
Publisher's PDF, also known as Version of record

License (if available):
CC BY

Link to published version (if available):
10.1109/ACCESS.2021.3072955

Link to publication record in Explore Bristol Research

This is the final published version of the article (version of record). It first appeared online via Institute of Electrical and Electronics Engineers at https://ieeexplore.ieee.org/document/9402733 . Please refer to any applicable terms of use of the publisher.

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available: http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/
Identifying Optimal Granularity Level of Modular Assembly Supply Chains Based on Complexity-Modularity Trade-Off

BUGRA ALKAN1, (Member, IEEE), SETH BULLOCK2, AND KEVIN GALVIN3

1School of Engineering, London South Bank University, London SE1 0AA, U.K.
2Department of Computer Science, University of Bristol, Bristol BS8 1UB, U.K.
3Thales UK, Berkshire RG2 6GF, U.K.

Corresponding author: Bugra Alkan (alkanb@lsbu.ac.uk)

This work was supported in part by the Thales Group and the University of Bristol, in part by the U.K. Engineering and Physical Sciences Research Council Research Grant Award entitled Thales-Bristol Partnership in Hybrid Autonomous Systems Engineering (T-B PHASE) under Grant EP/R004757/1, and in part by the open access fee through the University of Bristol.

ABSTRACT

Complexity has been argued to limit operational efficiency, hinder decision-making and induce disruption in supply chain networks. The main aim of this paper is to investigate the architectural trade-off between complexity and modularity in modular assembly supply chain networks. Towards this, an information-entropic complexity model is developed and applied to the domain of assembly supply chains and logistics. This approach characterises complexity as a combination of the intrinsic complexity of the system modules/interfaces and the influence of the topological composition of the network. The model is then used within an optimisation framework, where the optimal granularity level for assembly supply chain design solutions for a given assembly product can be automatically verified by considering the trade-off between complexity and network modularity. It is concluded that the proposed methodology could help to minimise the complexity of supply chain assembly configurations while maximising their modularity and thereby help to increase both the reliability and performance of supply chain networks.

INDEX TERMS

Complexity, modularity, supply chain management, assembly systems, assembly supply chains, network planning, optimization.

I. INTRODUCTION

A. RESEARCH BACKGROUND

An assembly supply chain (ASC) is a dynamic network in which different business actors, including goods, supplies, finance, and information, are distributed via upstream and downstream flows [1]. ASCs usually consist of many distinct manufacturing entities, each of which may combine several production inputs to produce an output [2]. In today’s manufacturing settings, most companies use modular ASC network models as they offer more flexibility than non-modular designs [3]. In modular configurations, the final assembler assigns the bulk of assembly tasks to intermediate sub-assemblers instead of doing all assembly work. As a result, a comparatively limited number of assembly operations are carried out by the final assembly line, reducing the complexity of the final assembly processes while spreading the risk and responsibilities to intermediate assemblers [2]. According to Baldwin et al. [4], complexity and modularity are considered closely related; meaning that complexity is expected to be reduced by an increase in system modularity. However, if an efficient complexity management strategy is not in place, high modularity may also lead to a complex system architecture with a large number of interfaces/relationships between modules [5]. This is true for ASC networks as highly modular supply networks may introduce more supply interfaces to be established and can therefore contribute to the overall system complexity significantly. Therefore, an optimal granularity level for ASC architectures should always be pursued to minimise excessive/harmful complexity.

B. RESEARCH HYPOTHESIS

It is hypothesised in this research work that a novel ASC complexity measure can be used to characterise the difference
between ASC architectures with a simple topology over a small number of complex assembly systems versus ASC architectures with a complex topology over a large number of relatively simple assembly systems. From this viewpoint, it is suggested that increasing ASC modularity could be a potential solution for managing complexity associated with increased product variety since complexity is distributed across relatively small and manageable parts (i.e., divide and conquer principle). However, it should also be remembered that high modularity may lead to an increase in the complexity of supply interfaces which can result in logistics problems and disruptions. To overcome this challenge, this paper proposes an optimisation framework to verify ASC network architectures for network modularity and complexity. This helps system designers to re-formulate the ASC design dilemma as a multi-objective architectural design optimisation in which ASC networks can be built for optimum modularity while minimising network complexity and related risks.

**C. RESEARCH APPROACH**

In this article, an information-entropic complexity definition for modular ASC networks is presented. The approach considers the complexity of ASCs as a function of both i) intra-module complexity (i.e., cumulative complexity of assembly systems) and ii) inter-module complexity which is a function of both a) the cumulative complexity of pair-wise supply interfaces between suppliers/assemblers and b) the effect of the network’s topological pattern. Inherent complexity of assembly systems and pair-wise supply interfaces are assumed to be related to the increased product that they support, and are measured through Shannon’s information entropy, whereas the topological effect of the network is assumed to be related to the network’s structural arrangement and is calculated through a graph energy metric. This complexity metric is then embedded within an optimisation framework, where the near-optimal granularity level for an ASC network realising a particular product with given assembly precedence relationships can be automatically verified with respect to the minimisation of both i) standard deviation of module complexities and ii) the overall inter-module complexity. This provides a Pareto-optimum frontier for the modularity and complexity distribution of the network and can be used to guide the selection of the optimal granularity level for the system architecture. The approach is demonstrated in two case studies. Results demonstrate that the approach can support decision-making activities in supply chain planning by optimally balancing modularity and complexity considerations.

**D. STRUCTURE OF THE PAPER**

The remainder of the paper is organised as follows. Section 2 reviews the literature on supply chain complexity. Section 3 presents the theoretical definition of supply chain complexity. Section 4 proposes the Pareto-optimisation framework for the configuration selection of ASCs considering both system modularity and complexity. Section 5 provides an implementation of the proposed optimisation approach on two case studies. Section 6 summarises the paper’s findings.

**II. LITERATURE REVIEW**

**A. SUPPLY CHAIN COMPLEXITY**

Supply chain networks are often characterised as dynamic social systems consisting of multiple interdependent components that are very vulnerable to the external environment [6]. According to Serdarasan [1], the complexity inherent in supply chains can be defined in three intertwined dimensions, i.e., structural, operational and decision-making. Structural complexity is related to a system’s time-independent properties, which relate to the sheer quantity and variation of system elements as well as the strength of their interactions [7]. Operational complexity is linked to the supply chain network’s time-dependent characteristics and concerns factors such as: time, latency, and uncertainty [8]. Finally, decision-making complexity involves facets of both structural and operational complexity and is related to variables such as organisational mechanisms, decision-making structures and IT networks [1]. According to Bode et al. [9], these variables are closely related. As both the quantity and the diversity of the information in a system increase, a larger number of interactions and a wider range of system behaviours can be observed.

In the literature, supply chain complexity drivers are classified according to two distinct perspectives [1]. The former groups the drivers of complexity based on their roots. These are i) physical situation (e.g., number and variety of products and processes), ii) operational behaviours (e.g., uncertainties associated with manufacturing and logistics operations), iii) dynamism (e.g., market/demand fluctuations), and iv) organisational structure (e.g., IT systems, decision-making hierarchy, etc.) [1]. The latter categorises complexity drivers based on their domains: i) internal, ii) interface, and iii) external drivers. According to Bozarth et al. [10], internal drivers emerge from both decision-making and operational considerations, including product and process architectures. Interface complexity drivers stem from factors related to both material and data flows between manufacturers, consumers and service providers. On the other hand, external complexity drivers such as environmental regulations, global geopolitics, etc., are inherently more difficult to control or manage.

**B. EMPIRICAL STUDIES**

There are diverse approaches to measuring complexity in supply chains, which can be grouped into i) empirical and ii) conceptual studies. An example of an empirical study, proposed by [11], investigated the relationship between supply chain performance, complexity and uncertainty, and proposed a formal categorisation for identifying risks and uncertainties in supply chain systems. A similar study, carried out by [12], examined the implications of complexity drivers on system performance, and proposed a framework.
for the classification of variety management strategies and managerial recommendations to reduce the complexity in a mass customization environment. Vachon and Klassen [13] proposed a two-dimensional framework that conceptualises the degree of complexity embedded in a supply chain along two major dimensions: the form of technology and the nature of information processing. They found a correlation between delivery performance and both the complicat-
edness of the product/process and the uncertainty of the management systems. A supply chain complexity model suggested by Bozarth et al. [10], was statistically validated using real-world data obtained from more than two-hundred plants in several countries. A negative correlation between the complex-
ity of the supply chains and the performance of the man-
ufacturing facilities was identified. Manuj and Sahin [14] presented a supply chain decision-making complexity model, which was developed using an interview-based “grounded theory analysis approach” to assess the implications of complexity for production performance. A method based on “lower-than-network” level metrics was proposed by Bode and Wagner [9] to analyze upstream supply chain disturbances. Their study is based on data from a survey of 396 firms from Germany, Switzerland and Austria and reveals a positive correlation between the drivers of com-
plexity in the supply chain and the intensity of upstream disruptions [9].

C. CONCEPTUAL STUDIES
Several conceptual studies also aim to quantify com-
plexity in assembly systems and supply chains. Frizelle and Woodcock [7] proposed a method to assess both structural and operational manufacturing systems complexity. Their method is based on Shannon’s information entropy [15] and considers complexity as uncertainty associated with both a system’s structural and operational aspects (e.g., queue lengths, deviations between actual and scheduled system states, e.g., busy, idle, etc.). Huaccho Huatuco et al. [16] employed Shannon’s entropy to characterise the interactions between complexity and efficiency in job scheduling in a bottle manufacturing company. In their study, operational complexity was evaluated using Shannon entropy, in terms of the deviation between actual and planned scheduling states. They found that operational complexity is related to fluctuations in customer demand and that it negatively impacts on organisational flexibility. Sivadasan et al. [17] presented an information entropic complexity model for supplier-customer networks. Their model is based on Shannon’s information entropy and relates complexity to the uncertainties associated with material and information flow within supply networks. Sivadasan et al. [18] examined the implications of operational complexity for inventory capacities using Shannon’s entropy. Operational complexity was found to have a significant impact on capacity scheduling in supply networks. The authors also suggested that operational complexity could be better handled using appropriate IT systems providing decision-making support for scheduling planning activities and better information exchange between various supply chain stakeholders. Zhu et al. [19] proposed a novel complexity quantification approach called “Operator Choice Complexity” (OCC) to assess complexity arising from increased product variety in mixed-model assembly production lines and ASCs. Their measure is another adaptation of Shannon’s entropy within the manufacturing domain and considers variety-induced complexity as uncertainty associated with choices occurring in manual assembly operations (e.g., tool choices, fixture choices, etc.). Similarly, Wang et al. [20] examined the impact of OCC on overall system throughput and found that variety-induced complexity can hinder the human cognitive efficiency and human reaction times in assembly operations, and hence affects the rework/scrap rate of assembly stations and disturbs assembly line throughput. Information entropy was also used alongside graph theory-based metrics (e.g., vertex degree) to quantify complexity in assembly supply chain networks [3], [21]–[23].

A summary of the literature review on supply chain complexity assessment is given in Table 1.

D. RESEARCH GAPS
Based on the literature review of complexity management in assembly supply chains, little attention has been given to the question of supply chain network design from a viewpoint aiming to find a balance between network modularity and supply chain complexity. Only a few studies investigated the implications of network configurations over complexity in modular ASCs [2], [3], [20], [21], [23]. However, these studies mostly depend on pen-and-paper based solutions and did not emphasise the modularity-complexity trade-off within a broader context of the network design optimisation problem. This paper addresses this knowledge gap in ASC architectural planning by implementing a multi-objective optimisation framework in which the optimal granularity level of ASC network configurations for a given product or product family can be automatically verified by considering standard deviations in the module-level complexities alongside the network’s overall inter-module complexity. This allows system designers to achieve an equitable allocation of variety-induced complexity across the system, thereby reducing supply interface complexity arising from module-to-module interactions and the network’s overall topology. The results of the study demonstrate that the approach helps designers in minimising supply chain risks associated with increased product variety by optimising the modularity of the ASC network.

III. THEORETICAL FRAMEWORK
The research presented here defines a modular assembly supply chain as a network consisting of several connected mixed-model assembly systems (i.e., modules). To formally define the complexity of modular ASCs, we adopt the following definition proposed by [32] as a base frame: “Complexity of a network-based system is a function of i) the complexities of individual components, ii) the complexities of pair-wise interactions, and iii) the effects of the system’s architectural
pattern, which makes the management of the system mentally difficult and error-prone”. Following the definition given above, the complexity of an ASC \((C)\) is defined as a combination of the inherent complexity of system entities (i.e., modules and interfaces) in isolation, and the effect of the network’s architectural topology, and is given as follows.

\[
C = f(C_1, C_2, C_3) \tag{1}
\]

Here, \(C_1\), \(C_2\) and \(C_3\) represent module complexity (i.e., the internal complexity of each assembler in the supply chain), supply interface complexity (i.e., the interaction complexity of each link between suppliers and assemblers in the supply chain), and topological complexity (i.e., the complexity due to the overall topology of the supply chain).

The term \(C_1\) represents the summation of intrinsic complexities of each assembly system \(a_i\) within the supply chain network, and is estimated as follows:

\[
C_1 = \sum_{i=1}^{N} a_i \tag{2}
\]

where, \(N\) represents the number of assemblers within the ASC.

The term \(C_2\) depicts the sum of pair-wise interface complexities between system modules, and is calculated as follows:

\[
C_2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij}s_{ij} \tag{3}
\]

where, \(\beta_{ij}\) is the pair-wise supply interface complexity between assembler \(i\) and supplier \(j\), and \(s_{ij}\) describes the connectivity matrix:

\[
s_{ij} = \begin{cases} 
1 & \text{if modules } i \text{ and } j \text{ are connected,} \\
0 & \text{else.} 
\end{cases} \tag{4}
\]

The term \(C_3\) represents the influence of the architectural topology on the system complexity and is defined as the ratio between the summation of singular values of the connectivity matrix \(S\) and the total number of modules within the ASC.

\[
E = \sum_{i=1}^{N_e} \sigma_i \tag{5}
\]

\[
C_3 = \frac{E}{N} \tag{6}
\]

In Equation 5, \(\sigma_i\) represents the \(i\)th singular value and \(N_e\) represents the total number of singular values. Please note that, the calculation of the term \(C_3\) requires complete knowledge of the entire system architecture (i.e., the complete topology of the network). Hence, it contributes to system complexity only at the system level [32]–[34]. According to [32], the term \(C_2C_3\), within the context of complex systems, can be referred as an overall indicator of required system management/integration effort.

In supply chain systems, both increased variety and number of interactions within a supply system can result in excessive interdependency between system modules which can negatively impact the cost and performance of the supply chain system [35]. By following the definition given above, we categorised the terms \(C_1\) and \(C_2C_3\) as the intra-module and inter-module complexity, respectively. In this context, the inter-module complexity of a supply chain system is a system-level complexity indicator reflecting the operational and logistics risks arising due to the architectural arrangement of the network modules and the material/information transmissions between them. It is known that inter-module complexity is a function of interface complexity and topological complexity. However, the interaction between these two elements is not directly perceptible. Although it is assumed that inter-module complexity represents the product of these

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Year</th>
<th>Domain</th>
<th>Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sivadasan et al. [17]</td>
<td>2002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blecker et al. [24]</td>
<td>2005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hoole [25]</td>
<td>2005</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sivadasan et al. [26]</td>
<td>2006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hu et al. [2]</td>
<td>2008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bozarth et al. [10]</td>
<td>2009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sivadasan et al. [18]</td>
<td>2006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isik [27]</td>
<td>2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wang and Cheng [28]</td>
<td>2010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wang et al. [29]</td>
<td>2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cheng et al. [30]</td>
<td>2014</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modrak and Soltysova [22]</td>
<td>2017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hamta et al. [31]</td>
<td>2018</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VOLUME 9, 2021
The term \( \alpha_i \) represents the inherent complexity of the \( i \)-th assembly system in the supply network. In this research, the OCC model proposed by [36] is adopted to assess the inherent complexity of assembly systems. OCC is a widely used information-theoretic complexity quantification model for mixed-model manual assembly systems [2], [19], [20], [28], [29], [31], [36]–[42] relating complexity to the variety-induced operational uncertainty associated with assembly choices that are made by assembly operators. In such a sense, OCC is only able to measure variety-induced complexity in manual assembly systems. In this study, this model is selected on the basis that the modularisation of a supply chain network is primarily to manage how an increase in product variety affects the complexity of both assembly system and supply chains. Owing to this relationship between product variety and complexity, the selection of the OCC model, which measures variety induced complexity in manual assembly systems, is justified. Please note that, in the domain of assembly systems, product variety is only one of many sources of complexity. Some other sources include: assembly design complexity [43]–[49], assembly task complexity [50], product complexity [51]–[57], assembly automation complexity [58], [59] and operational stochasticity [60].

In this approach, the inherent complexity \( \alpha_i \) of assembly system \( i \) composed of \( N^m_i \) sequential manual assembly stations is linked to both the number and ratio of product variants introduced to the assembly system, and is formulated as follows:

\[
\alpha_i = \sum_{q=1}^{N^m_i} \psi_q
\]

where \( \psi_q \) represents the complexity of assembly station \( q \). This term is formulated as follows:

\[
\psi_q = \psi_{qq} + \sum_{p \neq q} \psi_{pq}
\]

where, \( \psi_{qq} \) is the feed complexity associated with the feature variants assembled in station \( q \), and \( \psi_{pq} \) is the transfer complexity caused by the feature variants assembled at any upstream station \( p \). Here, \( \psi \) is linked to the uncertainty associated with performing the sequential activities (e.g., part selection, fixture selection, tool selection, etc.) that realise a particular assembly operation.

\[
\psi_{qq} = \sum_{k=1}^{K} \lambda_{qq}^k H_{qq}^k
\]

where, \( K \) is the total number of activities performed in the station, \( \lambda_{qq}^k \) is the relative activity difficulty coefficient (\( \lambda_{qq}^k \geq 0 \)), and \( H_{qq}^k \) is the information entropy associated with the variant mix ratio relevant to the \( k \)-th activity at station \( q \). Entropy \( H_{qq}^k \) is formulated as follows:

\[
H_{qq}^k = - \sum_{m=1}^{N^p_m} \psi_{qq}^k \log_2 \psi_{qq}^k
\]

where, \( \psi_{qq}^k \) is the probability of occurrence of a choice taking the \( m \)-th (\( m \in 1, 2, \ldots, N^p \)) outcome in the activity \( k \).

Please note that the propagation of the two types of complexity (i.e., feed and transfer complexity) results in an increase in the associated assembly system complexity, and affects overall supply chain complexity dramatically. One simple way to reduce operator choice complexity is to use flexible tools and fixtures within an assembly station [31]. Flexibility in tools, fixtures and procedures reduces the cognitive effort associated with the operator’s decision-making by reducing the number of selections that are required. This ultimately reduces the operator choice complexity and helps to improve the quality of the assembly and reduce human errors. Nevertheless, not all assembly processes may be simplified by such techniques; flexible equipment, specific fixtures or common assembly methods can entail major improvements in the station design and task planning, which are often expensive and may require a time-consuming ramp-up phase [61].

The term \( C_2 \) is defined as a function of three main factors: \( i \) number of pair-wise interfaces in the supply chain network, \( ii \) number of variants produced by each assembly system, and \( iii \) the demand uncertainty. Let’s assume that for every relation in the adjacency matrix \( S \) where \( S_{ij} = 1 \), the matrix \( Q \) represents the product mix ratio for modules \( i \) and \( j \) as follows:

\[
Q_{ij} = \begin{bmatrix}
\psi_{ij}^1 & \psi_{ij}^2 & \cdots & \psi_{ij}^{O_{j}} \\
\vdots & \vdots & \ddots & \vdots \\
\cdots & \cdots & \cdots & \psi_{ij}^{O_{j}} \\
\psi_{ij}^{O_{j}} & \psi_{ij}^{O_{j}} & \cdots & \psi_{ij}^{O_{j}}
\end{bmatrix}
\]

where \( O_j \) is the number of product variants arriving at module \( i \), \( O_i \) is the number of product variants produced at module \( j \), \( p_{uv} \) is the product ratio of variant \( u \) at module \( i \) to meet the demand of product variant \( v \) at module \( j \). Following this, the complexity of each supply interface within an assembly
supply chain can be defined in the following form:

\[
\beta_{ij} = - \sum_i O_i \sum_j O_j \sum_u \sum_v p_{uv}^{*ij} \log_2 p_{uv}^{*ij} \tag{14}
\]

\[
p_{uv}^{*ij} = \frac{p_{uv}^{ij}}{K} \tag{15}
\]

\[
K = \sum_i \sum_j \sum_u \sum_v p_{uv}^{ij} \tag{16}
\]

In this equation, \( p_{uv}^{*ij} \) is used to represent the normalised interaction factor between modules \( i \) and \( j \), which is a function of both the number of variants in module \( i \) and module \( j \) and the mix ratio of the variants of downstream module \( i \) [38]. In this way, the pair-wise interface complexity defines the level of uncertainty associated with the material flows between supplier and assemblers.

**C. TOPOLOGICAL COMPLEXITY**

Topological complexity of a network-based system stems from the architectural configuration of the network and depends on the nature of the connectivity. It attempts to capture the “intricateness” of structural dependency between system modules [62]. In this research, the topological complexity of an ASC is calculated using the graph energy metric \( E \) (see [63]), defined as the ratio between the sum of the singular values of the connectivity matrix of the system under consideration and the number of system modules (Eq. 5). According to [32], the graph energy of a system increases as the system moves from centralised architectures to more distributed topologies.

**Figure 1** shows the topological complexity of different synthetic ASC networks with varying topologies. It can be observed that topological complexity reduces for networks with increasingly centralised topologies. Please note that topological complexity may reach values above one as the network becomes maximally distributed. Since ASC networks are often trees, their topological complexity is expected to vary between \([0, 1]\). In a practical way, topological complexity allows us to distinguish between network architectures with a similar number of similarly complex modules that are nevertheless organised in different ways. Please note that, for systems such as distributed sensory networks, biological systems, autonomous systems, etc., topological complexity can take scores of more than two, as a result of high bi-directional connectivity.

**D. AN EXAMPLE COMPLEXITY ASSESSMENT**

An example complexity calculation for a supply chain network belonging to a particular product family is illustrated in **Figure 2**. The product has four parts (i.e., A, B, C and D) with multiple variants (e.g., A1 versus A2); which can be assembled into a maximum of 24 possible final products. There are eight modules in the supply chain network, where module 0 is the virtual supplier, modules 1, 2, 3 and 4 are suppliers in the most upstream echelon, modules 5 and 6 are intermediate assembly systems and module 7 is the final assembler. Each module assembles all the possible combinatorial variants provided by its suppliers and supplies a certain number of variants to a downstream module. The complexity of the supply chain configuration is calculated as follows.

- First, the connectivity matrix \( S_{ij} \) of the supply chain, including the virtual supplier, is defined as follows.

\[
S_{ij} = \begin{bmatrix}
0 & \cdots & 0 & 1111000 \\
0 & \cdots & 0 & 0000100 \\
0 & \cdots & 0 & 0000010 \\
0 & \cdots & 0 & 0000001 \\
0 & \cdots & 0 & 0000000 \\
0 & \cdots & 0 & 0000000 \\
0 & \cdots & 0 & 0000000 \\
0 & \cdots & 0 & 0000000
\end{bmatrix}
\tag{17}
\]

- Topological complexity is calculated by estimating the graph energy of the connectivity matrix \( S_{ij} \).

\[
C_3 = 0.7803, \quad [E = 6.2426, \ N = 8] \tag{18}
\]

- For each assembler module, assembly system complexity is calculated by assuming the inherent complexity of the virtual supplier is zero \( \alpha_{00} = 0 \). An example calculation is given for assembly system 5. Let’s consider
that this assembly system is composed of two assembly stations and one inspection station (Figure 3). In the first station, an assembly operator selects one of the two parts from the tray and places it onto the relevant fixture. Then, in the second station, an assembly operator selects one of two parts per given customer order and assembles it with the part fed from the previous station. In this operation, the operator has to select one of the two tools and assembly procedures as appropriate to the part being assembled. Here, selections of both tools and assembly procedures are impact on transfer complexity associated with the interface between stations one and two. Finally, an inspection operator inspects the final sub-assembly at station three by selecting the relevant tool, procedure and fixture relevant to the customer order. Hence, station complexities are calculated as follows:

\[
\psi_2 = H_1^1 + H_1^2 = 1.8366 \text{ bits}
\]

\[
\psi_3 = H_2^1 + H_2^2 + H_3^2 = 2.8316 \text{ bits}
\]

\[
\psi_3 = H_3^1 + H_3^2 + H_3^3 = 3.6182 \text{ bits}
\]

\[
\Psi_{5} = \psi_1^5 + \psi_2^5 + \psi_3^5 = 8.2864 \text{ bits}
\]

\[
\Psi_{ii} = \begin{bmatrix}
0 & 0 & 0 & 0 & 8.2864 \\
0 & 0 & 0 & 8.7855 \\
0 & 0 & 8.5889 \\
\end{bmatrix}
\]

\[
C_1 = 25.6608 \text{ bits}
\]

- Interface complexities are calculated as follows. Relationship matrix \( p_{uv} \) for each edge is developed. For example, the relationship between vertices 2 and 5 is written as follows.

\[
p_{25} = \begin{bmatrix}
6/24 & 0 & 5/24 & 0 & 3/24 \\
0 & 10/24 & 0 & 3/24 & 0 \\
\end{bmatrix}
\]

(22)

- Each relationship matrix is normalised with respect to, \( K \), total number of interactions within the supply chain (\( K=10 \) in our example). The normalised matrix for the relationship between vertices 2 and 5 is written as follows.

\[
p_{25}^* = \begin{bmatrix}
6/240 & 0 & 5/240 & 0 & 3/240 \\
0 & 10/240 & 0 & 3/240 & 0 \\
\end{bmatrix}
\]

(23)

- Finally, interaction complexity for each edge is calculated using Eq. 12. Then total interaction complexity is estimated using Eq. 3.

\[
\begin{bmatrix}
\beta_0 & 0.424 \\
\beta_1 & 0.432 \\
\beta_2 & 0.486 \\
\beta_3 & 0.430 \\
\beta_4 & 0.520 \\
\beta_5 & 0.520 \\
\beta_6 & 0.686 \\
\beta_7 & 0.686 \\
\beta_8 & 0.681 \\
\beta_9 & 0.681 \\
\end{bmatrix} = \begin{bmatrix}
0.424 \\
0.432 \\
0.486 \\
0.430 \\
0.520 \\
0.520 \\
0.686 \\
0.686 \\
0.681 \\
0.681 \\
\end{bmatrix}
\]

(24)

\[
C_2 = 5.546 \text{ bits}
\]

(25)
Finally, the overall supply chain complexity can be defined in a two-dimensional form as follows:

\[ C = [C_1 : 25.6608, C_2C_3 : 4.3275(5.546 	imes 0.7803)] \text{bits} \quad (26) \]

**IV. OPTIMISATION FRAMEWORK**

System decomposition refers to the decomposition of a system into smaller manageable sub-systems/modules [64]. In this context, one of the important factors that should be considered in managing supply chain risks is to check that the complexity across the supply chain is evenly distributed. Since the uneven distribution of complexity could indicate that one subsystem might be more costly in terms of management than the rest [65]. This underscores the importance of considering the even distribution of complexity across the system. It is to be understood that modularisation and functional encapsulation do not decrease intrinsic system complexity but instead effectively reallocate it in a way that enables the system to be more easily managed. Another point to note is that, as the information variety across the system increases, introducing modularity that carefully take account of system decomposability can often result in system architectures that remain workable. This study employs a multi-objective optimisation framework to obtain the optimal granularity level of ASC configurations with respect to the trade-off between variety-induced complexity allocation and the modularity of ASC network configurations. **Figure 4** shows the flow-chart of the proposed approach. Each step is explained below.

- **Step 1 Generate alternate configurations:**

  Generating all possible combinations of alternate supply chain configurations creates enormous combinatorial difficulties. In this research, an iterative decomposition algorithm, proposed by [31], is adopted to generate all valid supply chain network alternatives for a given number of suppliers at the most upstream echelon. The algorithm is written in the MATLAB programming language [66]. In this approach, the output of each node within the supply network is defined using a coding approach. As an example, \{P1P2\} indicates that the first and second product parts (P1 and P2) will be assembled at one assembly system, whereas \{P1P2P3\} indicates that the third product part will be added to the sub-assembly \{P1P2\} that is coming from an upstream assembly system. The followings are the steps for the algorithm:

  - **Step 1.1:** Firstly, all sub-assembly combinations are produced based on the total number of product parts. Given \( m \) product parts, \((m_1 + m_2 + \ldots + m_{m-1})\) possible sub-assemblies can be generated. As an example, the possible combinations for a product with three parts can be written as follows: \{P1\}, \{P2\}, \{P3\}, \{P1P2\}, \{P1P3\}, and \{P2P3\}.

  - **Step 1.2:** Next, using the sub-assemblies obtained from **Step 1.1**, combinations defining the final assembly are generated. In our cases, these combinations are as follows: \{P1\}P2P3\}, \{P2\}P1P3\}, \{P3\}P1P2\} and \{P1\}P2P3\}]. This step is vital, as product assembly precedence relations are checked with respect to the obtained final assembly definitions and infeasible designs are eliminated. In this step, assembly precedence relations are imported to the algorithm in the form of source and target node pairs defined by the designer.

- **Step 1.3:** If the final product description includes inner braces of more than one cardinality, the inner braces must have a sub-assembly relationship. In this case, the sub-assembly is treated as the finished product defined in **Step 1.1**. As a consequence, **Step 1.1** should be repeated for
that sub-assembly until the cardinality of all inner braces equals one, meaning that no further sub-assembly decomposition is necessary. In our example, the final assembly codes are generated as follows: \{\{P1\}\{P2\}\{P3\}\}, \{\{P2\}\{P1\}\{P3\}\}, \{\{P3\}\{P1\}\{P2\}\} and \{\{P1\}\{P2\}\{P3\}\}\). As depicted in Figure 5, the configuration for \{\{P2\}\{P1\}\{P3\}\} (Design 2) is infeasible as it does not satisfy the assembly precedence relations given in Figure 5.a. Note that all design alternatives are also checked against any plant constraints that require sub-assemblies to be assembled at specific nodes.

- **Step 2 Calculate supply chain complexity:**
  As soon as a network configuration is marked as feasible, the algorithm calculates its intra-module and inter-module complexities based on the approach presented in the previous section. Assembly station complexities in are calculated based on the uncertainty associated with the selection of i) parts, ii) fixtures, iii) tools and iv) assembly procedures. It is assumed that each assembler has a virtual assembly station that performs part and fixture selections followed by downstream a sequence of assembly stations, in which assembly operations are carried out involving selections of parts to be assembled, tools to be used, and procedures to be followed. In this context, all calculations are made based on the aggregation of the final product’s mix ratio. Here, the final product mix is assumed to be divided equally between the final product variants to produce the maximum demand entropy \(H_{\text{max}}\), defined by the following equation:

\[
H_{\text{max}} = -\sum_{i=1}^{N_p} \frac{1}{N_p} \log_2 \left( \frac{1}{N_p} \right) \quad (27)
\]

where \(N_p\) is total number of product variants at the final assembler. In a similar fashion, inter-module complexity of the configuration is calculated using pair-wise interface and topological complexity metrics. Once all calculations are completed, network information (i.e., layout information and detailed complexity scores, etc.) is recorded in a database with a unique ID tag.

- **Step 3 Pareto-optimality check:**
  In the last step, the algorithm identifies the non-dominated solution set from the design space database by considering the following objectives:

  - **Objective 1** is to minimise the standard deviation of intra-module complexities (i.e., assembly system complexity) \((\alpha_1...\alpha_N)\). From a system architecting and design perspective, an even distribution of module-level complexity is required in order to reduce the module-level variation in complexity. This result in better risk distribution across intermediate assemblers. Consequently, the first objective function \(f_1\) is defined as follows.

\[
f_1 = \min \left( \sqrt{\frac{\sum_{i=1}^{N} (\alpha_i - \mu_1)^2}{N}} \right) \quad (28)
\]

where, \(N\) is the number of assembly systems, \(\alpha_i\) is the inherent complexity of the \(i^{th}\) assembly

![FIGURE 4. The flow-chart of the proposed framework.](image-url)
system, and $\mu$ is the mean value of the assembly system complexities.

- **Objective 2** is to minimise the inter-module complexity ($C_2C_3$). The inter-module complexity captures the complexity associated with the integration of the modules and material/data flow within the network. The contribution of module integration to the total complexity is indicated by the relative magnitude of inter-module complexity. When the total structural complexity of a system is considered constant, a low inter-module complexity is often associated with high intra-module complexity (i.e., complexity embedded within the module) and vice-versa [32]. Objective function 2 is described as follows.

$$f_2 = \min\left(\sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij}S_{ij} \frac{\sum_{i=1}^{N_{t}} \sigma_i}{N}\right) \quad (29)$$

Our implementation of multi-objective optimisation (i.e., minimisation) involves searching the entire design space $D$ for the Pareto front solution set $R$ and sorting the non-dominated solution set. In this research, a modified Quick-sort algorithm (see [67]) is used to find the non-dominated solution set, due to its ability to quickly sort a large number of solutions. Once, this Pareto-optimal set is identified, the corresponding solutions could be analysed for other performance indicators, such as: cost, flexibility and sustainability, etc., within the context of broader supply chain Multi-Criteria Decision-Making (MCDM) framework. The details of this MCDM framework will be part of a future manuscript.

V. RESULTS AND DISCUSSION

This section discusses the application of the proposed framework in two case studies. The case studies presented here are deployed on a PC with AMD Ryzen 5 1500X Quad-Core Processor with 8 GB RAM. The first case involves a hypothetical scenario for the design of a supply chain consisting of eight suppliers. The second case study is derived from heavy industry, where the proposed optimisation framework is used to analyse unique supply chain configurations for bulldozer assembly logistics.

A. CASE STUDY ONE

In a hypothetical supply network consisting of eight suppliers at the most upstream echelon, each supplier delivers two variants of a single product part with identical demand. The final product is assumed to have no assembly precedence relation constraints, i.e., full integral architecture. Each assembly system produces a product sub-assembly with sequential flow, i.e., no parallelism. Once the assembly operations are completed, the quality of each sub-assembly is checked at a final inspection station. It is assumed that i) all assembly systems have a virtual assembly station at the start of the assembly
that selects the first part to be assembled and ii) assembly and inspection operations are carried out by human operators. Each assembly operation requires a unique assembly tool, assembly fixture, and assembly procedure and their selection probabilities are calculated based on the mix ratio of the input module from the upstream station. Figure 6 shows the Pareto-optimal front for this ASC network.

The Pareto-front, highlighted as red circles, identifies the non-dominated solutions that provide an optimal trade-off between the complexity induced by the modularity and the standard deviation of module complexities within the supply system. It can be observed that increasing the modularity of the supply chain leads to a better distribution of complexity over the modules resulting in low standard deviation values, but that it also increases the complexity associated with the supply interfaces and the topology of the network.

It is in accordance with our initial hypothesis that modularity of the supply chain network and average intra-module complexity conflict with each other. This is an interesting point to mull over when system architects need to decide on the design of a supply chain network. For example, a company might choose a supply network architecture with simpler and more balanced modules (i.e., lower complexity at the module level) if they have the confidence to manage the resulting high complexity of supplier-assembler interactions. From Figure 6, solution C represents this scenario. On the other hand, centralised architectures might be preferable when the industry has a reduced number of product variants that make it easier to handle the resulting complexity at the module level. Moreover, if it can be assumed that centralised architectures tend to be less expensive than decentralised architectures, ceteris paribus, solution A might be preferable, since it involves a smaller number of more centralised, high complexity assembly systems. From the Pareto-front, the majority of solutions that are located between the points B and C provide an optimum trade-off between both considered objectives and in certain situations, industries that do not have clear preferences for either of the objectives might prefer them as a safer option (i.e., solution B). This kind of decision is vital for complexity management strategies that might be put in place during the supply chain planning phases.

**B. CASE STUDY TWO**

The second case study considers a modular bulldozer assembly supply chain selected from heavy vehicle industry. A bulldozer is a strong tractor with a huge metal plate that is used in construction and conversion projects to clear large quantities of sand, soil or similar material. Bulldozers may be used in a wide range of settings from mines and quarries, to military bases, heavy industrial installations, manufacturing companies and farms. Product parts of a bulldozer can be combined into 18 major groups: frame assembly, case, brake, drive, plant carrier, platform, fender, roll-over, transmission, transmission casing, engine, fan, bogie assembly, pin assembly,
and track-roller frame assemblies (A and B) [35]. In this study, the product parts: track-roller frame assembly A and B, transmission, boogie assembly, engine, plant carrier, drive and case are assumed to have the following number of product variants: two, two, four, two, three, two, three and two; resulting in a total of 1152 product variants at the final assembler. **Table 2** shows the names and number of variants for each sub-assembly.

<table>
<thead>
<tr>
<th>Sub-assembly Name</th>
<th>Number of variants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Frame</td>
<td>1</td>
</tr>
<tr>
<td>2 Case</td>
<td>2</td>
</tr>
<tr>
<td>3 Box</td>
<td>1</td>
</tr>
<tr>
<td>4 Plant carrier</td>
<td>2</td>
</tr>
<tr>
<td>5 Transmission</td>
<td>4</td>
</tr>
<tr>
<td>6 Brake</td>
<td>1</td>
</tr>
<tr>
<td>7 Drive</td>
<td>3</td>
</tr>
<tr>
<td>8 Track-roller frame Part A</td>
<td>2</td>
</tr>
<tr>
<td>9 Track-roller frame Part B</td>
<td>2</td>
</tr>
<tr>
<td>10 Fender</td>
<td>1</td>
</tr>
<tr>
<td>11 Rollover</td>
<td>1</td>
</tr>
<tr>
<td>12 Platform</td>
<td>1</td>
</tr>
<tr>
<td>13 Engine</td>
<td>3</td>
</tr>
<tr>
<td>14 Fan</td>
<td>1</td>
</tr>
<tr>
<td>15 Boogie assembly</td>
<td>2</td>
</tr>
<tr>
<td>16 Pin assembly</td>
<td>1</td>
</tr>
</tbody>
</table>

It can be seen from **Figure 7** that a reduced number of solutions are presented on the Pareto-front. This is because the assembly precedence constraints define a feasible region of the design space. Despite this, as with the first case study, we find that an increase in standard deviation of the module complexity tends to be accompanied by a decrease in the inter-module complexity and vice-versa. Two solutions (A and B) that represent distinct strategies along the Pareto-front are chosen for further discussion. From the whole set of feasible solutions, it should be noted that Solution A represents one of the most preferable of all possible centralised architectures. Similarly, Solution B represents one of the most preferable of all decentralised architecture solutions. Solutions A and B each consist of three and five tiers, respectively. Both solutions are transformed to supply chain schematics in **Figure 8**, representing the various sub-assemblies at different stages of the assembly process. The ability of the approach to provide feasible solutions is validated by specialists in the field. It is also possible to make small modifications to the supply chain architecture to better fit with the industrial requirements. The alternate architectures should be compared by using certain performance measures that might be either qualitative or quantitative to arrive at the most preferable solution for the considered industry. Therefore, this methodology should be considered as a primary guide that filters from a set of solutions, the most suitable ones that then need to be further analysed.

### C. DISCUSSIONS

The framework introduced in this paper contributes to the area of supply chain complexity management by proposing a new approach with which to characterise and optimise assembly supply chain network configurations. This approach illustrates the use of a novel, multi-factor information-entropic complexity measure to guide multi-criteria optimisation in...
order to arrive at non-dominated solutions that provide a good compromise between architectural modularity and variety-induced supply chain complexity. Real-world supply chains can encompass a large number of different suppliers, assembly plant locations, etc. At any one point in time, many alternative configurations of this network may be possible. Moreover, the requirements of an assembly supply chain may also evolve over time as products are commissioned and decommissioned, requiring repeated refinements and re-design of the assembly supply chain or parts thereof. This results in a very challenging design challenge that is difficult to verify manually. Using the approach presented here it is possible to screen the design space automatically in order to candidate ASC network designs that optimise the trade-off between modularity and assembly system complexity while respecting the constraints on product assembly and reflecting the implications of the required product variant mix.

The approach, however, has certain limitations that need to be addressed. Firstly, the coding algorithm used to generate the feasible design space could be ineffective for network designs with a large number of suppliers. This is because the number of combinations of a supply network exponentially increases with the quantity of suppliers [23], [31]. Sophisticated clustering algorithms and meta-heuristics optimisation methods can be employed to offset the increased computational load within an acceptable time-frame. Currently, the effects of topological complexity on overall inter-module complexity is assumed to be linear due to lack of data. This is a crude assumption that should be further evaluated via case studies in which the relationships between architectural topology and performance indicators such as system reliability, operational performance and system cost can be established.

VI. CONCLUSION AND FUTURE WORK

The paper presents an optimisation approach to verify the trade-off between modularity and variety-induced complexity in assembly supply chain planning phases. The proposed approach offers automatic verification of network architectures; thereby, contrary to pen-and-paper based complexity quantification approaches, providing reduced complexity measurement effort and allowing designers to explore a wider design space in a relatively short time. As future work, the presented complexity definition will be further calibrated using a series of empirical studies, where the supply chain complexity model can be correlated with a series of system performance indicators observed at both design and operational stages. The presented approach is also planned to be integrated into a wider MCDM-based network design optimisation framework, where supply network designs are verified and optimised within the context of a wider set of decision criteria (e.g., cost, sustainability, flexibility, etc.).

AUTHOR CONTRIBUTIONS

Bugra Alkan conceived of the presented idea, developed the theory, performed the numerical simulations, and verified the analytical method. Seth Bullock and Kevin Galvin contributed to the interpretation of the results, and provided critical feedback and helped shape the research, analysis and manuscript.
BUGRA ALKAN (Member, IEEE) received the M.Sc. degree in mechanical engineering (robotics) from the Izmir Institute of Technology, in 2012, and the Ph.D. degree in engineering (intelligent systems) from the University of Warwick, in 2019. He is currently an Assistant Professor (Lecturer) in artificial intelligence and secure systems with manufacturers from aerospace, automotive, and defense industries. He has authored and coauthored several peer-reviewed academic journal articles and conference proceedings. His research interests include on developing methodologies that enable the creation of robust, self-adaptive and self-optimized cyber-physical production systems; especially focusing on management of system complexity, changes and disturbances in complex production systems, system-of-systems engineering, Industry 4.0, intelligent manufacturing processes and systems, bio-intelligent and bio-inspired manufacturing, industrial robotics, distributed production systems, digital factories, modeling and optimization of production processes and supply chains, sustainable manufacturing, production planning and control, machine learning techniques, ANNs, hybrid AI techniques and their technical and business applications. He received awards, such as Best Paper Award from the CIRP Conference on Assembly Technologies and Systems, Gothenburg, Sweden, in 2016.

SETH BULLOCK received the first degree in cognitive science and the Ph.D. degree in evolutionary simulation modeling from the University of Sussex, U.K., in 1993 and 1997, respectively. He is currently the Toshiba Chair of Data Science and Simulation, Department of Computer Science, University of Bristol, U.K. He has undertaken consultancy for the U.K. Government on Complexity in ICT, in 2004, and financial systems in 2012. He has published in international peer-reviewed journals spanning health, social sciences, biology, architecture, engineering, environmental science, computing, and physics. His research interest includes complex systems simulation modeling. He has won over £28m in research and infrastructure funding in this area. He was elected twice to the Board of Directors of the International Society for Artificial Life. He serves on the editorial boards of Adaptive Behaviour Journal, Artificial Life Journal, Frontiers in Robotics and AI Journal, and Swarm Intelligence Journal. He has given invited keynote lectures in London, Paris, Athens, Melbourne, Madrid, Granada, and Tokyo.

KEVIN GALVIN received the master’s degree in defense modeling and simulation from Cranfield University. He is currently a Systems Capability Researcher for Advanced Architecture Concepts with Thales Research, Technology and Innovation, U.K., and a Thales Expert in enterprise/systems architecture and command and control to simulation/autonomous systems interoperability. He is also conducting research into Hybrid Autonomous Systems in partnership with Bristol University to understand human–machine teaming and application of artificial intelligence and machine learning and how they architect for design of autonomous systems. This includes analysis of formalisms, like the NATO Architecture Framework Version 4 and the development of a supporting ontology. He spent almost 40 years at the British Army with seven years as the Army Operational Architect. He is a member of the British Computer Society, the Institute of Leadership and Management, and the International Council on Systems Engineering.