{c0}$ amplitude in $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay [7,8], the Mirandizing distribution for Run I data [3] have shown already a clear and huge CP asymmetry around the χ_{c0} invariant mass. This asymmetry suggests the presence of this resonance interfering with the nonresonant amplitude placed in this region.³ However, the distribution of CP asymmetry is much larger than the narrow width expected for this resonance, suggesting that part of the nonresonant background around the χ_{c0} peak comes from the same physical process that produces this resonance. Also, it is observed a change of sign in the CP asymmetry around the $D\bar{D}$ threshold, that can be assigned to the amplitude proposed in [10].

The discussion about the importance of charm loops as a source of important contribution in heavy decay processes is not new [16–20]. In particular, Colangelo et al. [18,19] calculated the $B^- \rightarrow K^- \chi_{c0}$ decay rate using a hadronic triangle loop in combination with QCD factorization (QCDF) approach and HM χ PT to describe the heavy-light mesons vertices, including the coupling between $D\bar{D} \rightarrow c\bar{c}$ resonances. They argue that only QCDF cannot predict correctly the experimental branching fractions of the $B \rightarrow K(c\bar{c})$ transition, and in this framework the $B^- \rightarrow K^- \chi_{c0}$ process is not allowed. Indeed other models of the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay amplitude were proposed in the literature using QCDF approach and none of them included χ_{c0} [21–24]. Contemporary to the present work, Ref. [25] also assumes that the open-charm threshold may play an important role in generating CP violation. They used the isobar approach in which only resonances above threshold are considered and dressed by the coupling to the $D\bar{D}$ intermediate state.

In this work, we explore the same mechanism used to describe the $B^\pm \rightarrow K^- K^+ K^\pm$ decays [10] (also applied to the rare $B_c^\pm \rightarrow K^+ K^- \pi^\pm$ decay [11]), namely, the hadronic charm loop and $D\bar{D}$ rescattering to light pseudoscalars, to investigate the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay, in an attempt to extract the main qualitative features observed in the high mass region ($M_{\pi\pi}^2 > 3 \text{ GeV}^2$) of the CPV Mirandizing data distribution. The present study brings one important novelty to the S-matrix model including the χ_{c0} resonance with mass $(3414.7 \pm 0.3) \text{ MeV}$ and width $(10.5 \pm 0.8) \text{ MeV}$ [26], suggested to be a tetraquark [27], in the s-wave scattering coupled channels $D\bar{D} - \pi\pi$. Furthermore, focusing on a mechanism that can generate CP violation in high mass regions, the hadronic charm loop with rescattering is added to a nonresonant amplitude carrying the weak phase, as will be explained and fully explored in what follows.

Decay amplitude model. A CPV process has to be described by a decay amplitude that must have two interfering contributions car-

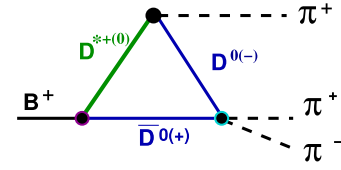


Fig. 1. Two different possibilities for the charm loop contribution to $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay.

rying different strong and weak phases. The standard mechanism at quark level to produce CP asymmetry is through the interference of tree and penguin amplitudes as proposed in BSS model [28]. In the case of $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$, we assume that the weak phase γ come from the tree level diagram with quark transition $b \rightarrow u$. For simplicity we neglect the suppressed penguin contribution $b \rightarrow d$ to the direct $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay process. The hadronic decay channel having as source tree or loop diagrams at the parton level can also contribute with a strong phase from the final state interaction or low energy resonances. Besides, the B decay in two charmed mesons have a hadronic penguin like topology, that together with the subsequent rescattering $D\bar{D} - \pi\pi$ is assumed to contribute with a strong phase.

Inspired by the isobar model description of three-body decays, the amplitude of $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay can be parametrised by two independent contributions as:

$$A_{B^\pm \rightarrow \pi^- \pi^+ \pi^\pm}(s_{12}, s_{23}) = A_{tree}^\pm(s_{12}, s_{23}) + A_{D\bar{D}}(s_{12}, s_{23}), \quad (1)$$

where we assume that $A_{D\bar{D}}$ amplitude is dominated by a charm hadronic loop, Fig. 1, and A_{tree}^\pm which is the dominant topology, has weak ($\pm\gamma$) and strong phases. Furthermore, the χ_{c0} will be introduced as a resonant state below threshold within the $D\bar{D}$ scattering amplitude. We will exploit the model in the high mass region of the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ phase space to find out the manifestation in the CP violation distribution of the $D\bar{D} \rightarrow \pi\pi$ rescattering, with χ_{c0} being a resonant state below the $D\bar{D}$ threshold.

A remark on the implication of CPT invariance to CP asymmetry for the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay in the present model is appropriate. In the framework developed by Wolfenstein [16] (see also [29]) where the hadronic final-state interactions [30] and the CPT constraint were considered together, the CP asymmetry seen in channels that can be coupled by strong QCD dynamics are related. The consequence of this framework is that the sum of the partial widths for those channels should be identical to the sum in the charge conjugated channels. Such result is more restrictive than the general CPT condition that gives equal lifetime for a particle and its anti-particle. The Wolfenstein formalism was further elaborated in [31], where it was considered the hadronic transition matrix of different channels coupled by FSI in the expansion of the CP violating B decay amplitude. Restricted to two channels the leading order formalism was applied to study the CP asymmetries seen in the $B^\pm \rightarrow K^- K^+ K^\pm$ and $B^\pm \rightarrow K^\pm \pi^- \pi^+$ in the mass region where the $K^+ K^-$ and $\pi^+ \pi^-$ channels are strongly coupled. It explained the remarkable opposite signs and the shape of the projected CP asymmetry. This mechanism was confirmed by the LHCb collaboration amplitude analyses for $B^\pm \rightarrow K^- K^+ \pi^\pm$ [9] which found 65% of asymmetry due to $KK \rightarrow \pi\pi$ with a different sign of the one observed in $B^\pm \rightarrow \pi^+ \pi^+ \pi^-$ decays [8,7], although with less intensity.

We observe that the leading order formalism also applies to the present model of the three-body B decay where the $B^\pm \rightarrow D\bar{D}\pi^\pm$ and $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ channels are coupled by the strong force and the associated $D\bar{D}$ and $\pi\pi$ S-matrix provides the FSI contribution to the decay amplitude. The CP asymmetry of the $B^\pm \rightarrow D\bar{D}\pi^\pm$ has to receive a corresponding contribution with opposite sign respecting CPT invariance if only this channel coupling is present.

³ Small amplitudes can be observed in the Dalitz plot when they interfere with large ones, even before their peculiar signature becomes clear.

However, the $D\bar{D}$ channel can also couple to KK as we already discussed in [11], suggesting that the CP asymmetry in $B^\pm \rightarrow D\bar{D}\pi^\pm$ would call for contributions from final state interaction involving more hadronic channels, a discussion that is much beyond the scope of the present work.

Hadronic charm loop. The charm rescattering contribution to the $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ decay can be described by a triangle loop of D mesons as the source for $D\bar{D}$, which makes a transition to $\pi^+ \pi^-$, for two possible charge states as depicted in the diagram at Fig. 1. In this case, because both possibilities are similar we consider only the neutral one, $B^+ \rightarrow D^0 \bar{D}^0 \pi^+$, which is similar to our previous study of the $B^\pm \rightarrow K^- K^+ K^\pm$ decay [10].

The technique to compute the triangle loop in Fig. 1 was already developed in previous works within the context of hadronic three-body decays [10,11,32,33]. For the sake of clarity, we repeat some of the steps required to formulate and compute the loop integral.

We assume factorisation to build the $B \rightarrow D\bar{D}\pi$ vertex in the loop diagram which is written as the product:

$$\Gamma_{B \rightarrow D\bar{D}\pi} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^* \langle \bar{D}^0 | V^\mu | B^+ \rangle g_{\mu\nu} \langle D^0 \pi^+ | V^\nu | 0 \rangle, \quad (2)$$

where G_F is the Fermi constant and V_{cb} and V_{cd}^* the matrix elements (m.e.) of the CKM matrix. The currents m.e.'s are described by form factors with the single pole approximation, and for convenience we introduce the notation $V_{BD}^\mu \equiv \langle \bar{D}^0 | V^\mu | B^+ \rangle$ and $V_{D\pi}^\mu \equiv \langle D^0 \pi^+ | V^\mu | 0 \rangle$. The former represents the vector current $B^+ \rightarrow \bar{D}^0$ transition and the latter takes into account the amplitude for the pair $D^0 \pi$ produced from the W boson excitation from the vacuum. From crossing one finds that $V_{D\pi}^\mu$ represents the vector current m.e. of the $D \rightarrow \pi$ transition obtained from the vector meson dominance (VMD), with a single D^* contribution.

The vector current m.e. for the transition $B^+ \rightarrow \bar{D}^0$ is written as:

$$V_{BD}^\mu = \left[p_B^\mu + p_{\bar{D}^0}^\mu - \frac{M_B^2 - M_{\bar{D}^0}^2}{\ell^2} \ell^\mu \right] F_+(\ell^2) + \frac{M_B^2 - M_{\bar{D}^0}^2}{\ell^2} \ell^\mu F_0(\ell^2), \quad (3)$$

where $\ell = p_B - p_{\bar{D}^0}$ is the momentum transfer and the vector and scalar form factors are, for simplicity, given by:

$$F_{+(0)}(\ell^2) = -m_{B^*}^2 \frac{F^{BD}(0)}{\Delta_{B^*}(\ell^2)}, \quad (4)$$

where $\Delta_{B^*}(k^2) = k^2 - m_{B^*}^2 + i\epsilon$ follows from describing the form factor as suggested by VMD, with the coupling of the weak current to B^* , namely the heavy meson vector ground state with mass m_{B^*} .

The amplitude for the $D^0 \pi$ pair produced from the vacuum through the resonance D^* is parametrized as:

$$V_{D\pi}^\mu = \left[p_\pi^\mu - p_{D^0}^\mu - \frac{M_\pi^2 - M_{D^0}^2}{\ell^2} \ell^\mu \right] F_{D^*}^+(\ell^2), \quad (5)$$

where the form factor is $F_{D^*}^+(\ell^2) = m_{D^*}^2 F^{D^*}(0) \Delta_{D^*}^{-1}(\ell^2)$ and $\Delta_{D^*}(\ell^2) = \ell^2 - D_{pole}^*$. The product of both currents in Eq. (2) is written as

$$V_{BD}^\mu V_{\mu D\pi} = m_{B^*}^2 m_{D^*}^2 N(\ell, p_\pi; P_B) \frac{F^{BD}(0) F^{D^*}(0)}{\Delta_{B^*}(\ell^2) \Delta_{D^*}(\ell^2)}, \quad (6)$$

where p_π is the bachelor pion momentum and the contraction of the Lorentz structure is given by the invariant:

$$N(\ell, p_\pi; P_B) = \Delta_{D^0}(p_{D^0}^2) + 2 \Delta_{\bar{D}^0}(p_{\bar{D}^0}^2) - 2s + 3M_\pi^2 + M_B^2 - \ell^2 \quad (7)$$

where $p_{D^0} = \ell - p_\pi$, $p_{\bar{D}^0} = P_B - \ell$ and $s = (P_B - p_\pi)^2$ is the mass of the pion pair in the transition $D\bar{D} \rightarrow \pi\pi$.

The full amplitude for the hadronic loop is obtained by integrating the momentum inside the triangle with off-shell propagators, taking into account the absorptive and dispersive part of the triangle. And the integral expression is given by:

$$A_{D\bar{D}}^B = i C_0 T_{D\bar{D} \rightarrow \pi\pi}(s) \times \int \frac{d^4 \ell}{(2\pi)^4} \frac{N(\ell, p_\pi; P_B)}{\Delta_{D^0}(p_{D^0}^2) \Delta_{\bar{D}^0}(p_{\bar{D}^0}^2) \Delta_{D^*}(\ell^2) \Delta_{B^*}(\ell^2)}, \quad (8)$$

with

$$C_0 = \frac{G_F}{\sqrt{2}} V_{cb} V_{cd}^* m_{B^*}^2 m_{D^*}^2 F^{BD}(0) F^{D^*}(0),$$

and $T_{D\bar{D} \rightarrow \pi\pi}(s)$ is the $D\bar{D} \rightarrow \pi\pi$ scattering amplitude, which will be discussed in the sequence. Because $T_{D\bar{D} \rightarrow \pi\pi}(s)$ acts on the s -wave and we assume minimal unitarity to describe the T-matrix, it is only a function of the invariant mass s and can be factorized out from the loop integral.

The loop integral is calculated using the Feynman technique, which gives:

$$A_{D\bar{D}}^B = i C_0 T_{D\bar{D} \rightarrow \pi\pi} \times \left[R \frac{I_{D_0 \bar{D}_0 B^*} - I_{D_0 \bar{D}_0 D^*}}{m_{B^*}^2 - D_{pole}^*} - I_{D_0 \bar{D}_0 D^*} + I_{\bar{D}_0 D^* B^*} + 2 I_{D_0 D^* B^*} \right], \quad (9)$$

where

$$R = M_B^2 + M_\pi^2 - 2s + M_{D_0}^2 + M_{\bar{D}_0}^2 - m_{B^*}^2, \quad D_{pole}^* = m_{D^*}^2 - i\Gamma_{D^*}. \quad (10)$$

The functions I_{xyz} are Feynman integrals defined in Appendix A, which are computed numerically with meson masses and widths from Ref. [26].

S-matrix and $D\bar{D} \rightarrow \pi\pi$ transition amplitude. We modify our previous phenomenological amplitude for $D\bar{D} \rightarrow KK$ [11] and generalize it for $D\bar{D} \rightarrow \pi\pi$. Furthermore, χ_{c0} is introduced as resonant state below the $D\bar{D}$ threshold. This is an improvement with respect to the previous approach and different from considering only the contribution of χ_{c0} to the $D\bar{D} \rightarrow \pi\pi$ transition as a Breit-Wigner resonance. Generically, a unitary two channel S-matrix can be parametrized as

$$S = \begin{pmatrix} \eta e^{2i\delta_1} & i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} \\ i\sqrt{1-\eta^2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} \quad (11)$$

where δ_1 and δ_2 are the phase-shifts of the $\pi\pi$ and $D\bar{D}$ elastic scattering. η is the inelasticity parameter, which accounts for the probability flux between the two channels. Unitarity demands that the off-diagonal S-matrix elements should have a magnitude lower than one, and its modulus square can be interpreted as the probability to occur the transition between the initial and final channels.

We introduce a parametrization for the phase-shifts and inelasticity parameter based on the reasonings presented in [10,11,34], brought to the context of $D\bar{D} \rightarrow \pi\pi$ transition to estimate $T_{D\bar{D} \rightarrow \pi\pi}(s)$, which is a key ingredient to the hadronic charm loop to form the pions in the final state. Of course one should, in principle, resort to the QCD theory to compute the S-matrix, which, is however, much beyond our work.

A proposal for the dependence of the transition probability with the two-meson invariant mass, s , in light-meson processes has been discussed in [34], in the context of $PV \rightarrow P'X'$ transitions, and here, these qualitative reasonings are brought in the light of the present case. In a naive description of the $\pi^+\pi^-$ inelastic collision amplitude, the pions annihilate into a quark-antiquark pair that propagates before recombining to produce the heavy-meson pair. The intermediate virtual state propagation of the quark pair scales roughly with the inverse of Mandelstam invariant s . The breakup of the pion into a quark-antiquark pair brings another factor of s^{-1} , and similarly for the formation of the D meson for $s \gg m_c^2$, with m_c the charm quark mass. That provides a damping factor of the off-diagonal S-matrix element of $\sim s^{-3}$, which combined with the threshold behaviour gives $\sqrt{s-s_{th}}/s^{2.5}$, keeps the asymptotic form for large s . Therefore, we write:

$$|S_{\pi\pi \rightarrow D\bar{D}}(s)| = \sqrt{1-\eta^2} \sim \mathcal{N} \sqrt{s/s_{thD\bar{D}}-1}/(s/s_{thD\bar{D}})^{2.5}, \quad (12)$$

where the normalization factor \mathcal{N} is chosen to keep the modulus of the S-matrix elements smaller than 1, as required by the unitarity constraint. If we chose $\mathcal{N} = \Lambda^6 = (1.24)^6$ in Eq. (12) then the maximum value reaches ~ 0.87 , at $\sqrt{s} = 1.08 \sqrt{s_{th}}$, which is close to example of the s -wave isospin zero $\pi\pi \rightarrow KK$, where the cross section drops fast and is relevant below $\sqrt{s} \sim 1.6$ GeV [35]. This qualitative formula is also consistent with one of the possible parametrizations for inelasticity parameter $\eta(s) = \sqrt{1 - |S_{\pi\pi \rightarrow KK}(s)|^2}$ presented in Ref. [36].

The magnitude of the off-diagonal S-matrix element is then written as Eq. (12), which is valid for $s > s_{thD\bar{D}}$. However, the three-body phase-space for the B decay has two pions below the $D\bar{D}$ threshold, which makes necessary the analytic continuation for $s < s_{thD\bar{D}}$ in the physical sheet of complex momentum, imposing that $k_2 \rightarrow ik_2$ with $\kappa_2 = \frac{1}{2}\sqrt{s_{thD\bar{D}}-s}$. Furthermore, the amplitude has to be regulated at low values of s . One phenomenological way to introduce an infrared cutoff in Eq. (12) is:

$$\sqrt{1-\eta^2} = \mathcal{N} \left(\frac{s}{s_{thD\bar{D}}}\right)^\alpha \sqrt{s/s_{thD\bar{D}}-1} \left(\frac{s_{thD\bar{D}}}{s+s_{QCD}}\right)^{2.5+\alpha}, \quad (13)$$

where s_{QCD} is an infrared scale of QCD estimated to be $s_{QCD} \sim \Lambda_{QCD}^2 \sim 0.2$ GeV². The factor s^α in the non-physical region, expresses that the coupling between the open channel and the off-mass-shell $D\bar{D}$ pair is damped in the non-physical region, but it does not change the large momentum power-law tail of the amplitude.

Next, we discuss the parametrization of the elastic phase-shift in the $\pi^+\pi^-$ channel that takes the form dictated by the effective range expansion:

$$e^{2i\delta_1} = \frac{c + bk_1^2 - ik_1}{c + bk_1^2 + ik_1}, \quad (14)$$

where $k_1 = \frac{1}{2}\sqrt{s-s_{th\pi\pi}}$ with the respective threshold of $s_{th\pi\pi} = 4M_\pi^2$. The parameters $b = 1$ GeV⁻¹ and $c = 0.2$ GeV come from our previous study [11].

The new aspect of the S-matrix parametrization with respect to our previous work is the introduction of the χ_{c0} as a resonant state of the $D\bar{D}$ system below threshold represented by a pole in diagonal term and to the phase δ_2 :

$$e^{2i\delta_2} = \frac{(a^{-1})^* - ik_2}{a^{-1} + ik_2}, \quad (15)$$

where $k_2 = \frac{1}{2}\sqrt{s-s_{thD\bar{D}}}$, and the $D\bar{D}$ threshold is $s_{thD\bar{D}} = (M_D + M_{\bar{D}})^2$. For $D\bar{D}$ channel, we choose a complex scattering length

dominated parametrization. We define the scattering length such that the elastic $D\bar{D}$ amplitude presents a pole in the complex plane. The real part below threshold accounts for the χ_{c0} mass and the width moves the pole into the complex plane:

$$a^{-1} - \kappa_{\chi_{c0}} = 0 \quad \text{with} \quad \kappa_{\chi_{c0}} = \frac{1}{2}\sqrt{s_{thD\bar{D}} - M_{\chi_{c0}}^2 + iM_{\chi_{c0}}\Gamma_{\chi_{c0}}}. \quad (16)$$

For $s < s_{thD\bar{D}}$ the transition amplitude becomes

$$T_{D\bar{D},\pi\pi}(s) = \left(\frac{s}{s_{thD\bar{D}}}\right)^\alpha \frac{2\kappa_2}{\sqrt{s_{thD\bar{D}}}} \left(\frac{s_{thD\bar{D}}}{s+s_{QCD}}\right)^{2.5+\alpha} F(k_1, ik_2), \quad (17)$$

where

$$F(k_1, k_2) = \mathcal{N} \left[\left(\frac{c + bk_1^2 - ik_1}{c + bk_1^2 + ik_1} \right) \left(\frac{\kappa_{\chi_{c0}}^* - ik_2}{\kappa_{\chi_{c0}} + ik_2} \right) \right]^{\frac{1}{2}},$$

with \mathcal{N} a normalization factor. The above formula respects the unitarity of the S-matrix model.

For $s \geq s_{thD\bar{D}}$ the transition amplitude is written as:

$$T_{D\bar{D},\pi\pi}(s) = -i \frac{2\kappa_2}{\sqrt{s_{thD\bar{D}}}} \left(\frac{s_{thD\bar{D}}}{s+s_{QCD}}\right)^{2.5} \left(\frac{s_{thD\bar{D}}}{2s-s_{thD\bar{D}}}\right)^\beta F(k_1, k_2), \quad (18)$$

where $\left(\frac{s_{thD\bar{D}}}{2s-s_{thD\bar{D}}}\right)^\beta$ was introduced to modulate the shape of the amplitude bump above the $D\bar{D}$ threshold as we have already used in the study of the $D\bar{D} \rightarrow KK$ [11]. Note that the χ_{c0} pole appears in Eq. (18) through function F , merged with the nonresonant structure of the $D\bar{D} \rightarrow \pi\pi$ amplitude. We should observe that our naive power counting can have corrections, and indeed this is the case as it will be shown in our numerical study.

In our naive modeling we left as free parameters in Eqs. (17) and (18), the exponents α and β , which can be determined by a fit to the data. As a theoretical exercise, we compare the transition amplitude obtained for the same set of parameters found in the study of $B_c^+ \rightarrow K^-K^+\pi^+$ (model I: $\alpha = 7$ and $\beta = 2$) [11], and vary α and β to find another set (model II: $\alpha = 4$ and $\beta = 0.5$), which seems more suitable to provide a qualitative description of the experimental data for CPV in the $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay for the high mass region. As shown in Fig. 2, the amplitude from Eqs. (17) and (18) plotted as a function of the $\pi\pi$ invariant mass, can have quantitative different signatures depending of the choice of the two exponents, but keeping three common features: (i) the χ_{c0} peak superposed to a wide bump below $D\bar{D}$ threshold; (ii) the zero at the threshold; (iii) a bump above the threshold; and (iv) a jump of the strong phase close to π when crossing the $D\bar{D}$ threshold. The parameters can only move the quantitative values of the transition amplitude magnitude, while keeping the qualitative features (i)-(iv). The phase is not affected by the particular choice of parameters α and β once it is connected to the dynamical choice of the amplitude. We just remind the reader that the $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay amplitude includes the loop integral.

Results for $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay and CPV. The total amplitude model for the $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay, Eq. (1), is the sum of a tree amplitude A_{tree}^\pm and the hadronic charm loop with $D\bar{D} \rightarrow \pi\pi$ rescattering, $A_{D\bar{D}}$. Thus the CP asymmetry in B^\pm decays will be the result of the interference between those two. In what follows, we are only interested in the dynamics above 3 GeV² where the low mass resonances contributions come mainly from their tails. Therefore, the amplitude A_{tree}^\pm can be approximated as a flat non-resonant (NR) amplitude with the constant weak phase, γ :

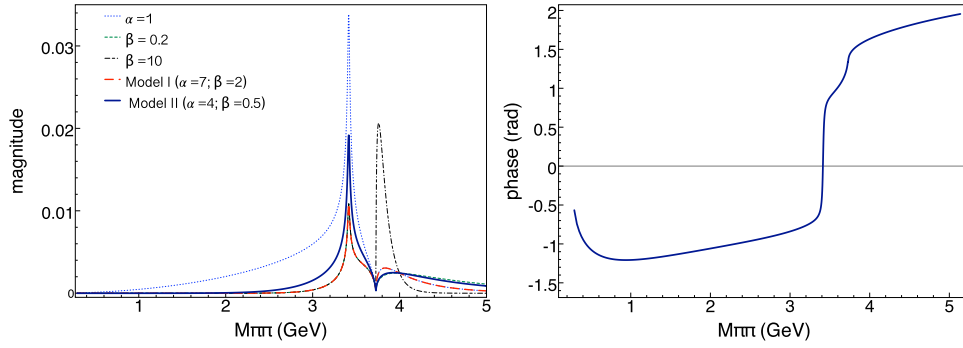


Fig. 2. Magnitude and phase of the decay amplitude from the charm hadronic loop with $D\bar{D} \rightarrow \pi\pi$ rescattering, Eq. (8), as a function of $m_{\pi\pi}$ (invariant mass of the $\pi\pi$ system). The results for the magnitude are presented for models I and II and some variations as indicated within the figure. The phase is not affected by these parameters.

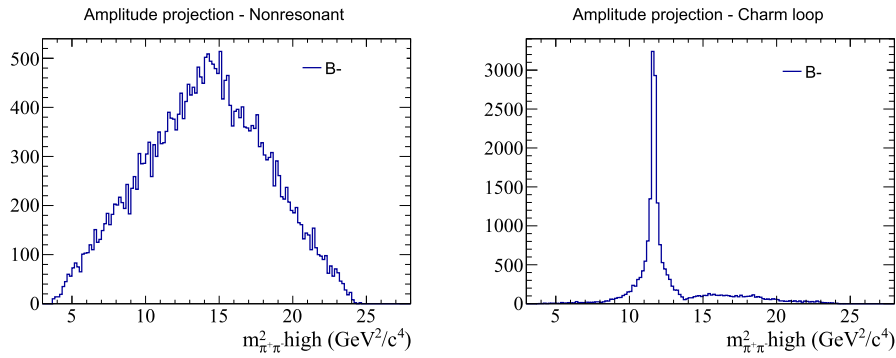


Fig. 3. LAURA++ Toy Monte-Carlo simulation: (left) only the flat nonresonant tree amplitude; (right) only the charm loop with rescattering amplitude (model II).

$$A_{tree}^{\pm} = a_0 e^{\pm i\gamma}, \quad (19)$$

where a_0 is complex to accommodate a strong phase.

The total amplitude was simulated using Laura++ software [37] with hundred thousands events. There are two main variables when two amplitudes interfere: the relative phase between them and the relative magnitude, in principle those quantities are fixed by a fit to data. In our toy model we have to choose a_0 and in order to have an insight on the typical results one gets by changing this quantity. We present a systematic study with model II.

To start our simulations, it is interesting to check the signature of each amplitude A_{tree}^{\pm} and $A_{D\bar{D}}$ alone in the phase-space projected on the $m_{\pi\pi}$ high invariant mass.⁴ We integrate in the $m_{\pi\pi}$ low invariant mass starting at $m_{\pi\pi}^2 = 3 \text{ GeV}^2$ to exclude the low energy interaction region. In Fig. 3, one can see the result from the flat NR amplitude deformed by the phase-space integral and the hadronic loop with model II. Each of them alone does not lead to CP violation, as expected.

In Fig. 4, we present the study of how the amplitudes interfere with different choices for a_0 . We set the relative magnitude for the NR to be twice the charm loop and change the relative global phase between them. As one can see, the different relative phases can result in completely different patterns, but with a clear mark at the resonance position. In the bottom left frame in Fig. 4, the phase difference of 180° eliminates the χ_{c0} peak and make it appears as a dip. Whereas with 0° phase the peak is enhanced.

In principle, we have the freedom to choose the relative phase and intensity of the decay amplitudes A_{tree}^{\pm} and $A_{D\bar{D}}$ besides the model parameters, which can be fitted to data. However, our goal in this study, is to check if the model is able to reproduce the main characteristics observed in the LHCb data [3]: a CP asymmetry (A_{CP}) positive above 3 GeV^2 until the region where the charm

channel opens and A_{CP} flips sign. We can retrieve such A_{CP} pattern with model II and a weak phase of $\gamma = 70^\circ$ [26] by choosing, guided by the study presented in Fig. 4, the relative phase to be 45° with magnitude of the NR amplitude twice the one for the hadronic charm loop with rescattering.

In the left frame of Fig. 5, we show that we can indeed produce the desired characteristics for A_{CP} described above for the projection in the three-body phase-space. We also checked the CP violation signature produced by the interference of the same flat NR amplitude with a simple Breit-Wigner representation of the χ_{c0} in an isobar model with the same relative phase and magnitude as above. We have found that the CP asymmetry is localized in a much smaller region around χ_{c0} compared what we have observed with the rescattering model.

In order to study the CPV signature between the B^+ and B^- in the high mass region, we use the Miranda technique [4] and present the CPV distribution in three-body phase-space on the right frame of Fig. 5. This can be compared to the same CPV Dalitz plot figure produced by LHCb data for $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decays [3]. From the projection in Fig. 5 it is clear the signature of the χ_{c0} peak coming from the $D\bar{D}$ resonant state below threshold widened by the $D\bar{D} \rightarrow \pi\pi$ rescattering. Whereas from the Dalitz plot in Fig. 5, one can see the red band for positive CP asymmetry in the χ_{c0} region followed by a blue band pointing to a change of sign around the $D\bar{D}$ threshold. A similar pattern can be identified in the experimental data [3]. We recall that there are other contributions that could spread the CP asymmetry of the $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay in the high mass region, which were not considered here, like the tails of the low mass resonances, the excited states of the D systems, still coupled to $\pi\pi$ channels, and/or three-body rescattering in the $D\bar{D}\pi$ channel.

Summary. We developed a model for the $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay amplitude, which has contribution from a tree $b \rightarrow u$ non-resonant amplitude and a hadronic charm loop with a s-wave $D\bar{D} \rightarrow \pi\pi$ rescattering, where χ_{c0} is introduced as a resonant

⁴ Defined as the higher one from the two possible pairs of $\pi^+\pi^-$ invariant masses.

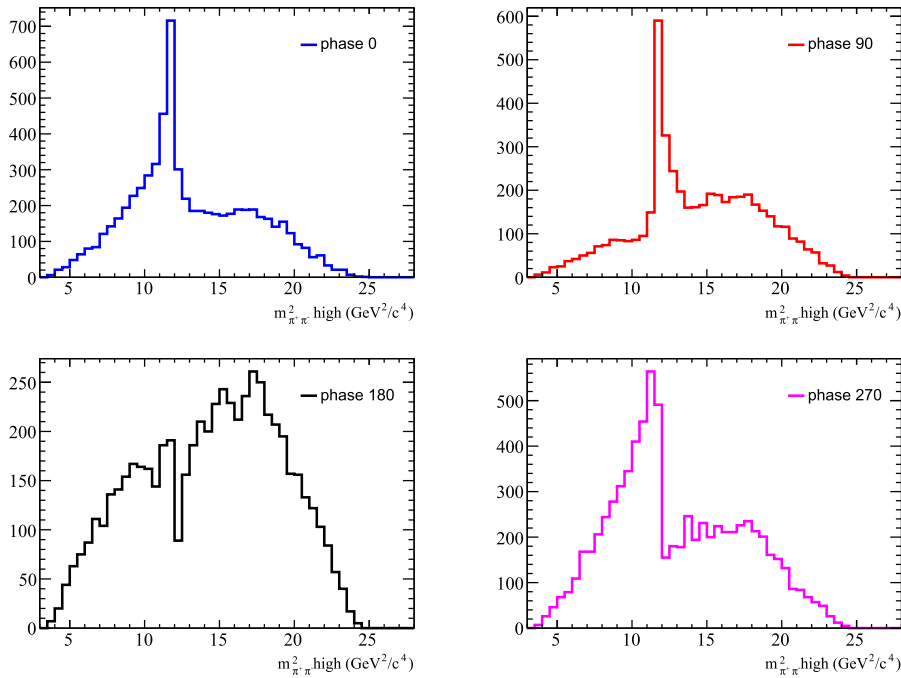


Fig. 4. Integrated decay rate from the full amplitude (model II) as a function of the $\pi^+\pi^-$ invariant mass. Variation of the relative phase between A_{tree}^\pm and $A_{D\bar{D}}$ with values taken from 0° up to 270° .

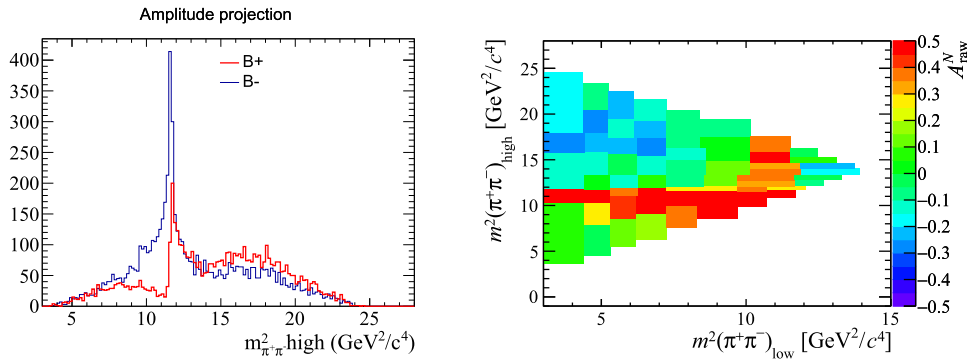


Fig. 5. Left: LAURA++ output for the integrated decay rate for model II with the NR amplitude having a strong phase of 45° and weak phase of 70° [26] with twice the magnitude the charm loop one. Right: Miranda technique [4] applied to expose the CP violation in different regions of the $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ phase-space for model II.

state of the $D\bar{D}$ system below threshold with the narrow experimental width. The χ_{c0} pole of the elastic $D\bar{D}$ scattering amplitude modifies the $D\bar{D} \rightarrow \pi\pi$ transition amplitude due to the assumed S-matrix unitarity of the two-channel model. With this simple model for $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ decay amplitude we were able to mimic qualitatively the CP asymmetry distribution reported by LHCb Run I data in the high mass region [3], giving a possible interpretation of the mechanism behind these challenging experimental results. Therefore, we strongly encourage the experimentalists to incorporate the present model in their amplitude analyses for the next data generation in order to improve our understanding of the nature of CP violation in charmless three-body B decays in the high mass region.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Charm loop integrals

A general triangle loop integral is written as the following form:

$$I_{xyz} = \int \frac{d^4l}{(2\pi)^4} \frac{1}{[(p_x-l)^2 - m_x^2 + i\epsilon]} \frac{1}{[(p_y-l)^2 - m_y^2 + i\epsilon]} \frac{1}{[(p_z+l)^2 - m_z^2]} \quad (\text{A.1})$$

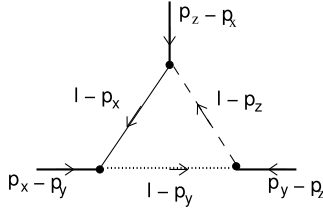


Fig. 6. A triangle loop integral.

where the momenta p_x , p_y and p_z are shown in Fig. 6 for the representation of the loop diagram.

The loop integral can be done using the standard Feynman technique:

$$I_{xyz} = -\frac{i}{(4\pi)^2} \int_0^1 da a \int_0^1 db \frac{1}{D_{xyz}}, \quad (\text{A.2})$$

where the denominator is given by:

$$D_{xyz} = \bar{a} m_x^2 + a \bar{b} m_y^2 + ab m_z^2 - a \bar{a} \bar{b} (p_x - p_y)^2 - a \bar{a} b (p_x - p_z)^2 - a^2 b \bar{b} (p_y - p_z)^2 - i\epsilon, \quad (\text{A.3})$$

where $\bar{a} = 1 - a$ and $\bar{b} = 1 - b$.

For the specific case of $B^\pm \rightarrow \pi^- \pi^+ \pi^\pm$ the four independent functions in Eq. (9), $I_{D_0 \bar{D}_0 B^*}$, $I_{D_0 \bar{D}_0 D^*}$, $I_{\bar{D}_0 D^* B^*}$ and $I_{D_0 D^* B^*}$, are obtained from the numerical integration of Eq. (A.2), with the denominators written explicitly as:

$$D_{D_0 \bar{D}_0 B^*} = M_B^2 (ab)^2 + ab (m_{B^*}^2 - M_{D_0}^2 + \bar{a}(s - M_\pi^2) - a M_B^2) + \bar{a} M_{D_0}^2 + a M_{D_0}^2 - \bar{a} a s - i\epsilon, \quad (\text{A.4})$$

$$D_{D_0 \bar{D}_0 D^*} = M_B^2 (ab)^2 + ab (D_{pole}^* - M_{D_0}^2 + \bar{a}(s - M_\pi^2) - a M_B^2) + \bar{a} M_{D_0}^2 + a M_{D_0}^2 - \bar{a} a s - i\epsilon, \quad (\text{A.5})$$

$$D_{\bar{D}_0 D^* B^*} = ab (D_{pole}^* - m_{B^*}^2) + a m_{B^*}^2 + \bar{a} M_{D_0}^2 - a \bar{a} M_B^2 - i\epsilon, \quad (\text{A.6})$$

$$D_{D_0 D^* B^*} = ab (D_{pole}^* - m_{B^*}^2) + a m_{B^*}^2 + \bar{a} M_{D_0}^2 - a \bar{a} M_\pi^2 - i\epsilon \quad (\text{A.7})$$

and for the numerical integration we use a finite value of $\epsilon = 0.01$ GeV.

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